Modeling/GIS, Risk Assessment, Economic Impact

Modeling Spatiotemporal Distribution of Mosquitoes Abundance With Unobservable Environmental Factors

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Abstract

Mosquito trap counts are heavily influenced by environmental factors such as temperature and precipitation. However, some important geographic factors, such as land use and elevation of a particular site, are often either not recorded or simply not observable. This is a major issue in building a predictive model for the mosquito trap counts over time across a particular region. The collective impact of all unobservable factors for one particular site is estimated by a hidden dimension method. Application to mosquito trap counts in Peel Region has shown that our model can significantly improve the modeling accuracy of the generalized linear model. This method may provide a significantly better characterization of the spatiotemporal distribution of mosquito (Diptera: Culicidae) abundance in areas with green lands or open spaces.

Key words: mosquito control, population modeling, mosquito-borne diseases

Mosquito-borne diseases (MBD) are serious threats to public health; the common MBD include malaria, dengue fever, West Nile virus (WNV), Rift Valley fever, chikungunya, etc. (World Health Organization 2017). The transmission of the mosquito-borne parasite may result in a wide range of symptoms from absent or very mild to severe symptoms and even death (Malaria Disease 2015). Mosquito abundance is heavily affected by environmental conditions such as temperature, precipitation, photoperiod, etc. (Reisen 1995, Robich and Denlinger 2005, Rubel et al. 2008, Yang et al. 2009, Jones et al. 2012, Brady et al. 2013, Denlinger and Armbruster 2014). There are several statistical studies about the relationship between mosquito abundance and environmental factors (Degatano 2005, Pecoraro et al. 2007, Trawinski and MacKay 2008, Walsh et al. 2008).

Since the arrival of WNV in Greater Toronto Area, Ontario has started the adult mosquito-monitoring program, and the weekly trap counts of *Culex pipiens/restuans* mosquito have been used to assess the risk of the human infection of the virus. Wang et al. (2011) built a statistical model for average *Cx. pipiens/restuans* mosquito trap count for the region. In their study, the weekly mosquito trap counts in Peel Region was assumed to follow a gamma distribution under the normal weather conditions, and a generalized linear model (GLM) was applied to correlate the mosquito trap counts with the daily temperature and precipitation. Their model has been used to predict average mosquito abundance and tested to match well of the averaged trap counts in the region. However, some environmental factors, such as land use, humidity, elevation, and other geographic

information, also affect the reproduction process of mosquitoes, and therefore have impact on the mosquito abundance (Norris 2004, DeGroot et al. 2007). But these environmental factors may not be observable or recorded due to technical difficulties, cost, or other reasons. In order to explain the variations of the trap counts across the whole region, it is essential to consider the impact of those unobservable critical factors in an improved statistical analysis. It remains a challenge due to the unavailability of the data and the difficulties in spatial modeling to count for the variations of the landscape features and their response regarding the mosquito reproduction.

The idea of hidden dimension can be viewed mathematically as modeling in a submanifold of the state space in which all variables reside on (Lee 2006). In physics, a hidden dimension method is used to model a complex high-dimensional phenomenon (Seiberg and Witten 1999, Witten 1995). In Bornn et al. (2012), the authors propose a method for modeling nonstationary processes through dimension expansion by projecting the geographic plane into higher dimensional space. Dimension expansion is achieved by examining the theoretical variogram in a transformed higher dimension.

Due to lack of information and observations, it is impossible to reconstruct all the hidden dimensions exactly. One can only hope to estimate the collective impact of all the unobservable factors with no prior information. The unobservable could be environmental factors or some other factors that are unknown. In this article, we generalize the dimension expansion method to model spatiotemporal distribution of mosquito population with unobservable information. We

estimate the collective impact of all the unobservable information of each trap by minimizing the cumulative fitting errors. Results from application to modeling the mosquito trap count in Peel Region demonstrate that our proposed method can achieve significant improvements in fitting accuracy for many monitoring sites located inside or close to areas with green lands or open spaces in comparison with the classical GLM by Wang et al. (2011).

Materials and Methods

Mosquito Data and Weather Data

In Southern Ontario, the two primary WNV vector mosquito species are *Cx. pipiens* L. and *Cx. restuans* Theobald. These two species are very difficult to distinguish (Degaetano 2005, Trawinski and MacKay 2008, Wang et al. 2011). They are merged as one group in the mosquito surveillance data set (Peel Public Health 2009), therefore are treated as one group *Cx. pipiens/restuans* in this study (Degaetano 2005, Trawinski and MacKay 2008).

