A data-efficient method for local noise power spectrum (NPS) estimation in FDK-reconstructed 3D cone-beam CT

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Purpose: For computed tomography (CT) systems in which noise is nonstationary, a local noise power spectrum (NPS) is often needed to characterize its noise property. We have previously developed a data-efficient radial NPS method to estimate the two-dimensional (2D) local NPS for filtered back projection (FBP)-reconstructed fan-beam CT utilizing the polar separability of CT NPS.¹ In this work, we extend this method to estimate three-dimensional (3D) local NPS for feldkamp-davis-kress (FDK)-reconstructed cone-beam CT (CBCT) volumes.

Methods: Starting from the 2D polar separability, we analyze the CBCT geometry and FDK image reconstruction process to derive the 3D expression of the polar separability for CBCT local NPS. With the polar separability, the 3D local NPS of CBCT can be decomposed into a 2D radial NPS shape function and a one-dimensional (1D) angular amplitude function with certain geometrical transforms. The 2D radial NPS shape function is a global function characterizing the noise correlation structure, while the 1D angular amplitude function is a local function reflecting the varying local noise amplitudes. The 3D radial local NPS method is constructed from the polar separability. We evaluate the accuracy of the 3D radial local NPS method using simulated and real CBCT data by comparing the radial local NPS estimates to a reference local NPS in terms of normalized mean squared error (NMSE) and a task-based performance metric (lesion detectability).

Results: In both simulated and physical CBCT examples, a very small NMSE (<5%) was achieved by the radial local NPS method from as few as two scans, while for the traditional local NPS method, about 20 scans were needed to reach this accuracy. The results also showed that the detectability-based system performances computed using the local NPS estimated with the NPS method developed in this work from two scans closely reflected the actual system performance.

Conclusions: The polar separability greatly reduces the data dimensionality of the 3D CBCT local NPS. The radial local NPS method developed based on this property is shown to be capable of estimating the 3D local NPS from only two CBCT scans with acceptable accuracy. The minimum data requirement indicates the potential utility of local NPS in CBCT applications even for clinical situations. *Published 2019. This article is a U.S. Government work and is in the public domain in the USA.* [<https://doi.org/10.1002/mp.13428>]

Key words: noise power spectrum, cone-beam CT, signal detectability, polar separability

1. INTRODUCTION

The noise property of an imaging system is often characterized by the Noise Power Spectrum (NPS) because it efficiently represents the noise covariance in the frequency domain.^{2–7} The noise information is also an important component in designing efficient model observers for detection and estimation tasks^{8–10} and in system optimization.^{9,11,12} By definition, for a zero-mean random process, its NPS is the expectation of the squared modulus of the

Fourier transform of the random process. A NPS calculated in this way is referred to as the traditional NPS in this paper.

The NPS is a standard image quality metric to measure computed tomography (CT) system performance among many others such as CT number, uniformity, contrast to noise ratio, and modulation transfer function. As CT noise is nonstationary, a local NPS is often needed. A faithful estimation of the local NPS for CT requires many repeated scans, because different regions within a single image are no longer

applicable since they sample different local NPS. Obtaining many repeated scans in CT is usually costly, especially in a clinical situation where x-ray exposure to patients is a major concern. To overcome the data cost, we developed a method to estimate the two-dimensional (2D) local NPS from a few scans for FBP-reconstructed 2D CT.¹ Using only a few scans to estimate a CT local NPS is possible because of the special properties that CT images possess as a result of the image formation process. This special property was named the polar separability in our 2D paper, as introduced in Ref. [1]. The property states that the 2D NPS of FBP-reconstructed slice CT images can be decomposed by a one-dimensional (1D) radial shape function and a 1D angular amplitude function in polar coordinates. The polar separability reduces the data dimensionality, so it is possible to extract the pertinent information for describing the CT local NPS from only a small number of repeats. Consequently, the radial local NPS method for 2D CT local NPS, developed based on the polar separability, has been shown to yield an acceptable local NPS estimate when obtained from as few as two repeats for 2D FBP-reconstructed CT.¹

In the work presented here, we extend our method to estimating the three-dimensional (3D) local NPS of cone-beam CT (CBCT) scans, particularly FDK-reconstructed CBCT. Initial work on this was previously presented in our conference proceedings.^{13,14} We first derive the 3D expression of the polar separability and then develop the 3D radial local NPS method for CBCT. The polar separability of the 3D CBCT NPS bears a similar form to that of the 2D fan-beam CT NPS, with modifications to account for the CBCT geometry instead of 1D. The accuracy of the 3D radial local NPS method is evaluated on both simulated and real CBCT data by comparing it to a reference local NPS in terms of NMSE and estimated system performance for detecting lesions in the images collected with the referred CBCT system.

