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Electromagnetic Imaging of Dielectric Objects Using a Multi-Directional Search-Based Simulated Annealing

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Abstract

In this paper, we introduce a global optimization method that is a novel combination of the simulated annealing method and the multi-directional search algorithm. We demonstrate the use of the algorithm for a microwave-imaging system to obtain the electrical properties of objects. The proposed global optimizer significantly improves the performance and speed of the simulated annealing method by utilizing a nonlinear simplex search, starting from an initial guess, and taking effective steps in obtaining the global solution of the minimization problem. Due to the efficient performance of the proposed global optimization method, we are able to obtain the shape, location, and material properties of the target without considering any a priori information about them. The accuracy and applicability of the proposed imaging method is demonstrated with some numerical results in which two-dimensional images of multiple objects are successfully reconstructed.

I. INTRODUCTION

Due to the non-contact penetration property and non-ionizing nature of electromagnetic waves, microwave imaging is currently of much interest, particularly in biomedical applications. The ultimate goal in microwave imaging is to identify the shape and location of the target object in addition to its electrical properties, such as conductivity and permittivity, in an investigation domain. The general framework in microwave imaging problem is to solve an inverse-scattering problem in which we reconstruct the desired properties of the investigation domain by illuminating the target by electromagnetic waves and collecting the scattered fields. Figure 1 demonstrates general configuration of this framework for two-dimensional (2-D) inverse-scattering problem. In this approach, we usually begin with an initial guess of the target's characteristics and minimize the mismatch between the measured scattered fields of the object and those of the initial guess. As a consequence of this approach, one of the major mathematical challenges in microwave imaging is to utilize an appropriate optimization algorithm to solve the inverse-scattering problem. The underlying inverse-scattering problem in the microwave imaging procedure, which is usually formulated using a scattered field volume integral equation, is highly non-linear.

Based on the optimization techniques utilized in the inverse- scattering problems, microwave imaging methods could be classified as two major groups, namely, local and global optimization methods. Due to the substantially higher speed of convergence in local optimization algorithms [1], they have been more popular in solving the inverse problems. Born iterative method, for example, is widely used as an effective imaging method to obtain

the conductivity and permittivity distribution of 2-D [2] and 3-D investigation domains [3], as is the distorted Born iterative method [4], [5]. Other related works include utilizing the steepest descent method local minimizer to solve the inverse scattering problems [6] and perfect electric conductors [7]. Due to the nonlinearity of the inverse problem, the main disadvantage of microwave imaging systems based on local optimization algorithms is the convergence to one of the multiple local minima of the cost function, which may not necessarily be the desired solution (global solution). In the aforementioned works, this undesirable convergence may be avoided by considering *a priori* information about the imaging problem and starting the algorithm from initial points ‘sufficiently’ close to the original solution of the inverse-scattering problem.

To overcome this drawback in local optimization algorithms, one solution is to perform a global search over the entire unknown domain of the inverse-scattering problem. Depending on the object properties and the formation of microwave imaging system, appropriate global multi-agent optimization methods, such as the genetic algorithm (GA) [8], particle swarm optimization PSO [9], and differential evolution (DE) [10], single-agent techniques such as simulated annealing method [11], and hybrid algorithms such as GA/CG method [12], memetic algorithm [13], [14], and hybrid DE [15] have been applied to solve the inverse-scattering problem. While these meta-heuristics have acceptable performance for small numbers of unknowns, they do not usually scale well with complexity. As a result, simplifications are often made to reduce the computational complexity of reaching the solution. Moreover, iterative multi-scaling techniques [16] as well as sparsity-based methods such as compressive sensing approaches [17] have been applied to overcome the issue of having large number of unknowns in meta-heuristics.

In this work, we present a novel global search algorithm that takes advantage of the speed of local optimization while ensuring convergence to a global minimum. More specifically, this is done by modifying the perturbation scheme in the simulated annealing method using a non-linear simplex search algorithm. This modification will enhance the performance of global search by choosing judicious perturbation directions. In Section II, we describe the model utilized in our microwave imaging system for solving the corresponding inverse-scattering problem. We then present the proposed global optimization algorithm for minimizing the cost function and provide a formal statement of the algorithm. The complexity of the method is studied by several numerical experiments. Numerical results in Section III show the successful performance of the proposed method in reconstructing dielectric regions. In addition, results of noise analysis follow in Section III.

