Published in final edited form as:

J Exp Child Psychol. 2019 March ; 179: 260–275. doi:10.1016/j.jecp.2018.11.017.

Operational Momentum for Magnitude Ordering in Preschool Children and Adults

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Abstract

When adding or subtracting quantities, adults tend to overestimate addition outcomes, and underestimate subtraction outcomes. They also shift visuospatial attention to the right when adding, and to the left when subtracting. These *operational momentum* phenomena are thought to reflect an underlying representation in which small magnitudes are associated with the left side of space, and large magnitudes with the right. Currently, there is limited research on operational momentum during early childhood, or for operations other than addition and subtraction. The current study tests whether English-speaking 3- and 4-year-olds and college-aged adults exhibit operational momentum when ordering quantities. Participants were presented with two experimental blocks. In one block of trials they were tasked with choosing the same quantity they had previously seen three times; in the other, they were asked to generate the next quantity in a doubling sequence composed of 3 ascending quantities. A bias to shift attention to the right after an ascending operation was found in both age groups, and a bias to overestimate the next sequential quantity during an ascending ordering operation was found in adults under conditions of uncertainty. These data suggest that, for children, the spatial biases during operating are more pronounced than the mis-estimation biases. These findings highlight the spatial underpinnings of operational momentum, and suggest that both very young children and adults conceptualize quantity along a horizontal continuum during ordering operations, even before formal schooling.

Keywords

Operational momentum; ordering operations; number; quantity; magnitude; preschool children; adults; number space mapping

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Adults (Cordes, Gelman, Gallistel, & Whalen., 2001), children (Halberda & Feigenson, 2008), preverbal infants (vanMarle & Wynn, 2006; Xu & Spelke, 2000) and non-human animals (Cantlon & Brannon, 2007; Garland, Low, & Burns, 2012) perceive and process non-symbolic quantities in an approximate fashion. This large-number system yields inexact, nonverbal representations of a given numerical magnitude as defined by Weber's Law: the ease with which two quantities are discriminable is proportional to their ratio, not their absolute difference (i.e. 10 is more discriminable from 4 than 110 is from 104, despite an identical absolute difference; Whalen, Gallistel & Gelman, 1999; Cordes et al., 2001).

Numerical magnitudes are represented on a spatially organized horizontal continuum, known as the mental number line (MNL) (Dehaene, 1992; Moyer & Landauer, 1967). For many people this mental number line is structured such that small values are mentally placed to one side of space and large values to the other. This predisposed lateralization is driven by evolved neurological architecture (Vallortigara, 2017). For instance, both rhesus macaques (*macaca mulatta*) and newborn chicks exhibit a propensity to map number onto space from left to right asymmetrically (Drucker & Brannon, 2014; Rugani, Kelly, Szelest, Regolin, Vallortigara, 2015). Further evidence for predisposed spatial-numerical associations comes from studies with human neonates (de Hevia, Veggiotti, Streri, & Bonn, 2017) and preverbal infants (Bulf, de Hevia, Macchi Cassia, 2016; de Hevia, Izard, Coubart, Spelke, Streri, 2014b), which also illustrate small-left and large-right mappings. This lateralization is, for many populations strengthened across the lifespan, namely among participants within Westernized cultures whose reading and writing direction is from left-to-right (Shaki, Fischer, & Gobel, 2012; see McCrink & Opfer, 2014 for a review). These culturally influenced processes are formed in early childhood and can reinforce the structure of spatial associations such as the mental number line (Dobel, Diesendruck, & Bolte, 2007).

Spatial associations between space and magnitude are recruited not only when discriminating two magnitudes, but also when operating over these magnitudes, resulting in a phenomenon known as operational momentum. According to one popular account of this phenomenon, the asymmetrically oriented spatial associations lead to arithmetic miscalculations and spatial shifts of attention when operating over sets of objects (McCrink, Dehaene, & Dehaene-Lambertz, 2007; Knops, Viarouge, & Dehaene, 2009b; Knops, Zitzmann, & McCrink, 2013; c.f. Chen & Verguts (2012) and Prather (2012) for alternate, non-spatial, accounts). Operational momentum is related to the perceptual phenomenon known as "representational momentum", in which participants have a tendency to anticipate the final location of a perceived moving target erroneously, displaced in the direction of target motion, in dimensions of space, action, or the pitch of sounds (Ashida, 2004; Freyd & Finke, 1984; Freyd, Kelly, & DeKay, 1990; Getzmann, Lewald, & Guski, 2004). Just as moving objects have physical thrust, mental calculations exhibit a momentum of their own.

When mentally adding, subtracting, multiplying, and dividing, people experience a forward displacement along their mental representation in the direction of the operation's outcome (i.e. toward large numbers for addition and multiplication problems, and toward small numbers for subtraction and division problems; McCrink et al., 2007; Katz & Knops, 2014). Thus, operational momentum is thought to be driven by attentional shifts along an internally represented horizontal continuum. In line with this view, Knops, Thirion, Hubbard, Michel,

and Dehaene (2009a) found evidence that the neural activity associated with rightward saccades is the same as that activated when viewing centrally-presented addition problems, suggesting that addition shifts attention toward the right side of the mental number line. Further, Knops et al. (2009b) found that during non-symbolic addition, adults preferentially select outcomes from the right side of a screen, whereas during subtraction, they are biased toward the left side of the screen. Operational momentum is proposed to be the result of a type of logical heuristic (McCrink & Wynn, 2009; McCrink & Hubbard, 2017), in which participants generate a rule of “if adding, more” and “if subtracting, less”, which is implemented via spatial shifts of attention. As with other heuristics (Kahneman & Tversky, 2013) operational momentum increases under conditions of estimation uncertainty, time pressure, and distraction (Knops et al., 2009b; McCrink et al., 2007; McCrink & Hubbard, 2017).

