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Cognitive Abilities and Mathematical Competencies at School Entry

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Abstract

The aim of this study was to identify mathematical competencies in early childhood and cognitive correlates of those competencies in a prospective longitudinal sample of children (N=1292) in predominantly low-income and nonurban communities in the United States. General mental ability (IQ), processing speed, vocabulary, and the working memory, inhibitory control, and cognitive flexibility components of executive function (EF) were assessed when children were ages 4 and 5. Math ability was assessed prior to school entry using a norm-referenced assessment. Exploratory factor analysis indicated that items from the math assessment loaded onto factors representing conceptual and procedural skill. IQ, processing speed, vocabulary, and a unitary EF composite all related to both conceptual and procedural skill. When EF components were examined separately, however, only the inhibitory control aspect of EF was related to conceptual skill and only the working memory aspect of EF was related to procedural skill.

Keywords

executive function; mathematics; cognition; education; early childhood

Basic numerical/mathematical competencies in early childhood are robust indicators of later math achievement (e.g., Duncan et al., 2007; Jordan, Kaplan, Ramineni, & Loduniak, 2009; LeFevre et al., 2010). Nevertheless, questions about the structure of math abilities as children enter formal schooling and the individual and contextual factors that support success in early math remain to be addressed. The aim of the present study was to better understand basic mathematical competencies that make up early math achievement, and to evaluate in what way domain-general cognitive abilities relate to performance in those competencies in a large prospective longitudinal sample of 5-year-old children. In particular, we considered indicators of children's general cognitive functioning, including IQ,

processing speed, and vocabulary, with a specific focus on executive function (EF) and its component parts as individual factors. Additionally, we investigated the role of contextual factors such as maternal education and family income.

Conceptual and Procedural Mathematics

The majority of large-scale assessment research on mathematical achievement treats math competencies as a unitary construct: Assessors typically obtain a single parameter score to reflect children's math achievement broadly. However, a closer look into basic research on numerical cognition and math education suggests that math is not a unitary construct, but made up of a series of competencies that can be differentiated into distinct components (e.g., LeFevre et al., 2010). Indeed, early math has been conceptualized a number of ways that range from a two-dimensional model that includes counting and magnitude comparison (e.g., Aunio, Ee, Lim, Hautamäki, & Van Luit, 2004; Aunio & Nirmivirta, 2010) to a five-dimensional model that includes non-symbolic magnitude comparison, symbolic comparison, symbolic labeling (e.g., numeral recognition and sequencing), rote counting forward and backward across the number line, and counting knowledge (e.g., conceptual understanding of counting) (e.g., Cirino, 2011). However, there is little agreement with respect to a specific operationalization of the domains of numeracy (e.g., Berch, 2005; Gersten, Jordan, & Flojo, 2005).

Furthermore, decades of research into underlying components of academic success in mathematics has emphasized conceptual and procedural skills as central to the teaching and learning of math (Bisanz & LeFevre, 1992; Hiebert, 2013; Rittle-Johnson, Siegler, & Alibali, 2001; Star, 2005). Conceptual knowledge, which is defined as implicit or explicit understanding of the principles and rules that govern mathematics, is generalizable and transferable across contexts. For example, the same rules always govern counting (e.g., no double counting, 1-1 correspondence, cardinal principle) regardless of the language in which the counting occurs. That is counting, "one, two, three" represents a set of "3" wherein each object was referred to once and only once. The skills that make up conceptual knowledge also include quantity representation, including the idea that a given quantity can be represented with a spoken word, a written numeral, or a drawn figure (e.g., "two" is the same as "2" which is the same as "••"). For young children, conceptual knowledge often pertains to principles that underlie children's ability to perform higher-order operations, and include identification of Arabic numerals, identifying and filling in breaks in patterns, and understanding ordinal relations between numbers.

In contrast, procedural knowledge refers to understanding of the specific sequence of steps taken to solve a problem (e.g. order of operations), is often implicated in solving multi-step problems, and is tied to specific problem types. Procedural knowledge does not generalize in the same way as principles understood as a part of conceptual knowledge, just as the process of carrying values in multi-digit addition and subtraction differs. For young children, procedural knowledge is often assessed as children's ability to identify the number of objects in a set and perform exact, formal arithmetic operations. Others have described procedural knowledge as an understanding of *what* is to be done to solve a given problem, whereas conceptual knowledge describes the underlying *how* and *why* each of those steps

are taken to solve the problem. Some problem types, such as multi-digit addition and subtraction, require both procedural and conceptual knowledge; whereas others require only conceptual or procedural knowledge.

