Design of a Tri-Axial Force Measurement Transducer for Plantar Force Measurements

Weizi Li

Jiao Tong University Joint Institute,
University of Michigan-Shanghai,
Shanghai 200240, China
e-mail: patronum1991@sjtu.edu.cn

Roberto Dugnani¹

Jiao Tong University Joint Institute,
University of Michigan-Shanghai,
Shanghai 200240, China
e-mail: roberto.dugnani@sjtu.edu.cn

Transducers for spatial plantar force measurements have numerous applications in biomechanics, rehabilitation medicine, and gait analysis. In this work, the design of a novel, tri-axial transducer for plantar force measurements was presented. The proposed design could resolve both the normal and the shear forces applied at the foot's sole. The novelty of the design consisted in using a rotating bump to translate the external loads into axial compressive forces which could be measured effectively by conventional pressure sensors. For the prototype presented, multilayer polydimethylsiloxane (PDMS) thin-film capacitive stacks were manufactured and used as sensing units, although in principle the design could be extended to various types of sensors. A quasi-static analytic solution to describe the behavior of the transducer was also derived and used to optimize the design. To characterize the performance of the transducer, a 3 cm diameter, 1 cm tall prototype was manufactured and tested under various combination of shear and normal loading scenarios. The tests confirmed the ability of the transducer to generate strong capacitive signals and measure both the magnitude and direction of the normal and shear loads in the dynamic range of interest. [DOI: 10.1115/1.4039205]

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1 Introduction

Force transducers play an important role in biomechanics since shear and normal forces at the base of the foot could shed light on both the structure and functionality of the foot. The force distribution on soles also plays an essential role in rehabilitation medicine, gait analysis, anatomical inspection, patients' diabetes status forecasting, and neuromuscular examination [1].

Plantar shear stress proved to be very useful in foot pathology [2]. Plantar shear stresses not only were shown to complement normal pressure data [3], but also to play a greater role in diagnosing various patients' medical conditions such as foot ulcers. Recent studies, for instance, have shown that pressure data alone cannot consistently predict the occurrence of foot ulcer [4]. Yavuz et al. presented strong evidence showing that plantar shear stress data were clinically significant in predicting ulceration, and when combined with normal stress data, they provided an effective way to identify ulcers even before they manifested [5]. Plantar shear stress measurements could also help investigate mechanical properties of soft tissue [6]. Pai and Ledoux [7] carried out extensive mechanical tests on plantar soft tissue shear properties and found significant differences between the shear performance of healthy and diabetic tissues.

Various methods to perform measurement on both shear and normal pressure could be found in the literature, but it was found that designs often displayed some limitations. Wertsch et al. [8], for instance, manufactured an array of 0.25 mm thick, effective conductive polymer pressure sensors and installed them on an insole. These pressure sensors were able to measure normal forces but could not measure shear forces. Heywood et al. [9] constructed a large platform to measure both normal and shear force

concurrently using capacitive sensing technology. Nevertheless, the proposed design was limited to laboratory testing as it proved too large to fit into a shoe. Dobrzynska and Gijs [10], Lee et al. [11], Roberts et al. [12], and Huang et al. [13] all proposed similar types of small, flexible sensors for normal and shear force measurements. In all cases, for each sensing unit, the designs utilized four electrode pairs and a combination of air-polydimethylsiloxane (PDMS) dielectrics. For instance, Lee et al. [11] used air as dielectric material and could measure pressures up to only 130 kPa. Huang et al. [13] used both air and PDMS as dielectrics. The sensors manufactured regrettably yielded small capacitive signals and could not uniquely be related to the applied stress field as the air gap chamber in the transducer could theoretically deform in an infinite number of configurations with identical capacitive responses. Although viable, all these transducer designs generally were applicable to relatively low force range (less than 200 kPa for normal pressure) and used relatively expensive fabrication methods such as etching and vacuum evaporation. Finally, the transducers produced by these authors proved to produce weak response signals with static capacitance less than 10 pF, and a change in capacitance for the pressure range of interest in the fF range.

1.1 Design Requirements and Approach. Since the reported normal pressure and shear stress range on a plantar sole during normal activity were approximately 350 kPa [14,15] and 80 kPa [16,17], respectively, and the target range for the current design was set as 0–500 kPa for normal pressure and 0–100 kPa for shear stress. Also, the typical adult gait period strolling is in the range of 1 s [18] although the useful frequency content of the pressure data is more plausibly in the tens of hertz. Hence, the proposed design had to show a good response to signals for frequencies higher than 1 Hz.

Although millimeter-size transducers were necessary for a reasonable spatial distribution of the plantar pressure, as a proof of concept, a transducer 3 cm in diameter and thinner than 1.5 cm

¹Corresponding author.

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Fig. 1 Three-dimensional (3D) rendering of the designed transducer with major components highlighted. Two of the three sensing stacks (hidden) are shown by dotted lines. In the top right corner, the transducer is shown with its protective cover in place.

was manufactured. The relatively large prototype's dimensions were based on the practical requirement that all components had to be large enough to be assembled by hand. The static capacitance goal was in the range of 100 pF (i.e., one order of magnitude larger than values found in the literature).