As utilization of CBCT increases for treatment planning and diagnosis, it is desirable to have an accurate description of the CBCT noise properties to better understand the system performance. As the field moves toward 3D image modalities, the data demand for estimating the noise property is expected to increase.¹⁵ In addition, the missing cone of the 3D local NPS along the z dimension¹⁶ adds more complexity in providing a thorough characterization of the nonstationarity of CBCT noise. However, the literature on the development of data efficient approaches to estimating 3D local NPS for CBCT is relatively sparse, with two publications found to be most relevant.^{4,16} In both Refs. [4 and 16], the authors started from the NPS of the projection views and worked through the reconstruction process to derive the NPS of the reconstructed CBCT volume. A parallel beam was assumed in Ref. [4], so this approach may not be able to characterize the actual CBCT NPS when the location is off the central axial plane. The work in Ref. [16] provides a more accurate approach to analyzing the 3D CBCT local NPS since divergent 3D beams are modeled. Our work shares somewhat similar analysis to that in Ref. [16]. However, our approach works directly on the reconstructed CBCT volumes like the

traditional NPS does without the need for knowing the object model, projection data statistics, or reconstruction kernel. Therefore, our approach is more practical. Moreover, our approach does not assume uniformity of the scanned object, as demonstrated in the Section 3 where a nonuniform numeric phantom was used to simulate CBCT data. We believe that the local NPS method presented in this work is a valuable contribution toward efficient and accurate evaluation of CBCT local noise behavior.

The remaining paper is organized as follows. In Section 2, we first explain the polar separability of 2D CT NPS and extend it to 3D CBCT; we then derive the 3D radial local NPS method from the polar separability and describe our approaches to assess its accuracy. In Section 3, we evaluate the accuracy of the 3D radial local NPS method using both simulated and real CBCT data. In Section 4, we discuss the results and the applicability of the radial local NPS method, followed by our conclusions at the end.

2. MATERIALS AND METHODS

We introduced the concept of polar separability and the 2D local radial NPS method in our previous publication on estimating 2D local NPS for fan-beam CT.¹ In this paper, we first extend this property from 2D CT to 3D CBCT, so we can apply the radial local NPS method to CBCT scans. To help with the explanation, we start with a description of the polar separability of CT noise in the simplest case, 2D parallel-beam CT, and the modifications to make the polar separability suitable for 2D fan-beam CT NPS. The description on the 2D case will be relatively brief as readers can always refer to our previous publication for a more detailed illustration. Then, we explain the form of the polar separability for 3D CBCT NPS and construct the 3D radial local NPS method for CBCT. Note that the polar separability is a property associated with CT images reconstructed with FBP/FDK-based reconstruction methods, not for nonlinearly or iteratively reconstructed images.

2.A. Polar separability for 2D CT NPS

As introduced in Ref. [1], the local NPS of a parallel-beam CT image ensemble has the following property:

The local NPS of parallel-beam CT has a constant radial profile shape along all angular spokes independent of location, with varying magnitude.

Mathematically, if we represent the local NPS at location l in the polar coordinates by $W_l(f_r, \beta)$, where f_r is the radial frequency and β the polar angle, then $W_l(f_r, \beta)$ can be decomposed into two 1D functions as follows,

$$W_l(f_r, \beta) = A_l(\beta)W_{\text{glb}}(f_r), \quad (1)$$

where $A_l(\beta)$ denotes a 1D angular amplitude function, varying with the location l , and $W_{\text{glb}}(f_r)$ denotes the 1D radial profile shape function that is global and not varying with angle and location. The polar separability is a consequence of the FBP reconstruction process, which reconstructs the CT

image by backprojecting the filtered projection views along their projection angles and summing them together.

The polar separability of 2D fan-beam CT takes a slightly different form compared to Eq. (1) due to the fan-shaped backprojection in its reconstruction process. When backprojecting data along a fan toward the x-ray source, the data sampling distance becomes smaller as the source location is approached. This corresponds to a shape expansion in the spectrum in the frequency domain. Second, when backprojecting the data along a ray that has a fan angle, the resulting NPS spoke rotates by the fan angle relative to the backprojected view direction because the resulting NPS spoke is always perpendicular to the ray path along which the data are backprojected. Therefore, the polar separability of fan-beam CT needs to be modified to account for these two geometric transforms as follows,

$$W_l(a_{l,\beta}f_r, \beta + \varphi_{l,\beta}) = A_l(\beta)W_{\text{glb}}(f_r), \quad (2)$$

where $\varphi_{l,\beta}$ and $a_{l,\beta}$ represent the rotation angle and the expansion factor, respectively. Readers can refer to our previously published paper¹ for more details and an illustration of the relationship between the NPS spoke and the contribution projection view in the parallel- and fan-beam cases.

2.B. Polar separability for 3D CBCT NPS

Similarly, the FDK-based CBCT reconstruction implies that CBCT NPS also possesses polar separability.

A typical FDK reconstruction is implemented as follows.¹⁷ Let us denote each raw projection view after a flat-field correction and a negative logarithm operation as $g_\theta(u, v)$, where θ represents the projection angle, u and v represent the two dimensions, fan and cone directions of the detector plane, respectively. First, each raw projection view is weighted by a weighting map to obtain a weighted projection view:

$$g_\theta'(u, v) = w_{\theta,u,v}g_\theta(u, v), \quad (3)$$

with the weighting map $w_{\theta,u,v}$ being precalculated based on the CBCT scan geometry. Second, filtering is applied to each projection view along the fan direction to obtain a filtered projection view:

$$g_\theta''(u, v) = g_\theta'(u, v) \otimes h(u), \quad (4)$$

where $\otimes$ denotes the convolution operator and $h(u)$ is the apodized ramp filter function similar to that in 2D FBP reconstruction. Sometimes, another filtering function may be applied along the cone direction to help reduce noise or artifacts in the reconstructed volume. Finally, the filtered projection views are backprojected and summed over all projection angles with a weighting factor to form the reconstructed value at each image voxel (x, y, z) :

$$f(x, y, z) = \sum_{\theta} L_{\theta,u,v}g_\theta''(u(x, y, z), v(x, y, z)), \quad (5)$$

with $g_\theta''(u(x, y, z), v(x, y, z))$ representing the ray from the projection angle θ that passes through the voxel to be reconstructed and $L_{\theta,u,v}$ being the weighting factor for the ray path.