Operational momentum for addition and subtraction has also been documented in infants (McCrink & Wynn, 2009), a population found to have a left-to-right oriented spatial-numerical mapping (Bulf et al., 2016; de Hevia, Girelli, Addabbo, & Macchi Cassia, 2014a). Infants who see four objects added to a set of six look reliably longer to an incorrect outcome of five (an underestimation) than of 20 (an overestimation) (McCrink & Wynn, 2009). Thus, infants appear to also experience the mis-estimation aspect of operational momentum. This suggests there may be an underlying common mechanism of shifting visuospatial attention (Bulf et al., 2016) while operating over magnitudes for both infants and adults (although to date no one has looked directly at spatial effects of operational momentum during adding and subtracting in infancy). Despite similarities in mis-estimation among infants and adults, there is not strong evidence for consistent overestimation and underestimation in childhood. For example, Knops et al. (2013) found that six- and seven-year-old children display either no operational momentum effect, or even an inverse effect in which they significantly overestimate subtraction problems rather than addition problems.

Given what is known about operational momentum for adding and subtracting, the effect should only be strengthened during the operation of ordering. Ordering underlies arithmetic reasoning, as it is the ability to recognize that in an ordered numerical series, one number is greater than another number, which allows participants to transform numerical quantities and anticipate outcomes as they operate (Gallistel & Gelman, 1992). The process of ordering information is especially evocative of lateralized spatial structuring; in adults, any well-learned ordinal sequence - such as the days of the week (Gevers, Reynvoet, & Fias, 2004), months of the year (Gevers, Reynvoet, & Fias, 2003), or even a newly-learned list of unrelated words (Previtali, de Hevia, & Girelli, 2010) - becomes represented on a horizontal continuum with initial information on one side of space, and final, on the other side. Importantly, such mapping of ordinal information into a directional space is functional prior to the acquisition of symbolic knowledge and language, as Italian 7-month-old infants learn rule-like patterns from visual sequences when the sequences are presented from left to right, but not from right to left (Bulf, de Hevia, Gariboldi, & Macchi Cassia, 2017). Therefore, preverbal infants, like adults, organize not only numerical order along a directional space (de Hevia et al., 2014b), but any kind of ordinal information (Bulf et al., 2017).

In particular, when it comes to ordering operations over magnitude, infants show evidence of operational momentum. Macchi Cassia, McCrink, de Hevia, Gariboldi, and Bulf (2016) and Macchi Cassia, Bulf, McCrink, and de Hevia (2017) have shown that, when 4- and 12-month-old infants are habituated to a set of objects that progressively increases or decreases in number (Macchi Cassia et al., 2017) or physical size (Macchi Cassia et al., 2016), and are subsequently presented with ordinal sequences whose magnitude is larger or smaller compared to the initial sequence, they look longer at sequences in which the direction violates the operational momentum experienced during habituation (the smaller sequence after viewing ascension, and the larger sequence after viewing descension).

While these findings in infancy indicate that operational momentum may reflect an underlying component of an early system of arithmetic transformations, the limited evidence for operational momentum during toddlerhood and early childhood casts doubt as to whether there is developmental continuity for this mechanism. In the current study, the presence of operational momentum was tested during an ordering operation in English-speaking 3- and 4-year-olds, and college-aged adults. In this experiment, participants were presented with an ascending sequence of magnitudes (Order trials) or a series of identical magnitudes (No Order trials), and asked to generate the next answer in the series by selecting a response from two laterally presented arrays. The correct answer was presented alongside over- and under-estimates of the correct outcome. In some cases, no correct answer was presented, and participants had to choose from two incorrect amounts. If participants experienced both the arithmetic and spatial components of operational momentum when ordering amounts, two overall patterns should emerge. First, when participants perform an ascending operation they will choose the overestimated outcome more often than if they did not just perform an ascending operation. Second, on trials in which participants perform an ascending operation, they will be more likely to choose the right-side array (e.g., the side associated with ‘more’ on the mental number line) compared to trials in which they must simply estimate an amount. Both of these effects are predicted to increase during trials in which no correct answer is present, due to greater use of heuristics under conditions of uncertainty.

Method

Participants

Fifty preschoolers ($M_{\text{age}} = 3.9$ years; range 3 years, 0 months – 4 years, 11 months) and 49 college-aged students participated in either a children’s museum (children) or a university laboratory (adults) in a major urban area. An additional 8 children participated, but were excluded from the final sample, due to an unwillingness to complete the experiment. An additional 4 adults participated, but were excluded from the final sample, due to right-to-left scripted language fluency (3) or computer error (1). Sample size was determined using effect sizes from previous research examining preschool space-number relations, which are in the moderate range (McCrink, Shaki, & Berkowitz 2014; Patro & Haman, 2012; effect size $f = .18$, with power $(1 - \beta \text{ err prob}) = .95$, and $\alpha = .05$), for a main experimental design for each population that has two within-subjects factors (block type, trial type) and one between-subjects factor (gender). Gender was included in the power analysis, pursuant to National Institutes of Health request that gender be included in all NIH-funded research designs,

analyses, and reporting (Notice 15–102). Additionally, there are some findings that suggest males may have a stronger linear representation of number (Bull et al., 2013; Hutchison, Lyons, & Ansari, 2018), which may potentially result in a larger OM effect for males relative to females.

