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Steady Displacement of Long Gas Bubbles in Channels and Tubes Filled by a Bingham Fluid

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Abstract

Bingham fluids behave like solids below a von Mises stress threshold, the yield stress, while above it they behave like Newtonian fluids. They are characterized by a dimensionless parameter, Bingham number (Bn), which is the ratio of the yield stress to a characteristic viscous stress. In this study, the non-inertial steady motion of a finite size gas bubble in both a plane 2D channel and an axi-symmetric tube filled by a Bingham fluid has been studied numerically. The Bingham number, Bn , is in the range $0 \leq Bn \leq 3$, where $Bn=0$ is the Newtonian case, while the Capillary number which is the ratio of a characteristic viscous force to the surface tension has values $Ca=0.05, 0.10$, and 0.25 . The volume of all axi-symmetric and 2D bubbles has been chosen to be identical for all parameter choices and large enough for the bubbles to be long compared to the channel/tube width/diameter. The Bingham fluid constitutive equation is approximated by a regularized equation. During the motion, the bubble interface is separated from the wall by a static liquid film. The film thickness scaled by the tube radius (axi-symmetric)/half of the channel height (2D) is the dimensionless film thickness, h . The results show that increasing Bn initially leads to an increase in h , however, the profile h versus Bn can be monotonic or non-monotonic depending on Ca values and 2D/axi-symmetric configurations. The yield stress also alters the shape of the front and rear of the bubble and suppresses the capillary waves at the rear of the bubble. The yield stress increases the magnitude of the wall shear stress and its gradient and therefore increases the potential for epithelial cell injuries in applications to lung airway mucus plugs. The topology of the yield surfaces as well the flow pattern in the bubble frame of reference varies significantly by Ca and Bn .

1-Introduction

Taylor flow refers to the motion of a train of long bubbles in a conduit [1]. Bubbles are separated from the conduit wall by a liquid film and the spaces between consecutive bubbles are filled by the liquid, i.e. the slug or plug region. In Taylor flow the bubbles move with a uniform speed and as a result the plug length remains constant. Taylor flow with long plugs

^aYield criterion in terms of von Mises stress is not applied to all yield stress fluids as some are following other criteria for yielding.

^bIn the literature for the axi-symmetric tubes instead of h , film thickness parameter [28] is usually used. Film thickness parameter is defined by $1 - r_S^2$ where r_S is the dimensionless r coordinate of the static film interface [28]. We, however, use h for both 2D channels and axi-symmetric tubes to be able to compare the results of the two geometries. [46] has called the same parameter as mass fraction.

is equivalent to the motion of a single bubble in a long conduit owing to minor interaction of the consecutive bubbles.

Studying Taylor flow in conduits filled by Newtonian or Non-Newtonian fluids has many applications in optimizing industrial processes and better understanding of biological systems. Some examples of Taylor flow include: catalyst reactors [2–3], enhanced oil recovery by gas injection [4]; Gas Assisted Injection Molding [5–6], displacement of penetrated gas bubbles in blood vessels during cardiac surgery or deep sea diving [7]; displacement of the liquid in respiratory airways [8–12] as occur in processes such as surfactant replacement therapy (SRT) [13], partial liquid ventilation (PLV) [14], pulmonary drug delivery [11,15], airway closure with post closure filling flows [16–17], and reopening of airways blocked by liquid plugs or mucus [12].

Taylor flow in conduits filled by Newtonian fluids has been studied analytically, numerically and experimentally for a couple of decades. More details can be found in a review by Angeli and Gavrilidis [18]. Early experiments by Fairbrother and Stubbs [19] showed that the middle part of a long bubble moving in a horizontal pipe is separated from the wall through a static liquid film with a dimensionless thickness, h , proportional to Capillary number, Ca , raised to some power. Using lubrication theory, Bretherton [20] proposed a formula for $h \sim Ca^{2/3}$, valid for small Ca in creeping motions. The analysis also predicts the presence of capillary waves in the rear of the bubble due to the surface tension effects. Ratulowski and Chang's [21] analytical work validates Bretherton's formula [20] for h up to $Ca=0.01$ and extends it to higher Ca for the creeping motions. For large values of Ca , the combined experimental and analytical studies by Taylor [22] and Cox [23] suggests that h asymptotes to a constant value when fluid inertia is neglected. The analytical work by De Ryck [24] includes the effects of fluid inertia, showing a non-monotonic profile of h in terms of Reynolds number, Re , for a range of Ca . Klaseboer et al. [25] recently extended Bretherton [20] model to larger Ca . The model proposed by them is valid for Ca up to 2. Aussillous and Quere [26] have proposed an empirical correlation for h in term of Ca up to $Ca=1.2$ where Taylor's [22] and their own data were used for curve-fitting. The computed values for h by Reinelt and Saffman [27], Giavedoni and Saita [28], Halpern and Gaver [29] and Heil [30] for creeping motion of a gas finger in a tube or a 2D channel are in good agreement with experimental and theoretical results of Ratulowski and Chang [21] and Taylor [22]. Moreover, Heil [30] shows a non-monotonic profile of h in terms of Re for some ranges of Ca , predicted by De Ryck's analysis [24] as well. These suggest that a gas finger can represent the front section of a Taylor bubble with a long slug.

Edvinsson and Irandoust [31], Kreutzer et al. [32], Feng [33–34], Gupta et al. [35], Taha and Cui [36], and Giavedoni and Saita [28,37] investigated numerically the steady state pressure and/or the buoyancy driven motion of a single bubble in a tube or in a 2D channel for different ranges of the relevant dimensionless numbers. The results show that for small Ca the bubble interface consists of two spherical (circular for a planar conduit) transitions in its front and rear [28,37] and a uniform section in the middle. In addition, there are capillary wave undulations [31,34,35,37] where the interface merges with the transition region in the rear. As Ca increases up to 1 (for the motion in a tube) the front transition becomes more slender [34,37]. Beyond this value the profile of the front transition does not change any

further [28]. The profile of the rear transition, however, undertakes large deformations by evolving from a convex shape for small values of Ca to a concave one for the large values of Ca [31,34,36,37]. Further increase in Ca leads to bubble break up due to a re-entrant jet at the rear as demonstrated by Lac and Sherwood [38]. The results also show that the amplitude of the interfacial waves at the rear of the bubble decreases with increasing Ca [31,37].