The exact expressions for the projection weighting map $w_{\theta,u,v}$ and the backprojection weighting factor $L_{\theta,u,v}$ can be found in Ref. [17]. They were omitted here since these weighting factors do not alter the correlation structure and thus do not affect the derivation of the polar separability of CBCT NPS.

Based on the FDK reconstruction process described above, the reconstructed CBCT volume is formed by backprojecting and summing the 2D projection views from all projection angles. Consequently, the 3D NPS of CBCT is composed of many 2D NPS planes, each resulting from the backprojection of a noisy 2D projection view; we call these as backprojected NPS planes. The NPS in each of the backprojected NPS planes follows the noise structure in the projection view, which has the same correlation structure due to the same filtering function used to filter the projection views in one FDK reconstruction. Therefore, it is natural to represent the CBCT NPS in the cylindrical coordinate system, which is defined by the polar angle and 2D radial planes, as shown in Fig. 1.

It is easier to start from the location in the CBCT system that has the simplest polar separability format: the 3D local NPS at the isocenter. The isocenter of a CT system is the point where the x-ray source and the detector rotate around. The center rays (passing through the isocenter) are always perpendicular to the projection view, so the backprojected NPS planes resulting from a backprojection along these rays have the same orientations as their contributing projection views. Each backprojected NPS plane is exactly one radial plane in the cylindrical coordinate system. In addition, the magnification factor is constant along each projection angle for the isocenter location, equal to $\frac{R}{D}$, the ratio of the x-ray rotation arm length R and the source to detector distance D . Therefore, the 3D local NPS at the isocenter, denoted as $W_0(f_r, f_z, \theta)$, can be decomposed into a 1D angular function and a 2D shape function, similar to the parallel-beam case,

FIG. 1. Illustration of a cylindrical coordinate system. Each point in this coordinate is described by three variables f_r , f_z , and β representing the three dimensions in a cylindrical coordinate system in the Fourier domain: the in-plane radial frequency, the z frequency, and the polar angle. The rectangle planes in the plot represent a few radial planes in the cylindrical coordinate system. [Color figure can be viewed at wileyonlinelibrary.com]

$$W_0(f_r, f_z, \theta) = A_l(\theta) W_{glb}(f_r, f_z), \quad (6)$$

The 2D radial plane shape function may be further decomposed into two 1D shape functions as $W_{glb}(f_r, f_z) = W_{glb1}(f_r) W_{glb2}(f_z)$, one representing the noise correlation in the fan direction and the other representing the noise correlation in the z-direction since the filtering operations along the two dimensions are usually separable.

When reconstructing the value of an off-center location, backprojection is usually along noncentered rays, which result in three geometrical transforms to each backprojected NPS plane: shape expansion, a rotation by the fan angle [illustrated in Fig. 2(a)] and a tilting by the cone angle of the ray [illustrated in Fig. 2(b)]. The first two are the same as the expansion and rotation in the fan-beam case as explained in Section 2.A. The additional transform, tilting, is caused by the cone angle. After a tilting, the backprojected NPS plane is no longer on a radial plane. One can imagine that a radial sweep of a tilted NPS plane by all the projection angles forms a missing cone around the z-axis in the NPS of the location off the central axial plane [Fig. 2(b)]. The missing cone is a consequence of the fact that no x ray passes through the location in the cone for a circular CBCT scan trajectory.¹⁸ In terms of data transfer, this is a null cone.

Therefore, for an off-center location, the left side of the polar separability is modified to incorporate the geometric transforms as follows,

$$W_l\left(T_{\phi_{l,\theta};\phi_{l,\theta}}(a_{l\theta}f, a_{l\theta}f_z, \theta)\right) = A_l(\theta) W_{glb}(f_r, f_z), \quad (7)$$

where $T(\cdot)$ is a geometric transform function consisting of: first, a rotation by fan angle $\phi_{l\theta}$ in the x-y plane, and then a tilting by cone angle $\phi_{l\theta}$ away from the x-y plane. $a_{l\theta}$ is the shape expansion factor. The transform parameters $(\phi_{l,\theta}, \phi_{l\theta}, a_{l\theta})$ are fully determined by the location $l:(x,y,z)$ and the projection angle θ for a given CBCT geometry:

$$\phi_{l,\theta} = \text{atan}\left\{\frac{x\cos\theta + y\sin\theta}{R - (y\cos\theta - x\sin\theta)}\right\} \quad (8)$$

$$\phi_{l,\theta} = \text{atan}\left\{\frac{z}{\sqrt{(x - R\sin\theta)^2 + (y - R\cos\theta)^2}}\right\} \quad (9)$$

$$a_{l,\theta} = \frac{R - (y\cos\theta - x\sin\theta)}{R} \quad (10)$$

Note that R is rotation arm length (distance from the x-ray source to the isocenter) of the CBCT geometry and the magnification factor $a_{l,\theta}$ is normalized to the magnification of the isocenter location.