In Canada, the Ontario Ministry of Health and Long-Term Care initiated the mosquito surveillance program in southern Ontario in 2001. As in Wang et al. (2011), we focus on the surveillance program in Peel region of Ontario. There are 31 permanent, fixed monitoring traps in the region. Mosquitoes were collected weekly on Tuesdays or Wednesdays from mid-June to early October, then the trapped mosquitoes were brought to the laboratory where they were identified to species level and tested for WNV. Note that the mosquitoes are collected only one time a week, and for each collection, the trap is placed in the site for 1 d. We use mosquito trap count data from 2006 to 2012 from the 30 traps in Peel Region, Ontario (We ignore one trap since it missed the mosquito data in the year 2006). The raw data, however, are very noisy due to various uncontrollable factors. Therefore, we apply the 3 wk moving average method to smooth the raw data in order to study the trends. This technique is well accepted in the literature since the smoothed data are more robust than the raw data. Fig. 1 shows the average mosquito trap counts of the 30 traps and mosquito trap counts of four traps of Peel Region in the year 2006.

The land use information of the 30 traps are extracted from GeoBase Land Cover Product (Land Cover, circa 2000-Vector) which is available through the website of Natural Resources Canada (Topographic Information 2018). For weather data, we use reanalysis data of ERA-Interim (Dee et al. 2011) as our historical weather data (temperature and precipitation) of Peel Region. As shown in

Fig. 1. *Cx. pipiens/restuans* mosquito trap counts of four traps and the average mosquito trap counts of the 30 traps of Peel Region in the year 2006.

Fig. 2, there are 13 grid points in and very close to Peel Region in ERA-Interim. So, we use an interpolating thin-plate spline (TPS) to interpolate each of the weather observation, add observations of temperature and precipitation from $n = 13$ sites to the 30 mosquito trap locations. The TPS method, introduced by Duchon (1976, 1977), was first applied to weather data by Wahba and Wendelberger (1980). Many researchers have shown that it is an accurate, operationally straightforward, and computationally efficient solution to the problem of spatial interpolation of weather data (Hutchinson 1995, Tait et al. 2006). What's more, in a comparison of different interpolation methods conducted by Jarvis and Stuart (2001), TPS achieved the best accuracy compared with ordinary kriging and inverse distance-weighting method.

Methods

Let $\{y_1, \dots, y_n\}$ be n independent observations of a response, where y_i is treated as a realization of a random variable Y_i . Assume that the observations come from a distribution in exponential family with the following density function,

$$f(y_i) = \exp \left\{ \frac{y_i \theta_i - b(\theta_i)}{a_i(\phi)} + c(y_i, \phi) \right\}, \quad (1)$$

where θ_i and ϕ are parameters, and $a_i(\phi)$, $b(\theta_i)$ and $c(y_i, \phi)$ are known functions. $a_i(\phi)$ has the following form:

$$a_i(\phi) = \frac{\phi}{p_i}, \quad (2)$$

where p_i is a known prior weight. If Y_i has a distribution in the exponential family, it has mean and variance

$$E(Y_i) = \mu_i = b'(\theta_i); \quad \text{Var}(Y_i) = \sigma_i^2 = b''(\theta_i)a_i(\phi).$$

Fig. 2. Trap locations in Peel Region and grid points in ERA-Interim.

For GLM (McCullagh and Nelder 1989), one defines a link function $g(\mu_i) = \eta_i$ which describes how the mean $E(Y_i) = \mu_i$ depends on the linear predictor. Therefore, the GLM with spatiotemporal setting can be defined as

$$\begin{aligned} E(Y_{k,t}) &= \mu_{k,t}, \\ g(\mu_t) &= \eta_{k,t}, \\ \eta_{k,t} &= \beta_0 + \beta_1 x_{k,t}^1 + \dots + \beta_p x_{k,t}^p, \end{aligned} \quad (3)$$

where k in $Y_{k,t}$ is a location index ($1 \leq k \leq K$) and t is a time index ($1 \leq t \leq T$). Let $\mathbf{x}^j = (x_{k,t}^j)_{K \times T}$ be the matrix corresponding to j th predictor, $1 \leq j \leq p$.

When confronted by incomplete information, we assume that the collection of all independent variables includes $\mathbf{x}^1, \mathbf{x}^2, \dots, \mathbf{x}^p, \mathbf{u}^1, \dots, \mathbf{u}^m$. We can only observe $\mathbf{x}^1, \mathbf{x}^2, \dots, \mathbf{x}^p$, while $\mathbf{u}^1, \dots, \mathbf{u}^m$ are not observable.