In our approach, a capacitive-type transducer was chosen because of its relatively simple structure and its ability to measure both static and dynamic forces [19]. Resistive transducer might prove to yield better frequency response, but the use of resistive transducer was left for future work. Rather than attempting to measure shear loads and normal loads separately as proposed in other transducers' designs, in the proposed design the capacitive sensors only needed to measure three normal compressive forces to be able to resolve the tri-axial force vector. Also, our design used three sensing units to extrapolate the three force components, whereas other designs found in the literature often used four sensing units resulting in unnecessary redundancies in the measurements, larger than necessary transducers, and increased costs.

Figure 1 illustrates the structure of the proposed transducer and highlights its four major components: (1) a rigid cover with a

bump connected by curved flexure beams to the protective enclosure, (2) three circular plungers axially pressing on the capacitance stacks (shown in dotted lines), (3) three sensing units (i.e., the multilayer capacitive stacks), and (4) a printed circuit board base. One of the characteristic features of the design was that the flexure beams provided high in-plane stiffness and high out-of-plane compliance, hence, effectively allowing the bump to rotate under the action of external loads.

2 Analytical Model

In this section, a static analytical model is introduced. The goal of the analytical model was to both describe and help optimize the behavior of the transducer as external loads were applied to the surface of the bump. The developed analytical model was necessary both at a design stage and later to compute the external loads based on the sensors' electrical signals. A schematic diagram of the transducer's structure, key dimensions, and load application points is shown in both Figs. 1 and 2. As shown in Fig. 2, the external loads were applied on the bump's upper surface (i.e., the

Fig. 2 Schematic view of the analyzed transducer including the key dimensions and location of both external and reaction loads. The suffices 1, 2, and 3 refer to capacitance stacks 1, 2, and 3, respectively.

surface in contact with the base of the foot). Generally, the external loads included the normal pressure, N , and two shear components T_x and T_y . The average reaction forces from the capacitance stacks were denoted by P_1 , P_2 , and P_3 as shown in Fig. 2. D stood for the bump's diameter, H was the height of the bump, and l is the radial distance to the center of the capacitance stacks. For the prototype presented, a symmetric layout with $\alpha = 30^\circ$ was chosen. Different values of α , could be selected to provide different directional sensitivity. This could prove to be useful as the general direction of the shear loads known and relative small deviations from such direction were expected.

The static equilibrium equations were used to relate the axial loads on the transducers, P_1 , P_2 , and P_3 to the magnitude of the normal compressive force N and of the two components of the shear forces T_x and T_y . The deformation of the bump and plungers were assumed insignificant; hence, the static equilibrium equations could be written as

$$P_1 + P_2 + P_3 + P_{\text{edge}} - N = 0 \quad (1)$$

$$T_x H + T_{c,y} + M_{Fy} = 0 \quad (2)$$

$$T_y H + T_{c,x} + M_{Fx} = 0 \quad (3)$$

The force P_{edge} in Eq. (1) indicated the vertical force imparted by the bending of the flexure beams along the perimeter of the bump. Although the reaction loads from the flexure beams were truly localized loads, it was assumed for simplicity that the reaction forces were uniformly distributed around the perimeter of the bump. $T_{c,y}$ and $T_{c,x}$ were the bending moments about the y -axis and the x -axis resulting from integration of the stress distribution on the surface of each capacitor. Similarly, M_{Fx} and M_{Fy} were the bending moments reacting the bending of the flexure beams. The derivation of the bending moment terms assumed that the bump and plungers moved as a rigid body and therefore the surface at the interface between the capacitor stacks and the plungers remained planar. In order to calculate P_{edge} , the equation of the plane describing the displacement of the bottom surface of the plungers was expressed in the form

$$Ax + By + Cz = D \quad (4)$$

A schematic view of the deformation and the loads induced by the transducers stacks and the linear springs approximating the vertical reaction of the flexural beams is shown in Fig. 3.

If the vertical deflection at the center of each plunger were denoted by u_1 , u_2 , and u_3 , respectively, two vectors on the deflection plane could be defined, i.e., $C_1 C_2 (-l \cos \alpha, -l \sin \alpha - l, u_2 - u_1)$ and $C_1 C_3 (l \cos \alpha, -l \sin \alpha - l, u_3 - u_1)$, and the equation of the deflection plane could be obtained by the cross product of these vectors. Carrying out the cross product, the coefficients of the plane in Eq. (4) could be determined

$$A = -(l \sin \alpha + l)(u_3 - u_2) \quad (5a)$$

Fig. 3 Schematic representation of displacements at the perimeter of the bump, $z(\theta)$, and at the center of each transducer stack: u_1 , u_2 , and u_3

$$B = (u_2 + u_3 - 2u_1)l \cos \alpha \quad (5b)$$

$$C = 2l \cos \alpha (l \sin \alpha + l) \quad (5c)$$

$$D = (u_2 + u_3)l^2 \cos \alpha + 2u_1 l^2 \cos \alpha \sin \alpha \quad (5d)$$

The average displacement of the plane, z_{average} , was obtained by letting $x = y = 0$, and its magnitude was used to determine the amount of stretching of the equivalent flexural springs

$$z_{\text{average}} = \frac{(u_2 + u_3) + 2u_1 \sin \alpha}{2 + 2 \sin \alpha} \quad (6)$$