Design

Each participant received two experimental blocks of trials. Each block was preceded by 5 training trials to ensure that the participants could understand the task. In the *No Order* block (8 trials), subjects were tasked with choosing a numerosity that matched an identical number of objects shown for three preceding priming slides (i.e. a non-ordered sequence). In the *Order* block (18 trials), participants were tasked with choosing the numerosity that would arithmetically come next after viewing three priming slides of dots which doubled in numerosity (e.g., 4, then 8, then 16). After the presentation of these sequences, participants were shown two choice options, one on the left and one on the right side of the screen. These choice options were either a) correct, b) an overestimate of the correct answer, or c) an underestimate of the correct answer. There were 4 test trial types: Correct vs. Overestimate, Correct vs. Underestimate, Overestimate vs. Underestimate, and Tie Trials. In the *Order* block, there were three different versions of Tie Trials (Underestimate vs. Underestimate, Correct vs. Correct, and Overestimate vs. Overestimate), and in the *No Order* block there was solely the Correct vs. Correct version. This design allowed us to have the same magnitudes, on average, across the blocks, but keep the overall number of trials down for the children. Using this design, we could calculate the prevalence of overestimation (choice of Overestimates in the Correct vs. Overestimate trials and Overestimate vs. Underestimate trials, choice of Correct in the Correct vs. Underestimate Trials) and the prevalence of right-side bias (choice of the right-side test array for the Tie Trials). Tables I, II, III, and IV provide the exact values used, and the side of the screen on which they were displayed, for both the training and testing trials. The order of blocks was counterbalanced between participants, and the presentation of trials within each block was randomized. There were fewer *No Order* trials than *Order* trials; although an earlier design had identical amounts of trials for both blocks, initial piloting revealed that the younger preschoolers were unable to tolerate more than ~24 trials plus the training for each block. Thus, the side of presentation for the correct answer in the *No Order* block was counterbalanced between trial types, and for the *Order* block both within and between trial types.

Stimuli and Procedure

All stimuli consisted of dots .4 cm in diameter, presented in a black frame that was 6.1 cm by 6.1 cm during priming, and during the choice slide. The visual stimuli were created on Keynote Presentation Software, and presented with Superlab 5 on a 13-inch Hewlett Packard touch-screen laptop computer.

No Order block

Training trials.: Participants received five training trials, each consisting of three priming slides (a non-ordered sequence) presented by the experimenter, and then two choice arrays in which a correct answer was present. The first trial was a demonstration performed by the

experimenter, in order to provide a background story and orient participants to the computer touch screen. In this trial, the experimenter presented two cartoon dancers inside a black square frame (the “dance floor”). As the experimenter flipped to the next slide, which showed the same two cartoon dancers, this time in a new spatial layout within the black square frame, she explained that the two dancers were still dancing. The third slide again showed two dancers *still* dancing. The experimenter then posed a question to the participant: “If our dancers are still dancing, which dance floor do you think will be ours?” The experimenter presented a screen with two choices and used the talk aloud strategy to acclimate the participant to the logic behind her choice, explaining that she would touch the dance floor she thought was correct. When she selected the correct box, after touching a small cross in the middle of the screen, a picture of a salsa dancer flashed on the screen, along with a celebratory noise.

The four remaining training trials were performed by the participant, with guidance from the experimenter in order to solidify the participant’s understanding of the task at hand (e.g., choosing the same number). These trials consisted of stimuli that were not pictures but rather dots; the participants were told that these dots were the tops of the dancers’ heads. When the participant made the correct selection after touching the cross in the middle of the screen, a picture of a cartoon salsa dancer flashed on the screen with a celebratory noise. If participants chose incorrectly, a buzz sound occurred and the choice slide remained on the screen until the participant selected the correct choice square. After successfully completing all four training trials, participants were able to move on to the experimental test trials. In these training trials, children were presented serially with three arrays of 56 dots. There were two comparisons, each presented twice (yielding 4 training trials done by each child): a correct choice option vs. underestimate (56 vs 20), and a correct choice option vs. overestimate (56 vs 157). In this way, children saw comparisons that largely mirrored what they would be exposed to during the test trials, with each trial type probing for the acceptance of a correct outcome relative to an overestimated or underestimated outcome. The correct answer was presented at each spatial location (top, bottom, left, and right) once during the training trials. This was done to ensure that the children did not associate a particular side of the screen with a correct response. These training trial contrasts were chosen to be a relatively easy discrimination (~3:1 ratio of magnitudes).

Experimental Trials.: The experimental trials were identical to practice trials in their procedure and story, with the sole difference that no feedback was offered to participants as to whether their answers were right or wrong. Participants sat ~40 cm away from the laptop computer, with the experimenter oriented diagonally behind the participant on these trials to avoid inadvertent cueing of the answer. The participants viewed three priming slides of the same numerosity for 1250 ms each, followed by a set of choice slides, presented until the participant indicated their choice. The spatial arrangement within each priming set changed between slides, even when the number of items remained the same. There were four types of trials in the experimental blocks, and participants were presented with priming slides of 32-32-32 (50% of the trials) or 96-96-96 (50% of the trials). In the Correct vs. Underestimated trials, children saw the correct answer contrasted with an underestimated answer. For example, a participant who viewed slides of 32, 32, and 32 would see a choice

slide containing 32 dots (the correct answer) and 20 dots (an incorrect, underestimated answer). In the Correct vs. Overestimated trials, participants saw the correct answer contrasted with an overestimated answer (e.g., 32 vs. 52 after being shown 32, 32, and 32 as primes). In the Underestimated vs. Overestimated trials, there was no correct answer displayed. Instead, participants were shown two incorrect answers, one of which was overestimated and one underestimated (e.g., 20 vs. 52 after seeing 16, 16, and 16 as primes). Finally, in the Tie Trials, participants saw two identical numerosities (Correct vs. Correct). All trials were preceded by a small cross in the middle of the screen, which the participant pressed to orient to a neutral starting point.

The side of screen showing the correct answer was counterbalanced within subject; 50% of the time it appeared on the right, and 50% on the left. There were 8 experimental trials in the No Order block (2 Correct vs. Underestimated, 2 Correct vs. Overestimated, 2 Underestimated vs. Overestimated, and 2 Tie Trials). The order of trial presentation within blocks was randomized, as was the absolute placement of objects within each array. The ratio of the incorrect outcomes to the correct outcomes on the choice slide comparisons was 1.6:1 (a relatively challenging, but still discriminable, contrast for children this age; Halberda & Feigenson, 2008). After completing one block, participants had a brief break before the experiment resumed with the next block.

