Numerical study of hydromagnetic axisymmetric peristaltic flow at high Reynolds number and wave number

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Abstract

The computational study of MHD peristaltic motion is investigated for axisymmetric flow problem. The developed model is present in the form of partial differential equations. Then obtained partial differential equations are transferred into stream-vorticity ($\psi-\omega$) form. Then Galerkin Finite element method is used to find the computational results of governing problem. The current study is compared with the existing well-known results at low Reynolds number and wave number. It is revealed that the present results are in well agreement with existing results in the literature. So, it is effective for higher values of Reynolds number and wave number. The variations of streamline are present graphically against high Reynolds number. It concludes that high Reynolds number and Hartmann number increase pressure rise per unit wavelength in positive pumping region sharply.

Keywords Peristaltic motion · High Reynolds numbers · MHD · Finite element method

Introduction

Peristalsis is a phenomenon of contraction and relaxation of wave incorporated by the movement of walls of flexible tube/channel. The applications of peristaltic motion are found in many physiological and industrial systems. In the human body, flow of chyme in small intestine, the transportation of fluid like propel/mix, the contents of a tube as in the ureter, spermatid fluid transport in female reproductive tract, blood flow through arteries gastrointestinal, blood flow through capillaries, and the swallowed food through the esophagus are examples in physiological flow. Transport in corrosive fluid in the nuclear industry, heart lung machine, Diabetes pumps, and pharmacological delivery systems also follow this phenomenon. The comprehensive investigation on peristaltic motion was provided by Latham (1966) in his master thesis. After this, Fung and Yih (1968) investigated peristaltic motion in fixed frame of reference without employing lubrication approach. They concluded that if the pressure gradient is smaller than the critical values of reflux, then the backward flow (reflux) does not exist. Shapiro et al. (1969) study the peristaltic motion in

mathematical point of view. They used lubrication theory to investigate the peristaltic motion for two-dimensional channel and axisymmetric flow in wave frame of reference. They model the problem by taking Reynolds number small enough to neglect the inertial effect and long wavelength assumption. They obtained theoretical results in both two-dimensional channel and axisymmetric flow by taking range of amplitude ratios from zero to one. They concluded that by increasing time mean flow rate, pressure rise per wavelength linearly decreases. The approach of Shapiro et al. (1969) was much simpler than the approach of Fung and Yih (1968). This is because the nonlinear terms in the Navier-Stokes equation vanish under the lubrication approach and provide in a linear equation. The lubrication approach does not make possible to study the inertial effects and wave number on various flow characteristics. That way, Shapiro et al. (1969) investigate peristaltic pumping under the condition in which Reynolds number takes small enough to neglect the inertia effects and diameter ratio of wavelength large enough in which the pressure to be considered uniform over the cross-section. The applicability of such assumption is narrow in physiological and industrial peristaltic flow because the characterizing of Reynolds number in such flow is very small. Moreover, in such flows, the wavelength is quite large in comparison with the radius of the organ. A typical example of such a flow is found in the small intestine. Lew et al. (1971) reported that the Reynolds number of such flow is much less than unity. In addition to that, the wavelength of wave is nearly 0.125 cm/s and the radius of the intestine is 0.008 cm. This also

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allows the applicability of long wavelength assumption to model peristaltic flow of chyme in small intestine. Jaffrin and Shapiro (1971) discussed the reflux and trapping limits of peristaltic pumping using the perturbation technique. They concluded that Reynolds number rapidly increases the domain of reflux region and decreases the domain of trapping. They also discussed the domain of reflux and trapping for the case of long wavelength and low Reynolds number assumption in two-dimensional plane and axisymmetric tube. They concluded that domain of reflux or trapping occur is larger than in plane case as compared with axisymmetric tube. Jaffrin (1973) studies the peristaltic pumping in two dimension tube using perturbation method. He concludes that increasing wall curvature also increases the pumping performance and decreases inertial effects except at high squeeze.