The magnitude of P_{edge} could then be obtained multiplying the average vertical displacement, z_{average} , times the vertical stiffness of the flexure beams, $k_{f,v}$. Equation (1) could then be written in terms of the relative vertical stiffness of the capacitor stacks, $k_{s,v}$ and of the curved beams $k_{f,v}$

$$\left(1 + \frac{\sin \alpha}{1 + \sin \alpha} * \frac{k_{f,v}}{k_{s,v}}\right) P_1 + \left(1 + \frac{1}{2 + 2 \sin \alpha} * \frac{k_{f,v}}{k_{s,v}}\right) (P_2 + P_3) = N \quad (7)$$

The bending moments from the flexure beams could also be calculated assuming a rigid motion of the bump and plungers, and assuming that the load imparted by the flexure beams was uniformly distributed around the circumference of the bump. The infinitesimal bending reaction load, dM_x and dM_y , could be written as

$$dM_x = dF \cdot \left(\frac{D}{2}\right) \sin \theta = f(\theta) d\theta \left(\frac{D}{2}\right) \cdot \left(\frac{D}{2}\right) \sin \theta \quad (8a)$$

$$dM_y = dF \cdot \left(\frac{D}{2}\right) \cos \theta = f(\theta) d\theta \left(\frac{D}{2}\right) \cdot \left(\frac{D}{2}\right) \cos \theta \quad (8b)$$

where $f(\theta)$ represented the force per unit length along the perimeter of the bump and the angle θ measured with respect to the positive x -axis. Since for a rigid body motion of the plungers and linear elastic behavior of the flexures, $f(\theta)$ changed linearly, it follows that M_{Fx} and M_{Fy} could be easily calculated from integration of the relevant infinitesimal bending reactions about the x -axis and the y -axis, respectively,

$$M_{Fx} = -2 \int_0^\pi f(\theta) * \left(\frac{D}{2}\right) * d\theta * \left(\frac{D}{2}\right) * \sin \theta \quad (9)$$

$$M_{Fy} = -2 \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} f(\theta) * \left(\frac{D}{2}\right) * d\theta * \left(\frac{D}{2}\right) * \cos \theta \quad (10)$$

with $f(\theta) = k_{f,v} z(\theta) / \pi D$ defined as the force per unit length along the circumference. Since the displacement $z(\theta)$ was described by Eq. (4) with coefficients as in Eqs. (5a)–(5c), the integrals in Eqs. (9) and (10) could be computed, leading to the equations

$$M_{Fy} = \frac{k_{f,v}}{8k_{s,v}} \frac{D^2}{2l \cos \alpha} \cdot (P_2 - P_3) \quad (11)$$

$$M_{Fx} = -\frac{k_{f,v}}{8k_{s,v}} \frac{D^2}{2l(1 + \sin \alpha)} \cdot (2P_1 - P_2 - P_3) \quad (12)$$

Similarly, $T_{c,y}$ and $T_{c,x}$ could be obtained from direct integration of the stress on the sensor's surfaces

$$T_{c,x} = -\oint_{C_1} y \cdot \sigma(x, y) dA - 2 \oint_{C_2} y \cdot \sigma(x, y) dA \quad (13)$$

$$T_{c,y} = -\oint_{C_2} x \cdot \sigma(x, y) dA - 2\oint_{C_3} x \cdot \sigma(x, y) dA \quad (14)$$

The equations for the stress, $\sigma(x, y)$, could be obtained from knowledge of the displacement, i.e., Eq. (4) with coefficients as in Eqs. (5a)–(5c), the value of the average stress at the center of each transducer stack, $\sigma_1, \sigma_2, \sigma_3$, and the coordinate system for these points. The average stress at the center of each transducer stack i was assumed to be equal to $4P_i/(\pi d)$. Integration of Eqs. (13) and (14) led to the expressions

$$T_{c,x} = -\sigma_1 \pi r^2 l * \frac{\frac{3}{4} \frac{r^2}{l^2} + \sin \alpha + 1}{\sin \alpha + 1} + (\sigma_2 + \sigma_3) \pi r^2 l * \frac{\frac{3}{8} \frac{r^2}{l^2} + \sin^2 \alpha + \sin \alpha}{\sin \alpha + 1} \quad (15)$$

$$T_{c,y} = (\sigma_2 - \sigma_3) \pi r^2 l * \left(\frac{r^2}{l^2} \frac{1}{4 \cos \alpha} + \cos \alpha \right) \quad (16)$$

Finally, the reaction forces P_1, P_2 , and P_3 were obtained by substituting the computed moments into Eqs. (2) and (3), and solving. Using conventional linear algebra's methods, it could be shown that

$$\begin{cases} P_1 = D_1/D_0 \\ P_2 = D_2/D_0 \\ P_3 = D_3/D_0 \end{cases} \quad (17)$$

where the constants D_0, D_1, D_2 , and D_3 were defined as

$$D_0 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}; \quad D_1 = \begin{vmatrix} N & a_{12} & a_{13} \\ -T_x H & a_{22} & a_{23} \\ -T_y H & a_{32} & a_{33} \end{vmatrix}; \quad (18)$$

$$D_2 = \begin{vmatrix} a_{11} & N & a_{13} \\ a_{21} & -T_x H & a_{23} \\ a_{31} & -T_y H & a_{33} \end{vmatrix}; \quad D_3 = \begin{vmatrix} a_{11} & a_{12} & N \\ a_{21} & a_{22} & -T_x H \\ a_{31} & a_{32} & -T_y H \end{vmatrix}$$