In early childhood, most mathematical problem types are limited to only conceptual knowledge (e.g., recognizing Arabic numerals) or procedural knowledge (e.g., identifying the number of respondents for a given group on a bar chart), and measurement of conceptual and procedural knowledge typically reflects that. For instance, Gilmore, Keeble, Richardson, and Cragg's (*in press*) investigation of the development of mathematical skills 6-year-old students defined procedural knowledge as formal arithmetic and counting, whereas conceptual knowledge was measured as children's more informal understanding of principles by detecting errors in watching an experimenter count or perform operations. Other investigations with like-aged children have attempted to isolate the conceptual and procedural demands of a single skill, such as counting. LeFevre and colleagues (2006) tested the development of children's conceptual and procedural understanding of counting by asking 5- to 6-year-old students to detect errors about principles of counting (e.g., double counting, skipped object, repeated value) and to count and report the number of objects in a set, respectively. While there is extensive debate as to whether conceptual knowledge precedes procedural knowledge or vice versa (Gilmore et al., *in press*; Rittle-Johnson & Alibali, 1999), or whether there is a bidirectional relation between the two (Rittle-Johnson, Schneider, & Star, 2015; Rittle-Johnson, Siegler, & Alibali, 2001), it has been well established that both conceptual and procedural knowledge contribute unique variance to math abilities throughout the lifespan (Gilmore et al., *in press*; Hiebert, 2013; Rittle-Johnson & Siegler, 1998).

Although the conceptual-procedural distinction is well established in research on mathematics ability, it is not generally reflected in the way that math is assessed in young children. Usually, the single parameter score researchers, practitioners, and policy makers use when discussing children's math even before they enter formal schooling reflects a combination of conceptual and procedural knowledge, especially as many items in large-scale assessment tests of children's math achievement require both competencies. However, it is also important to consider the development of each of these competencies individually, particularly given that these competencies reflect the way that mathematics is taught to children (Baker et al., 2010). It remains uncertain, though, the individual- and contextual-level factors that contribute to the development of individual mathematical competencies before children enter kindergarten.

Executive Function

Just as math can be considered in terms of component areas and conceptual and procedural parts on learning in these areas, so too can EF be considered as a composite of distinct but related abilities. EF broadly encompasses cognitive skills effortfully engaged in the achievement of goals, and includes manipulation and integration of information, and conscious control of attention, thoughts, and behavior, and is considered a product of inhibitory control (IC), working memory (WM), cognitive flexibility (CF) (Diamond, 2013; Garon, Bryson, & Smith, 2008; Hughes, Ensor, Wilson, & Graham, 2009). IC is

characterized as the ability to inhibit an automatic or prepotent response in order to perform a less automatic, but more correct, action. WM, or updating, describes the ability to maintain and manipulate multiple pieces of information in mind at a given time. Finally, CF, or shifting, is defined as flexibly shifting attention between distinct but related aspects of a given set of stimuli. Although confirmatory factor models indicate that EF in early childhood is best considered as unitary (Weibe, Espy, & Charak, 2008), two and sometimes three factor solutions tend to fit as well as a single factor solutions (Miller, Müller, Giesbrecht, Carpendale, & Kerns, 2013). For reasons of parsimony, single factor models are adopted, though there remains debate as to whether each component of EF contributes unique variance to the development of various academic skills.

Executive Function and Mathematics

It has been well established that EF is strongly related to the development of math skills throughout the lifespan (e.g., Bull & Lee, 2014; Cragg & Gilmore, 2014). However, from the perspective of education and the development of competencies in mathematics, it may be beneficial to distinguish among components of EF when coming to conclusions about the cognitive demand of different domains of mathematics.

From an educational standpoint, if children are struggling with a specific aspect of mathematical ability, more precise information about the cognitive skills that contribute to that ability may allow for more targeted assessment and support. For example, prior investigations have found that for adults, working memory training may—at least in some cases—transfer to arithmetic skills (e.g., Bergman-Nutley & Klingberg, 2014) and in children, may transfer to numerical comparisons, but not to counting skills (Kroesbergen, van't Noordende, & Kolkman, 2012); however meta-analyses have shown that training effects generally do not transfer to mathematical skills (e.g., Melby-Lervåg & Hulme, 2013). Others have proposed IC as the component of EF that contributes the most unique variance to math achievement in early years (Espy, McDiarmid, Cwik, Stalets, Hamby, & Senn, 2004).