To take into account the impact of the unobservable factors, we proposed a new GLM with the linear predictor as following,

$$\hat{\eta}_{k,t} = \beta_0 + \beta_1 x_{k,t}^1 + \dots + \beta_p x_{k,t}^p + z_k, \quad (4)$$

where z_k represents the collective impact of all unobserved information $\mathbf{u}_{k,t}^1, \dots, \mathbf{u}_{k,t}^m$ for location k .

In order to find z_k , it is desirable to have the new predicted output $g^{-1}(\hat{\eta}_{k,t})$ to be as close as possible to observations $y_{k,t}$. The closeness, however, can be arbitrarily defined. Denote the loss function by $\psi(y_{k,t}, \hat{\eta}_{k,t})$. The collective impacts z_k can then be estimated by the following:

$$\mathbf{z} = \arg \min_{\mathbf{z}} \sum_{k,t} \psi(y_{k,t}, \hat{\eta}_{k,t}), \quad (5)$$

where $\mathbf{z} = (z_1, z_2, \dots, z_K)^T$.

It is clear that the estimated z_k 's depend on the chosen loss function. With the purpose of improving model fitting accuracy, we want the distance of observations $y_{k,t}$ and predicted output $g^{-1}(\hat{\eta}_{k,t})$ to be as small as possible, so we choose the loss function in this study as follows:

$$\psi(y_{k,t}, \hat{\eta}_{k,t}) = |y_{k,t} - g^{-1}(\hat{\eta}_{k,t})|. \quad (6)$$

Application to Mosquito Abundance in Peel Region, Ontario

Current Mosquito Model

Wang et al. (2011) used growing degree-days to describe the effect of temperature on mosquito abundance in their model. dd_t , the growing degree-day on date t , is defined as follows:

$$dd_t = \begin{cases} 0^\circ\text{C} & \text{if } Tm_t \leq 9^\circ\text{C}, \\ Tm_t - 9^\circ\text{C} & \text{if } Tm_t > 9^\circ\text{C}, \end{cases}$$

where Tm_t is daily mean temperature for date t . Base temperature of 9°C is chosen because below this temperature the immature *Cx. pipiens/restuans* mosquito development will slow down or stop entirely.

Wang et al. (2011) proposed a GLM to analyze mosquito abundance data and their model has the following form,

$$\begin{aligned} E(\rho_t) &= \mu_t, \\ g(\mu_t) &= \log(\mu_t) = \eta_t, \\ \eta_t &= \beta_0 + \beta_1 (ddm_t^{[n]})^2 + \beta_2 ddm_t^{[n]} + \beta_3 (ppm_t^{[m]})^2 + \beta_4 ppm_t^{[m]}, \end{aligned} \quad (7)$$

where ρ_t is the arithmetic mean of mosquito counts per trap night in Peel Region at the week t (note that Peel Region collect mosquitoes one time a week, and for each collection, they put the trap in the site for 1 d), and it is assumed to follow a gamma distribution,

$$\rho_t \sim \text{Gamma}(\alpha, \beta); \quad ddm_t^{[n]} = \frac{1}{n} \sum_{i=t-n}^{t-1} dd_i \quad \text{and} \quad ppm_t^{[m]} = \frac{1}{m} \sum_{i=t-m}^{t-1} pp_i,$$

pp_i is the total precipitation on day i . To get the time-spans of n and m which are lags measured in days, they tried with different combinations of n ($1 \leq n \leq 60$ days) and m ($1 \leq m \leq 60$ days). After comparing all combinations of $ddm_t^{[n]}$'s and $ppm_t^{[m]}$'s (3,600 combinations) through likelihood ratio test statistics, they find that the model with $ddm_t^{[11]}$ and $ppm_t^{[35]}$ achieve the highest test statistic. So, $ddm_t^{[11]}$ and $ppm_t^{[35]}$ are used in the model defined by equation 7.

The GLM is used to study the relationship between adult female mosquito abundance and weather conditions (temperature and precipitation). There are four stages for mosquito life cycle including eggs, larva, pupa, and adult mosquito. For *Cx. pipiens/restuans* mosquitoes, the stages of eggs, larva, and pupa are all in water. All these stages need to accumulate enough heat to progress into the next stage. Thus, the stages of eggs, larva, and pupa are heavily affected by precipitation and temperature and there is a delayed impact for temperature and precipitation.

When applying the model defined by equation 7 on average mosquito abundance in Peel Region of 2006 to 2012, we use deviance as a statistic to compare different models with combinations of $ddm_t^{[n]}$'s and $ppm_t^{[m]}$'s. We find that the model with $ddm_t^{[12]}$ and $ppm_t^{[42]}$ achieves the lowest deviance (smaller deviance represents a better model) in all 3,600 models, so we will use $ddm_t^{[12]}$ and $ppm_t^{[42]}$ in this study.