The first experimental study of peristaltic problem was investigated by Weinberg et al. (1971). They visualized the trapping and reflux phenomena for two-dimensional plane channel and compared the obtained result with the analytical result. Yin and Fung (1971) give a comparison between the theoretical and experimental results for two-dimensional peristaltic pumping. They observed 20 percent differences between theory and experiment result when amplitude ratio is equal to 0.41. They also observed that time mean flow rate is independent of Reynolds number when $Re < 2.5$. Brown and Hung (1977) study two-dimensional peristaltic motions experimentally and compare with the numerical simulations. They concluded that pressure reduces in the pumping region against time mean flow rate. They also concluded that by increasing Reynolds number from 2.3 to 251 yields uncertain increase in the ratio of flow rate to Reynolds numbers but a considerable increase in the shear stress. After these studies, many scientist and mathematician study the peristaltic motion for both Newtonian and Non-Newtonian fluid in channel and tube.

Mekheimer (2004) study the MHD peristaltic motion in inclined symmetric channel using regular perturbation method. His study was based on without long wavelength and low Reynolds number assumption. He concludes that by increasing MHD, the bolus exists and fluid particle is moving along the wave wall. El Naby et al. (2006) investigate the hydromagnetic peristaltic flow of Newtonian fluid through a uniform tube. They concluded that pressure rise per wavelength increases in the pumping region by increasing Hartmann number and amplitude ratio. Hayat and Ali (2006) study MHD peristaltic flow for third-grade fluid confined in a deformable tube under long wavelength and low Reynolds number assumption. They accomplished that axial velocity decreases, while pressures increase in the pumping region by increasing Hartmann number. Hayat and Ali (2007) also study peristaltic motion for Non-Newtonian fluid in a tube. They use perturbation technique to find the analytic solution after using long wavelength and low Reynolds number assumption. They concluded that by increasing Hartmann numbers, the pressure

rise per wavelength increases nonlinear in Non-Newtonian fluid. The nonlinear peristaltic motion in asymmetric inclined channel is investigated by Srinivas and Pushparaj (2008). They find the analytical solution by considering Reynolds number small enough to neglect the inertial effect and long wavelength assumption. They concluded that increasing wave amplitude ratio at lower wall, the stress on the lower wall enhances in the presence of MHD and reduces in the absence of MHD. Yildirim and Sezer (2010) study the partial slip effects of MHD peristaltic flow for Newtonian fluid in an asymmetric channel. They find an analytic solution using homotopy perturbation method (HPM) under long wavelength assumption. They determined that pressure rise per wavelength increases by increasing Hartman number for small-time mean flow rate less than unity and decreases when time mean flow rate greater than unity. Ali et al. (2008) study the slip effects of MHD peristaltic flow in the presence of variable viscosity in channel. They find series of solution for magnetohydrodynamic fluid under long wavelength and low Reynolds assumption. They concluded that pressure rise per wavelength decreases in positive pumping region by increasing the slip parameter. Mekheimer and Al-Arabi (2003) investigated the MHD peristaltic flow through a porous medium in a channel using perturbation method. In their investigation, a remarkable increase is observed in pressure rise per wavelength by increasing Hartmann number. Ebaid (2008) finds numerical solution for MHD peristaltic flow of a bio-fluid in a circular cylindrical tube with variable viscosity under long wavelength and low Reynolds number assumption. He used Adomian decomposition method (ADM) to solve the governing partial differential equation. He concluded that by increasing viscosity parameter, the pressure rise slightly decreases. Moreover, Hartmann number helps to enhance pressure rise per wavelength against time mean flow rate in the pumping region.

Computational or numerical studies against high Reynolds number and short wavelength of peristaltic motion in channel/tube are always challenging for researchers. First of all, Dennis and Chang (1969) solve Navier-Stokes equation for two-dimensional flow numerically using successive approximations against zero time mean flow. Then, the comprehensive computational study of peristaltic flow in the channel was carried out by Takabatake and Ayukawa (1982) with influences of wave amplitude, wavelength, and Reynolds number. They solve Navier-Stokes equation in stream-vorticity ($\psi - \omega$) form using an upwind finite difference technique with SOR method. They explained the features of peristaltic motion at moderate Reynolds number and discussed the restrictions of perturbation results given by Zien and Ostrach (1970) and Jaffrin (1973). They discussed velocity and streamline phenomena upto $Re = 210$ against zero time mean flow. Takabatake et al. (1988) study the peristaltic motion in an