Generally speaking, however, prior analyses of relations between cognitive ability and mathematics ability have not attended to distinctions among both math and EF simultaneously. Many studies have examined the relations of various dimensions of cognitive ability, including specific aspects of EF to broad mathematics ability composites. Findings from these studies clearly indicate a unique role for specific cognitive abilities, including in many cases specific aspects of EF (Blair & Razza, 2007; Bull, Espy & Weibe, 2008; Clark, Pritchard, & Woodward, 2010; Espy et al., 2004; Kroesbergen, Van Luit, Van Lieshout, Van Loosbroek, & Van de Rijt, 2009; St. Clair-Thompson & Gathercole, 2006) over and above measures of general intelligence, processing speed, and vocabulary.

Only a small number of studies have examined relations of components of cognitive ability, including components of EF to components of mathematical ability (e.g., Fuchs et al., 2006). In a recent study, Purpura and colleagues (2017) found that in a sample of 3- to 5-year-old children, IC was uniquely related to basic math skills including recitation of the verbal count sequence, subitizing, and one-to-one correspondence and thus more conceptual aspects of

mathematics, whereas WM related to more advanced math skills such as ordinal comparisons and operations and thus rather procedural aspects; CF only related uniquely to numeral identification, though also overlapped with WM and IC in its relation to cardinality and number ordering. However, that study had several limitations: First, measures of EF and math were taken concurrently. Second, assessment of math skills was limited to relatively few items chosen a priori by investigators to represent different domains of math, in such a way in which a single task was used to assess each construct. While all assessed constructs represent important building blocks and standards in mathematics (e.g., verbal counting, subitizing, set comparison, cardinality, ordinality, numeral identification), they did not assess larger broader divisions in which math educators conceptualize mathematics (i.e., conceptual and procedural mathematics), and did not consider items that might correlate to standardized assessments of math achievement.

Current Study

Given the limited research base on the unique contribution of specific aspects of cognitive ability to distinct aspects of math learning in early childhood, the aim of the present study was to understand the unique relations of different aspects of cognitive ability to distinct aspects of mathematical competency at school entry. We pursued this aim in a two-staged procedure. We first employed an exploratory approach to establish the presence of distinct basic mathematical competencies within a large-scale, standardized test of early math ability. We then evaluated relations of multiple aspects of cognitive ability to the distinct mathematical competencies. We expected, in keeping with the recent findings of Purpura et al. (2017) that the IC component of EF will be related specifically to more conceptual aspects, whereas WM will be related to more procedural aspects of mathematics which frequently require students to remember and enact multiple steps in order. We explore whether CF will relate to either aspect of mathematics.

Method

Participants

The Family Life Project (FLP) was designed to study families living in two of the four major geographical areas of high child rural poverty (Dill, 2001). Complex sampling procedures were used to recruit a representative sample of 1,292 families at the time of the target child's birth, with oversampling of low-income families and families of African American ethnicity. Further details on the Family Life Project sampling plan and recruitment procedures are available in Vernon-Feagans, Cox, and the Family Life Project Investigators (2013). Seventy percent of families had an average income of less than 200% of the poverty line. Additionally, forty percent of mothers had a high school education (12 years of schooling) or less, while only sixteen percent had at least 4 years of post-secondary education. A little more than half of the sample is White (57%) with the remainder of African American descent.

Procedures

Families were visited in their homes for data collection at child ages 7, 15, 24, 36, 48, and 60 months and the primary caregiver (the mother in almost all instances) responded to a number of questionnaires including one on family and child demographics (e.g., child race, sex, maternal education, household income). In addition to a number of other procedures, at the 36-, 48-, and 60-month visits children were administered a battery of EF tasks. At the 36-month visit, children were administered a short-form IQ test. Academic abilities and measures of speed of processing and vocabulary were measured when children were approximately 60 months ($M_{age} = 60.16$ months, $SD = 3.29$) and identified as being in pre-kindergarten (i.e., the year before children entered formal schooling). These assessments took place in school settings when possible, or in home settings in cases that children were not enrolled in center- or school-based care.

Measures

Math Ability—Math ability was assessed using the Early Childhood Longitudinal Study-Kindergarten Cohort (ECLS-K) math assessment. The reliability and validity of measure are well established (Rock & Pollack, 2002). The ECLS-K uses a routing system in which all children receive a series of 14 routing items. Scores of 8 or lower are routed to a low difficulty block of items while scores higher than 8 are routed medium or high difficulty items. The majority of participants in our sample were routed to the “low” block ($N = 849$). Because few participants were routed to the “medium” block ($N = 50$), or “high” block ($N = 12$), only data from participants who were routed to the “low” block were included in analyses. Those participants in the “low” block responded a total of 32 items, including the 14 routing items and 18 items on the “low” block.