II. INVERSE-SCATTERING PROBLEM

The inverse-scattering problem entails illuminating the investigation domain with electromagnetic waves from different directions, measuring the scattered fields (measured data), comparing the measured data with the predicted scattered field obtained from the latest “iteration” (the repetition of the optimization procedure) of the unknown (simulated data), and finally, minimizing the mismatch between the measured and simulated data by properly updating the unknown distribution. Mathematically, we describe the proposed

microwave imaging system as a nonlinear optimization problem in which we minimize a cost function defined as

$$C(X) = \sum_{i=1}^M \|e_i(X)\|^2 = \sum_{i=1}^M \|S_i(X) - m_i\|^2, \quad (1)$$

where x denotes the vector of unknowns that model the distribution of the electrical properties of the investigation domain. The residual vector $e(x) = S(x) - m$ represents the mismatch between the simulated (or predicted) measured data $S(x)$ in the reconstructed domain and the measured data m in the original domain. The index i denotes the measurement index and M is the number of measurements. In the next section, we present the proposed global search algorithm to find the optimum solution of the cost function defined in (1).

A. Multi-Directional Search Based Simulated Annealing

The conventional form of the simulated annealing method for optimization of functions with continuous variables, presented in [18], is based on a heuristic optimization scheme with adaptive moves along coordinate directions in the domain of unknowns. The procedure starts with an initial solution (initial guess) of the unknown x , sets a parameter called temperature T_0 , and follows by an iteratively decreasing series of temperature and random perturbations of the initial solution to reach the global optimum of the problem. According to the Corana algorithm [18], these random perturbations are obtained by sequentially moving the current solution point along the coordinate directions in the domain of variables. In other words, each perturbation is performed on one of the variables to obtain a new candidate solution x' . The candidate solution x' will be accepted and replaced by x based on the Metropolis Criterion [19], that:

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if  $C(x) < C(x')$  then
    Accept the new candidate solution  $x'$ .
else
    Accept the new candidate solution  $x'$  by the probability
     $e^{-\Delta C(x)/T_k}$ .
end if

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where T_k is the temperature parameter at the current iteration k and $C(x)$ denotes $C(x')$ — $C(x)$. At each temperature, the process of perturbing currently accepted solution and replacing the perturbed solution will be repeated until the function values of the sequence solution points reach a stable value. At ‘high’ temperatures most of the solutions would be accepted as the criterion is usually satisfied, while at ‘low’ temperatures solutions that reduce the cost function would be accepted as the probability of accepting worse solutions significantly decreases. It has been shown that by reducing the temperature sufficiently slowly and keeping the number of perturbations at each temperature sufficiently high, the

simulated annealing algorithm always converges to a global minimum of the cost function [20]. While the simulated annealing method in [18] has good performance in the optimization problems with small number of variables, such as the inversion scheme presented in [21], increasing the number of unknowns extends the duration of algorithm's convergence to a global solution significantly. Furthermore, as a result of the sequential perturbation scheme proposed in [18], convergence of the method to a satisfactory solution may not be achievable in a reasonable amount of time. Therefore, employing an efficient scheme to move the current solution along the directions in which we expect significant reduction in the cost function will enhance the performance of the algorithm in optimizing functions with large number of continuous variables, such as the cost function in an inverse-scattering problem.

For this purpose, we combine the non-linear simplex search scheme proposed in [22] with the simulated annealing method to accelerate the convergence and improve the performance of this global search method. We start with an initial guess of the unknown variables and produce a non-degenerate simplex -- in which no lower dimensional hyperspace can be found that contains all vertices of the simplex -- by randomly perturbing the current solution along coordinate directions of the unknowns. Similar to the simulated annealing scheme in [18], we will generate new perturbed points based on the currently accepted solution, but instead of sequentially perturbing the accepted solution along the coordinates, we obtain the perturbed solutions using the vertices of the evolving simplex in the multi-directional search algorithm. At the l th iteration of the multi-directional search algorithm, for a given simplex with vertices $\{v_0^l, v_1^l, \dots, v_n^l\}$, we update the solution points, which are the vertices of this simplex, by reflecting, expanding, and contracting the simplex based on the following scheme:

Perform reflection step:

$$v_i^{l+1} = v_0^l + r \cdot (v_0^l - v_i^l) \quad i = 1, \dots, n$$

- Calculate $f(v_i^{l+1})$.

if $\min\{f(v_i^{l+1}), i = 1, \dots, n\} < f(v_0^l)$ **then**

Perform expansion step:

$$v_{e_i}^l = v_0^l + 2r \cdot (v_i^l - v_0^l) \quad i = 1, \dots, n$$

- Calculate $f(v_{e_i}^l)$.

if $\min_i \{f(v_{e_i}^l)\} < \min_i \{f(v_i^{l+1})\}$ **then**

Replace $v_{e_i}^l$ with v_i^{l+1} for $i = 1, \dots, n$.

end if

else

Perform contraction step:

$$v_i^{l+1} = v_0^l + r/2 \cdot (v_0^l - v_i^l) \quad i = 1, \dots, n$$

- Calculate $f(v_i^{l+1})$.

end if

However, in the proposed method, instead of applying deterministic movements of the simplex vertices in the given directions of the reflection, expansion, and contraction steps, we generate random perturbations in these directions to develop our heuristic scheme ensuring that the process does not stop at a local minimum. As illustrated in Figure 2, the main difference between simulated annealing and the proposed optimization algorithm is the perturbation scheme, in which we obtain more effective updates to the solution, manifested in an accelerated rate of cost function reduction.

A detailed description of the algorithm is described in the next section. The flow charts in Figure 2 illustrate the similarities and differences between the classical simulated annealing algorithm and the proposed hybrid method.

B. Inversion Algorithm

In this section, we present a formal statement of the algorithm to minimize the cost function $f(x)$, which is a function of n variables.

Initialization Step:

- Set the initial temperature parameter T_0 and the rate of temperature reduction r_T .
- Set the parameters related to internal cycles of algorithm N_r , N_s , and N_{mut} .
- Choose an initial-guess point x_0 and initialize the size of step lengths s_i for $i = 1, 2, \dots, n$.
- Evaluate the cost function at x_0 , $f_0 = f(x_0)$, and initialize the accepted and optimum points:

$$x_{accept} = x_0, \quad f_{accept} = f_0$$

$$x_{opt} = x_0, \quad f_{opt} = f_0$$

Iterations:

for $m = 1, 2, \dots$ **do**

- Check the stopping criteria of the algorithm.

for $j = 1 \rightarrow N_r$ **do**

- Set $n_{sj} = 0$, $i = 1, 2, \dots, n$.

for $k = 1 \rightarrow N_s$ **do**

- Set $l=1$.

- Set $v_0^l = x_{accept}$.

- Randomly generate the vertices $\langle v_0^l, \dots, v_n^1 \rangle$ to provide a non-degenerate simplex using the following process:

$$v_i^1 = v_0 + r \cdot s_i \cdot e_i \quad i = 1, 2, \dots, n,$$

where r is a random number generated in $[0,1]$ and e_i denotes the vector with a 1 in the i th coordinate and 0's elsewhere.

for $l=1,2,\dots$ **do**

- Check the stopping criteria of the random multi-directional search.

Perform random reflection step:

for $i=1,\dots,n$ **do**

$$v_i^{l+1} = v_0^l + r \cdot (v_0^l - v_i^l),$$

where r is a random number generated in $[0,1]$.

- Calculate $f(v_i^{l+1})$.

- Accept or reject the candidate point v_i^{l+1} according to the Metropolis Criterion.

if $l=1$ **then**

if v_i^{l+1} is accepted, **then**

Add 1 to n_{si}

end if

end if

end for

if $\{f(v_i^{l+1}), i=1, \dots, n\} < f(v_0^l)$ **then**

Perform random expansion step:

for $i=1,\dots,n$ **do**

$$v_{e_i}^l = v_0^l + 2r \cdot (v_i^l - v_0^l),$$

where r is a random number generated in $[0,1]$.

- Calculate $f(v_{e_i}^l)$.