axisymmetric tube under finite wavelengths, Reynolds number, and amplitude ratio using FDM with SOR method. They concluded that peristaltic mixing and transport are higher in circular cylindrical tube than in a plane channel. Takabatake et al. (1987) also study the peristaltic motion in two-dimensional peristaltic channel using finite element method. They concluded that the inertia effects of the fluid and the effects of large wall slop increase the backward flow from the narrowest region of the channel. Takabatake (1990) also extended it and find the peristaltic pumping against large wave amplitude and wave-wall slop. They concluded that against large wall slop, the pressure rise per wavelength increases remarkably for zero time mean flow, and fluid inertia effects decrease for small amplitude and increase for large amplitude ratio. After this, Kumar and Naidu (1994) also solve Navier-Stokes equation for two-dimensional peristaltic flow using stream-vorticity ($\psi - \omega$) formulation. The main difference from the earlier study is that they obtained well-convergent results at high Reynolds number up to 100 against small time value of time mean flow, i.e., $Q = 0.05$. They concluded that progressive sinusoidal waves with low wave number and high amplitude generate peristaltic flows with high shear stress variations. Their analyses also include the effects of constant applied magnetic field on the peristaltic for non-zero Reynolds numbers and wave numbers and they conclude that wall shear stresses decrease under her influence of external magnetic field. Xiao and Damodaran (2002) find the numerical solution of peristaltic motion in an axisymmetric tube by considering zero time mean flow rate assumption using finite volume method. They discussed trapping phenomena at high inertial effects at any time mean flow rate. They also discussed trapping phenomena by taking time mean flow rate 0.6 and Reynolds number $Re \leq 1$. They concluded that time mean flow rate must be zero at higher inertial effects.

In the background of the above literature review, the aim of the present study is to analyze the peristaltic motion in an axisymmetric tube in the presence of MHD without using any assumption. To discuss the peristaltic motion in a circular tube, the numerical technique like finite difference method, finite volume method, Adomian decomposition method, etc. is not good enough to achieve the accuracy for a moderate Reynolds number because higher-order non-linearity appears in the equation. That's why, we need an efficient technique to handle higher-order non-linearity. For this purpose, we evaluate the governing partial differential equation using the Galerkin finite element method. The effect of velocity profile, temperature profile, and pressure rise against moderate Reynolds number are observed graphically. The trapping phenomena are also discussed against different values of parameters. To validate the obtained results, obtained through the present scheme is also compared with results

based on previously available studies of Shapiro et al. (1969) and numerical result of Xiao and Damodaran (2002).

Mathematical modeling

Consider that the axisymmetric incompressible peristaltic flow is inscribed in an inclined tube of length $2a$ with inclination angle γ filled with the Newtonian fluid. The motion is considered sinusoidal wave with constant speed c along the wall. The uniform magnetic field B_0 is applied in the transverse direction of the flow. The geometry of the flow is presented in Fig. 1.

The moment of the boundary in fixed frame define as

$$H(Z, T) = a - b \cos \left[\frac{2\pi(Z - cT)}{\lambda} \right] \quad (1)$$

where a is the mean distance of the wall from the central axis, b is the wave amplitude and λ is the wavelength, c is the velocity and t is the time. The governing model in a fixed frame is presented as set partial differential equations define as

$$\frac{1}{R} \frac{\partial(RU)}{\partial R} + \frac{\partial W}{\partial Z} = 0 \quad (2)$$

$$\rho \left(U \frac{\partial U}{\partial R} + W \frac{\partial U}{\partial Z} \right) = - \frac{\partial P}{\partial R} + \frac{\partial}{\partial R} \left(\frac{1}{R} \frac{\partial(RU)}{\partial R} \right) + \frac{\partial^2 U}{\partial Z^2} - \rho g \cos \gamma \quad (3)$$

$$\rho \left(U \frac{\partial W}{\partial R} + W \frac{\partial W}{\partial Z} \right) = - \frac{\partial P}{\partial Z} + \frac{1}{R} \frac{\partial}{\partial R} \left(R \frac{\partial W}{\partial R} \right) + \frac{\partial^2 W}{\partial Z^2} - \sigma B_0^2 W + \rho g \sin \gamma \quad (4)$$