Executive function—The EF battery contained six tasks. All tasks were administered on an open spiral-bound notebook by a trained research assistant. These tasks are described in detail and evaluated elsewhere (Willoughby, Blair, Wirth, Greenberg, & Family Life Project Investigators, 2010; Willoughby & Blair, 2011; Willoughby et al., 2012) and described only briefly here. Data from administrations at 48 and 60 months are included in this analysis.

Working Memory Span (working memory): Children were shown a line drawing of an animal and a color inside an outline of a house and asked to keep both the animal and the color in mind, and to recall one of them (e.g., animal name) when prompted. Task difficulty increased by adding items to successive trials: Children received one 1-house trial, two 2-house trials, two 3-house trials, and two 4-house trials. Responses were summarized as the number of items answered correctly.

Pick the Picture Game (working memory): Children were presented with a series of 2, 3, 4, and 6 pictures and instructed to continue picking pictures until each picture had “received a turn”. This task requires children to remember which pictures they have already touched.

Silly Sounds Stroop (inhibitory control): Children were asked to make the sound opposite of that associated with pictures of dogs and cats (e.g., meow when shown a picture of a dog).

Spatial Conflict Arrows (inhibitory control): Children were asked to touch the “button” (black circles) presented at the bottom left and right of the page corresponding to the direction in which an arrow presented to the left or right at the top of the page was pointing. The task proceeds in three blocks: the first 8 trials depict all stimuli centrally, items 9–22 present left-facing arrows on the left and right-facing arrows on the right of the page (congruent trials), and items 23–35 present a mix of congruent and incongruent trials (direction of the arrow and response location are not congruent).

Animal Go/No-Go (inhibitory control): Children were instructed to push a button (which emitted a sound) whenever they saw an animal appear, except when the animal was a pig. The number of go-trials before a no-go trial varied, in a standard order, of 1-go, 3-go, 3-go, 5-go, 1-go, 1-go, and 3-go trials.

Something’s the Same Game (cognitive flexibility): Children were shown two pictures that were similar on a single criterion (e.g., the same color; the same size), and were then shown a third picture that was similar to one of the first two pictures along a second criterion. Participants were asked to identify which of the first two pictures was similar to the new picture.

Item Scoring: Item response theory (IRT) scoring was used for each task. A single composite variable that reflected the mean of IRT scores on each of the 6 tasks at each time point was created as well as composites reflecting the inhibitory control (Silly Sounds Stroop, Spatial Conflict Arrows, Animal Go/No-Go), working memory (Working Memory Span, Pick the Picture Game), and cognitive flexibility (Something’s the Same Game) aspects of EF. Only tasks from 48- and 60-month visits were included in the present investigation.

Covariates

Individual- and family-level covariates were included in final models of analyses. These covariates included continuous variables for years of maternal education, income-to-need ratio, processing speed, IQ, and receptive vocabulary.

Processing Speed—At the 60-month school visit, processing speed was measured using the symbol search and coding subscales of the Wechsler Preschool and Primary Scales of Intelligence (WPPSI; Wechsler, 2002).

IQ—At the 36-month home visit, children completed the block design and receptive vocabulary subtests of the Wechsler Preschool and Primary Scales of Intelligence (WPPSI; Wechsler, 2002). A full-scale IQ score was estimated.

Receptive Vocabulary—At the 60-month school visit, receptive vocabulary was measured using the Peabody Picture Vocabulary Test-4th Edition (PPVT; Dunn & Dunn, 2007), a norm-referenced assessment commonly used with children of this age.

Analytic Strategy

To establish the presence of separate mathematical competencies, items from the ECLS-K were subjected to an exploratory factor analysis (EFA). Based on the results of the EFA, sum scores were created to reflect each of the resulting constructs. Scores reflecting individual mathematical competencies were then regressed onto EF as a unidimensional construct, and in a separate equation, onto components of EF. All models were estimated using Mplus 8 (Muthen & Muthen, 2017) and took the complex sampling design of the Family Life Project into account with sample weights and stratification. In all models, coefficients represent the unique variance attributable to each variable, adjusted for all other variables in the model. Correlations between outcome variables and between predictor variables were estimated in all models.