FIG. 2. Illustration of the rotating and tilting transforms of the resulting NPS plane (marked by red parallelograms) when backprojecting a view along a non-centered ray. (a) Rotating by the fan angle ϕ when backprojecting along a ray passing a location l on the y-axis. For this location, only rotating happens since the cone angle is 0. (b) Tilting by the cone angle ϕ when backprojecting along a ray passing a location l on the z-axis. For this location, only tilting happens since the fan angle is 0. [Color figure can be viewed at wileyonlinelibrary.com]

2.C. 3D radial local NPS method for CBCT

Based on the polar separability, a local 3D NPS for CBCT can be described by a 2D global radial NPS shape and a 1D angular amplitude function. The 3D radial local NPS method estimates the global radial NPS shape and the angular amplitude function separately and then combines them to obtain the final local NPS estimate. It contains three major steps as described below.

The first step is to estimate the global 2D NPS shape. It can be estimated by averaging over the backprojected NPS planes (not necessary a radial plane) in the traditional local NPS at multiple locations. The aforementioned transforms should be applied to find the backprojected NPS planes for a correct average. Interpolation is usually needed to find the values on those planes. However, if we select certain locations and angles such that the backprojected NPS planes happen to be a vertical or horizontal plane in the Cartesian coordinates, a direct average can be performed without the need for interpolation. In addition, we can also estimate the fan direction and z-direction shape function separately since they are usually separable. We will elaborate in the next paragraph on how to simplify the implementation of estimating the 2D global NPS shape by taking advantages of certain locations and separating the 2D shape functions into two 1D shape estimations. *The second step* is to estimate the angular amplitudes of the local NPS for a location of interest. The amplitude for each angle can be found by fitting the backprojected 2D NPS plane (scaled by the magnification factor) of the traditional local NPS of the specified location to the estimated global 2D NPS shape. The estimated amplitude curve may look noisy, but a smoothness window function can be used to filter it to reduce the estimation noise. For the results we present later in Section 3, a Hanning window of length 40° was applied to smooth the estimated amplitude curve. *Finally*, the estimated global 2D NPS shape and the angular amplitudes are combined to form the final estimate of the 3D local NPS of the specified location according to Eq. (4), which is represented in cylindrical coordinates. Interpolation can be used to convert the final local NPS estimate to the Cartesian coordinates. One may repeat the second and third step to estimate the local NPS for another location since the global NPS shape estimate stays the same for the same dataset. The three steps are the same as the 2D local NPS method described in Ref. [1], although the implementation is more complicated due to the need to extract the tilted plane from a 3D volume.

This paragraph elaborates the first step of the radial local NPS method. Since the 2D global radial shape function can be separated into two 1D shape functions: fan and z direction, we can estimate them separately. If we use certain locations, we may perform a simple average without the need of finding rotated or tilted NPS planes to estimate the two 1D shape functions. For example, for a location on the z-axis, the fan angle of a ray passing through that location from any projection angle is zero. The magnification factors associated with all projection angles are the same and equal to that of the

isocenter location. Therefore, we can take an average of the central axial slice of the 3D local NPS of the locations on z-axis and then perform a radial binning to obtain an estimate of the 1D fan shape function. Figure 3 illustrates this implementation. For the locations on the x-axis, the cone angles of the rays from the 0° projection angle passing these locations are zero, and the magnification factors for all locations on the x-axis from the 0° projection angle are the same and equal to that of the isocenter. Although the fan angles of those rays are not zero, they should be very small if the locations are close to the isocenter; hence, the backprojected NPS plane is at almost the central coronal slice. Therefore, we can simply take an average over the central coronal slices of the 3D local NPS at these locations and then perform an average over the x-axis to obtain an estimate of the 1D z direction shape function, as illustrated in Fig. 4. Similarly, this can be applied to the locations on the y-axis to improve the estimation accuracy of the z direction shape function except that the central sagittal slice of the 3D local NPS of those locations on y-axis should be used. At last, an estimate of the 2D radial NPS shape function can be obtained by taking a tensor product of the estimated 1D fan shape and 1D z direction shape functions.

2.D. Accuracy evaluation

Two quantitative metrics were used to assess the accuracy of the local NPS estimation method, as listed below.

- (1) NMSE relative to a 3D reference local NPS

$$\text{NMSE} = \frac{\|\widehat{\text{NPS}}(f) - \text{NPS}_{\text{ref}}(f)\|_2^2}{\|\text{NPS}_{\text{ref}}(f)\|_2^2} \times 100\%, \quad (11)$$

where $\widehat{\text{NPS}}$ represents a 3D local NPS estimate, and NPS_{ref} represents a 3D reference local NPS. The reference local NPS is computed traditionally using many images. NMSE is one way to directly measure how close the estimated local NPS is to the reference local NPS.

- (2) Detectability index d'

$$d' = \sqrt{\int_f \frac{|S(f)|^2}{\text{NPS}(f)} df} \quad (12)$$

where $S(f)$ denotes the Fourier transform of a noiseless CBCT volume of a signal to be detected. This quantity characterizes the capability of a linear system for detecting a signal from its noisy background by an optimal linear model observer.¹¹ The more accurate the estimated local NPS is, the more closely the calculated d' value reflects the actual system performance. Therefore, this metric indirectly evaluates the accuracy of a local NPS estimate based on its utility in quantifying a system detection performance. In this study, detecting spherical objects was considered. The noiseless CBCT images of those objects were created using our simulated CT scanner as described next. For this evaluation, the reference

FIG. 3. Illustration of the process to estimate the one-dimensional fan-direction shape function (b) using the local noise power spectrum of multiple locations on the z-axis (a). [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 4. Illustration of the process to estimate the one-dimensional z-direction shape function (b) using the local noise power spectrum of multiple locations on the x-axis (a). [Color figure can be viewed at wileyonlinelibrary.com]

detectabilities were computed using the reference local NPS. The d' values evaluated using the radial local NPS and the reference local NPS will be compared.