Order block

Training Trials.: The training trials for the Order block retained the format of the No Order block; an introductory, experimenter-run trial followed by 4 trials in which a correct answer was presented against an incorrect one. The story was slightly different: the experimenter explained that this time a single dancer was dancing by herself, but she was lonely, so she left and soon returned with a partner. The next slide showed two cartoon dancers dancing as a pair. As the experimenter flipped to the next slide, she described how the two dancers left and each returned with a new friend. The third slide showed two pairs of cartoon dancers. The experimenter then asked: “If everyone leaves and comes back with a new friend, which dance floor do you think will be the dance floor at our party?” When the experimenter selected the correct box, a picture of a salsa dancing pair flashed on the screen, along with a celebratory noise. In the training trials, the values doubled across the priming sequence (7-14-28), yielding a correct test choice answer that was identical to the No Order value (56). The choices presented after the sequence were identical to that of the No Order training trials: correct choice option vs. underestimated choice (56 vs 20); correct choice option vs. overestimated choice on (56 vs 157); underestimated choice option vs. correct choice option (20 vs 56); and overestimated choice option vs. correct choice option (157 vs 56).

Experimental Trials.: The experimental trials for the Order block were identical to those in the No Order block, with respect to the type of trials presented (Overestimate vs. Correct, Underestimate vs. Correct, Overestimate vs. Underestimate, and Tie), and the counterbalancing of the side-of-screen for the correct answer. However, the Order block had an ordering sequence before the test choices were presented of either 4-8-16 (50% of the trials), or 12-24-48 (50% of the trials). There were also more Order trials overall, with 6 Tie trials (2 Correct vs. Correct, 2 Overestimate vs. Overestimate, 2 Underestimate vs.

Underestimate), and 4 of each other trial type (Overestimate vs. Correct, Underestimate vs. Correct, Overestimate vs. Underestimate). See Figure 1 for a schematic of the experimental trials presented to the participants.

Results

Description of Analyses and Measures

We performed different sets of analyses to test for the estimation biases and spatial displacements that are characteristic of operational momentum. In the first Estimation Bias analysis (i.e., under conditions of certainty), we analyze data from the trials in which a correct answer is paired with an incorrect answer at test (the Correct vs. Overestimate, and Correct vs. Underestimate trials). The measure for this analysis is how frequently the correct answer was chosen when paired with foils that are larger or smaller than the correct answer, and how this frequency changes as a function of Ordering block. In the second Estimation Bias analysis (i.e., under conditions of uncertainty), we analyze data from the trials in which there is high uncertainty; no correct answer is present, and an overestimate is paired with an underestimate at test. The measure for this analysis is how frequently the overestimate was chosen, and whether this changes as a function of Ordering block. In the Spatial Biases analysis, we analyze data from Tie Trials. Here, the measurement drawn was the frequency of choosing the right-side array, and the question of interest was whether this choice of right-side array differed as a function of Ordering. For all analyses, each participant's score was an average of all trials of that particular type (i.e., a participant who chose the overestimate 2 out of the 4 possible trials in which an overestimated outcome was presented alongside the correct outcome would get a score of .50).

Estimation Bias Under Conditions of Certainty

This analysis looks for evidence of estimation bias in trials in which there is an objectively correct answer (the Correct vs. Overestimate trials, and the Correct vs. Underestimate trials). Thus, the measure for this analysis is how frequently the correct answer was chosen when paired with foils that are larger or smaller than the correct answer. In these trials, if participants are experiencing operational momentum, they should be less accurate for the Overestimate vs. Correct trials in the Order block than the No Order block. For example, a participant who generates an outcome for the 4-8-16 trials of 40 (an overestimate of the correct answer, 32) would state that 52 and 32 are equally good answers. This would not be the case for the No Order block; their generated answer would be centered around 32, with the foil of 52 readily rejected. Conversely, for the Correct vs. Underestimate trials, if participants are experiencing operational momentum, they should be more accurate for the Order block relative to the No Order block. A participant who generates an outcome of 40 for a 4-8-16 sequence, and sees the correct answer of 32 paired with 20, should more easily reject 20 as an answer. Together, this pattern predicts an interaction of the type of trial (correct answer being compared to an under- or an over-estimate) and type of block (whether the participant performed an ordering operation, or not). To test this prediction, we calculated the percentage of time the participants chose the correct answer, and entered it into a repeated-measures ANOVA with block (Order, No Order) and trial type (Overestimate

vs. Correct, Underestimate vs. Correct) as within-subject factors, and gender (male, female) as a between-subjects factor.

For preschoolers, there was no main effect of block, $F(1,48) = .28, p = .60$. Children chose the correct answer 51.3% ($SEM = 2.1$) of the time in the Order block and 49.2% of the time ($SEM = 3.2$) in the No Order block. There was no main effect of trial type, $F(1,48) = .96, p = .33$, or gender, $F(1,48) = .05, p = .82$. The predicted Block x Trial Type interaction was not present, $F(1,48) = .17, p = .68$. There was a significant interaction of gender and trial type, $F(1,48) = 4.54, p = .038$, partial $\eta^2 = .09$. Pairwise comparisons, Bonferroni-corrected for multiple comparisons, indicate that girls were less accurate than boys in the Correct. Vs. Underestimate trials (39.1% ($SEM = 4.7$) vs. 54.6% ($SEM = 6.0$); $p = .046$). Girls also performed more poorly on the Correct vs. Underestimate trial (39.1%, $SEM = 4.7$) than the Correct vs. Overestimate trials (60.5%, $SEM = 5.0$; $p = .015$).