with the matrix terms $a_{i,j}$ given by

$$\begin{cases} a_{11} = 1 + \frac{\sin \alpha}{1 + \sin \alpha} \frac{k_{f,v}}{k_{s,v}}, a_{12} = a_{13} = 1 + \frac{1}{2 + 2 \sin \alpha} \frac{k_{f,v}}{k_{s,v}} \\ a_{21} = 0, a_{22} = -a_{23} = \frac{k_{f,v}}{8 k_{s,v}} \frac{D^2}{2 \cos \alpha} + \left(\frac{r^2}{l^2} \frac{1}{4 \cos \alpha} + \cos \alpha \right) l \\ a_{31} = -\frac{2 k_{f,v}}{8 k_{s,v}} \frac{D^2}{2 l (1 + \sin \alpha)} - l \frac{\frac{3}{4} \frac{r^2}{l^2} + \sin \alpha + 1}{\sin \alpha + 1} \\ a_{32} = a_{33} = \frac{k_{f,v}}{8 k_{s,v}} \frac{D^2}{2 l (1 + \sin \alpha)} + l \frac{\frac{3}{8} \frac{r^2}{l^2} + \sin^2 \alpha + \sin \alpha}{\sin \alpha + 1} \end{cases} \quad (19)$$

Although not immediately obvious from the direct inspection of Eqs. (18) and (19), it could be shown that the magnitude of the reaction loads P_1, P_2 , and P_3 were related to the ratio $k_{f,v}/k_{s,v}$ in such a way that when the ratio increased, the reaction loads also decreased. Hence, as also expected by intuition, by reducing the vertical stiffness of the flexure beams, $k_{f,v}$, the sensitivity of the transducer could be increased.

3 Sensor Design and Manufacturing

The first part of this section describes the design and manufacturing steps carried out for manufacturing the sensing capacitor

stacks. In the second part of this section, the relevant information about the dimensions and design features of the bump, the enclosure, and the flexure beams were discussed.

3.1 Multilayer Capacitor Stacks. The first part of this section is dedicated to describe the design and manufacturing of the capacitor stack used as pressure sensing units. In the capacitor stacks, when a normal force was applied axially, the thickness of the dielectrics changed, which in turns changed the system's capacitance. The load-free capacitance of an individual dielectric disk, C_0 , and the change in capacitance due to axial loading, ΔC , could be expressed as

$$C_0 \equiv \frac{\epsilon_r \epsilon_0 A}{d_0} \quad \text{and} \quad \Delta C = C_0 \frac{1}{1 - \frac{\Delta d}{d_0}} = C_0 \frac{\Delta d}{d_0} \left[1 + \frac{\Delta d}{d_0} + \left(\frac{\Delta d}{d_0} \right)^2 + \dots \right] \approx \Delta P \frac{\epsilon_0 A}{d_0} \frac{\epsilon_r}{E} \quad (20)$$

where ϵ_r was the relative dielectric constant, ϵ_0 was the dielectric constant of air, A was the electrode area, d_0 was the stress-free thickness of the dielectric film, ΔP was the axial pressure applied, and E was the Young's modulus (or the tangent modulus for non-linear materials) of the dielectric film. Based on Eq. (20), it was clear that to increase the sensitivity of the transducer, i.e., $\Delta C/\Delta P$, a material with high ϵ_r and low tangent modulus should be selected and manufactured in thin films (i.e., small d_0). Among various commercially available dielectrics considered in this work, PDMS displayed the highest ratio ϵ_r/E , and could be easily manufactured into thin films. Furthermore PDMS was known to resist the pressure requirements (yield stress, 1 MPa [20]) for the our design application. The major drawback from using PDMS was its viscoelastic behavior which inevitably was expected to introduce inaccuracies in the predicted loads.

To increase the effective area of the transducer, the PDMS films were sandwiched between flexible printed circuit electrodes, as shown in Fig. 4. Large PDMS sheets (approximately 65 μm thick) were first manufactured by spin coating following a technique similar to what described in the work of Ryoo et al. [21] and subsequently cut into 6 mm diameter disks using a steel punch. Electrical wires were soldered at the flexible printed circuit's terminal vias and electrically connected the thin film capacitors in parallel. The flexible printed circuits were disposed in a staggered fashion to minimize the chances of electrical shorting. Stacks were assembled with terminals in a "V-shape," 120 deg apart, to achieve a relatively compact design at assembly (Fig. 3). In order to achieve 100 pF static capacitance's goal, 25 layers of 6 mm diameter PDMS film disks were used in each stack. The final assembly thickness of each stack was approximately 4 mm. The radial distance between the center of each capacitor stack and the transducer's center (i.e., l) was set equal to 4 mm.

3.2 Bump Sizing. The bump structure was manufactured by standard 3D printing technology utilizing acrylonitrile butadiene styrene as the printing material. The structure consisted of a hexagonal enclosure (with a removable protective cover), a set of flexible curved beams, and a rigid bump/plunger structure at the center. The protective closure provided a rigid structure around the transducer and protected it from the external environment. The flexure beams were intended to provide a stiff in-plane connecting link to the enclosure while remaining compliant out-of-plane. The bump protruded outside the protective enclosure by means of an opening on the top cover and hence the bump's top surface could be loaded by external forces. A schematic view of the design and its major components is shown in Fig. 1.

Fig. 4 Schematic representation of the multilayer structure and assembly of the capacitor stack units (left). Image of capacitor stack during the assembling stage (top right). In the lower part of the figure, the assembly configuration of the three capacitor stacks with the terminals 120deg apart is also shown schematically.