The front and rear transitions were shown to become more slender and more flattened by the fluid inertia, Re , respectively [30,31,34]. In addition, the amplitude of the interfacial waves at the rear of the bubble increases with Re [31,34]. Inertia was also shown to alter the pressure distribution around the front and rear tips significantly [30,32]. Major differences in bubble shape and flow patterns were observed between the pressure and the buoyancy-driven motions in a quiescent liquid owing to the differences in the background flow of these two mechanisms [33–34]. Also, gravity was shown to alter h and induce a gradient of axial velocity in the radial direction inside of the liquid film [31,33].

Non-Newtonian fluids with a yield stress behave like solids below a von Mises stress a threshold, the yield stress, τ_y , while above it they behave like fluids in Couette flow. They exist in the human body and are utilized widely in industrial processes. Blood, mucus, ketchup, mayonnaise, gels, pastes, paints and nuclear waste suspensions are a few examples.

The macroscopic behavior of yield stress fluids can be analyzed via some proposed constitutive equations. Among them the simplest is the constitutive equation for a Bingham fluid, i.e. the Bingham equation. It assumes a linear relation between the viscous stress and the rate of strain tensors for stresses above τ_y , while for stresses below τ_y it assumes that the material behaves like a solid. Depending on the local value of the stresses, this divides the material domain into the yielded and un-yielded regions with liquid and solid like behaviors, respectively. The Bingham equation introduces a new dimensionless parameter, Bingham number, Bn , which is an indicator of the ratio of τ_y to a characteristic viscous stress.

Motion of gas bubbles in conduits filled by yield stress fluids mimics some major transport phenomena. Despite these wide applications there are few published works on this subject. Dubash and Frigaard [39] studied analytically the rise of a small bubble in a tube filled with a quiescent Herschel-Bulkley [40] fluid. They found a critical τ_y in terms of the bubble size and the bubble and medium properties below which buoyancy causes the bubble motion. Potapov et al. [41], Singh and Denn [42], and Tsamopoulos et al. [43] investigated computationally buoyancy-driven motion of a bubble/drop in a Bingham fluid and found qualitatively similar phenomena. Lavrenteva et al. [44] observed similar results in their experiments. Potapov et al. [41] showed that the presence of a second drop within a critical distance extends the yielded regions and eases the motion. Lavrenteva et al. [44], and Singh and Denn [42] found this easing phenomenon for a bundle of bubbles/drops.

Dimakopoulos and Tsamopoulos [5] numerically studied the transient motion of an advancing gas finger into a constricted tube filled by a Bingham fluid. Similar to the Newtonian fluids, they showed that a layer of the fluid is deposited on the tube wall as the gas advances throughout the conduit. They showed that the thickness of this layer is a

function of Ca , Re and Bn . The change of the finger shape and the topology of the yielded and the un-yielded regions were discussed in detail in the study. The work was repeated for the conduits with more complex geometry by the same group [6].

Allouche et al. [45] studied a Bingham fluid finger propagating into a vertical channel filled with a second Bingham fluid which is displaced. They restricted their analytical and numerical investigations to creeping motion and negligible surface tension. Interestingly, they found a range of parameter values where the finger leaves a zero-thickness trailing film. Outside of this parameter range they predicted an upper-bound for the thickness of the trailing film for each set of parameters.

de Sousa et al. [46] investigated numerically the steady creeping motion of an advancing gas finger in a tube filled by power law or Bingham fluids. The results show that, similar to the Newtonian fluids, a static liquid film is formed adjacent to the wall with a thickness which is a function of Ca and a dimensionless yield stress. The results also indicate that the regime of the flow in front of the finger (viewed from a frame of reference attached to the finger) can be affected significantly by the value of the dimensionless yield stress. The results were discussed further by the same group, Thompson et al. [47], through inclusion of the location of the yield surfaces for some of the computations. It was shown that the un-yielded regions are extended with the dimensionless yield stress. The locations of the yield-surfaces for some of the dimensionless parameters, however, were not consistent with the Bingham fluid equation. They attributed this to inadequate values of the regularization parameter used in those computations. Jalaal and Balmforth [48] through lubrication theory computed the shape of the front and rear menisci of a long bubble travelling in 2D channel and axisymmetric tubes filled by a viscoplastic fluid. For the film thickness, they have proposed an expression covering larger Ca values. They have also presented the results of a few numerical simulations for small Ca , $Ca \ll 0.01$, (which do not overlap with the range of Ca in this study, $0.05 \leq Ca \leq 0.25$). Their analytic and numerical results for the film thickness are in agreement up to a value of a dimensionless yield stress.

Zamankhan et al. [49], numerically studied the creeping steady motion of Bingham plugs in 2D channels for $0.025 < Ca < 0.1$ and $0 < Bn < 1.5$. They showed that increasing the yield stress increases the gradient of wall shear stress significantly and the shear stress up to 30% for the range of studied parameters. They also showed that h increases with increase in Bn while the amplitude of the capillary waves decreases with increasing Bn . They also showed that the plug length mostly affects the topology of the yield surfaces.

The propagation of long gas bubbles in conduits filled by Bingham fluids can resemble the advancing/receding motions of gas fingers into occluded airways by mucus or diseased alveolar studied by Douville et al. [50]. Figure 1 shows the schematics of a bubble, a plug, advancing and receding gas fingers. The front/back and back/front of a long bubble/plug represent the advancing and receding motion of a gas fingers into conduits filled by liquid, respectively.

In most previous works related to Bingham fluids, either the bubble is small compared to the conduit size or there is a gas finger which is only advancing, as opposed to receding. The

advancing finger is similar to the front end of a long bubble. However, the shape of the front and rear of a bubble might differ significantly. This would lead to some major differences between the flow patterns and the profile of wall stresses near to the two regions. The numerical simulations by Jalaal and Balmforth [48] reveals some flow features but do not focus on wall shear stress, and it is limited to $Ca < 0.01$.

For the particular case of the lung where the airways are flexible, the blocked airways can turn into a flattened elliptical cross section [51–52] shown in Figure 2 with an aspect ratio between 2 to 20. Because of this, both 2D channels and axi-symmetric tubes are analyzed in this study. The results show the effects of Ca and Bn on the bubble shape, film thickness, velocity components, pressure and from them, stress fields and the topology of the yield surfaces. The effects of the dimensionless numbers on the profile of the shear stress along the walls with applications in the respiratory system are also discussed. We also compare the flow features in 2D channel and axi-symmetric tube models. In this work in addition to the inclusion of axi-symmetric model, we have significantly extended the ranges of Bn and Ca used in our previous study for liquid plugs [49]. In addition, we provide information about the location of the yield surfaces for a bubble in an infinite tube which cannot be extracted from our previous study dedicated to short plugs.