For adults, there was a main effect of block, $F(1, 47) = 43.14, p < .001$, partial $\eta^2 = .48$, as participants were less accurate in the Order block (70.3%, $SEM = 2.0$) than the No Order block (87.8%, $SEM = 2.1$). There was also a main effect of trial type, $F(1,47) = 23.27, p < .001$, partial $\eta^2 = .33$. Adults were overall more accurate in the Overestimate vs. Correct trials (87.5%, $SEM = 1.9$) than the Underestimate vs. Correct trials (70.6%, $SEM = 2.7$). We did not find the predicted Block x Trial interaction, $F(1,47) = .15, p = .70$, that would indicate shifting over- or under-estimation as a function of an ordering operation. There was no significant effect of gender, $F(1,47) = 1.14, p = .29$.

Estimation Bias Under Conditions of Uncertainty

Previous work on operational momentum for addition and subtraction indicates that this bias is heightened under conditions of estimation uncertainty (Charras, Brod, & Lupiáñez, 2012; Knops et al., 2009b; McCrink et al., 2007). The evaluation of whether there is an estimation bias under conditions of uncertainty applies most clearly to the trials in which the overestimated amount is contrasted with the underestimated amount; here, there is no correct answer presented. If participants are experiencing operational momentum after an ordering operation, they should be more likely to choose the overestimated amount in the Order block than the No Order block. To test this, we calculated how frequently the overestimate was chosen when an over- and under-estimate were presented as test choices. The percentage of time the participant chose the overestimate was calculated, and placed into a repeated-measures ANOVA with block type (Order, No Order) as a within-subjects factor, and gender (male, female) as a between-subjects factor.

For preschoolers, there was no main effect of block, $F(1, 48) = .01, p = .98$; children were equally likely to choose the overestimate in the Order block (47.9%, $SEM = 4.9$) and No Order block (48.1%, $SEM = 5.8$). There was no main effect of, or interactions with, gender (see Figure 2). For adults, there was a main effect of block, $F(1, 47) = 8.03, p = .007$, partial $\eta^2 = .06$; adults were more likely to choose the overestimate in the Order block (44.9%, $SEM = 4.1$) than the No Order block (26.4%, $SEM = 5.1$) (see Figure 2). There was no main effect of, or interactions with, gender.

Spatial Biases

The evaluation of whether there are spatial biases prompted by the ordering operation pertains primarily to the Tie Trials, in which there is no magnitude information that could influence the participant's choice (since both sides of the screen display the same value). If participants have a general bias to attend to the right side of space after computing ascension values, they should be more likely to indicate that right-side answers are more acceptable than left-side answers for the Ordering block (relative to the No Order block). To test this prediction, participants' percentage choice of right-side answers was placed into a repeated-measures ANOVA with block type (Order, No Order) as a within-subjects factor, and gender (male, female) as a between-subjects factor. To reiterate, the measurement here is the frequency of choosing the right-side array for each type of trial.

For preschoolers, there was a significant main effect of Block type, $F(1, 48) = 4.32$, $p = .04$, partial $\eta^2 = .08$. For these Tie trials, children chose the right-side answer more in the Order block (64.4%, $SEM = 4.4$) than the No Order block (49.3%, $SEM = 5.8$). For adults, there was a significant main effect of Block type, $F(1, 47) = 4.09$, $p = .048$, partial $\eta^2 = .08$. For these Tie trials, right-answers were chosen more frequently in the Order block (56.1%, $SEM = 3.6$) than the No Order block (45.2%, $SEM = 4.7$) (see Figure 3).

Exploratory Analysis: Examining High-Performing Preschoolers

The data from the preschoolers indicate that they did not do very well overall in the task. The question arises if children would look more like adults if they were more-competent at the task overall. Here, we grouped children into low-performers ($< 50\%$; $N = 12$), mid-performers (50% ; $N = 23$), and high-performers ($> 50\%$; $N = 15$). Because there is a general bias to underestimate large sets of objects (Izard et al., 2008), it is difficult to define chance performance. A large population of children were at 50% exactly, and this striation allows us to have roughly equal N s in each of the groups. In this way, we can pull out the children with varying levels of performance when a correct answer *was* present, and analyze them to see if they vary with respect to their operational momentum biases for those trials in which a correct answer was *not* present. A series of paired-sample t-tests were performed for these populations, and Bonferroni-corrected for multiple comparisons. Low-performers showed no right-side bias for Order block over No Order block in the Over vs. Underestimate trials (64.6%, $SEM = 6.6$ vs. 58.3%, $SEM = 8.4$; $p = .55$) or Tie Trials (62.5% in both blocks, $SEMs = 9.0, 11.2$; $p = 1.0$). Mid-performers also showed no right-side bias for Order block over No Order block in the Over- vs. Underestimate trials (60.9%, $SEM = 4.8$ vs. 58.7%, $SEM = 6.1$; $p = .77$) or Tie Trials (58.7%, $SEM = 5.9$ vs. 56.5%, $SEM = 8.1$; $p = .84$). High-performers did not show a significant right-side preference for Order block over No Order block in the Over- vs. Underestimate trials (56.7%, $SEM = 5.9$ vs. 46.7%, $SEM = 7.5$; $p = .29$), but this effect does emerge in the Tie trials (68.8%, $SEM = 8.0$ vs. 33.3%, $SEM = 10.0$; $p = .009$).

Exploratory Analysis: Detecting Age Effects

One additional question of interest was whether the strength of the effects found differed as a function of whether the participant was an adult or a child. To address this question in the context of overestimation effects, we looked further at participants' tendency to choose an

Overestimate when comparing an Overestimate vs. Underestimate, in the Order and No Order blocks. These scores were entered into a repeated-measures ANOVA with block (Order, No Order) as a within-subjects factor and age (child, adult) as a between-subjects factor. There was a significant effect of age, $F(1, 97) = 4.49$, $p = .04$, partial $\eta^2 = .04$, with children generally more likely to choose the overestimate more than the adults. There was an interaction between Block and Age, $F(1, 97) = 4.03$, $p = .047$, partial $\eta^2 = .04$, with adults, but not children, choosing the overestimate more in the Order block relative to the No Order block. To address the question of age differences in the context of spatial bias effects, we examined right-side choices for the Tie Trials, in the Order and No Order blocks. These scores were entered into a repeated-measures ANOVA with block (Order, No Order) as a within-subjects factor and age (child, adult) as a between-subjects factor. There was a main effect of Block, $F(1, 97) = 6.32$, $p = .01$, partial $\eta^2 = .06$, with right-side choices being made more frequently during the Order block than the No Order block. There was no significant interaction with age, $F(1, 97) = .01$, $p = .96$.