We tested the detectability accuracy under two conditions. One was detecting a low-contrast spherical object with varying radius ranging from 1 to 3.5 mm with a 0.5 mm increment at a certain location. The contrast of the signals was fixed to be 10 HU. This condition tested the NPS accuracy

for characterizing system performance across varying signal sizes. The second test was to compute the detectability map for detecting a certain signal across the whole image region. For this test, a 2 mm radius spherical object with 10 HU contrast was considered. Detectability maps were evaluated at two slices, one at the central slice and one at an off-center slice. This condition tested the NPS accuracy for characterizing system performance across different locations.

We evaluated the radial local NPS method on two datasets. One set was generated with a simulated CBCT scanner and one set was acquired on a real CBCT system.

2.D.1. Simulated CBCT dataset

We used a virtual CBCT scanner to generate CBCT volumes of a 3D nonuniform numerical phantom composed of ellipsoids, spheres, and cylinders. The x-ray source of the virtual CBCT scanner had an infinitely small focal spot. The detector of the virtual CBCT scanner has a pixel size of 0.85 mm when mapped at the isocenter plane. The source to isocenter distance was 541 mm and to detector distance was 949 mm. The virtual CBCT scanner acquired 300 projection views in one rotation. Analytical projection views were computed through ray tracing of geometrically defined objects in the phantom. They were then degraded with Poisson noise modeled by an air exposure of 10^5 photons per ray. Finally, the projection views were sent to a FDK reconstruction engine to obtain reconstructed volumes. The CBCT simulation and the reconstruction code were implemented in MATLAB and were based on the publicly available CT reconstruction package available at <http://web.eecs.umich.edu/~fessler/code/index.html> developed by Dr Fessler's group. The reconstruction volume was $256 \times 240 \times 80$ with a voxel size of 1 mm in all three dimensions. The numerical phantom was designed to mimic a simple lung. It contained a large ellipsoid (attenuation of water, axes of 100, 75, and 380 mm in the x, y, and z dimension, respectively) embedded with two smaller ellipsoids (attenuation of air, axes of 30, 50, and 380 mm, respectively) to simulate the thorax and the lungs. A few higher attenuated small thin ellipsoids and a long cylinder were placed around the thorax boundary to simulate rib bones and a spinal bone. The nonuniform structure of this phantom was selected in our evaluation to show that the proposed radial NPS method is flexible to process an arbitrary object. One hundred CBCT volumes of the numerical phantom were simulated. Figure 5a shows one slice of a noisy volume. All 100 volumes were used to calculate a reference local NPS. A small number of volumes (2–20) were used in the proposed radial local NPS method for accuracy evaluation. The VOIs for calculating the local NPS had a size of $32 \times 32 \times 32$ in the three dimensions.

2.D.2. Real CBCT dataset

Physical phantom data were collected on a Varian CBCT system (Truebeam, Varian Medical System, Palo Alto, CA) at the Radiation Oncology department in the National Cancer Institute. The phantom we used was the CCT 191 MITA CT IQ Low Contrast Phantom (the Phantom Laboratory, Inc., Greenwich, NY). The scans were collected at 120 kV and 535 mAs. The distances from the x-ray source to the isocenter and the detector were 1000 and 1500 mm, respectively. The reconstruction filter “Scan Option” was set to “smooth.” The ring artifact suppression was set to “strong.” Each scan

provided a 3D volume of $512 \times 512 \times 186$ voxels with a voxel size of $0.5112 \text{ mm} \times 0.5112 \text{ mm} \times 0.9949 \text{ mm}$. It took about 3 min to scan, reconstruct, and transfer one volume to the workstation. Figure 5(b) shows a few slices of one CBCT scan. A total of 45 scans were collected and utilized in the evaluation. The VOIs for calculating the local NPS had a size of $32 \times 32 \times 32$ in the three dimensions.

3. RESULTS

3.A. Simulated CBCT dataset

Figure 6 shows the 3D local NPS images of location (90, 90, 20) in the numerical phantom [pixel location marked in Fig. 5(a)], including the reference local NPS computed from 100 scans, the radial local NPS, and the traditional local NPS estimated from 2, 4, 6, and 20 scans. It can be seen from Fig. 6 that the traditional local NPS images look very noisy even with 20 scans while the radial local NPS images already show resemblance to the reference NPS images with similar shape and intensity pattern with only two scans, although they appear slightly blocky. As the number of scans increases, the blocky appearance quickly improves. Visual differences between the 6- and 20-scan-based radial local NPS images are almost unperceivable, indicating that a convergence of the local radial NPS estimate might have reached at six scans.

To check how the NMSE accuracy changes with the number of scans used in the local NPS estimation, we plotted the NMSE of the local NPS estimate as a function of the number of scans for the same location (90, 90, 20) in the lung-mimicking phantom in Fig. 7. The mean and error bar associated with each point in the plot were obtained from ten estimates, each time using a set of randomly chosen scans. It is shown in this figure that the NMSE of the radial local NPS method is generally small. With only two scans, the NMSE is already smaller than 5% and it converges quickly at about 4–6 scans. In contrast, for the traditional local NPS method, at least 20 scans are needed to reach this accuracy.