2-Bingham equation

In the Bingham constitutive equation the fluid tolerates any level of stress less than a threshold, yield stress, without any strain rate (the elastic deformation below the yield stress is neglected in this study). Above the yield stress it behaves as a generalized Newtonian fluid. The constitutive equation was formulated by Oldroyd [53–54] as;

$$\begin{cases} \underline{\underline{\tau}}^* = \left(\mu_p + \frac{\tau_y}{\dot{\gamma}^*} \right) \underline{\underline{D}}^*, & \text{if } |\tau^*| > \tau_y, \\ \underline{\underline{D}}^* = 0, & \text{if } |\tau^*| \leq \tau_y, \end{cases} \quad (1)$$

where; $\underline{\underline{D}}^* = \left(\nabla \vec{V}^* + (\nabla \vec{V}^*)^T \right)$, $\dot{\gamma}^* = \sqrt{D_{ij}^* D_{ij}^* / 2}$, and $|\tau^*| = \sqrt{\tau_{ij}^* \tau_{ij}^* / 2}$

In Equation (1) $\underline{\underline{\tau}}^*$ is the viscous stress tensor, $\underline{\underline{D}}^*$ is the rate of strain tensor, $\vec{V}^*$ is the fluid velocity vector, μ_p is the plastic viscosity, τ_y is the yield stress, $\dot{\gamma}^* = \sqrt{D_{ij}^* D_{ij}^* / 2}$ is the strain rate, and $|\tau^*| = \sqrt{\tau_{ij}^* \tau_{ij}^* / 2}$ is the von Mises stress.

Transition from a rigid body to a viscous fluid on a surface that is not known *a priori* produces significant difficulties in numerical simulations of the Bingham equation. Therefore computational studies commonly use regularized constitutive equations instead of the Bingham equation to ease the difficulties. The regularized equations are continuous and are characterized by a regularization parameter. In this study we use a regularized method suggested by Papanastasiou [55] through the following equations;

$$\underline{\underline{\tau}}^* = \eta^* \underline{\underline{D}}^*, \quad (2)$$

$$\eta^* = \mu_p + \frac{\tau_y}{\dot{\gamma}^*} (1 - \exp(-m^* \dot{\gamma}^*)), \quad (3)$$

where η^* the apparent viscosity and m^* is the regularization parameter. For large enough values of m^* the regularized method mimics the behavior of the Bingham equation. The yield criterion for the regularized method is given by the following relation

$$\begin{cases} |\tau^*| \leq \tau_y, & \text{unyielded,} \\ |\tau^*| > \tau_y, & \text{yielded.} \end{cases} \quad (4)$$

The criterion is used to determine the boundaries between the yielded and the un-yielded regions, i.e. the yield surfaces. The obtained velocity fields and free surfaces profiles from the regularized method converge as $m^* \rightarrow \infty$. However in general, there is no guarantee for the convergence of the location of the yield surfaces [56–57].

The augmented Lagrangian method [58–59] is another approach to deal with the Bingham equation. It resolves the location of the yield surfaces better than the regularized method [60–63]. To the best of our knowledge, however, it has not yet been used for free surface flows of Bingham fluids including surface tension.

3-The governing equations and boundary conditions

Figure 3 shows a schematic of the domain of calculation. The bubble is moving from the right to the left with a constant speed, v_{tip} in a 2D channel/axi-symmetric tube filled by a Bingham fluid. Neglecting the gravitational force, the lower half of the domain is sufficient for the analysis. To work with the steady form of the governing equations the frame of reference is attached to the tip of the bubble. Due to the small values of the gas-liquid density and viscosity ratios, the effects of the gas phase on the motion of the liquid phase are negligible. Therefore the conservation equations are not solved for the gas phase. The velocity components are scaled by v_{tip} ; the length dimensions by the half width of the channel/radius of the tube, b ; the pressure and stresses by $\mu_p V_{tip}/b$ where μ_p is the plastic viscosity; the strain rate by V_{tip}/b and the regularization parameter by b/V_{tip} . Then the dimensionless forms of the governing equations for the liquid phase with constant properties are given by;

Continuity:

$$\nabla \cdot \vec{V} = 0, \quad (5)$$

Cauchy momentum:

$$-\nabla p + \nabla \cdot \underline{\underline{\tau}} = 0 \quad (6)$$

where the viscous stress tensor is calculated via Papanastasiou [55] regularized equations given by;

$$\underline{\underline{\tau}} = \eta \underline{\underline{D}}, \quad (7)$$

$$\eta = 1 + \frac{\text{Bn}}{\dot{\gamma}} (1 - \exp(-m\dot{\gamma})). \quad (8)$$

In the above equations p is the pressure and $\text{Bn} = \tau_y b / \mu_p V_{tip}$ is the Bingham number, which represents the ratio of the yield stress to a characteristic viscous stress.

The boundary conditions along the bubble surface are:

Kinematic:

$$\vec{V} \cdot \vec{n} = 0, \quad (9)$$

Stress:

$$-p \vec{n} + \eta \left(\nabla \vec{V} + \left(\nabla \vec{V} \right)^T \right) \cdot \vec{n} = \frac{\kappa}{\text{Ca}} \vec{n}, \quad (10)$$

where $\vec{n}$ is the normal unit vector, κ is the local curvature, and $\text{Ca} = \mu_p V_{tip} / \sigma$ is the capillary number which represents the ratio of a characteristic viscous stress to the surface tension, σ . In writing Equation (10) the pressure in the gas phase is assumed to be zero as a reference.

At the wall the velocity components are specified by:

$$2\text{D}: V_x = 1, \text{ Axi-symmetric}: V_z = 1, \quad (11)$$

$$2\text{D}: V_y = 0, \text{ Axi-symmetric}: V_r = 0. \quad (12)$$

where V_x and V_y are the velocity components in x and y directions in a 2D channel while V_s and V_r are the velocity components in the axial and the radial directions in an axi-symmetric tube.

Along the plane of symmetry, the boundary conditions are:

$$2D: \frac{\partial V_x}{\partial y} = 0, \text{ Axi-symmetric: } \frac{\partial V_z}{\partial r} = 0, \quad (13)$$

$$2D: V_y = 0, \text{ Axi-symmetric: } V_r = 0, \quad (14)$$

We have assumed the bubble intersects the plane/axis of symmetry with a right angle at the front and the rear tips.