Discussion

This study investigated operational momentum in preschoolers and adults during ordering. To date, the evidence for this spatial-numerical phenomenon in childhood is mixed, and has been examined exclusively for addition and subtraction operations (Knops et al., 2013). Results showed that adults, but not preschoolers, exhibited a greater tendency to overestimate the next magnitude in a series of magnitudes when performing an ascending operation, compared to simply estimating a magnitude. Importantly, this was the case only in the face of uncertainty (during Overestimate vs. Underestimate trials). Both adults and preschoolers selected a right-side answer more frequently when performing an ascending operation, compared to a simple estimation task. Thus, we found some evidence for developmental continuity in the presence of lateralized spatial associations during the operation of ordering. However, in spite of the fact that both adults and children exhibit the spatial biases characteristic of operational momentum, only adults exhibit the predicted overestimation bias (and even then, only under conditions of estimation uncertainty).

In some respects, the lack of overestimation after ordering in the preschoolers is surprising. Not only is this finding distinct from the direct adult comparison within the study, but it also differs from findings on operational momentum during infancy. Macchi Cassia et al. (2017) tested 4-month-old infants in a version of an ordering task similar to that used here. Recall that in this study infants were habituated to ascending or descending series of magnitudes, and then shown test sequences of ascending or descending magnitudes that were smaller or larger than the habituated sequences. Infants who habituated to ascending series looked longer, on average, to a test ascending series that was less-numerous to what they had learned (and the opposite was found for infants habituated to descending sequences). This pattern is thought to reflect a novelty response, prompted by the generation and acclimation of overestimates during the ordering task. Thus, while there is evidence that infants generate misestimates when ordering magnitudes, the present study reveals that preschoolers do not.

In other respects, however, the current finding of a lack of overestimation after ordering in the preschoolers accords with other work on OM in childhood. Knops et al. (2013) tested

adults and young elementary school children for operational momentum during addition and subtraction. As in this study, these children did not show misestimation effects as a group (e.g., overestimating addition problems, and underestimating subtraction problems), but the adults did. In a separate visual spatial attention task that was part of the same study, children were tested on their ability to overcome an invalid cue when shifting attention (Knops et al., 2013). Tellingly, the degree to which the children exhibited adult-like spatial attention patterns was correlated with their degree of exhibited operational momentum (Knops et al., 2013).¹ This led Knops et al. (2013) to suggest that it is the maturation of the spatial attention networks that leads to shifting of spatial attention along the number line when operating.

Given that current theories posit that it is the spatial shifts of attention along an internally represented horizontal continuum that prompts mis-calculation of arithmetic outcomes (c.f. McCrink & Hubbard, 2017), how can we reconcile the presence of one without the other? One line of interpretation stems from the methodology of the present study. It is possible that there were indeed miscalculations arising from spatial shifts in the preschoolers, similar to the infants, but the study's design dampened them for the preschoolers. For instance, in the infant study (Macchi Cassia et al., 2017), an infant-controlled habituation procedure was employed, which gave each single participant ample time to encode and form a robust representation of the ordering operation. Moreover, during habituation infants were given multiple examples of the same ordering rule. In the current study, instead, children saw just one example sequence before being probed for their test choice, and presentation times were fixed and rather short. Thus, it is reasonable to assume that the representation of the ordering rule formed by the preschoolers was not as strong as that formed by the infants; as a result, the spatial shifts of attention might have been present but too weak to actually influence their calculations. Additionally, the infants in Macchi Cassia et al. (2017) received ordered *sequences* (e.g., three ordered slides) at test, whereas the preschoolers in the current study received two quantities from which to choose an answer. This difference in the two testing paradigms may have resulted in a more-apparent match between the priming/habituation ordering operation and the test operation for the infants compared to the preschoolers, which in turn increased operational momentum for the infants.

Another possibility is that there were genuinely no miscalculation biases in children, because they did not fully grasp the task. Indeed, overall children performed poorly at the task. It follows logically that if one cannot grasp the task, then one will not generate a correct outcome to begin with, and further, will not experience an added overestimation error from operational momentum. In order to intuit the ordinal relationship between the amounts, the children had to very quickly learn one type of relation for one block of trials (i.e. either estimate the same amount for the No Order block, or double the last value in the sequence for the Order block), and then abandon that relation for the second block of trials. However,

¹To investigate if that is the case here, we calculated a difference score for how often participants chose the right-side answer in the Order Block vs. No Order block for the Tie trials. This gave us a measure of spatial displacement generated by the operation for each participant. We also calculated a difference score for how often participants chose the overestimated amount in the Order block vs. No Order block, for the over- vs. underestimate comparison. For both adults and children, there was no relation between the strength of spatial displacement tendency and over-estimation tendency. However, we must be cautious in drawing conclusions from this analysis as it is exploratory, and the experimental design was not crafted with it in mind (e.g., uneven numbers of trials in the Order and No Order blocks).

this performance account seems unlikely if one examines data from the higher-performing preschoolers in the study. This population was able to reliably choose the correct answer when it was present, and showed spatial shifts in their responses in accord with operational momentum. Yet, even the higher performing preschoolers did not show a stronger tendency to overestimate after an ordering task compared to an estimation task.