The appropriate boundary conditions are

$$W = 0, \quad \frac{\partial U}{\partial Z} = 0 \quad \text{at} \quad Z = 0 \quad (5)$$

$$U = 0, \quad W = \frac{\partial H}{\partial t} \quad \text{at} \quad Z = H \quad (6)$$

Fig. 1 Geometry of the peristaltic flow in the tube

The Neumann boundary conditions on U at $R=0$ arise for symmetric flow and $U=0$ at $R=H$ is no-slip condition at the wall. The condition $W=0$ at $R=0$ means that transvers velocity zero at the center of the channel and condition $W=\partial H/\partial t$ at $R=H$ represents that the normal velocity at the wall is equal to the normal velocity of the fluid. First, transform governing equation from lab frame to wave frame using the relation

$$r^* = R, \quad z^* = Z - cT, \quad w^* = W - c, \quad u^* = U \quad (7)$$

where (Z, R) are axial and radial component of velocity in fixed frame and (z^*, r^*) in moving frames of reference. As both planes $r^*=0$ and $r^*=\eta(z^*)$ constitute the streamline in the moving frame, reference volume flow rate q^* in the moving frame is constant at all cross-section of the channel. Thus, the following boundary conditions are obtained

$$\psi^* = 0 \text{ on } r^* = 0, \psi^* = q^* \text{ on } r = \eta(z^*) \quad (8)$$

The relation between time mean flow rate in wave and laboratory frame is defined as $q^* = Q^* - ca^2 \left(1 + \frac{\phi^2}{2}\right)$. Where q^* and Q^* are time mean flow rate in moving frame and fixed frame respectively and ψ^* is stream function. The dimensionless variables are define as follows

$$z = \frac{z^*}{\lambda}, \quad r = \frac{r^*}{a}, \quad w = \frac{w^*}{c}, \quad u = \frac{u^*}{c},$$

$$p = \frac{a^2}{\lambda \mu c} p^*(z), \quad \psi = \frac{\psi^*}{ca}, \quad q = \frac{q^*}{ca}, \quad \eta = \frac{\eta^*}{a}, \quad h = \frac{h(x^*)}{a},$$

$$Re = \frac{ca}{\nu} \alpha, \quad \alpha = \frac{a}{\lambda}.$$

Eliminate pressure gradient and introducing stream function relation $u = \frac{a}{r} \partial \psi / \partial z$ and $w = -\frac{1}{r} \partial \psi / \partial r$, the final form of governing equations and boundary conditions in terms of stream-vorticity form are as follows

$$\frac{\alpha^2}{r} \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \psi}{\partial r} \right) = -\omega \quad (9)$$

$$Re \left(\frac{\partial \psi}{\partial r} \frac{\partial}{\partial z} \left(\frac{\omega}{r} \right) - \frac{\partial \psi}{\partial z} \frac{\partial}{\partial r} \left(\frac{\omega}{r} \right) \right) = \frac{1}{r} D^2(r\omega) \quad (10)$$

where $D^2 = \alpha^2 \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r}$ is modified Laplacian and the boundary conditions becomes

$$\psi = 0, \quad \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \psi}{\partial r} \right) = 0, \quad \frac{\partial \psi}{\partial z} = 0 \quad \text{on } r = 0,$$

$$\psi = q, \quad \frac{1}{r} \frac{\partial \psi}{\partial r} = -1, \quad \frac{\partial \psi}{\partial z} = 2\pi \phi \sin 2\pi z \quad \text{on } r = \eta(z) \quad (11)$$