The boundary conditions at the inflow and the outflow are:

$$2D: \begin{cases} V_x = 1 + \frac{1}{2}G(y^2 - 2y) + Bn y, & \text{for } y \leq y_{yield} \\ V_x = 1 + \frac{1}{2}G(y_{yield}^2 - 2y_{yield}) + Bn y_{yield}, & \text{for } y > y_{yield} \end{cases}$$

$$y_{yield} = 1 - \frac{Bn}{G}.$$

$$\text{Axi-symmetric: } \begin{cases} V_z = 1 - \frac{1}{4}G(1 - r_{yield}^2) - Bn(r_{yield} - 1), & \text{for } r \leq r_{yield} \\ V_z = 1 - \frac{1}{4}G(1 - r^2) - Bn(r - 1), & \text{for } r > r_{yield} \end{cases}$$

$$r_{yield} = 2\frac{Bn}{G}.$$
(15)

$$V_y = 0, V_r = 0. \quad (16)$$

Equation (15) is the 2D/axi-symmetric version of the Bingham-Poiseuille equation. It provides the fully developed velocity profile for a pressure-driven flow in a 2D channel, V_x , or an axi-symmetric tube, V_z , filled by a Bingham fluid. G is the fully developed pressure gradient and y_{yield} or r_{yield} are y or r coordinate of the yield surface in the fully developed region of a 2D channel or an axi-symmetric tube, respectively. The values of G and y_{yield} or r_{yield} are obtained during the calculation with an iterative method described in the next section. The inflow and the outflow boundaries are placed in the fully-developed regions of the channel. The Papanastasiou model [55] and Equation (15) provide almost the same profiles for V_x or V_z in the fully developed regions of a pressure driven flow. For example in a 2D case with $Bn=1$, $G=4$, and $m=1000$ the maximum difference between the two profiles was less than 0.02%. The pressure contours, on the other hand, slightly deviate of being vertical lines in regions close to the inflow and outflow, if the boundary condition given by Equation (15) is used. The deviation, however, occurs only in the un-yielded strip adjacent to the plane or axis of symmetry and therefore it does not affect any key flow features.

The bubble aspect ratio, AR, is the ratio of equivalent radius of the bubble to b . The equivalent radius of the bubble is the radius of a sphere with the same volume of a bubble. For bubbles with AR more than 1.1 [38] an increase in the aspect ratio merely leads to an increase in the bubble length. Therefore AR is not an additional parameter in this study which focuses only on long bubbles.

Bn and Ca are the input parameters of the problem, while the interface shape along with the velocity and the pressure fields are to be calculated. m is 1000 in all the simulations while the effect of its value on the results is also discussed.

4-The numerical schemes

For numerical simulations a commercial package, ANSYS-FIDAP, is used. The governing equations are discretized by mixed-discontinuous standard Galerkin formulation. In the utilized elements $\vec{V}$ and p are interpolated with quadratic and linear interpolation functions, respectively. To resolve the free surface, the method of spines [64] is used. The resulting non-linear system of equations is solved by a Newton- Raphson method. The inverse of the sparse Jacobian matrix is computed by a skyline Gaussian elimination method. Using a Newton-Raphson scheme along with the method of spines requires a very good initial guess for the solution variables so that convergence is achieved. This, however, can be eased significantly if the steady solution is obtained through solving the transient equations. Therefore, in this study the steady solutions are obtained through solving the time dependent equations. G and y_{yield} or r_{yield} in Equation (15) are updated after every five to ten time steps so that the steady-state kinematic boundary condition, Equation(9), is satisfied. Equations (15) and (16) preserve the bubble aspect ratio during the calculations. The computational grid consists of about 15,000–18,000 elements and 60,000 to 70,000 nodes depending on the values of the dimensionless parameters. The regularized constitutive equation and adjustment of V_x or V_z profiles at the inflow and outflow are introduced through two ANSYS-FIDAP user subroutines. More information regarding to the problem setup in ANSYS- FIDAP can be found in ANSYS- FIDAP user manual [65].

5-Range of dimensionless numbers for respiratory airways

Weibel [66] has introduced a model for the anatomy of the respiratory airways. The model assumes that generation n includes 2^n airways with a diameter of $d_n = d_0 2^{-n/3}$ where d_0 is the trachea diameter. Assuming that the anatomy of the respiratory airways is determined by this model, for a steady motion Bn and Ca in different generations of the respiratory airway can be represented by the following relations where Q is the average breathing rate:

$$Bn(n) = \tau_y \pi d_0^3 / \mu_p Q, \quad (17.1)$$

$$Ca(n) = 4\mu_p Q / \pi 2^{n/3} d_0^2 \sigma. \quad (17.2)$$

In derivation of the above relations we have assumed that the liquid plug velocity is the same as average air velocity in each airway.

Above equations indicate that Bn is constant throughout of the respiratory tract while Ca decreases with increasing n . Mucus has a yield stress ranging between 400–800 dyne/cm² (40/80 Pa) depending on the health condition where the larger values are associated with disease in general.

Through using above equations and mucus properties given by Bush et al. [67], for moderate breathing rate of an average person, Bn is 1.5/3 for a yield stress of 400/800 dyne/cm² (40/80 Pa). Ca for generations 1, 10, and 15 is 2.19, 0.27, and 0.086 respectively. Therefore the range of dimensionless number in this study i.e. $Bn = 3, Ca = 0.25$ covers diseased mucus for generations greater than 10. This is an extension to our previous work [49] limited to healthy mucus and it was for generations greater than 14.

6-Results and discussions

In this section the results of a parametric study are presented. To assure that the locations of the boundaries do not affect the results, the inlet and outlet are at least 2.5 times tube diameter or channel width away from the front and rear tips. The front tip is at $x=3$ or $z=3$ in all figures. AR is about 1.3 and 1.4 for 2D and axi-symmetric bubbles in all simulations, respectively.

6A-Bubble shape and the film thickness

Figure 4(a) shows the profile of an axi-symmetric bubble moving in a Newtonian fluid for different Ca values. For all values of Ca the middle of the bubble is separated from the wall by a liquid film. In the liquid film h is nearly uniform and V_z is nearly equal to the wall velocity, therefore it is called the static film.

For $Ca=0.01$ the transitions at the front and rear are nearly symmetric. As Ca increases the bubble takes a bullet shape with pointed and flat transitions at the front and rear, respectively. Capillary waves are present at the rear of the bubble. The amplitude of the waves decreases as Ca increases, predicted by Bretherton [20] as well as Ratulowski and Chang [21].