The second line of interpretation for why there are stronger spatial biases than overestimation effects is theoretical. Perhaps, for young children, the emergence of OM reflects a protracted developmental entwining of visuospatial attention and mental calculations. As the findings from Knops et al. (2013) suggest, the spatial and mis-estimation effects of OM may emerge only in the context of a fully developed attentional system and a strong interface between visuospatial attention and the MNL. Pinheiro-Chagas, Didino, Haase, Wood, & Knops (2018) recently found no evidence of OM among 8-year-olds, but an emergent OM effect during addition and subtraction among children 9 through 12. Perhaps, as visuospatial attention systems become more adult like, these shifts of attention begin to interface with the number line, and this leads to OM emerging in older children. Given that addition/subtraction and ordering are both arithmetic operations, it is possible that the ordering operations investigated in the present study also follow a protracted developmental trajectory.

How, then, do we reconcile this account – a protracted interface of spatial attention and the mental number line, with spatial shifts emerging before estimation biases - with the findings on OM in infancy? For both ordering OM and addition/subtraction OM, there is evidence that infants are miscalculating outcomes in accord with the “direction” of the arithmetic operation (Macchi Cassia et al., 2017; McCrink & Wynn, 2009). Yet, this bias isn’t apparent in childhood (Knops et al., 2013; Pinheiro-Chagas et al. 2018; current study). One possibility is that the underlying construct of associations between magnitude and space in an infant’s mind is qualitatively different than that in an adult’s mind, and early childhood is when this shift occurs (see McCrink & de Hevia, 2018 for a detailed version of this account). There has been a recent spate of data strongly suggesting that even newborns and non-human animals possess some sort of mental number line, with small quantities associated with the left and large to the right (de Hevia et al., 2017; Rugani et al., 2015; Bulf et al., 2016). At the same time, there are findings from preschool age and up that children’s associations between magnitudes and lateral space are sensitive to the spatial customs of their environment (Opfer & Furlong, 2010; Goebel et al., 2018; Shaki et al., 2012). Toddlerhood and very early childhood, then, must logically be a transitional period in which associations between magnitude and space move from an innate construct to one that is culturally sensitive and mature. Part of this maturity involves a rebuilding of the spatial associations to reflect the spatial biases of the child’s culture, and then the integration of this mature representation with a slowly developing visuospatial attention system. Put simply, we may observe differing findings on OM in infancy and childhood because the underlying basis of OM – associations between magnitude and space – differs meaningfully between infants and children.

Beyond the issue of differences in spatial and calculation OM, one striking aspect to the current results is the degree of underestimation found throughout the study, and for both the

adults and children. Although we observe that adults are more likely to choose an overestimated outcome after ordering (compared to estimating), under conditions of certainty (i.e., when the correct amount was available), both populations generally avoided choosing the larger test amount throughout. Both children and adults were less accurate for the Underestimated vs. Correct trials than the Overestimated vs. Correct trials. That is, participants were systematically more likely to select underestimated outcomes relative to correct responses, compared to overestimated outcomes relative to correct responses. This pattern is not without precedent in the literature. Izard and Dehaene (2008) found that when observers were asked to estimate the approximate numerosity of dot arrays, in the absence of calibration, they most often underestimated. Classic studies on numerosity estimation also found that participants are poor at spontaneously approximating non-symbolic numerosities (Krueger, 1982, 1984; Minturn & Reese, 1951). For example, Minturn and Reese (1951) (as cited in Izard and Dehaene, 2008) reported responses diverging from the true numerosity by a factor as high as four (e.g. responses range from 50–700 for a stimulus containing 200 dots). Importantly, these studies involved adults viewing dot arrays, and then generating a number word without counting, while the present study did not involve number words at all. With regards to the current study, there was clearly a tendency to underestimate for all participants, which may or not be related to the same phenomenon in the symbolic to non-symbolic mapping literature. This tendency dampened the overall results within the Order block, and required us to focus on comparisons between the Order block and the No Order block.

Another notable aspect to the current findings is that, insofar as we observe the phenomenon of operational momentum here, we see it only under conditions of uncertainty. When the same magnitude was presented at test on either side of the screen, or both an over- and under-estimate were presented together, miscalculation and spatial biases characteristic of operational momentum emerged. This result supports the idea that operational momentum is at least partially a type of spatial heuristic: when adding, ordering, or multiplying, we generate a logical rule of “accept more than the initial amount”, and further, this concept of ‘more’ is associated with ‘right’. For adults, at least, this rule is not the only concept in play, as it has been observed that they generate arithmetic addition outcomes that skew just slightly more than the correct answer, and do successfully reject outcomes that are much larger than the correct answer (Knops et al., 2009b; McCrink et al., 2007). Further, Fischer and colleagues have delineated additional types of heuristics that can lead to OM, such as anchoring on a particular starting value for an equation, or even just seeing the symbolic operation sign + or – (Pinhas & Fischer, 2008; Pinhas, Shaki, & Fischer, 2014). For adults, operational momentum is heightened under cognitive load, as is also the case for other, non-arithmetic logical heuristics (Kahneman, 2013). For example, operational momentum effects are heightened when participants are operating over larger, more-imprecise values (Knops et al., 2009b), are under time constraints and cannot accurately count (McCrink et al., 2007), or when their attention is drawn to a distractor task (McCrink & Hubbard, 2017). The implementation in the current study of the Tie trials, and Overestimated vs. Underestimated trials, elicited operational momentum in a way that conditions of more certainty did not. This provides additional evidence for a spatial heuristics account of operational momentum,

and establishes that these logical-spatial heuristics are present as early as the preschool years.