Unlike the designs found in the literature, the most obvious advantage of the proposed design was that it could react the external loads by loading each sensing stack axially in compression. This was achieved by letting the bump rotate with minimal in-plane translation. In this section, the conditions under which rotation of the bump/plungers could be achieved were presented. This concept was better illustrated by considering a simple two-dimensional model and extending the physical principle to the 3D model (see Fig. 5).

Let an external shear force T_x be applied on the top of bump, and let us assume that the horizontal reaction force from the flexural beams be $F_{f,h}$ and the effective flexure in-plane stiffness, $k_{f,h}$

$$F_{f,h} = k_{f,h} \cdot u_h \quad (21)$$

where u_h is the horizontal displacement of the cover frame. Under such loading scenario, the cover moved horizontally as the capacitor stacks deformed in shear (no slip at the interface was assumed). Assuming small displacements, the shear force on the capacitor stacks, T^* , could be written as

$$T^* = A \cdot \tau_s \quad (22)$$

Fig. 5 Illustration of two-dimensional analytical model including equivalent horizontal and vertical stiffness for the capacitor stacks and flexure beams

$$\tau_s = G_s \gamma = G_s \cdot \arctan\left(\frac{u_h}{t_s}\right) \approx G_s \frac{u_h}{t_s} \quad (23)$$

where G_s was the effective shear modulus, and t_s is the total thickness of the capacitor stacks. Re-arranging Eqs. (22) and (23)

$$T^* = \left(\frac{G_s A}{t_s}\right) u_h = k_{s,h} u_h \quad (24)$$

From the static force equilibrium in the horizontal direction

$$T + 3T^* + F_{f,h} = 0 \quad (25)$$

By substituting Eq. (24) into Eq. (25)

$$T^* = -T / \left(3 + \frac{k_{f,h}}{k_{s,h}}\right) \quad (26)$$

Equation (26) indicated that for $k_{f,h} \gg k_{s,h}$, the shear loads on the capacitor stacks became insignificant, i.e., $|T^*| \ll T/3$ and the external shear forces could only be counteracted by vertical forces between the plungers and the capacitors' stacks (i.e., the bump and plungers rotate and translates vertically). Such condition could be achieved if the flexure beams connecting the enclosure to the bump/plungers displayed high in-plane stiffness compared to the shear stiffness of the capacitor stacks.

Based on the analytical model, the goal for the configuration considered reduced to designing a structure where: (1) the vertical stiffness ratio ($k_{f,v}/k_{s,v}$) was as small as possible (to improve the transducer's sensitivity) and (2) the horizontal stiffness ratio ($k_{f,h}/k_{s,h}$) was as large as possible to both allow the bump's rotation and minimize the shear loads on the capacitor stacks. In terms of the design, these goals were practically achieved using long, slender beams (i.e., flexible in the vertical direction and stiff in-plane) to connect the bump/plungers to the hexagonal enclosure.

3.3 Flexure Beams. To increase the effective beam length (and reduce the vertical stiffness), curved sections were used. The cross section of beams was kept as small as possible to increase the flexural compliance. Due to the 3D printing limitation, the cross section had to be larger than 1 mm and hence the prototype

Fig. 6 Detailed sketch of the curved, flexure beams including some relevant dimension

design finally used 1.1mm diameter for the flexure beams. Figure 6 shows a schematic view of the curved flexure beams including some of the relevant dimensions.

Flexure beams of different geometries and curvature were initially considered and virtually tested by means of finite element software. The in-plane stiffness and the vertical stiffness of the beams were computed by virtually applying either a vertical load or a horizontal load at the bump. The value of Young's modulus for the acrylonitrile butadiene styrene material was set at 1.6 GPa [22,23] in all simulations. When analyzing the vertical stiffness, the bottom surface of the enclosure was fixed, and a vertical load was applied on the top surface of the bump. The vertical stiffness was then approximated as $k_{f,v} = u_z/N$. Similarly, for the in-plane stiffness analysis, the bottom surfaces of the plungers were prevented from moving in the vertical direction (i.e., rollers boundary conditions), and the horizontal stiffness was approximated as $k_{f,h} = u_h/T$. In general, the vertical and horizontal stiffness depended on the magnitude of the applied load although only weakly. From the numerical simulation, the average $k_{f,v}$ was found to be 22 N/m, and the average $k_{f,h}$ was 18 kN/mm. For $N=20$ N, it was found that $(k_{f,v}/k_{s,v}) \cong 0.35$, and for $T=5$ N, we had $(k_{f,h}/k_{s,h}) \cong 9 \times 10^3$ which appeared to be reasonable to achieve the current design's goals.

3.4 Bump Design. The diameter of the bump was chosen based on both geometric constraints and the response of the transducers. Mechanical testing on the manufactured capacitor stacks indicated that the sensitivity of the sensors decreased for pressure

levels corresponding to loads higher than 4 N as also shown in Fig. 7. Taking $P_1 = P_2 = P_3 = 5$ N as the highest desirable load magnitude, and using $(k_{f,v}/k_{s,v}) \cong 2.5$ from the mechanical testing, allowed us to compute the corresponding maximum desired magnitude of the axial load $N \cong 27.5$ N. As the maximum expected plantar pressure was 500 kPa [15,24], it followed that the diameter of bump, D , should be larger than 4.3 mm. The value selected for the prototype was rounded to 4 mm.