Figure 4(b) shows an axi-symmetric and a 2D bubble with $Bn=0$ and $Ca=0.1$. h for the 2D bubble is slightly greater. The shapes of the front of the two bubbles are indistinguishable on the scale of Figure 4(b), as are the shapes of their rears. The rear bubble surfaces are shown again in the $z>9$ region to compare their shapes.

Table 1 shows computed h in this study in terms of Ca in comparison with previous works [20,26–27] for an advancing gas finger. The agreement is excellent.

Figure 5(a–c) shows the profile of an axi-symmetric bubble for $Bn=0$ and 3 and $Ca=0.05$, 0.1, and 0.25. Figure 6(a–c), on the other hand, zooms on the rear of the bubble for the same Ca values and $Bn=0$ to 3. For all Ca values the amplitude of the capillary waves at the rear of the bubble decreases with increasing Bn . The effect is more profound for larger Ca as

$Bn \times Ca$, which is the ratio of the yield stress to the surface tension, increases. For example Figures 6(a–c) show for ($Ca=0.1$, $Bn=2$) and ($Ca=0.25$, $Bn=1$) the waves nearly disappear from the rear of the bubble, while they are present for $Ca=0.05$ and $Bn=3$. This behavior is consistent with the suppression of the waves in Newtonian fluids with increasing Ca , since the yield stress is a component of the viscous stress. Increasing the yield stress enhances the viscous effects against the surface tension and therefore waves are suppressed. The length of the static film increases with increasing Bn for all the three Ca values.

Figure 7 shows the value of G in terms of Bn and Ca for axi-symmetric and 2D bubbles. The symbols and the lines in the figure represent the computed data points and linear fits through them. For both configurations G varies almost linearly with Bn for all Ca values, and G decreases with increase in Ca . G of the axi-symmetric case is roughly 2.2 times larger than that of the 2D. G has significant effect on trend of h in terms of Ca and Bn which is discussed next.

Table 2 shows the computed h for the axi-symmetric bubble in this work along with the numerical results of de Sousa et al. [46] for axi-symmetric advancing gas fingers. The agreement is good. We need to emphasize that Ca in their study has been defined based on the apparent viscosity rather than the plastic viscosity used by us. As a result, Ca in their study is equivalent to $Ca \times (1+Bn)$ in our work. We therefore in Table 2, have converted their dimensionless inputs so that they become consistent with ours. Their data were extracted through a graph which inherently might include small reading errors.

Figure 8(a) shows the value of h in terms of Bn for $Ca=0.05$, 0.1 and 0.25 for axi-symmetric and 2D bubbles. For $Ca=0.05$ and 0.1 , the trend of h with Bn is similar for both configurations. For $Ca=0.05$, h increases monotonically with Bn , while for $Ca=0.1$ there is a local maximum for h as Bn varies. For $Ca=0.25$, however, h for the 2D attains a maximum at $Bn=0.4$ while it increases monotonically for the axi-symmetric bubble. To investigate the non-monotonic trend of h with Bn for the axisymmetric bubble with $Ca=0.1$, we have reduced the increment of Bn from 0.4 to 0.1 to compute h in interval of $1.2 < Bn < 2.5$. We have extracted G for this interval by assuming that G varies piecewise linearly with Bn between the points shown in Figure 7. We have used Equation (15) to compute h . These extra points are shown by a diamond symbol in Figure 8(a) where they follow the same trend as the square symbols for the axi-symmetric bubble with $Ca=0.1$. Through this we can conclude that the almost linear trend of G with Bn for the axisymmetric bubble with $Ca=0.1$, leads to non-monotonic trend of h with Bn for this Ca value. Similar investigation should show that non-monotonic trend of the two other set of data points in Figure 8(a) is due to the linear variation of G with Bn shown in Figure 7. h of 2D is larger than the axi-symmetric for $Ca=0.05$ and 0.1 while for $Ca=0.25$ it becomes smaller for $Bn > 1.5$.

The existence of the static film causes the area-average fluid axial velocity upstream/downstream of the bubble to be always smaller than the bubble velocity in the wall frame of reference. This difference increases with increasing Ca owing to the increase of the film thickness. Since h of Bingham is greater than Newtonian, the bubble velocity relative to the average fluid axial velocity is greater in the Bingham fluid compared to the Newtonian fluid for a given Ca in the wall frame of reference. Figure 8(b) shows the average fluid axial

velocity upstream/downstream of the axi-symmetric bubble in the wall frame of reference in terms of Bn and Ca . The average velocity values are computed through Equation (15) where G is known from Figure 7. Owing to one to one relationship between h and the average velocity, the trend is non-monotonic in terms of Bn for $Ca=0.1$.

6B-Profile of D_{12} along the wall

The advancing and receding motion of gas fingers in occluded airways induce normal and shear stresses along the airways wall. The stresses can be damaging to epithelial cells covering the inner side of the walls. The shear stress at a horizontal wall is the shear rate times the apparent viscosity. The trends of the shear rate and shear stress, however, can be different since the apparent viscosity is a flow dependent parameter. We therefore discuss them separately in the followings.

Figures 9(a–c) show the profile of D_{12} (For the components of the rate of strain and viscous stress tensors, indices 1 and 2 stand for x and y directions in a 2D channel and z and r directions in an axi-symmetric tube) along the wall for the axi-symmetric bubble with different Bn values and $Ca=0.05, 0.1$ and 0.25 . In the fully developed regions, $z = 0$ or $z = 12$, D_{12} is a negative constant and its magnitude, $|D_{12}|$, increases with Bn . The maxima of $|D_{12}|$ beneath the front and rear transitions all decrease with increasing Bn . This effect is more profound at larger Ca where the ratio of the yield stress to the surface tension increases. D_{12} is almost zero beneath the flat section of the bubble in the middle. This region is extended with increasing Bn .

The insets in Figures 9(a–b) magnify the region where D_{12} changes the sign along the wall. For the Newtonian fluid the sign is changed in a single point, the stagnation point. On the other hand for the Bingham fluid the change of sign occurs at the two sides of a line segment where D_{12} is almost zero. The line segment according to the yield criteria is an un-yielded segment. Hence, in the Bingham fluids instead of a stagnation point at the wall there is an un-yielded line segment. The length of the segment increases with increasing Bn .

As Bn increases, the global maximum of $|D_{12}|$ along the wall can shift from a point beneath the rear of the bubble to another beneath the front or to a point in the fully developed region. Figure 9(d) shows the distribution of D_{12} along the wall for the 2D and the axi-symmetric bubbles with $Ca=0.1$ and $Bn=2.0$. The global maximum of $|D_{12}|$ for the 2D is greater than that of the axi-symmetric. The same holds for $|D_{12}|$ in the fully developed regions.