- Accept or reject the candidate point $v_{e_i}^l$ according to the Metropolis Criterion.

end for

if $\min_i \{f(v_{e_i}^l)\} < \min_i \{f(v_i^{l+1})\}$ **then**

Replace $v_{e_i}^l$ with v_i^{l+1} for $i=1,\dots,n$.

end if

else

Perform random contraction step:

for $i=1,\dots,n$ **do**

$$v_i^{l+1} = v_0^l + r/2 \cdot (v_0^l - v_i^l),$$

where r is a random number generated in $[0,1]$.

- Calculate $f(v_i^{l+1})$.

- Accept or reject the candidate point v_i^{l+1} according to the Metropolis Criterion.

end for

end if

end for

Update the step length (presented in [18]):

$$s_i = \begin{cases} s_i \left(1 + 2 \frac{\frac{ns_i}{N_s} - 0.6}{0.4} \right), & \text{if } n_{s_i} > 0.6N_s \\ s_i \frac{1}{\frac{0.4 - \frac{ns_i}{N_s}}{1 + 2 \frac{\frac{ns_i}{N_s} - 0.6}{0.4}}}, & \text{if } n_{s_i} < 0.4N_s \\ s_i, & \text{otherwise} \end{cases}$$

end for

end for

- Reduce the temperature for the next iteration, $T_{m+1} = r_T \times T_m$

- Reset the accepted point and its cost function by currently obtained optimum solution:

$$x_{accept} = x_{opt}, \quad f_{accept} = f_{opt}$$

end for

The parameter N_l denotes the number of step length adjustments at each iteration. In addition, since we initialize the multi-directional search algorithm by randomly perturbing the current solution along coordinate directions of the unknowns (sequential perturbations), we employ the random multi-directional search N_s times to investigate the individual effect of each function variable separately. This is done by tracing the first iteration of the multi-directional search, which only includes sequential perturbations of the variables. After obtaining the ratio of the accepted solutions to the total perturbations of each variable, we utilize the step-length adjustment approach similar to the one provided in [18].

We consider two stopping criteria for the random multi-directional search algorithm. As the first stopping criterion, we consider the following test proposed by Woods[23] at k th iteration

$$\frac{1}{\Delta} \max_{1 \leq i \leq n} \|v_i^k - v_0^k\| < \epsilon, \quad (2)$$

where $\Delta = \max(1, \|v_0^k\|)$, ϵ is a preset tolerance, and $\|\cdot\|$ indicates the second norm. In addition to the stopping test shown in Eq. 2, which measures the size of the simplex and how far it can move in the next iteration, we consider another stopping criterion, namely one that restricts the maximum number of iterations for this random search algorithm.

III. NUMERICAL RESULTS

To investigate the performance of the proposed method, we have applied the optimization algorithm to various test functions and compared the optimization results with the standard simulated annealing method. For this purpose, we have considered 2- and 4-dimensional Rosenbrock test functions [24], which are

$$f_2(x_1, x_2) = 100(x_2 - x_1^2)^2 - (1 - x_1)^2, f_4(x_1, x_2, x_3, x_4) = \sum_{i=1}^3 100(x_{i+1} - x_i^2)^2 - (1 - x_i)^2. \quad (3)$$

The global minimum of the n -dimensional Rosenbrock function is at $x_n = [1, \dots, 1]$, which is located inside a long, narrow, parabolic-shaped flat valley, making it difficult to be found. To provide a reasonable comparison with the standard SA, we have used the same algorithm parameters suggested in [18]. Therefore, we set the initial temperature to $T_0 = 10$ and apply the exponential cooling schedule of $T_k = (0.85)^k T_0$ for the k th iteration. In addition, N_t is chosen to be 10 and 20 for the 2- and 4-dimensional test functions, respectively, and N_s is chosen to be 20 for both cases. Moreover, we restrict the maximum number of iterations in the multi-directional search algorithm, N_{mult} , to be 100 and set the parameter $\epsilon = 0.0001$ in Eq. 2. Table I provides the optimization results of test functions using both the proposed optimizer and the the simulated annealing method. The optimization process is repeated for different starting points. As provided in Table I, the number of function evaluations is significantly lower using our proposed optimization method compared to the standard SA.