The height of bump, H , was selected based on the analytical solution and by considering the magnitude of the expected external loads. The imposed criteria for the selection were: (1) the capacitor stack had to be in compression for all the expected loading conditions and (2) the shear force on each capacitor's stack, T^* , should be smaller than the static friction force between the face of the plungers and the capacitor stacks. Namely, the design's constraints were

$$\begin{cases} P_1 > 0; & P_2 > 0; & P_3 > 0 \\ T^* < \min(\mu P_1, \mu P_2, \mu P_3) \end{cases} \quad (27)$$

where μ was the static friction coefficient between the plunger and the capacitor (estimated as $\mu \cong 0.3$). Applying the conditions outlined in Eq. (22) to the equilibrium equations and conservatively assuming $|T|/N < 0.4$, it was concluded that $H/l < 1.5$. Since, based on the analytical solution, the sensitivity of the transducer was higher for higher value of the ratio, H/l , the largest possible value satisfying the design's constraints was selected.

4 Model Validation and Mechanical Testing

The first part of this section was dedicated to the verification of the analytical solution by numerical model. The second part describes some of the mechanical testing performed to characterize the performance of the transducer.

4.1 Analytical Model Validation. To validate the analytical model, numerical simulations for the geometry described in Fig. 2 were run, and the results were compared with the analytical predictions. The finite element analysis software used in the validation was COMSOL 3.5. The software was used to build a rigid 3D model consisting of three cylinders (capacitor) and one disk with a bump (cover) in a configuration analogous to the proposed design. The height of cylinders was 4 mm and their radii 3 mm. The radial distance l to the center of the capacitors was set equal to 3.6 mm, the diameter of bump was 4 mm, and the height of the bump was varied in the range 2–3 mm. Also, various values of the

Fig. 7 Loading wedge and alignment pin used in the shear/tension test (left). Shear loading test setup for the transducer assembly.

vertical stiffness ratio $k_{f,v}/k_{s,v}$ were considered. In the model, the center of bump was fixed in the horizontal direction as in the analytical solution it was assumed ($k_{f,h}/k_{s,h} \gg 1$).

4.2 Capacitive Units Validation. The electrical and mechanical characterization of the capacitor stacks performance was carried out through mechanical testing using a standard MTS C45.504 universal test machine. According to manufacturing specifications, the load cell resolution was 0.01 N. The capacitance signal was converted into voltage using a standard CAV444 capacitance–voltage transformation circuit [25], and the voltage data were recorded by Yokogawa DL850 scope-coder. A circular rod was used to apply force directly on the surface of the capacitance stacks. The loading was applied in displacement control following a sinusoidal curve with a peak displacement magnitude of 0.5 mm and loading frequencies 0.1/0.2/0.4/0.8 Hz.

4.3 Transducer Assembly Validation. To verify the behavior of the assembly when concurrently loaded by normal and shear loads, a double wedge setup configuration was used to load the transducer concurrently in shear and in tension. A schematic view of the test setup including the loading wedges is shown in Fig. 7. For each combination of normal and shear load, two separate loading cases were analyzed, the first with the shear force applied along the positive y -axis and the second with the shear force applied along the x -axis. The ratio of the shear to normal force was controlled by changing the angle of the loading wedge. The angles considered were 11.3 deg and 16.7 deg corresponding to ratios between the magnitude of the normal force and the shear force equal to 0.2 and 0.3, respectively. To prevent slipping during loading of the transducer, a 1.5 mm diameter steel dowel pin was placed at the center of the wedges and bump. Axial pressure tests with no shear loads were also carried out.

5 Results and Discussion

The accuracy of the proposed analytical model was verified for various scenarios by comparing the values predicted by the analytical solution with the results from numerical simulations. The recorded loads on the transducers P_1 , P_2 , and P_3 obtained from the numerical simulation were used as the input parameter in the analytical model to compare how well the values of T_x , T_y , and N predicted by the analytical model matched the input values. The parameters varied in the verification were the relative vertical stiffness between the connecting beams, $k_{f,v}$, and the transducer stacks, $k_{s,v}$, and the height of the bump, H . The relative vertical stiffness was varied by changing the magnitude of Young's modulus for the capacitance. The results are summarized in Table 1. In general, it was found that the analytical model agreed with the numerical simulation's trends although some differences were present due to the simplifications implemented of the analytical solution. The largest error in the predicted loads was 2.6% for the normal force and 20% for the shear force, while the error for the shear angle (with respect to the x -axis) was ± 8 deg. Nonetheless,

it should be noted that for small values of the ratio $k_{f,v}/k_{s,v}$ the predictions were considerably more accurate.

5.1 Capacitance Stack Characterization. Test results for the measured capacitance of one capacitor stack (C_1) versus the applied compressive normal force in the 0–450 kPa range are shown in Fig. 8(b). Two different cycling frequencies are shown in the plot (i.e., 0.4 Hz and 0.8 Hz). Limitations in the available equipment did not allow testing at higher frequencies in the pressure range of interest. For the pressure range considered, the minimum recorded capacitance was approximately 115 pF, and the maximum capacitance was 120 pF. It was also found that the sensitivity of the device changed considerably with the applied stress. At low pressure (i.e., 0–100 kPa), the sensitivity was approximately 40 pF/MPa, while for larger pressure the sensitivity was about 4 pF/MPa. It also appeared that the capacitance measured by the stacks was function of the frequency. The capacitance measured at 0.8 Hz was consistently larger than that measured at 0.4 Hz for the same force even though the displacements were similar. This was likely due to the viscoelastic effects in the PDMS layers. A well-defined hysteresis loop was especially evident at higher frequencies. For the 0.8 Hz loading cycles, a difference of up to 0.5 pF between the loading and unloading portion of the signals were also recorded.