Final models are limited to children for whom a direct assessment was conducted during at least one time point. One hundred-forty-four children were excluded from analyses for having no available direct assessment data, leaving a total of 1,148 participants. For those participants who completed at least one direct assessment, missing data was accounted for using Full Information Maximum Likelihood estimation. Full Information Maximum Likelihood estimation takes into account the covariance matrix for all available data on the independent variables to estimate parameters and standard errors. This approach provides more accurate estimates of regression coefficients than do listwise deletion or mean replacement (Enders, 2001).

Results

Missing Data

Data were analyzed to test whether children who did not complete the math assessment differed from those who did on any characteristics. Independent-sample t-tests suggested children without a math assessment had lower EF, $t(1058) = 3.06, p < .01$; IQ, $t(1033) = 4.62, p < .001$; and less well educated mothers, $t(1084) = 3.42, p = .001$.

Exploratory Factor Analysis of Math Items

Following best practices, a parallel analysis using 1000 datasets was conducted and compared to eigenvalues extracted from a principle components analysis, and the scree plots of both the raw and the simulated data were analyzed (O'Connor, 2000). Both determined two underlying factors fit the data. Data were then subjected to an EFA to extract two factors using a CF-EQUAMAX rotation, which assumes correlations between factors and has previously been shown to accurately estimate factor structures with dichotomous data (Finch, 2011). Items that did not load onto any one factor ($N = 1$) were dropped from the model, as were items with a loading of $|\lambda| < .3$ ($N = 7$), and items that cross loaded onto both factors ($N = 2$). Thus, although many of these items probably are reasonable representations of mathematic ability, their content appears to incorporate more than one competency. Descriptive statistics and factor loading for all items from the ECLS-K assessment are shown in Table 1.

Items that loaded onto the first factor assessed skills including Arabic numeral recognition and ordinal relations, and thus more conceptual aspects of mathematics, whereas items that loaded onto the second factor assessed skills such as children's abilities to perform basic arithmetic operations and read and interpret graphs, and thus rather procedural aspects. For example, of the 5 items that loaded most strongly onto the "conceptual" factor (items 6, 5, 19, 20, and 10; λ s = .83, .77, .73, .73, .72, respectively), four required participants to read Arabic numerals, and one required students to fill in the blank item in a pattern (i.e., 5, 10, 15, __, 25). Of the 5 items that loaded most strongly onto the "procedural" factors (items 31, 30, 16, 15, and 17; λ s = .90, .87, .54, .46, .45, respectively), two required participants to read and interpret a graph (i.e., indicate how many were in the named category in a bar graph), two required participants to use procedural counting to identify the number of objects on a page, and one required participants to identify the group of objects that contained the requested quantity (which again required procedural counting). Sum scores were created for both factors to reflect the 12 items that loaded onto the conceptual factor and the 10 items onto the procedural factor. Both had reasonable internal consistency (conceptual: $\alpha = 0.72$; procedural: $\alpha = 0.67$), and were moderately correlated with one another ($r = .48, p < .001$), suggesting that while the two are related they do indeed represent separate constructs.

Descriptive Statistics

Unweighted descriptive statistics for all variables, including conceptual and procedural scales, are presented in Table 2. Correlations among variables in the analysis are presented in Table 3. Both conceptual and procedural skills were correlated with maternal education ($r_{Conceptual} = .33, p < .001$; $r_{Procedural} = .26, p < .001$), as well as child IQ ($r_{Conceptual} = .40, p < .001$; $r_{Procedural} = .45, p < .001$), processing speed ($r_{Conceptual} = .42, p < .001$; $r_{Procedural} = .42, p < .001$), and receptive vocabulary ($r_{Conceptual} = .43, p < .001$; $r_{Procedural} = .55, p < .001$).

Does EF Predict Different Mathematical Competencies?

The sum scores of conceptual and procedural items were regressed onto EF as a unidimensional construct. Regression coefficients represent the unique effect of each variable in the model. Results of Model 1 in Table 4 indicate that EF was positively related to children's performance on both conceptual and procedural items ($\beta = .11, p < .01$; $\beta = .13, p < .001$, respectively). Net of other covariates, IQ at age 3, processing speed, and receptive vocabulary measured concurrently were related to both conceptual and procedural skills. Additionally, maternal education related to conceptual ($\beta = .14, p < .001$), but not procedural skills.

Do Components of EF Differentially Relate to Mathematical Competencies?

Conceptual and procedural skills were then regressed onto the three different components of EF. Results are presented in Model 2 of Table 4. Inhibitory control was related to conceptual ($\beta = .11, p < .01$), but not procedural skills, and working memory was related to procedural skills ($\beta = .07, p = .041$), but not Conceptual skills. Cognitive flexibility was related to neither conceptual nor procedural skills. Net of EF components, coefficients on other cognitive variables were of a similar magnitude to the model in which EF was included as a

unitary construct. IQ, processing speed, and receptive vocabulary were again related to conceptual and procedural skills. Of contextual variables, maternal education remained related to conceptual, but not procedural skills.