Numerical analysis

In present study, the governing Eqs. (9) and (10) with the boundary conditions (11) are solved numerically. The present studies are based on without applying any assumption which leads the governing model into higher order nonlinear PDE. The exact or analytical solution fails to produce good result. Therefore, a rapid convergent, more accurate, and efficient numerical method is used to solve higher-order nonlinear PDE. The Galerkin finite element method is used to solve governing Eqs. (9) and (10). The domain is discretized in terms of non-uniform meshing and quadratic triangle elements by using pde tool in MATLAB. In all the cases, a highly convergent result has been obtained using Newton Raphson method with tolerance of $10e-15$ into maximum 3–5 numbers of iterations. The dependent variables, stream function, and vorticity are approximated as follows

$$\psi = \sum_{k=1}^n N_k \psi_k, \quad \omega = \sum_{k=1}^n N_k \omega_k, \quad (12)$$

where N_k , ψ_k , and ω_k are shape function, element nodal approximation of stream function, and vorticity respectively. The weak formulation is applied to governing Eqs. (9) and (10) as follows

$$\int_{\Omega} w_1 \left(\frac{\alpha^2}{r} \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \psi}{\partial r} \right) + \omega \right) d\Omega = 0 \quad (13)$$

$$\int_{\Omega} w_2 Re \left(\frac{\partial \psi}{\partial r} \frac{\partial}{\partial z} \left(\frac{\omega}{r} \right) - \frac{\partial \psi}{\partial z} \frac{\partial}{\partial r} \left(\frac{\omega}{r} \right) - \frac{1}{r} D^2(r\omega) \right) d\Omega = 0 \quad (14)$$

where $d\Omega = 2\pi r dr dz$ is domain of the problem; w_1 and w_2 are weight functions. Simplify Eqs. (13) and (14)

$$\int_{\Omega} \left(\frac{\alpha^2}{r} \frac{\partial w_1}{\partial z} \frac{\partial \psi}{\partial z} + \frac{\partial w_1}{\partial r} \left(\frac{1}{r} \frac{\partial \psi}{\partial r} \right) - w_1 \omega \right) d\Omega = \int_{\Gamma} w_1 \frac{\partial \psi}{\partial n} d\Gamma \quad (15)$$

$$\int_{\Omega} w_2 Re \left(\frac{\partial \psi}{\partial r} \frac{\partial}{\partial z} \left(\frac{\omega}{r} \right) - \frac{\partial \psi}{\partial z} \frac{\partial}{\partial r} \left(\frac{\omega}{r} \right) \right) d\Omega + M^2 \int_{\Omega} \left(\frac{\partial w_2}{\partial r} \frac{1}{r} \frac{\partial \psi}{\partial r} \right) d\Omega +$$

$$\int_{\Omega} \left(\frac{\partial w_2}{\partial r} \frac{1}{r} \frac{\partial(r\omega)}{\partial r} + \frac{\alpha^2}{r} \frac{\partial w_2}{\partial z} \frac{\partial(r\omega)}{\partial z} \right) d\Omega = \int_{\Gamma} w_2 \frac{\partial \psi}{\partial n} d\Gamma + M^2 \int_{\Gamma} w_2 \frac{\partial \psi}{\partial n} d\Gamma \quad (16)$$

where $d\Gamma = \pi r dr dz$. Introducing Eq. (12) into Eqs. (15) and (16) and considering the discretized domain, we have

$$-\sum_i B_{ki}^e \omega_i + \sum_i A_{ki}^e \psi_i = S_n^{k^e} \quad (17)$$

$$Re \sum_i C_{kij}^e \psi_i \omega_i + \sum_i A_{ki}^e \omega_i + M^2 \sum_i D_{ki}^e \psi_i = M^2 S_n^{k^e} \quad (18)$$

where

$$A_{ki}^e = \int_{\Omega^e} \left(\frac{\alpha^2}{r} \frac{\partial N_k}{\partial z} \frac{\partial N_i}{\partial z} + \frac{\partial N_k}{\partial r} \left(\frac{1}{r} \frac{\partial N_i}{\partial r} \right) \right) d\Omega \quad (19)$$

$$B_{ki}^e = \int_{\Omega^e} N_k N_i d\Omega \quad (20)$$

$$C_{kij}^e = \int_{\Omega} N_k \left(\frac{\partial N_i}{\partial r} \frac{\partial}{\partial z} \left(\frac{N_j}{r} \right) - \frac{\partial N_i}{\partial z} \frac{\partial}{\partial r} \left(\frac{N_j}{r} \right) \right) d\Omega \quad (21)$$