Three capacitor stacks were independently tested during one full loading–unloading cycle under compressive normal force. It was found that although the signal was stable for each stack, the individual capacitance stacks displayed significant variations. Such variations were likely due to the geometric (i.e., film thickness) and possibly even mechanical performance variations of the PDMS films used in the manufacturing of each of the transducers. For instance, C_1 , C_2 , and C_3 displayed quite different average sensitivities for the stress levels considered 0–450 kPa (approximately 5.6 pF/MPa, 3.3 pF/MPa, and 1.8 pF/MPa, respectively, during the sensor loading).

Figure 8(a) shows the force–displacement curves for capacitor C_1 for a cross-head displacement up to 0.4 mm. Two different loading frequencies (i.e., 0.4 Hz and 0.8 Hz) are shown in the figure. It was clear from the recorded data that the capacitor stacks significantly stiffened upon loading. The stiffening might be a result of both the martial stiffening and better adhesion of the PDMS films as normal pressure was applied. Figure 8(a) also shows that the stiffness of stacks was relatively consistent with marginal hysteresis especially at high frequencies. The variation in the recorded stiffness between the loading and unloading cycle at equivalent displacements was less than 1% for the 0.8 Hz loading case. Noticeably, both the stiffness and the sensitivity of the stacks decreased for axial compressive loads greater than approximately 4 N.

5.2 Transducer Assembly Testing. To verify the behavior of the transducer assembly when concurrently loaded by normal and shear loads, a double wedge compressive test setup configuration

Table 1 Comparison of analytical model and simulated results (forces in N, angles in deg)

$k_{f,v}/k_{s,v}$	H (mm)	N (simulated)	N (analytical)	$ T /N$ (simulated)	$ T /N$ (analytical)	angle (simulated)	angle (analytical)
0.1	3	20.4	20.0	0	0.00	—	—
		20.4	19.9	0.2	0.18	0	0
		20.4	20.1	0.2	0.20	90	89
2	3	20.4	20.1	0	0.01	—	—
		20.4	20.1	0.2	0.18	0	5
		20.4	20.1	0.2	0.21	90	88
1	2	20.4	19.8	0.2	0.01	—	—
		20.4	20.1	0.2	0.24	0	–8
		20.4	19.7	0.2	0.16	90	91

Fig. 8 (a) Compressive axial force versus recorded displacement for C_1 capacitance stack (25 layers; loading frequencies 0.4 Hz and 0.8 Hz). (b) Corresponding compressive axial force versus capacitance for the same sensor and loading frequencies.

was used (Fig. 7) under different loading scenarios. Before each test, the transducer was loaded at low stress levels for ten cycles to let all components set in. Due to the viscoelastic nature of the PDMS film, it was found that the peak normal load decreased at an approximate rate of 0.3% per cycle even for low stress levels and low frequencies ($N \leq 10$ N, 0.35 Hz). In general, higher testing frequencies displayed more significant decrease in magnitude.

To estimate the magnitude of N , and the relative magnitude/direction of the shear forces T_x/N and T_y/N , the analytical solution was used in conjunction with the individual capacitor stack's calibration curves. To determine the applied loads, first the capacitance versus force calibration curves (Fig. 8(b)) were used to convert capacitance reading into axial load. Next, the axial loads on the capacitance stacks were used to estimate the external loading conditions by means of the analytical model described in Sec. 2.

To use the analytical solution, the ratio of the vertical stiffness of the flexure beams over the stiffness of the capacitance stacks had to be determined. Although the approximate value was obtained by numerical simulation, a more accurate value for the ratio, $k_{f,v}/k_{s,v}$, was measured directly from a load versus displacement test. Two tests were necessary, the first on the transducer without the capacitance stacks, to estimate $k_{f,v}$, and the second on the transducer with the capacitance stacks to calculate the ratio, $k_{f,v}/k_{s,v}$. For this last case, the vertical equilibrium equation (Eq. (28)) was utilized

$$3k_{s,v} \cdot u_z \cdot \left(1 + \frac{k_{f,v}}{3k_{s,v}}\right) = N \quad (28)$$

Average values of $k_{f,v} = 27.5$ N/mm and $k_{f,v}/k_{s,v} = 2.5$ were obtained at a constant loading rate 30 mm/min. The vertical stiffness ratio was found to be one order of magnitude higher than the expected value at $N = 20$ N obtained in the numerical simulation (i.e., $k_{f,v}/k_{s,v} \cong 0.35$). The differences could likely be attributed to the nonlinear material behavior of the capacitor stacks. Due to the viscoelastic nature of the capacitor stacks, stiffening occurred as

the load increased. The flexure beams also were expected to display stiffening as the membrane stress became more significant for large deflections.