Sensitivity Analysis

Analyses described above were conducted again limited only to participants who had complete data on math variables ($N = 849$). Coefficients on predictor variables in both analyses were unchanged.

Discussion

The goals of this study were to i) confirm the presence of distinct aspects of mathematical competency in a large-scale, standardized assessment of mathematical ability with a large, diverse sample of 5-year-old children, and ii) to evaluate the relation of EF, both as a unitary construct and as a construct composed of component abilities to the distinct aspects of mathematical competency. The results largely support our hypotheses: Results of the EFA revealed two moderately correlated, yet distinct constructs onto which the items in the ECLS-K assessment nearly evenly split. Upon analysis of the common skills required to solve the items that loaded onto each factor, the two factors were labeled as conceptual and procedural skills. Notably, this was the first investigation to our knowledge to establish that items within a widely-used, norm-referenced assessment of mathematics for young children tapped distinct, separable aspects of mathematical ability. Prior investigations of conceptual and procedural skills (e.g., Gilmore et al., *in press*; Hiebert, 2013; LeFevre et al., 2013; Rittle-Johnson & Siegler, 1998) have used items specifically designed to assess one or the other, rather than capitalizing on items that make up an assessment of math achievement from which a single score characterizing overall ability is derived.

EF as a unitary construct has been shown to be uniquely related to both conceptual and procedural knowledge in 10-year-old children (LeFevre et al., 2013). Ours is the first analysis of which we are aware to examine distinct relations of EF components to procedural and conceptual math ability. When EF was analyzed as a multivariate construct with three components (inhibitory control, working memory, and cognitive flexibility), we found that only IC was related to conceptual math ability, and only WM was related to procedural math ability. CF was related to neither, however it is possible that CF may be engaged only for items that require shifting between and engaging both conceptual and procedural skill, but not for understanding the conceptual and procedural aspects of math themselves. These relations of IC to conceptual math knowledge and WM to procedural math knowledge are supported by the prior literature (e.g., Cragg & Gilmore, 2014; LeFevre et al., 2006; McClean & Hitch, 1999), but have been reported largely for older children (e.g., Robinson & Dubé, 2013; Hecht, Close, & Santisi, 2003). By limiting the present investigation to early childhood—a period during which children are rapidly developing an understanding of conceptual and procedural skills that underlie much of their later mathematical understanding—we are able to extend findings from prior investigations to suggest that different facets of EF might support the learning of different skills—or be engaged in the solving of different problem types—even within the same academic domain.

These differential associations are not unexpected: Previously, Purpura et al. (2017) found that IC was related to basic mathematical competencies such as knowledge of the count list, subitizing, and 1-1 correspondence, and thus more conceptual aspects of mathematics. Similarly, WM was related to more advanced competencies including operations and more procedural aspects of mathematics. That investigation described no clear differentiation for CF, which related to a narrow range of math skills that spanned aspects of conceptual and procedural knowledge. In contrast, we did not find an association between CF and individual math components. While that investigation did not have the breadth of assessment to establish conceptual versus procedural skills, the skills conceptualized as “basic” and those conceptualized as “more advanced” correspond loosely to the conceptual and procedural distinction made here.

Commentaries on prior investigations of the relations between EF and math ability in early and middle childhood have taken issue with the treatment of EF and with the exclusion of potentially important covariates. Bull and Lee (2014) noted many of the investigations of EF and math have found a unique role for cognitive flexibility (referred to in their article as shifting), and for working memory (referred to as updating), but mixed evidence for a unique role of inhibition. The authors claim many of the existing investigations of the relations between EF and math ability are complicated by the absence of relevant covariates, and that much of the variance attributed to EF in its relation to math ability (especially for inhibition and shifting) might in fact be attributable to processing speed. The present investigation attempted to disambiguate the role of separate components of EF from other cognitive covariates by including individual- and context-level variables that have been shown to relate to the development of math ability: processing speed, receptive vocabulary, IQ, maternal education, and family income. Net of these comprehensive covariates, we still find differential relations between components of EF and mathematical competencies.