In the next numerical experiment, we investigate dependence of the proposed heuristic on two algorithm parameters N_t and N_s . For this purpose, we apply our optimization method to find the global solution of 2-dimensional Rosenbrock function for different values of N_t and N_s . Figure 3 shows the Root-mean-square error (RMSE) in the obtained solution after 10 iterations for each of the N_t and N_s values changing from 1 to 30. The initial temperature and cooling schedule are same as in the previous experiment and the number of iterations for the multi-directional search algorithm is fixed at 20. Since we are dealing with a heuristic method with non-deterministic steps, we have repeated the optimization process 10 different times, presenting here the average RMSE for each set of N_t and N_s . As evident in Fig. 3, for N_t and N_s values larger than or equal to 10, the error in the obtained solutions drops significantly and the algorithm converges to the global solution. Although these results can provide reasonable ranges for choosing N_t and N_s values and obtaining the global solution of this problem, applying the presented heuristic to other optimization problems may lead to other feasible ranges of N_t and N_s values.

we performed several numerical simulations to validate the proposed heuristic in the inverse scattering problems. In the first numerical experiment, we retrieve the permittivity distribution of the investigation domain D shown in Fig. 4 (a), which is a $\lambda \times \lambda$ region

discretized by $\lambda/7 \times \lambda/7$ pixels, where λ is the free space wavelength, and clearly, the number of pixels is equal to the number of unknown variables in all experiments. To illuminate the target and collect the scattered fields, we have 7 transmitting/receiving antennas that are symmetrically located on a circle of 4λ diameter, which provides a total number of 49 measurements. For solving the forward electromagnetic scattering problem, we have utilized a two-dimensional method of moment-based formulation [25].

Fig. 4 (a) illustrates the investigation domain D in which we have 2 different objects with permittivities of 2 and 7. We set the initial temperature to $T_0 = 10$ and apply the exponential cooling schedule of $T_k = (0.85)^k T_0$ for the k th iteration for all of the simulations. In addition, the values of N_t and N_s parameters are chosen to be 10 and 2, respectively. In addition, we restrict the maximum number of iterations in multi-directional search algorithm, N_{mul} , to be 100 and set the parameter $\epsilon = 0.01$ in Eq. 2. We have chosen these values so that the number of function evaluations at each iteration of the proposed algorithm would be comparable to the classical simulated annealing method. Moreover, based on the empirical observations, the presented values for the algorithm parameters provide the most efficient performance of the proposed method. As it is evident in Fig. 4 (b)-(c), for noise-free measurements, the permittivity distribution of the investigation domain is reconstructed nearly perfectly and the cost function converges to zero, which are the indications of reaching the global solution. To investigate the noise effect, white Gaussian noise was then added to the measured data to get 10 dB, 20 dB, and 30 dB signal-to-noise ratio in the measurements. Figures 4(d)-(i) illustrate the imaging results obtained when the white Gaussian noise contaminates the measurements. While we cannot perfectly reconstruct the permittivity distribution in the presence of noise in the measurements, the retrieved image provides the precise information about the shape and location of the objects in addition of obtaining the permittivity values close to the original distribution.

Figures 5(a)-(b) demonstrate the reconstruction results of the same experiment using the standard simulated annealing method. In addition, the values of N_t and N_s parameters are chosen to be 10 and 20, respectively. The simulated annealing method has reconstructed the investigation domain in 65 iterations and 2258 seconds, while the proposed global optimization method has reached the same solution with the same accuracy in five iterations and 552 seconds, which illustrates the effectiveness of the presented method in searching for the global solution.

In the second experiment, we present the results of reconstructing the permittivity distribution of a more complicated target. As shown in Fig. 6 (a), we have 3 different objects with the permittivity distribution range between 2 to 7. The configuration of antennas, the size of investigation domain, the values of the algorithm parameters (N_t , N_s , and N_{mul}), and the resolution of the pixels are same as the first experiment. As shown in Fig. 6 (b)-(c), for noise-free measurements, the cost function converges to zero and the permittivity distribution of the investigation domain is perfectly reconstructed. Figures 6 (d)-(i) illustrate the imaging results in the presence of noise in the measurements. Although we do not have an exact reconstruction of the permittivity distribution, it is observed that the retrieved image provides the shape and location of the target precisely in addition of the permittivity values close to the original distribution.