Once reasonable value for $k_{f,v}$ and for the stiffness ratio $k_{f,v}/k_{s,v}$ were established, the analytical model was utilized in conjunction with the transducer calibration curves to estimate the magnitude and direction of the external loads. Various scenarios were tested to compare the prediction estimation to the known applied external loads. The cases considered were $T_x/N = 0, 0.2$, and 0.3 (with $T_y = 0$), and $T_y/N = 0, 0.2$, and 0.3 (with $T_x = 0$). In general, the predictions for the normal load N were reasonably accurate for values of $N > 5$ N, whereas the shear load and direction were relatively inaccurate even at high loads. For the case $T_x/N = 0.2$ and $T_y/N = 0.3$, the average error found for the magnitude of the normal load (for $N > 5$ N) was 6% during loading, and 16% during the unloading. For the case $T_x/N = 0.2$ and $T_y/N = 0.3$, the errors were 7% for loading and 11% for unloading.

For the shear load predictions, the average errors at high normal loads were 27%, 28%, 23%, and 41% for $T_x/N = 0.2$, $T_x/N = 0.3$, $T_y/N = 0.2$, and $T_y/N = 0.3$, respectively. In general, better estimations for the shear force were achieved when shear loads were applied along the x -axis. The behavior was mostly due to a better response of the transducer aligned along the x -axis direction and was solely related to manufacturing issues. The difference in stiffness among the different capacitor stacks was likely responsible for the shear stress estimation errors. It is reasonable to believe that more consistent manufacturing of the stacks resulting in comparable stiffness behavior among the sensors could reduce the shear prediction errors to within 20%. Figure 9(b) shows the predicted axial load N for $T_x/N = 0.2$ based on the analytical solution. Solid marks were used for the predicted values at high compressive axial loads (i.e., $N > 5$ N), while the open marks were used for low loads (i.e., $N < 5$ N). In general, both the shape and magnitude of the predicted axial compressive load were found to track the actual values accurately. Figure 9(b) also shows the predicted value of the ratio $T_x/N = 0.2$. Also in this case, solid marks were used for the predicted values at high compressive axial loads, while the open marks were used for low loads. It was clear from

Fig. 9 Predicted forces and forces direction for the full transducer mechanical test ($|T|/N = 0.2$). Filled markers represent data corresponding to axial loads higher than 5 N. Open markers represent data for low axial loads (less than 5 N).

the data that the model was able to approximate the magnitude of the shear stress only grossly.

Figure 9(a) shows the predicted shear force angle (with respect to the x -axis). In the figure, the solid marks refer to prediction for high compressive axial loads (i.e., $N > 5$ N), while the open marks refer to low loads (i.e., $N < 5$ N). The direction of the shear load was found to be poorly predicted at low loads. The maximum error for high loads nonetheless was 29 deg, while the average error percentage was less than 10%.

In general, the predicted values were found to overestimate the shear loads. Various reasons could be attributed to the discrepancy between the predicted data and the applied forces. First, the stiffness of the capacitance stacks and flexural beams were nonlinear, unlike what assumed in the analytical model. Second, the variations in stiffness among the different capacitance stacks were significant mostly due to the current limitations in the prototype manufacturing methods. Such differences were not considered in the analytical solution. Clearly, different stiffness will result in different amounts of rotation angles during compared to what predicted when shear loads are applied. The flexures were also expected to display some degree of nonlinearity especially for large deflections. The analytical solution also incorrectly assumed that the vertical displacement was linearly proportional to the loads when predicting the transducer's rotation. Finally, the PDMS material used for the capacitors displays nonlinearity, hysteresis, and frequency dependent viscoelastic behavior which were not accounted for in the calibration. In order to improve the prediction capacitor stacks with consistent axial stiffness values should be utilized. Alternatively, different materials should be considered to manufacture capacitive films (or resistive sensors). Otherwise a more thorough description of the PDMS films' behavior would be required.

6 Conclusion

In this work, the design of a novel, tri-axial force measurement transducer using three multilayer PDMS thin films capacitive stacks as sensing units was introduced. The novelty of the design consisted in using a 3D printed rotating cover to translate shear forces into axial loads which could then be easily resolved. The

proposed design displayed a static capacitance greater than 100 pF (i.e., one order of magnitude larger than most values found in the literature), and could concurrently measure the normal and shear force magnitudes and directions. The transducer's sensitivity was found to depend on the loading condition but in general it was consistently larger than 5 pF/MPa. A quasi-static analytic solution to describe the behavior of the transducer was derived and used: (1) at the design stage to optimize the transducer's performance, and (2) during testing to predict the external loading conditions.

To characterize the performance of the transducer, a 3 cm diameter, 1 cm tall prototype was manufactured and dynamically tested at approximately 1 Hz. The mechanical performance testing confirmed the ability of the transducer to generate significant output capacitive signals and to consistently measure both the magnitude and direction of normal and shear loads in the loading range of interest. For axial loads higher than 5 N, the normal compressive loads from testing were found to be predicted within 6–16% of the actual magnitude, while the shear loads were within 40%. The shear load estimation error was expected to reduce to approximately 20% from the actual magnitude were the capacitor stacks manufactured more consistently. Finally, the predicted directions of the shear forces were within 8% of the actual directions for axial loads higher than 5 N. Significant improvements in the system are expected from a more consistent manufacturing of the capacitor stacks. Future work should be carried out to farther miniaturize the device and alleviate some of the dynamic effects and hysteresis by using piezoelectric sensing or resistive sensing technologies.

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