6C-Profile of τ_{12} along the wall

Figures 10(a–c) show the distribution of the shear stress, τ_{12} , along the wall for the axi-symmetric bubbles with different Bn and $Ca=0.05, 0.10$, and 0.25 . From Equations (7) and (8) we see that the wall shear stress in the yielded regions for large values of m is

$$\begin{aligned}\tau_{12} &\cong D_{12} + Bn, & \tau_{12} &\geq Bn, \\ \tau_{12} &\cong D_{12} - Bn, & \tau_{12} &\leq -Bn.\end{aligned}\quad (17)$$

At any point in the yielded regions, then, the dimensionless wall shear stress is the sum of Bn ($-Bn$ for the negative stresses) and the shearing due to D_{12} . This leads to some qualitative differences in behavior of D_{12} and τ_{12} with Bn , described in the following.

All maxima of $|D_{12}|$ beneath the front and rear transitions of the bubble decrease with increasing Bn for all Ca values. On the other hand, the local maximum of the magnitude of τ_{12} , $|\tau_{12}|$, beneath the front transition increases with increasing Bn for all Ca values. This indicates that the increase in shear stress by the yield stress dominates the reduction by the shearing for this local maximum.

Beneath the rear transition and for the Newtonian fluid there are two maxima for $Ca=0.05$, 0.1 , and $Ca=0.25$. For these cases, the second maximum (which is closer to the z -coordinate of the rear tip) disappears for a critical Bn number. For Bn less than the critical value the amplitude of this maximum increases with increasing Bn . The value of the first maximum remains unchanged or decreases with increasing Bn depending on the Ca value.

$|\tau_{12}|$ in the fully developed regions of the wall increases monotonically with increasing Bn for all Ca values. The effect is more profound compared to the other locations, since $|D_{12}|$ increases monotonically with Bn in the fully developed regions.

The location of the global maximum of $|\tau_{12}|$ along the wall can be at a point beneath the front or rear transitions or in the fully developed region of the wall depending on the values of Bn and Ca . For example for $Ca=0.05$ the global maximum of $|\tau_{12}|$ shifts from point A in Figure 10(a) beneath the rear transition for the Newtonian case to point B beneath the front for $Bn=3.0$. For $Ca=0.1$ and 0.25 the global maximum shifts from points C/D beneath the front in Figure 10 (b/c) for the Newtonian to the fully developed region for $Bn=3.0$.

For $Ca=0.05$, 0.1 , and 0.25 the global maximum of $|\tau_{12}|$ for $Bn=3.0$, respectively, is 1.24 , 1.64 , and 2.2 times more than that of the Newtonian fluid. We showed in our previous study on liquid plugs [49] that for $Ca=0.1$ the global maximum of $|\tau_{12}|$ at $Bn=1.5$, associated with healthy mucus, is 1.27 times more than that of the Newtonian fluid. For that case the location of the global maximum was at a point beneath the trailing meniscus (equivalent to the front of the bubble). These indicate that the value of yield stress has a profound effect on the magnitude and the distribution of the wall shear stress. In addition, the motion of diseased mucus plugs (with higher yield stress) is more harmful to the epithelial cells covering the inner side of the airway respiratory walls. Figure 10(d) compares $|\tau_{12}|$ along the wall in an axi-symmetric and a 2D bubble with $Ca=0.1$ and $Bn=2.0$. While the trend of τ_{12} is the same for both configurations, the maxima of $|\tau_{12}|$ for the 2D case is slightly greater.

The magnitude of the gradient of τ_{12} along the wall, $|d\tau_{12}/dz|$, is another important quantity that can attribute to the airway wall epithelial cells injuries [68]. Figures 11(a–c) show a close-up of τ_{12} in the region of the wall where $|d\tau_{12}/dz|$ attains its global maximum. For $Ca=0.05$ and $Bn=1_3$, the global maximum occurs in the un-yielded line segment where the magnitude of slope of τ_{12} is significantly larger than that for the entire wall. In the segment τ_{12} roughly varies linearly from Bn to $-Bn$ from one end to the other. Therefore the slope can be approximated by $2 Bn/L_{us}$ where L_{us} is the length of the un-yielded segment. The global maximum of $|d\tau_{12}/dz|$ for $Bn=1_3$ is considerably larger than that of the Newtonian

fluid. This suggests that increase in the yield stress even further increases the chance of airway epithelial cell injuries during the motion. The same holds for $Ca=0.1$ and $Bn=1_2$.

For $Ca=0.1$ and $Bn=3.0$, D_{12} does not change its sign along the wall. Therefore the global maximum of $|d\tau_{12}/dz|$ cannot be estimated by $2 Bn/L_{us}$. The examination of the data shows that $|d\tau_{12}/dz|$ attains its maximum value at point with $z=8.7$ in Figure 11(b) which is a point inside of an un-yielded region.

Figure 11(c) shows a close up of τ_{12} for 2D and axi-symmetric bubbles with $Ca=0.1$ and $Bn=2.0$. The global maximum of the gradient of wall shear stress in each configuration is in an un-yielded line segment. The value of the global maximum in the 2D case is slightly greater as the slopes of τ_{12} in the un-yielded line segments in Figure 11(c) suggest.

6D-Pressure along the wall

Figures 12(a–b) show the variation of the pressure along the wall for a 2D channel in terms of Ca and Bn for Newtonian and Bingham fluids, respectively. In all cases the profile is linear with a slope of G downstream and upstream of the bubble where the flow is fully developed. Beneath the rear and the front transitions the variation is dramatic due to the surface tension effects and the bubble geometry. Beneath the static film, where the shear stress is small, the profile is nearly flat. The pressure drop between the inlet and the outlet of the channel decreases and increases with Ca and Bn , respectively, due to the surface tension effects and the yield stress. The broken lines represent the pressure along the wall for Poiseuille (Newtonian fluid) and Bingham- Poiseuille (Bingham fluid) equations in a channel with the same length, delivering the same volumetric flow rate. This figure shows that the presence of the bubble can reduce the required pressure drop for the same delivery depending on Ca , Bn , and AR values. It should be noted that the pressure drop induced by the bubble is mainly due to the front and the rear transitions, since the middle region is mostly static. For the same delivery, the induced pressure drop by the bubble can be less than the pressure drop in a single phase flow across a channel with a length the same as the bubble-total length. In this case, the presence of the bubble facilitates the delivery. However, if bubbles are present, the liquid delivery decreases significantly in the periods when the gas phase leaves the conduit. Therefore the Taylor flow with short slugs is associated with high frequency oscillations in delivery of the liquids.