Since the results presented above showed that the radial local NPS method could provide a local NPS estimate with very small NMSE error with only two scans, next we examined accuracy of using the local NPS estimated from two scans in computing d' . Figure 8 shows three curves of d' as a function of the signal radius for location (128,80,20) [pixel location marked in Fig. 5(a)]: the reference d' curve, the one evaluated using the radial local NPS from two scans, and the one evaluated using the traditional local NPS from two scans. It can be seen that a large gap exists between the d' curve evaluated using the traditional local NPS with two scans and the reference curve, which is expected as a traditional local NPS estimated from two scans would be very noisy so not useful to characterize the system performance at all. However, with the radial local NPS estimate, the d' curve is almost identical to the reference detectability curve.

We further tested the accuracy of using the radial local NPS estimate to compute a d' map for a 2 mm radius spherical signal at slice 20 and slice 40. Note that slice 40

FIG. 5. (a) One noisy cone-beam computed tomography (CBCT) slice of the simulated lung-mimicking numerical phantom with the yellow and red cross marking the pixel location of (90, 90) and (128, 80), respectively. (b) A few slices of one CBCT scan of the physical phantom with the yellow cross at the center slice marking the pixel location of (180, 180). [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 6. Various versions of the three-dimensional local noise power spectrum (NPS) images of location (90, 90, 20) [pixel location marked by the yellow cross in Fig. 5(a)] in the simulated phantom scans for visual comparison. The top row shows the three central planes of the reference local NPS computed from 100 scans using the traditional method. Following the top row are the corresponding traditional and radial local NPS images estimated with 2, 4, 6, and 20 scans going downward. [Color figure can be viewed at wileyonlinelibrary.com]

corresponds to the isocenter plane and slice 20 was the axial plane 2 cm away from the center. The radial local NPS was again estimated from two randomly selected scans. We evaluated the detectability every other four pixels horizontally and

vertically. The maps covered the pixel range of [30:230] in the x -axis and [45:195] in the y -axis, reaching about the object boundary as shown in Fig. 9(a). There are four detectability maps in Fig. 9, two being the reference

FIG. 7. The normalized mean squared error (NMSE) curve vs the number of scans used in estimating the local noise power spectrum (NPS) (the radial method in red star and the traditional method in blue circle) for location (90, 90, 20). (Note that the first point of the NMSE curve of the traditional local NPS method does not appear in the figure because its value is out of the displayed y-axis range). [Color figure can be viewed at wileyonlinelibrary.com

FIG. 8. Signal detectability curve at location (128, 80, 20) as a function of the signal size evaluated using traditional local noise power spectrum (NPS) estimated from two scans (blue triangle), radial local NPS estimated from two scans (red circle), and the reference local NPS (black dot). [Color figure can be viewed at wileyonlinelibrary.com

detectability maps and the other two evaluated using the radial local NPS. It can be seen that the detectability maps evaluated using the radial local NPS have similar patterns to the reference maps. For example, for slice 40, both appear darker (lower detectability) at the simulated muscle region and relatively brighter (higher detectability) at the simulated lung region; for slice 20, both maps present ring patterns on top of the darker and bright zones similar to those appearing

in slice 40. We will discuss the reasons for these patterns in Section 4.

3.B. Real CBCT dataset

Figure 10 shows local NPS images estimated for the physical phantom scans at location (180,180,100) [pixel location marked in Fig. 5(b)]. The reference local NPS was computed from 45 scans. The radial local NPS and the traditional local NPS showed in this figure were estimated using two scans. We can see again that the radial local NPS images visually resemble the reference images and look much less noisy than the traditional local NPS images. The NMSE of the radial local NPS method and the traditional local NPS were 3.92% and 85.19%, respectively. The accuracy improvement was again more than 20 folds, similar to the improvement found in the simulated dataset.

Figure 11 shows the NMSE curves as a function of the number of scans used in the local NPS estimate for location (180, 180, 100). Similar to the results in the simulated data, a very small NMSE error was achieved with the radial local NPS method from only two scans, showing a great accuracy improvement over the traditional local NPS method which had almost 100% NMSE error (not shown in the plot since it is out of the displayed y-axis range). Again, the traditional local NPS method would need about 20 repeats to reach the accuracy of the radial local NPS estimated from two scans.

3.C. Null cone effect

As a final validation on the 3D CBCT local NPS method, we checked if it could capture the widening trend of the null cone as the location moved away from the center along the rotation axis. We selected three locations in the physical phantom volume on the rotation axis with their distances to the isocenter of 0, 35, and 70 mm, respectively. The null cone angle of a local NPS can be analytically calculated using Eq. (9) and simplified to $\arctan(\frac{z}{R})$ when the point is located on the z-axis (i.e., $x = y = 0$). Therefore, the null cone angles of the three local NPSs were 0°, 2.0°, and 4.0°, respectively. Figure 12 shows the y-z plane of the radial local NPSs from two scans together with the reference local NPSs for the three locations. Although it is difficult to identify the exact null cone angles from the plots, the estimated radial local NPS does present a widening dark cone in the bottom three NPS images from left to right as illustrated by the yellow triangles in Fig. 12, reflecting the null cone phenomenon.