Modeling Through Dimension Expansion

The model of Wang et al. (2011) is for average mosquito abundance in Peel Region, Ontario, Canada. We adjust it and construct a model for mosquito abundance of each trap in Peel Region. There are 30 traps with available data, so we have $K = 30$. From 2006 to 2012, we take mosquito data from week 25 to week 39. For each year, we use one model to study mosquito abundance, so we have $T = 15$ wk and get the following Gamma GLM for mosquito abundance of each trap for a certain year,

$$\begin{aligned} E(\rho_{k,t}) &= \mu_{k,t}, \\ g(\mu_{k,t}) &= \log(\mu_{k,t}) = \eta_{k,t}, \\ \eta_{k,t} &= \beta_0 + \beta_1 (ddm_{k,t}^{[12]})^2 + \beta_2 ddm_{k,t}^{[12]} + \beta_3 (ppm_{k,t}^{[42]})^2 \\ &\quad + \beta_4 ppm_{k,t}^{[42]} + \beta_5 ddm_{k,t}^{[12]} ppm_{k,t}^{[42]}, \end{aligned} \quad (8)$$

where $\rho_{k,t}$ is the captured mosquito number of trap k at week t , $1 \leq k \leq 30$, $1 \leq t \leq 15$, we assume $\rho_{k,t} \sim \text{Gamma}(\alpha, \beta)$. $ddm_{k,t}^{[12]}$ is 12 d of arithmetic means of daily growing degree-day before mosquito collection of week t at trap k ; $ppm_{k,t}^{[42]}$ is 42 d of arithmetic means of daily precipitation before mosquito collection of week t at trap k . We add the interaction term of $ddm_{k,t}^{[12]} ppm_{k,t}^{[42]}$ since we believe interaction of temperature and precipitation also has an impact on mosquito abundance.

The model of Wang et al. (2011) works well for average mosquito abundance of Peel Region, but for mosquito abundance in each trap, the model defined by equation 8 has large error compared with surveillance data. For simplicity, we call the model defined by equation 8 the partial model. To improve the accuracy of the partial model and have a better understanding of the spatial distribution of mosquito population, we apply our method. We assume that missing information for trap k can be represented by one variable z_k , so

we get a new linear predictor $\hat{\eta}_{k,t}$, and the model with dimension expansion for one year can be defined as follows,

$$\begin{aligned} E(\rho_{k,t}) &= \mu_{k,t}, \\ g(\mu_{k,t}) &= \log(\mu_{k,t}) = \hat{\eta}_{k,t}, \\ \hat{\eta}_{k,t} &= \beta_0 + \beta_1 (ddm_{k,t}^{[12]})^2 + \beta_2 ddm_{k,t}^{[12]} + \beta_3 (ppm_{k,t}^{[42]})^2 + \\ &\quad \beta_4 ppm_{k,t}^{[42]} + \beta_5 ddm_{k,t}^{[12]} ppm_{k,t}^{[42]} + z_k \end{aligned} \quad (9)$$

where $\mathbf{z} = (z_1, z_2, \dots, z_k)'$ can be estimated by

$$\mathbf{z} = \arg \min_{\mathbf{z}} \sum_{k,t} |y_{k,t} - g^{-1}(\hat{\eta}_{k,t})|, \quad (10)$$

$y_{k,t}$ is the observed number of mosquito caught in trap k at week t for the year, $\mathbf{z}$ is estimated for each year so it could be different for different years.

Results

We calculate errors of modeled mosquito abundance from 2006 to 2012 and show them in Table 1.

In Table 1, we have $E_0 = \frac{1}{TK} \sum_{k,t} |\hat{y}_{k,t} - y_{k,t}|$ for each year,

$y_{k,t}$ is observed mosquito count in trap k at week t , where $\hat{y}_{k,t}$ is modeled mosquito count of trap k at week t from the GLM.

$E_1 = \frac{1}{TK} \sum_{k,t} |\hat{y}_{k,t}^{[1]} - y_{k,t}|$, where $\hat{y}_{k,t}^{[1]}$ is modeled mosquito count in trap k at week t from our model. Note that the E_0 and E_1 are averaged errors over all week and all traps, respectively.