6E-Yield surfaces

Figures 13(a–h), show the contours of the von Mises stress for $Ca=0.1$ and varying Bn between 0 to 3. The un-yielded regions in each figure are in white. The area beneath the flat section of the bubble is entirely un-yielded. This un-yielded region is extended by increasing Bn .

For the Bingham fluid there are un-yielded strips adjacent to the axis of symmetry upstream and downstream of the bubble. Depending on the value of Bn these un-yielded regions are separated from the bubble tips by the yielded fluid. As Bn increases up to 1.8, the un-yielded strips become thicker and get closer to the bubble tips. At $Bn=1.8$, the two un-yielded strips in the front and back of the bubble attach to its tips. At $Bn=1.9$, the un-yielded strip in front

of the bubble is separated from the tip by a yielded fluid region while an un-yielded lobe attached to the back of the bubble is formed.

At $Bn=2.0$, the un-yielded lobe at the back shrinks while the distance between the un-yielded strip in the front and the tip increases. At $Bn=2.2$, the lobe in the back disappears and therefore the un-yielded strips in the front and the back are separated from the tips by yielded fluid. At $Bn=3.0$, the distances between the un-yielded strips in the front and back and the two tips even increase further. The thickness of the un-yielded strips increases monotonically with increasing Bn . The rate of increase however becomes slower as Bn increases.

Far upstream/downstream of the bubble (e.g. inlet/outlet boundaries) the flow is fully developed with an axial velocity profile given by Equation (15). The profile is flat in the strips adjacent to the centerline. For small to moderate Ca , the axial velocity in this region has a negative value for $Bn=0$ (in the bubble frame of reference). The axial velocity increases with increasing Bn . For a Bn where the axial velocity becomes zero, the un-yielded strips extend from the inlet/outlet to the front and the rear tips, e.g. Figure 13(d). By further increase in Bn , the axial velocity in the strip becomes positive. This disconnects the un-yielded strips from the tips owing to the non-zero velocity gradient between the strips and the tips, e.g. Figures 13(e–h). For large enough Ca , the axial velocity is positive even for $Bn=0$. In this situation the un-yielded strips are always disconnected from the tips. Therefore, it suggests that for small and moderate Ca , the un-yielded strips starting from inlet/outlet extend in the axial direction with increasing Bn up to a critical value. At this critical value, the un-yielded strips are attached to the bubble tips. As Bn exceeds the critical value, the un-yielded strips are disconnected from the two tips. Figure 14 shows the value of the axial velocity in the un-yielded strips attached to the centerline in terms of Bn for different Ca values. The values are given for the bubble frame of reference.

The topologies of the un-yielded regions in the front and back of the bubble are not similar. This indicates that the location of the yield surfaces in the back of a bubble cannot be determined through analyzing of an advancing gas finger in general.

The insets in Figures 13 (b,f) show a distinct un-yielded triangular region attached to the wall beneath the back of the bubble. The aforementioned un-yielded line segments in section 6B are the bottom edges of the triangles. This indicates that in Bingham fluids instead of a single stagnation point at the wall, there is an un-yielded region attached to the wall. By increasing Bn the region becomes larger in both z and r directions. For large enough Bn , this distinct un-yielded region disappears. In this situation, D_{12} does not change its sign along the wall. We should mention that this distinct un-yielded region also exists for the other cases with $Bn < 2$ but they have not been plotted in Figure 13 panels.

In some simulations presented by Thompson et al. [47] for an advancing gas finger, the un-yielded regions beneath the middle (static film) and front of the bubble are connected, e.g. a case where $Ca=0.29$ and dimensionless yield-stress= 0.75^c . Those contradict Bingham

^cWe have introduced the relationship between Ca in their study and ours on page 23. Dimensionless yield-stress utilized by them as a parameter, is equal to $Bn/Bn+1$.

Equation, given by Equation (1) owing to significant velocity gradient between points in the region, e.g. tip and static film velocities are respectively 0 and 1 in the bubble frame of reference. They [47] attributed this to possibly an inadequate value for the regularized parameter used by them. We repeated one of those simulations (by converting their dimensionless parameters to ours) where $Ca=0.0725$ and $Bn=3.0$ and we used $m=1000$. Figure 15 shows the contours of von-Mises stress for this case where the aforementioned un-yielded regions are disconnected. This indicates that the computed von-Mises contours by us for this case do not contradict the Bingham Equation. While the Bingham Equation and the regularized method are not exactly the same but it is expected that the results obtained from them to be at least in a qualitative agreement.

Finally, we should mention that we repeated the computations for two cases where $Ca=0.1$, $Bn=1.8$ (critical value) and $Bn=3$ with $m=1000$, 2000, and 3000. We observed that the location of the yield-surfaces showed a converging trend as m increased.

6F-Streamline patterns

Figures 16(a–c) show fluid path-lines and contours of V_z for axi-symmetric bubbles with $Ca=0.1$ and varying Bn in the bubble frame of reference. At $Bn=0$ which is Newtonian there are two different patterns for the fluid path-lines. In the front and back of the bubble there are open vortices at the core. Near to the wall, however, the flow moves from the left to right. At $Bn=1.8$ which is a critical value, the flow almost vanishes in two strips starting from inflow/outflow and ending at the bubble front/ rear tips. The rest of the fluid path-lines are straight in the film and fully developed regions and curved in the areas near to the transient sections of the bubble. At $Bn=3$ there are neither open vortices nor vanishing strips in the flow. The fluid path-lines are straight in the fully developed and film regions while they are curved close to the transitions.

As we described in section 6E, the profile of V_z in the fully developed regions includes a flat (Constant V_z) and a parabolic section. For a given Ca , depending on the value of Bn the value of V_z in the flat section can be negative, zero, or positive while the value increases by increasing Bn . For the negative values there are open vortices in the front and back of the bubble. For a critical Bn where $V_z=0$ in the flat section, the two strips with vanishing flow in the front and back of the bubble are formed. As Figure 14 shows, the critical Bn decreases with increasing Ca and for large enough value of Ca , it does not exist because V_z in the flat section is positive even for $Bn=0$.

7-Summary and conclusions

The non-inertial steady motion of a long bubble inside of a 2D channel filled by a Bingham fluid was studied numerically using a regularized constitutive equation. The governing equations are discretized through a mixed-discontinuous finite element formulation embedded with the method of spines for resolving the free surfaces.