In the third simulation, we present the results of reconstructing the permittivity distribution of the region shown in Fig. 7 (a). The resolution of the image is increased by using $\lambda/10 \times \lambda/10$ pixels and 10 transmitting/receiving antennas have been utilized to obtain the measurements. In addition, we choose the values of N_t and N_s parameters to be 20 and 2, respectively, and restrict the maximum number of iterations in multi-directional search algorithm, N_{mul} , to be 200. As shown in Fig. 7 (b)-(c), for noise-free measurements, the cost function converges to zero and the permittivity distribution of the investigation domain is perfectly reconstructed. Figures 7 (d)-(i) demonstrate the impact of adding white Gaussian noise to the measured data.

In the last numerical experiment, we try to reconstruct the permittivity distribution of a $\lambda \times \lambda$ investigation domain shown in Fig. 8 (a) in which we have two objects with different permittivity values. The resolution of the image is increased by using $\lambda/15 \times \lambda/15$ pixels. In addition, we set the parameters of the algorithm by $N_t = 30$, $N_s = 15$, and $N_{mul} = 200$. To collect the scattered fields, we have assumed 15 transmitting/receiving antennas to keep the number of unknowns and measurements equal. Although we present the results of reconstructing the domain by using 15 transmitting/receiving antennas, we obtain quite similar and acceptable reconstruction results by reducing the number of transmitting/receiving antennas to 7.

Figures 8 (b)-(c) demonstrate the reconstruction results for noise-free measurements, in which the algorithm successfully converges to the global optimum solution. The solution correctly reflects the shape and the location of the target. The existing error in the results is due to the significant reduction in the performance of the simulated annealing method in obtaining new candidate solution points in low temperatures. Figures 8 (d)-(i) illustrate the sensitivity of this method to noise, where we add white Gaussian noise to the measurements. As a consequence of adding noise to the measurements, the accuracy of the reconstructed contrast decreases, yet it provides information about the shape and location of the objects in the investigation domain.

In the last three simulations, applying the standard simulated annealing method does not lead to a successful reconstruction of the investigation domain. The main reason for this is inability of the sequential perturbation scheme to generate candidate solutions close to the global solution. As a consequence, the searching domain for finding new candidate solutions is not large enough to encompass the global optimum solution. Therefore, by reducing the temperature, the algorithm is forced to get trapped in a solution that is not necessarily the desired solution of the inverse problem.

To sum up the presented inversion results, Table II shows the CPU-time needed for the convergence of each simulation and the root-mean-square error (RMSE) between the reconstructed permittivity domain and the original target using pixel-by-pixel values of the regions. As evident in Table II, by increasing the resolution of the image, the RMSE of the reconstructed images would be higher compared to the low resolution images. In addition, by increasing the noise level in the measurements, the error in the pixels of reconstructed domain will increase accordingly.

Here we briefly investigate the performance of the proposed method with respect to the number of unknown variables using a numerical experiment. For this purpose, we consider the same configuration of antennas, investigation domain ($\lambda \times \lambda$ region discretized by $\lambda/7 \times \lambda/7$ pixels), and parameters for the global optimizer as the first experiment. However, we assume that only some of the pixels in the investigation domain are unknown variables of the optimization problem. our goal is to obtain the number of function evaluations required to solve the inverse problem for different number of variables.

Figure 9 demonstrates the number of function evaluations required for convergence of the algorithm for different number of unknown variables. It is observed that we have an almost linear increase as the number of variables in the optimization problem is raised. Since the proposed method is non- deterministic, we have repeated each case for 100 times and taken their average in Figure 9.

IV. SUMMARY AND CONCLUSION

We proposed a global optimization technique using a multi-directional random search based on the simulated annealing method to reconstruct the electromagnetic properties of two-dimensional dielectric targets. As we perform random searches in multiple effective directions instead of sequential perturbations in the standard simulated annealing method, the proposed optimization approach works more successfully compared to the classical simulated annealing. It is worth mentioning that the presented optimization method has not been compared to other global optimization algorithms, such as GA, PSO, and DE, but that would be a worthwhile effort to take on in the future. The numerical results verify the capability of this method in obtaining the permittivity distribution without considering any specific constraint on the initial guess or *a priori* information about the size, location, or permittivity values of the target. Although a higher resolution (via reducing the pixel size) will result in slower convergence, applying the proposed imaging technique leads to the successful reconstruction of investigation domain. The proposed method can apply equally well to other imaging modalities that use inverse scattering, such as acoustic/ultrasound imaging. The difference is the employed forward model in these imaging techniques, which provide different types of measurements from the target.