Implications

These findings have implications for research and practice. A substantial amount of research on the relations between EF and math have drawn conclusions based on limited measurement of EF, e.g., measurement does not capture all three components of EF (Blair & Razza, 2007; Brock, Rimm-Kaufman, Nathanson, & Grimm, 2009; Mazzocco & Kover, 2007), or limited measurement of separate math competencies, e.g., measurement does not capture math ability differentially (Bull & Scerif, 2001; Purpura et al., 2017). Further, the finding that a number of child-level covariates are related—indeed, more strongly than is EF—to conceptual and procedural skills, there are implications for other avenues of intervention. Prior investigations have found that both processing speed and vocabulary may be amenable to change through successful intervention (e.g., Mackey, Hill, Stone, & Bunge, 2011; Marulis & Neuman, 2010). Additionally, the association of maternal education with conceptual but not procedural skills might suggest that heterogeneity in conceptual skills might arise as a result of instruction in the home, and may be a point of intervention for school settings. As well, given investigations that have suggested distinct math competencies as the focus of curriculum historically (e.g., Baker et al., 2010; Blair et al., 2007), it is crucial that future research continues to investigate abilities that support the development of separate mathematical skills.

Limitations and Conclusions

There are several limitations that must be addressed in the context of this investigation. First, it is important to note that while this study is longitudinal in nature, causality cannot be inferred. Second, the reported association of EF and its component parts to math is not large. However, we feel there is value added in these findings: IC was related to 10% of a standard deviation in conceptual knowledge, which was equal to the full magnitude of the association of EF as a unitary construct with conceptual knowledge. Additionally, the present sample is limited to only two regions of the United States, and results may not generalize to others. The current findings may only apply to children from non-urban areas of the US, or to children born to low-income families. Finally, there are limitations with regard to measurement. Individual-level predictors (e.g., IQ, processing speed, and receptive vocabulary, EF) were measured at separate times: Processing speed and receptive vocabulary were assessed at age 5, concurrently with measures of math skills, while IQ was assessed at age 3, and EF was an aggregate of scores from ages 4 and 5 in order to more effectively capture stability in EF over time. It is also important to note that assessment of math ability was limited to research assistant administered standardized assessments. While performance on the included math assessment is generally correlated with performance on formative and summative assessments in school contexts, it is likely it captures only some aspects of math achievement.

Despite these limitations, results from the present investigation are relevant to the way researchers and educators investigate and foster the development of mathematical skills in early childhood. Results support the idea that additional research is needed on the distinct contributions of distinct components of EF to distinct components of mathematics knowledge. Further research is needed to better understand assessment of different domains of knowledge within standardized assessments and to understand the validity of extracting distinct yet correlated domains of mathematics from broad assessments of mathematical skills. By better understanding the structures of these assessments and the underlying cognitive skills that support success on them, educators can be better prepared to teach the domains of knowledge required to successfully perform on academic assessments, even at young ages.

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Table 1

Descriptive analyses for items from ECLS-K math assessment

Item #	% Correct	SD	Required Skills	Item Type	Factor 1	Factor 2
1	.53	.50	Procedural Counting	Procedural	0.18*	0.38*
2	.73	.45	Procedural Counting	Procedural	0.19*	0.42*
3	.41	.49	--	Excluded ^a	0.26*	0.18*
4	.28	.45	1-1 Correspondence	Conceptual	0.59*	0.25*
5	.43	.50	Arabic Numerals	Conceptual	0.77*	0.15*
6	.11	.32	Arabic Numerals	Conceptual	0.83*	-0.10
7	.11	.31	Ordinal Positioning	Conceptual	0.61*	0.17*
8	.58	.49	--	Excluded ^b	0.30*	0.27*
9	.14	.35	Fill in Blank: Pattern	Conceptual	0.60*	0.10
10	.01	.12	Fill in Blank: Pattern	Conceptual	0.72*	-0.19
11	.16	.37	--	Excluded ^b	0.33*	0.32*
12	.15	.36	--	Excluded ^a	0.17*	0.06
13	.05	.23	Informal Addition	Conceptual	0.36*	-0.07
14	.02	.14	Informal Addition	Conceptual	0.67*	-0.19
15	.94	.23	Procedural Counting	Procedural	0.17	0.46*
16	.86	.35	Procedural Counting	Procedural	0.14*	0.53*
17	.31	.46	Procedural Counting	Procedural	0.10	0.45*
18	.76	.43	Arabic Numerals	Conceptual	0.68*	0.35*
19	.63	.48	Arabic Numerals	Conceptual	0.73*	0.27*
20	.20	.40	Arabic Numerals	Conceptual	0.73*	0.11
21	.90	.30	Shape Recognition	Conceptual	0.36*	0.18
22	.44	.50	--	Excluded ^a	0.21*	0.20*
23	.33	.47	--	Excluded ^a	0.20*	0.22*
24	.38	.48	Pattern Recognition	Procedural	-0.04	0.31*
25	.18	.39	--	Excluded ^a	0.06	0.18*
26	.42	.49	Subtraction (with manipulatives)	Procedural	0.13*	0.42*
27	.04	.20	--	Excluded ^a	0.07	0.28*