Going forward, future studies should elaborate on the present design and examine operational momentum in a descending ordering task as a comparison point for the ascending ordering operation offered here. Given the constraints of such a young population (with children as young as 37 months of age), the adoption of a between-subjects design would have proved necessary to include a descending operation, as they lose interest in the task rather quickly. However, the infants tested in Macchi Cassia et al. (2017) showed an underestimation tendency after habituation to descending numerical sequences which was not dissimilar from the overestimation tendency they showed after habituation to ascending sequences. Therefore, testing for the generalizability of the obtained findings to descending ordering operations in young children would be worth further investigation. Second, researchers should explore how calibration may impact operational momentum. For instance, Izard and Dehaene (2008) found that presentation of a concrete reference number prior to estimation enabled participants to approximate numerosities more precisely. While the present study did not involve number words and only involved non-symbolic quantities, future studies could extend this work by examining how calibration may impact operational momentum during ascending and descending ordering. Additionally, the current study used a sample of children from a largely English-speaking country. Future work could examine populations in other countries, such as Israel or Iran, whose languages moderate the direction of the mental number line early in childhood (Shaki, Fischer, & Gobel, 2012).

It is important to note that, in this experiment, we did not control for spatial extent cues that co-vary with numerical magnitude. As the number of items in each array increased, so did the area, perimeter, density, and convex hull of the set. Thus, the conclusions that we can draw from this design are limited to that of relations between a lateralized spatial continuum and quantity. The quantity representations here – what we and many others casually call “number” in the context of spatial-numerical relations – are computed over bounded, discrete objects, but they may ultimately be underlain by either spatial processing *or* abstract numerical processing. That is, the ordering operation prompting this mental ‘momentum’ may be performed by participants over spatial magnitudes (e.g., Macchi Cassia et al., 2016) or abstract numerical magnitudes (e.g., McCrink et al., 2007). Given that Macchi Cassia et al. (2017) found that, with infants, OM for ordering arose only when both spatial and numerical characteristics were ordered, we believed that the first investigation of ordering OM with children and adults should preserve both these characteristics of magnitude representations. Future experiments on this topic should address the question of whether ordering OM in these populations is truly a spatial-numerical interaction, or rather a spatial-magnitude interaction (via careful stimulus controls, or cross-modal methods which circumvent visual stimuli confounds).

In summary, the results of the present study suggest that both adults and preschool children tap into magnitude-space associations to support their ordering calculations. The fact that this occurs with preschoolers, who have yet to enter formal schooling, implies that this mechanism arises without instruction, and is part of an intuitive logical system which capitalizes on early-developing links between spatial structure and quantity representations.

Using a spatial scaffold to represent number is an important part of enriching mathematical cognition (Gunderson, Ramirez, Beilock, & Levine, 2012), and has implications for the etiology of dyscalculia and typically developing children's developmental trajectories in formal math (Huber, Sury, Moeller, Rubinsten, Nuerk, 2015). The current child-friendly paradigm offers a promising avenue to further explore the development of the phenomenon of operational momentum, and the findings presented here lead to novel predictions related to the relationship of space, magnitude, and visuospatial attention throughout development.

Acknowledgments

We wish to thank the Children's Museum of Manhattan and the Brooklyn Children's Museum for their continued support of our research. This work was funded by a grant to XX from the Eunice Kennedy Shriver National Institute of Child Health and Human Development (1R15 HD077518-01A1).

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Highlights:

- Preschool children (ages 3 and 4) and adults shift attention to the right after performing an ascending ordering operation.
- Adults overestimate quantities after viewing an ascending ordered sequence.
- The data highlight a phenomenon of spatial-numerical cognition known as operational momentum.
- The findings provide evidence for a spatial account of operational momentum, and suggest that adults and children (even prior to formal schooling) operate along a horizontal continuum when ordering quantities.

Figure 1.

A schematic illustrating the priming slides and test comparisons for the 4-8-16 Order Block trials. Note that for the Tie Trials, the Order block had three different versions, only one of which (Correct vs. Correct) is depicted here. The other Tie Trial versions were Overestimate vs. Overestimate, and Underestimate vs. Underestimate.

Figure 2.

Percentage of overestimate choice by preschoolers and adults, as a function of block (Order, or No Order), for those trials in which an overestimate was contrasted with an underestimate (i.e. estimation uncertainty). Within-subjects error bars indicate ± 1 SEM (after Cosineau (2005)).

Figure 3.

Percentage of right-side choices by preschoolers and adults, as a function of block (Order, or No Order), for trials in which the values presented on the left and right side of the screen were identical (i.e. Tie Trials). Within-subjects error bars indicate ± 1 *SEM* (after Cosineau (2005)).

Table I.

Experimental trials with priming 4, 8, 16

Trial Type	Left Side Probe	Right Side Probe
Correct vs. Underestimate	20	32 *
	32 *	20
Correct vs. Overestimate	52 *	32
	32	52 *
Underestimate vs. Overestimate	20	52 *
	52 *	20
Underestimate vs. Underestimate (Tie)	20	20 *
Correct vs. Correct (Tie)	32	32 *
Overestimate vs. Overestimate (Tie)	52	52 *

Note.

* indicates test option that accords with the direction of operational momentum (either spatially or arithmetically) for an ascending operation.

Table II.

Experimental trials for Order Block with priming 12, 24, 48

Trial Type	Left Side Probe	Right Side Probe
Correct vs. Underestimate	60	96 [*]
	96 [*]	60
Correct vs. Overestimate	154 [*]	96
	96	154 [*]
Underestimate vs. Overestimate	60	154 [*]
	154 [*]	60
Underestimate vs. Underestimate (Tie)	60	60 [*]
Correct vs. Correct (Tie)	96	96 [*]
Overestimate vs. Overestimate (Tie)	154	154 [*]

Note.

^{*} indicates test option that accords with the direction of operational momentum for an ascending operation.

Table III.

Experimental trials for No Order block with priming 32, 32, 32

Trial Type	Left Side Probe	Right Side Probe
Correct vs. Underestimate	20	32
Correct vs. Overestimate	32	52
Underestimate vs. Overestimate	52	20
Correct vs. Correct (Tie)	32	32

Table IV.

Experimental trials for No Order block with priming 96, 96, 96

Trial Type	Left Side Probe	Right Side Probe
Correct vs. Underestimate	96	60
Correct vs. Overestimate	154	96
Underestimate vs. Overestimate	60	154
Correct vs. Correct (Tie)	96	96