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Fig. 1.
General configuration of 2-D inverse-scattering problem in which we illuminate the target and collect the scattered fields.

Fig. 2.

(a) Flowchart of the conventional form of the simulated annealing method using the sequential perturbation scheme. (b) Flowchart of the multidirectional-search based simulated annealing.

Fig. 3. Root-mean-square error (RMSE) in the optimization of 2 dimensional Rosenbrock function for different values of N_t and N_s .

Fig. 4.

(a) Permittivity distribution of the investigation domain (b) Reconstruction of the target for noise-free measurements and (c) the corresponding cost function. (d) Reconstruction of the target in the presence of noise (30 dB signal-to-noise ratio) and (e) the corresponding cost function. (f) Reconstruction of the target in the presence of noise (20 dB signal-to-noise ratio) and (g) the corresponding cost function. (h) Reconstruction of the target in the presence of noise (10 dB signal-to-noise ratio) and (i) the corresponding cost function.

Fig. 5.

(a) Reconstruction of the target for noise-free measurements by using the standard simulated annealing method and (b) the corresponding cost function.

Fig. 6.

(a) Permittivity distribution of the investigation domain (b) Reconstruction of the target for noise-free measurements and (c) the corresponding cost function. (d) Reconstruction of the target in the presence of noise (30 dB signal-to-noise ratio) and (e) the corresponding cost function. (f) Reconstruction of the target in the presence of noise (20 dB signal-to-noise ratio) and (g) the corresponding cost function. (h) Reconstruction of the target in the presence of noise (10 dB signal-to-noise ratio) and (i) the corresponding cost function.

Fig. 7.

(a) Permittivity distribution of the investigation domain (b) Reconstruction of the target for noise-free measurements and (c) the corresponding cost function. (d) Reconstruction of the target in the presence of noise (30 dB signal-to-noise ratio) and (e) the corresponding cost function. (f) Reconstruction of the target in the presence of noise (20 dB signal-to-noise ratio) and (g) the corresponding cost function. (h) Reconstruction of the target in the presence of noise (10 dB signal-to-noise ratio) and (i) the corresponding cost function.

Fig. 8.

(a) Permittivity distribution of the investigation domain (b) Reconstruction of the target for noise-free measurements and (c) the corresponding cost function. (d) Reconstruction of the target in the presence of noise (30 dB signal-to-noise ratio) and (e) the corresponding cost function. (f) Reconstruction of the target in the presence of noise (20 dB signal-to-noise ratio) and (g) the corresponding cost function. (h) Reconstruction of the target in the presence of noise (10 dB signal-to-noise ratio) and (i) the corresponding cost function.

Fig. 9. Required number of function evaluations for the convergence of the proposed optimization algorithm for different number of function variables.

TABLE I:

A COMPARISON BETWEEN THE PROPOSED OPTIMIZATION SCHEME AND SIMULATED ANNEALING METHOD ON ROSENBROCK FUNCTIONS IN 2 AND 4 DIMENSIONS.

	Starting point	Simulated annealing		Proposed Optimizer	
		Final function value	Number of function evaluations	Final function value	Number of function evaluations
2-D	1001,1001	1.8E-10	500001	9.9E-11	42151
	1001,-999	2.6E-9	508001	1.3E-10	67882
	1443,1	1.5E-8	492001	1.0E-9	49770
4-D	101,101, 101,101	5E-5	1288001	5.0E-6	467040
	201,0, 0,0	7.5E-7	1288001	1.6E-8	196762
	-99,-99, -99 -99	3.3E-7	1304001	1.9E-7	295162

TABLE II:

ROOT-MEAN-SQUARE ERROR (RMSE) AND CPU-TIME (SECONDS) OF THE RECONSTRUCTED IMAGES IN THE NUMERICAL EXPERIMENTS.

	Noise free		30 dB	20 dB	10 dB
	CPU-time	Root-mean-square error			
Experiment 1	552	0.014	0.068	0.082	0.088
Experiment 2	986	0.025	0.076	0.294	2.135
Experiment 3	34239	0.021	0.094	0.183	0.396
Experiment 4	180510	0.259	0.489	0.608	0.636