$$D_{ki}^e = \int_{\Omega} \left(\frac{\partial N_k}{\partial r} \frac{1}{r} \frac{\partial N_i}{\partial r} \right) d\Omega \quad (22)$$

$$S_n^{ke} = \int_{\Gamma} N_k \bar{S}_k d\Gamma \quad (23)$$

The global system in matrix form is defined as

$$KA = F \quad (24)$$

where

$$K_{ij} = \begin{bmatrix} -B_{ki}^e & A_{ki}^e \\ A_{ki}^e & Re C_{kij}^e \omega_i + M^2 D_{ki}^e \end{bmatrix}, A_k = \begin{bmatrix} \omega_k \\ \psi_k \end{bmatrix}, F_k = \begin{bmatrix} S_n^{ke} \\ M^2 S_n^{ke} \end{bmatrix} \quad (25)$$

The system of Eq. (25) solve simple inversion method to obtain the required convergence.

Pressure analysis

One of the important part of peristaltic motion is pressure rise per unit wavelength (ΔP_λ) which is obtained from numerical integral of pressure gradient. Since peristaltic motion is based on infinite train sinusoidal wave, it is sufficient to calculate the pressure only middle part ($y = 0$) central of the unit wave domain. The pressure rise per unit wavelength (ΔP_λ) can be computed by using the following expression

$$\Delta p_\lambda = \int_0^\lambda \frac{dP}{dz} dz \quad (26)$$

where $\frac{dp}{dz}$ is easily obtained directly from Navier-Stokes equation in the form of stream-vorticity ($\psi - \omega$) directly.

Result and discussion

This section provides the detail of computation results made in terms velocity profile at $z = 0$ cross-section, streamlines in wave frame, pressure rise per wavelength for related parameters including Reynolds number (Re), amplitude ratio (ϕ), volume flow rate (Q), and Hartmann number (M). All obtained results are prepared graphically.

Validation and pressure

To ensure the validity of our computed results for axisymmetric flow, it is compared with the famous result of Shapiro et al. (1969) as the limiting case and found in good argument. Shapiro et al. (1969) results are based for long wavelength and low Reynolds numbers assumption so it is not valid for moderate Reynolds number. The compression of present pressure rise per wavelength with Shapiro et al. (1969) results against time mean flow Q is shown in Fig. 2a. It is observed that the present computational results at low Reynolds number and long wavelength assumption for different amplitude ratio are matched well with Shapiro et al. (1969). The present pressure gradient is also compared with the numerical result of Xiao and Domodaran (2002). Xiao and Damodaran (2002) solve peristaltic motion in tube using finite difference method with fixed time mean flow rate at zero. Figure 2b shows that

Fig. 2 Comparison of **a** pressure rise per unit length against Q (—) with that of Shapiro et al. (1969) (—) and **b** pressure gradient distribution at $Re = 0.01$ and $\alpha = 0.01$ with numerical solution of Xiao and Damodaran (2002)

Fig. 3 Pressure rise per unit length **a** different value of Re and **b** different values of M

the present pressure gradient is well matched with the result of Xiao and Damodaran (2002). Therefore, it builds confidence that the present study is valid for moderate values of Reynolds number Re and wave number α . Figure 3a shows that the

Fig. 4 Maximum velocity at the center of the tube against Reynolds number for different values of Hartmann number with $\alpha = 0.3$, $\phi = 0.5$, and $Q = 1.2$

pressure rise per unit wavelength against volume flow rate Q for different value of Reynolds numbers. It is noted that pressure increases in positive pumping region by increasing Reynolds number (Re), while it decreases in augmented pumping region. Also, the pressure becomes linear by neglecting inertia effect and the curvature effects are dominant in the positive pumping region against large inertia effect. Figure 3b shows the pressure rise per unit wavelength against time mean flow rate for different Hartman number. It is noted that pressure increases in the interval $0 < Q \leq 0.75$ and decreases in the interval $0.75 < Q < 1$.