From our results a number of conclusions can be drawn. The yield stress modifies the shape of the bubble. More specifically it leads to an increase in the thickness of the static film and the length of the bubble, compared to the Newtonian fluid. The increase of the thickness

with increasing Bn for a given Ca can be monotonic/non-monotonic depending on Ca and the conduit configuration. Yield stress also smooths the wavy part of the interface located in the rear transition. Also, the yield stress magnifies the global maximum of the magnitude of the wall shear stress and its gradient. Therefore the propagation of bubbles and plugs in the blood vessels and the human airways filled by yield stress fluids can increase the chance of damage of the endothelial and epithelial cells covering the inner surface of those passages. The location of the global maximum for the magnitude of the wall shear stress depends on the Bn and Ca values. The location can shift from the front to the rear, and vice versa, or be in the fully developed section of the wall because of the yield stress and the surface tension effects. The maximum magnitude of gradient of the wall shear stress along the wall decreases with increase in Ca . The increase with increasing Bn , however, is not monotonic while its magnitude is more than that of the Newtonian.

The un-yielded regions appear beneath the middle, static film, and downstream and upstream of the bubble. The static film extends with increase in Bn . For a given Ca , the upstream and downstream un-yielded strips also extend from the inlet and outlet toward the tips with increase in Bn up to a critical value where they attach the bubble. By further increase in Bn , the un-yielded strips are detached from the bubble. For a frame of reference attached to the bubble, the flow pattern also bifurcates at this critical value. For Bn less than the critical value, there are two open vortices in the middle of the tube upstream and downstream of the bubble. At the critical value the vortices are evolved to two strips where the flow vanishes. For Bn larger than the critical value, there are neither the open vortices nor the strip in which the fluid velocity is zero. For cases where D_{12} on the wall changes the sign, there is a disconnected un-yielded region beneath the rear of the bubble which is attached to the wall.

Depending on the value of the dimensionless parameters and the bubble aspect ratio, the presence of the bubble can ease the flow delivery. This potential is enhanced by increasing the yield stress, the bubble aspect ratio and reduction in the air-liquid surface tension.

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Figure 1. Schematics of (a): A bubble. (b) A liquid plug. (c) An advancing (left) and a receding (right) gas finger.

Figure 2.
A schematic from a deformed airway conduit.

Figure 3.

A schematic of the domain of calculation for steady motion of a bubble in a 2D/axis-symmetric channel.

(a)

(b)

Figure 4. The shape of a bubble moving in a Newtonian fluid for different Ca values. (a) Axi-symmetric. (b) Axi-symmetric versus 2D.

(a)

(b)

(c)

Figure 5.

(a–c) The shape of an axi-symmetric bubble for different Ca and Bn . (d) Comparison between 2D and axi-symmetric.

Figure 6.

(a–c) Close up from the rear of axis-symmetric bubbles for different Bn and Ca .

Figure 7.
 G in terms of Bn for different Ca values.

Figure 8.(a): h in terms of Bn for different Ca values.(b): Area average of the fluid axial velocity upstream/downstream of the bubble in terms of Bn for different Ca values. The velocity values are given in the wall frame of reference.

Figure 9.

Variation of D_{12} along the wall with Bn for different Ca values. (a) $Ca=0.05$. (b) $Ca=0.10$. (c) $Ca=0.25$. (d) Comparison between axi-symmetric and 2D.

Figure 10. Variation of τ_{12} along the wall with Bn for different Ca values. (a) Ca=0.05. (b) Ca=0.10. (c) Ca=0.25. (d) Comparison between axi-symmetric and 2D for Ca=0.1 and Bn=2.0.

Figure 11.

Distribution of τ_{12} around a region of the wall where $|d\tau_{12}/dz|$ is maximum. (a) $Ca=0.05$. (b) $Ca=0.1$. (c) Comparison between axi-symmetric and 2D configurations.

Figure 12.

Variation of pressure on the wall in a 2D channel (a) With Ca . (b) With Bn . The broken lines show the wall pressure for the Poiseuille and Bingham-Poiseuille equations with the same volumetric flow rate delivery.

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

Figure 13.

von-Mises stress for $Ca=0.1$ and varying Bn . The un-yielded regions are in white. The insets in panels (b) and (f) magnify the area around the distinct un-yielded region beneath the back of the bubble.

Figure 14.

Axial velocity in the un-yielded strips attached to the tube centerline in terms of Bn for different Ca . The values are given for the bubble frame of reference.

Figure 15.

Contours of von-Mises stress for $Ca=0.0725$ and $Bn=3.0$. The un-yielded regions, white color, in the front/back and the middle are disconnected.

Figure 16.

Fluid path-lines and contours of V_z for the axis-symmetric bubble with $Ca=0.1$ and varying Bn . (Top): Fluid path-lines. (Bottom): Contours of V_z .

Table 1

h^* in terms of Ca for a bubble moving in a Newtonian fluid. The current results are compared with previous works. The numbers in parenthesis show the difference in percentage with the previous works. For Ca=1/14000, 1/7000, and 1/3500, the comparison with Bretherton's formula [20] is made for 2D results of this study.

Ca	Current study		Bretherton [20]	Reinelt & Saffman [27]		Aussilous & Quere [26]
	2D	Axi-sym.	Axi-symmetric	2D	Axi-sym.	Axi-symmetric
1/14000	0.00228	–	0.002304(–%0.6)	–	–	0.002308(–%1.2)
1/7000	0.00362	–	0.003657(–%1.0)	–	–	0.003650(–%0.8)
1/3500	0.00571	–	0.005805(–%1.5)	–	–	0.005760(–%0.9)
1/100	0.05430	0.053000	0.062100(–%14.7)	0.054000(%0.6)	–	0.053971(%0.6)
1/10	0.17620	0.167300	0.288200(–%42.0)	0.176000(%0.1)	0.167600(–%0.2)	0.167826(–%0.3)
1/4	0.24530	0.230200	0.530836(–%56.6)	0.244160(%0.5)	0.230000(%0.1)	0.228376(%0.8)

Table 2

h in terms of Ca for the axi-symmetric bubble, $Bn=1$. The current results are compared with a previous work, de Sousa et al. [46], for advancing gas fingers. The numbers in parenthesis show the difference in percentage with their work.

Ca	Current study	de Sousa et al. [46]
0.1394	0.215	0.220(−%2.3)
0.1893	0.230	0.236(−%2.2)
0.2570	0.251	0.249(%0.8)