4. DISCUSSION

In this work, we extended the radial local NPS method from 2D CT to 3D CBCT. This is feasible because the polar separability is also preserved in the image noise structure in CBCT with FDK reconstruction. The essential part in the extension is to understand the geometric transforms of the resulted NPS plane with respect to the contributing projection view caused by the magnification, the fan angle, and the cone

FIG. 9. Detectability maps (d' map) evaluated within the indicated boundary [yellow box in (a)] for detecting a spherical signal of 2 mm radius at slice 40 (center slice) and slice 20 of the simulated phantom scan. The reference maps, evaluated using the reference local noise power spectrum (NPS), was shown in (b). The d' maps evaluated using the radial local NPS estimated from two randomly selected cone-beam computed tomography scans were shown in (c). [Color figure can be viewed at wileyonlinelibrary.com]

angle when backprojecting along a ray not crossing the isocenter. Once the transformed NPS planes were identified, a global 2D NPS shape can be estimated by averaging them from the traditional computed local NPS of multiple locations. Although the traditional local NPS computed using only a few scans is noisy, a fairly good 2D global NPS shape can be obtained after properly averaging over locations and angles. Then, the angular noise amplitude for each angle of any specified location within the scanned object can be estimated by fitting the corresponding 2D NPS plane to the 2D global NPS shape. The estimation uncertainty in the angular noise amplitudes may be further reduced by applying a smooth filtering to the amplitude curve.

We evaluated the 3D radial local NPS method with both simulated and real CBCT data. In both datasets, good accuracy was demonstrated by small NMSE values. The results showed that less than 5% NMSE was possible with only two scans, which is the minimum number of scans that are required for noise analysis. The usefulness of the radial local NPS estimated from two scans was further demonstrated in evaluating d' , a detection task-based system performance. The d' values computed using the radial local NPS estimated from two scans were close to those computed using the reference local NPS, indicating that the radial NPS method is

capable of characterizing the local noise property with only two scans.

We also evaluated the detectability maps for detecting a spherical signal across the simulated lung region in the central slice (slice 40) and a slice (slice 20) 2 cm off the center in the simulated CBCT data. The detectability maps evaluated using the radial local NPS estimated from two scans were similar to the corresponding reference detectability maps. The appearance of the detectability maps revealed some interesting observations. First, at slice 40 (the central plane), the maps showed that the detectability was relatively lower in the muscle region and relatively higher at the lung region. This is mainly due to the location-varying noise level, higher at the muscle region and lower in the lung region, as expected. Second, at slice 20 (2 cm away from the center), the detectability maps also present similar darker and brighter areas related to the location of muscle and lung. However, there were additional ring patterns on top of it, with the center at this slice appearing the brightest. Interestingly, the detectability at the center of slice 20 is even higher than that at the center of slice 40 ($d' = 3.1$ vs $d' = 2.1$), which indicates that if the sphere was located off the isocenter plane, it might be easier to detect. We found that this was mainly caused by the location-varying system transfer property of

FIG. 10. Images of (a) the reference local noise power spectrum (NPS) from 45 scans, (b) the radial local NPS from two scans, and (c) the traditional local NPS from two scans using the physical cone-beam computed tomography data for location (180, 180, 100).

FIG. 11. The normalized mean squared error (NMSE) curve vs the number of scans used in the local noise power spectrum (NPS) estimate (the radial NPS method in red star and the traditional local NPS method in blue circle) for location (180, 180, 100) in the real cone-beam computed tomography volume. (Note that the first point of the NMSE curve of the traditional local NPS method is not shown in this figure because its value is out of the displayed y-axis range.). [Color figure can be viewed at wileyonlinelibrary.com

CBCT. Take the center location as an example. When the signal was located 2 cm off the isocenter along the z-axis, it was cut finer in the longitudinal direction due to the x-ray cone

angle. This resulted in one more slice with a visible image of the signal, compared to the CBCT images of the sphere when located at the isocenter, as shown in Fig. 13. The location-varying system transfer causes the changing of the energy of the signal's image over location, as seen in the signal energy maps for the isocenter plane and the $z = 2$ cm axial plane in Fig. 14. Here, the signal energy was simply calculated as the sum of squared intensity values within the spherical signal's CBCT volume. It is clearly seen in Fig. 14 that the signal energy was more constant within the isocenter plane, while within the $z = 2$ cm axial plane, the lesion energy varied greatly with its pixel locations and contained an oscillating ring pattern. However, in general, the signal energy of the spherical object in the non-isocenter plane [Fig. 14(b)] slowly decays as the object moved away from the center due to the slight increase of the null cone. Regarding the ring pattern, a simple intuitive explanation is found by considering that the reconstructed value of one point in the object is formed by summing all the ray path lengths with appropriate weights through that point. For non-isocenter planes, the combination of ray path lengths passing through a point in the object changes with the object's location where the weighted summation of all the ray path lengths can present an oscillating pattern as the object shifts away from the center, and hence, the reconstructed image values are similarly impacted. The ring pattern of the energy map resulted in a similar ring pattern in the detectability map of slice 20 (Fig. 9). In summary, the location-varying noise level caused by the nonuniform anatomy structure and the location-varying system transfer due to the cone-beam scan geometry together resulted in the observed patterns in the detectability maps in Fig. 9.