Table 1 shows that the average absolute error of our model is significantly less than that of the GLM (E_1 is almost half of E_0 for each year). This demonstrates the benefit of using our method compared with the original GLM. Figs. 3 and 4 show the observed mosquito trap counts and modeled mosquito trap counts for the 30 traps of the year 2006 and the year 2012. In Figs. 3 and 4, the blue line shows the observed mosquito trap counts, the dashed black line presents modeled mosquito trap counts by the GLM, the green line represents modeled mosquito trap count from our model. Note that the mosquitoes are only present in the summer. Therefore, the plots are the weeks in the summers only.

Discussion and Conclusion

Discussion

In general, our method works well across all 30 traps in Peel region. However, the magnitude of improvement varies from one spatial

Table 1. Comparison of fitting accuracy of different models

Year	E_0 (GLM)	E_1 (our model)
2006	4.87	2.97 (0.39)
2007	7.64	4.17 (0.45)
2008	15.23	7.76 (0.49)
2009	16.28	7.93 (0.51)
2010	13.62	6.22 (0.54)
2011	10.78	5.11 (0.53)
2012	14.19	6.66 (0.55)

The numbers in bracket after E_1 are the proportion of absolute error reduced compared with absolute error of the GLM, which is $(E_0 - E_1)/E_0$.

location to the next. The histogram of average error reductions from 2006 to 2012 for each trap is shown in Fig. 5.

There is a gap at 0.6 which naturally divides the 30 traps into two subgroups. We call the upper group the high-improvement group with 11 traps and the other one the low-improvement group with 19 traps.

Fig. 6 shows the locations of traps of the two groups and identification of group memberships in Peel Region of Ontario, Canada.

As shown in Fig. 6, the high-improvement traps tend to locate in the low-built environment while low-improvement traps tend to be in high-built areas. Proportions of built environment are relatively low in the high-improvement group (with mean 0.4473), in comparison with those in the low-improvement group (with mean 0.7532). To confirm the built environment proportion differences, we do a permutation test with the null hypothesis that the land use data (built environment proportion l_k) of traps in high-improvement group and low-improvement group are from populations with the same mean. The test gives a p value of 0.0056 which rejects the null hypothesis and indicates that the differences are significant. This indicates that mosquitoes in the low-built environment area have different patterns compared with mosquitoes in high-built environment area. The GLM works well in an urban area with more buildings. Our method could significantly improve the GLM in areas with more green land and open areas.

To better understand the different performance of the two groups, we also compare the missing information z_k found in high-improvement group and low-improvement group by combining all the values across the 7 yr of observation period (we have $7 * 11 = 77$ z_k 's for high-improvement traps and $7 * 19 = 133$ z_k 's for low-improvement traps).

We perform a permutation test with the null hypothesis that the z_k 's found for traps in high-improvement group and low-improvement group are from populations with the same mean. The permutation test shows a significant difference between z_k 's found in the high-improvement group and the low-improvement group (with $p < 0.0001$). The missing information found for traps in high-improvement group has negative value with mean -1.3524 (which is significantly different from 0 with $p < 0.0001$ from permutation test), which indicates that the partial model tends to over-estimate for the traps in high-improvement group.

Conclusion

In this study, we provide a generalized of dimension expansion method and apply it to GLM setting to study the spatiotemporal mosquito population distribution with limited available data. The collective impact of all unobservable environmental factors of a particular site is estimated by minimizing the cumulative modeling errors. The method is a valuable tool to extend the application of GLM to data with information that are not observable especially in a site with a significant portion of green lands or open spaces.

Application to modeling the mosquito trap counts in Peel Region has shown that our model can significantly improve the model accuracy of the GLM by Wang et al. (2011). We find that the GLM method by Wang et al. (2011) works well in an urban area with many buildings. In contrast, our method can significantly improve their model in areas with more green lands and open spaces.

The limitation of our study is that we assume that the estimated collective impact of all unobservable factors for a particular site is fixed over a period of time. This is not applicable to trap efficiency which could vary over time due to different weather conditions. For example, extreme wind or heavy rain for a short period of time may

Year 2006

Fig. 3. Observed mosquito trap counts (blue solid line) and modeled mosquito trap counts of each trap for the year 2006. The modeled mosquito trap counts by the GLM described by dashed black line, modeled mosquito trap counts from our model is represented by green dash line with plus mark.

Year 2012

Fig. 4. Observed mosquito trap counts (blue solid) and modeled mosquito trap counts (back dash) of each trap for year 2012.

Fig. 5. Histogram of average error reduced by our method for each trap.

Fig. 6. Locations of traps of two groups and proportion of built environment in Peel Region.

have significant negative effect on mosquito activity and therefore reduce the trap efficiency.

Future research studies are therefore needed to extend our method to model the time-dependent nature of those unobservable impacts. This would help us to have a better understanding of the spatiotemporal distribution of the mosquito abundance.

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