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Item #	% Correct	SD	Required Skills	Item Type	Factor 1	Factor 2
28	.11	.32	--	Excluded ^c	-0.09	0.10
29	.49	.50	Measurement	Procedural	0.22*	0.39*
30	.48	.50	Graph Interpretation	Procedural	-0.12*	0.87*
31	.57	.50	Graph Interpretation	Procedural	-0.08*	0.90*
32	.37	.48	--	Excluded ^a	0.20*	0.06

^aitem loading α |.3|

^bitem cross-loads onto both factors

^citem does not significantly load onto either factor

Table 2

Descriptive statistics for all variables

	N	Minimum	Maximum	Mean	SD
Conceptual	849	0	9	3.66	2.12
Procedural	849	0	10	5.71	2.27
EF Composite	1060	-2.14	1.17	0.08	0.46
Inhibitory Control	1070	-1.98	1.20	0.15	0.49
Working Memory	1069	-2.14	1.33	0.08	0.55
Cognitive Flexibility	1052	-2.54	2.25	-0.02	0.73
Processing Speed	846	65	132.5	96.04	12.61
IQ	1035	45	142	93.57	16.51
Receptive Vocabulary	964	43	138	93.90	15.87
Mom Years Education	1086	1	22	14.97	2.83
INR	1148	0	17.76	1.99	1.62

Table 3

Correlations among all variables

	1	2	3	4	5	6	7	8	9	10	11
1 Conceptual	-										
2 Procedural	.48***	-									
3 EF Composite	.41***	.47***	-								
4 Inhibitory Control	.34***	.35***	.84***	-							
5 Working Memory	.30***	.40***	.77***	.41***	-						
6 Cognitive Flexibility	.32***	.35***	.70***	.43***	.41***	-					
7 Processing Speed	.42***	.42***	.45***	.32***	.39***	.33***	-				
8 IQ	.40***	.45***	.56***	.39***	.50***	.43***	.42***	-			
9 Receptive Vocabulary	.43***	.55***	.60***	.45***	.52***	.43***	.42***	.62***	-		
10 Mom Education	.33***	.26***	.26***	.17***	.24***	.24***	.24***	.31***	.33***	-	
11 INR	.29***	.27***	.27***	.17***	.26***	.23***	.23***	.34***	.38***	.46***	-

N = 1148;

*
 $p < .05$;**
 $p < .01$;***
 $p < .001$

EF = Executive Function; INR = Income-to-Needs Ratio

Table 4

Regression estimates predicting math competencies

		Model 1		Model 2	
		β	95% CI	β	95% CI
Conceptual	EF Composite	0.11**	0.03–0.19		
	Inhibitory Control			0.11**	0.04–0.17
	Working Memory			–0.03	–0.11–0.04
	Cognitive Flexibility			0.05	–0.02–0.12
	IQ	0.08	0.00–0.16	0.09*	0.01–0.17
	Receptive Vocabulary	0.16***	0.07–0.24	0.17***	0.08–0.25
	Processing Speed	0.25***	0.17–0.32	0.25***	0.18–0.32
	Maternal Education	0.14***	0.07–0.21	0.14***	0.07–0.21
	INR	0.07	–0.01–0.14	0.07	–0.00–0.14
R^2			.33		.33
Procedural	EF Composite	0.13**	0.06–0.21		
	Inhibitory Control			0.05	–0.02–0.11
	Working Memory			0.07*	0.00–0.14
	Cognitive Flexibility			0.04	–0.03–0.11
	IQ	0.11**	0.03–0.18	0.11**	0.03–0.18
	Receptive Vocabulary	0.32***	0.24–0.40	0.32***	0.25–0.40
	Processing Speed	0.19***	0.12–0.26	0.19***	0.12–0.26
	Maternal Education	0.03	–0.04–0.09	0.02	–0.04–0.09
	INR	0.03	–0.04–0.10	0.03	–0.04–0.09
R^2			.40		.40

N = 1,148;

*
 $p < .05$;**
 $p < .01$;***
 $p < .001$

EF = Executive Function; INR = Income-to-Needs Ratio

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