When evaluating the accuracy of the radial local NPS, we compared it to a reference local NPS computed from a relatively large number of scans using the traditional local NPS method. It should be noted that the reference local NPS computed in this way approaches the true local NPS, but it always contains uncertainty and is not the true local NPS due to the finite number of scans being averaged. Therefore, although the NMSE relative to the reference local NPS we evaluated in this work appeared to converge to a value larger than zero (Figs. 7 and 11), the converged values do not necessarily represent the bias of the radial local NPS because the values were a sum of the actual bias and the remaining gap between the reference and the true local NPS. The bias of the radial NPS method may be evaluated with simulation by computing the reference local NPS with many more scans or by calculating a theoretical local NPS with an analytical method. However, we did not further investigate this question since the NMSE of the radial local NPS estimated from only two scans was already very small for both the simulated and real CBCT data.

The radial local NPS estimation method starts from a traditional local NPS. It is worth mentioning that the noise in the initial local NPS estimate can impact the accuracy of the final radial local NPS. There are two points to discuss regarding the initial local NPS estimate. First, the noise-only image was obtained by subtracting the average of the N available

FIG. 12. The y-z plane of the local noise power spectrum (NPSs) of the physical phantom scans at the three locations on the rotation axis with their distances to the isocenter of 0, 35, and 70 mm. At the top row are the reference local NPSs estimated from the entire 45 scans using the traditional method. At the bottom row are the radial local NPSs estimated from two scans. The yellow triangles help illustrate the widening trend of null cone from left to right. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 13. Noiseless cone-beam computed tomography images of a spherical signal (4 mm diameter) when located at (a) the isocenter (slice 40) and (b) 2 cm off the isocenter along the z-axis (slice 20). There is one more slice in (b) with a visible image of the signal.

FIG. 14. Signal energy maps of a sphere in the reconstruction space across (a) the isocenter plane and (b) the $z = 2$ cm axial plane.

scans from each scan and then multiplying by $\sqrt{N/(N-1)}$ to adjust the noise magnitude to be the same as that in the data. Another way to create noise-only images is by pairing two scans and taking a difference between the pair. The direct difference approach is slightly simpler but may not use the data efficiently when the number of scans is small or the number of scans is not even because it yields a lower number of noise images, which may result in a larger variation in the initial local NPS estimate compared to the subtracting the average approach used in this paper. Therefore, the difference approach to creating noise images may be less preferable in the radial local NPS method. Second, the initial local NPS was computed directly as the average of the modulus square of the Fourier transform of the noise image ROIs. It is common to apply an apodization window like a Hanning window to the noise image ROIs before taking the Fourier transform to reduce variance in the NPS.^{11,19} We believe the radial local NPS method when using a few scans can benefit from a less noisy initial local NPS estimate with the help of an apodization window. As the number of scans becomes larger, the benefit of applying apodization may be less significant; our results indicated that the NMSE error became quite small after six scans, indicating that the use of an apodization step may be negligible for this number of scans or more.

The 3D radial local NPS method greatly reduces the data burden for CBCT noise characterization. This can benefit CBCT manufacturers seeking to validate their device performance for regulatory purposes, since NPS is a commonly requested image quality metric in the bench testing performance for CT. The radial local NPS method may find its utilization in clinics as well. Being able to estimate local NPS with a reasonable accuracy from as few as two scans implies the possibility of obtaining the noise characteristics for clinical patient scans, for example, from two low-dose scans. The local NPS may be used to prewhiten the correlated image noise to help model observers (or computer-aided detection tools) to increase lesion detectability and estimation accuracy.^{8,20}

The developed radial local method does not assume uniformity of the object and does not need prior information about the reconstruction kernel. As long as the projection data or the processed projection data that are fed to the FDK reconstruction are uncorrelated or correlate the same way spatially, the polar separability is valid and the radial local NPS method works. The radial local NPS is applicable to common CBCT systems, such as the onboard imager in a radiotherapy system, breast CBCT, and dental CBCT if FDK-based reconstruction method is used. This method may also be extended to limited angle CBCT systems such as digital breast tomosynthesis, but geometrical transform parameters need to be calculated differently because the tube and detector movement is different from a concentrically rotating CBCT system. It should be noted that if nonlinear reconstruction methods like iterative reconstruction are used to create the CBCT volume, then the polar separability breaks down, and in theory, the radial local NPS method does not apply to such nonlinearly reconstructed CBCT images.

5. CONCLUSIONS

In this work, we extended the radial local NPS method from 2D CT to 3D CBCT. The radial local NPS method was developed based on the polar separability, a noise property that CT possesses when the images are reconstructed with FBP- or FDK-based methods, as derived in this paper. We showed that a good accuracy can be achieved with the radial local NPS method using a few scans in CBCT. Even with only two scans, the method reached a NMSE of less than 5% and the detectability computed using the two-scan-based local NPS already accurately quantified the system performance. The radial local NPS method was demonstrated to be highly data efficient. Therefore, it is possible to compute and use the local NPS in clinical CT settings. The radial local NPS method may also be beneficial in reducing the data burden on CT manufacturers seeking to validate their device performance for regulatory purposes.

CONFLICTS OF INTEREST

The authors have no conflicts to disclose.

DISCLAIMER

The mention of commercial entities or commercial products, their sources, or their use in connection with material reported herein is not to be construed as either an actual or implied endorsement of such entities or products by the Department of Health and Human Services or the U.S. Food and Drug Administration.

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