Fig. 5 Velocity profile against r for **a** Reynolds number (Re), **b** volume flow rate (Q), and **c** Hartmann number (M)

Fig. 6 Streamline effects at different values of Reynolds number against $\alpha = 0.1$, $M = 2$, $\phi = 0.7$, and $Q = 1.4$

Velocity

Figures 4 and 5 show the longitudinal velocity w for different value volume flow rate Q , Reynolds number Re , and magnetic parameter $z = 0$ cross-section. Figure 4 shows the variation of velocity at $z = 0$ cross-section and center of the tube for

different magnetic parameter when time mean flow rate fixed at $Q = 1.4$. It is observed that the velocity is maximum at Reynolds number $Re = 18$ for small Hartmann number. When $Re > 18$, the velocity at center of the tube ($r = 0$) decreases and becomes stable for larger Re . It is also observed that by increasing Magnetic field, the velocity increases at the

center of the tube in the range of $0 < Re < 18$. The velocity profile at different Reynolds number at $z = 0$ cross-section is shown in Fig. 5a. It determines that for large Reynolds number, the velocity increases sharply at the center of the tube ($r = 0$) and reduces near the wall. It concludes that the inertia forces are helpful to enhance the velocity of the fluid. The velocity against different volume flow rates is shown in Fig. 5b. It determines that velocity increases in the entire region of peristaltic wave. Also, less curvature effects are observed against small-volume flow rate. In Fig. 5c, the minor change is observed in velocity profile against Hartman number, that is, by

Fig. 7 Streamline effects at different values of Reynolds number against $\alpha = 0.1$, $Re = 5$, $\phi = 0.7$, and $Q = 1.4$

increasing magnetic field, the velocity enhances near the center of the tube and reduces its strength near the wall.

Stream line and trapping

If a part of fluid motion enclosed by a streamline separated from the axis in the wave frame called trapping. Figures 6 and 7 show the trapping phenomenon against inertia and magnetic effects. It is observed that large number of bolus appear against small inertia effects and number of bolus reduces at high inertia effects when amplitude ratio is fixed at $\phi = 0.7$ and time mean flow rate at $Q = 1.4$. Also the streamline near the center of the tube tends to parallel to z -axis for a large value of Reynolds number. It concludes that for high Reynolds number, the flow of fluid becomes laminar at the center of the tube. The magnetic effects at amplitude ratio $\phi = 0.7$ and time mean flow rate $Q = 1.4$ are observed in Fig. 7. From the figure, it is observed that the trapping bolus appears near at the middle section of the peristaltic wave. It is also noted that more bolus strengthen appear near the convergent part of peristaltic wave. Moreover, when Hartman parameter increases, the behavior of streamlines is the same but small change is observed in boluses that is a number of boluses are almost the same in size. It determines that magnetic field is helpful to enhance the velocity of the fluid inside the peristaltic tube.

Conclusion

The computational study of MHD peristaltic motion for Newtonian fluid in n axisymmetric tube is carried out in this chapter. The finite element technique is used to find the numerical solution without using any assumption and discusses velocity profile, pressure rise, and streamline behavior at moderate Reynold number and wave number which is not available in the earlier study. The present study also compared with analytical result of Shapiro et al. (1969) at $Re = 0$ and $\alpha = 0$ and numerical result of Xiao and Damodaran (2002) for validation and found good agreement. The main findings of the present study are

- Maximum Longitude velocity at $z = 0$ and $r = 0$ achieved at $Re = 18$ for low magnetic effect. After increasing Reynolds number, velocity decreases and becomes stable at high Reynolds number.
- Longitude velocity increases by increasing magnetic effects at the center of the tube decreases near the wall.
- Longitude velocity increases at $z = 0$ by increasing time mean flow rate Q .
- Pressure rise per unit wavelength sharply increases in the positive pumping region by increasing Reynolds number and Hartmann number and decreases in co-pumping region.

- More trapping boluses appeared near the diverging part of the wave by increasing Hartmann number

Compliance with ethical standards

Conflict of interest A. H. Hamid declares that he has no conflict of interest. Tariq Javed declares that he has no conflict of interest. N. Ali declares that he has no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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