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Vertical Profile of Cloud Optical Parameters Derived from Airborne Measurements Above, Inside and Below Clouds

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Abstract

Past strategies for retrieving cloud optical properties from remote sensing assumed significant limits for desired parameters such as semi-infinite optical thickness, single scattering albedo equaling unity (non-absorbing scattering), absence of spectral dependence of the optical thickness, etc., and only one optical parameter could be retrieved (either optical thickness or single scattering albedo). Here, we demonstrate a new method based on asymptotic theory for thick atmospheres, and the presence of a diffusion domain within the clouds that does not put restrictions and makes it possible to get two or even three optical parameters (optical thickness, single scattering albedo and phase function asymmetry parameter) for every wavelength independently. We applied this method to measurements of angular distribution of solar radiation above, inside and below clouds, obtained with NASA's Cloud Absorption Radiometer (CAR) over two cases of marine stratocumulus clouds; first case, offshore of Namibia and the second case, offshore of California. The observational and retrieval errors are accounted for by regularization, which allows stable and smooth solutions. Results show good potential for parameterization of the shortwave radiative properties (reflection, transmission, radiative divergence and heating rate) of water clouds.

Keywords

solar radiance; airborne observations; inverse problem; cloud optical thickness; single scattering albedo; asymmetry parameter

1. Introduction

This paper discusses new strategies for retrieving cloud optical properties from measurements of angular distribution of solar radiation above, inside and below clouds, obtained with NASA's Cloud Absorption Radiometer (CAR) (King et al., 1986; Gatebe and King, 2016). This should help characterize vertical profiles of cloud optical parameters currently lacking in the literature. Clouds are major regulators of the Earth's radiation budget as they are known to reflect a great portion of the incoming visible radiation back to outer space during the day and serve as a blanket to protect the Earth surface against cooling especially at night.

According to Hansen and Hovenier (1974) the radiative impact of clouds is mainly controlled by two parameters, namely, the optical thickness and the effective radius. For this reason, most satellite retrievals focus on deriving these two parameters. In the last four decades, we have witnessed a lot of progress in determining cloud properties remotely from aircraft and satellites measurements (e.g., Hansen and Pollack, 1970; Twomey and Cocks, 1982; 1989; Nakajima and King, 1990; Minnis et al., 1998; Baum et al., 2000; Rozanov and Kokhanovsky, 2004). The underlying principle of most of the approaches is based on the spectral dependence of atmospheric radiation on water and ice extinction. So retrieval algorithms use wavelengths at which absorption by water vapor and other gases is minimal, and at which the scattering and absorption by cloud particles are sensitive to the cloud particle size distribution. For example, Nakajima and King (1990) used the reflection function of clouds at a non-absorbing channel in the visible wavelength region, which is primarily a function of the cloud optical thickness, and the reflection function at a water (or ice) absorbing channel in the near-infrared (e.g., 1610 nm channel), which is primarily a function of cloud particle size.

In passive remote sensing, there are two methods that are commonly used for cloud optical and microphysical properties, the infrared (IR) window (split-window) (Inoue, 1985; Parol et al., 1991; Giraud et al. 1997) and visible/shortwave IR (VIS/SWIR) bispectral (Hansen and Pollack, 1970; Twomey and Cocks, 1989; Nakajima and King 1990) approaches. The IR window cloud retrieval is mainly used for optically thin high clouds with cloud optical thickness from 0.1–5 (e.g., Garnier et al., 2012), and the VIS/SWIR method is suitable for optically thick clouds with cloud optical thickness > 1 (Nakajima and King, 1990; Platnick et al., 2003; 2017). Other methods include the IR method which is used to retrieve COT and CER by using the 8.5, 11, and 12 μm bands of the moderate resolution imaging spectroradiometer (MODIS) onboard the Terra and Aqua satellites (Iwabuchi et al., 2014). McBride et al. (2011) introduced a new spectral method for the retrieval of optical thickness and effective radius from cloud transmittance that relies on the spectral slope of the normalized transmittance between 1565 nm and 1634 nm, and on cloud transmittance at a visible wavelength. Most of these retrieval methods assume significant limits on desired parameters, e.g. semi-infinite optical thickness, single scattering albedo equal to unity (conservative scattering), absence of spectral dependence of the optical thickness, etc., and only one or two optical parameters are retrieved (either optical thickness or single scattering albedo or effective radius). In this study, we use an analytical approach that does not apply any such restrictions, making it possible to retrieve optical parameters for every wavelength independently using aircraft measurements of reflected and transmitted solar radiation.

Two cases of marine stratocumulus clouds are considered; first case, offshore of Namibia and the second case offshore of California. Stratocumulus clouds are generally low lying and characterized by an almost uniform base and form over large parts of the world's oceans that are dominated by strong inversion at the top of the boundary layer. These clouds are maintained by turbulent mixing caused by longwave cooling at the cloud top and then sustained by a delicate balance between moisture supply from the surface and entrainment of dry air from the free atmosphere (e.g. Bretherton et al, 2004; Woods, 2012). Their albedo effect dominates their emitted thermal radiation effect and plays an essential role in the climate system, as well as the global water cycle. The absorption of solar radiation and

drizzle formation plays some role in the state of these clouds (Hartmann et al., 1992). Thus, the needs for understanding the interactions between marine stratocumulus clouds and radiation, and how their radiative properties are controlled by their water content distribution, the particle size and their phase, are critical to our comprehension of weather and climate.

2. Experimental Data

2.1 Cloud Absorption Radiometer

The CAR is an airborne multi-wavelength scanning radiometer that measures scattered light in fourteen spectral bands between 0.34 and 2.30 μm (Table 1) (Gatebe and King, 2016).

Data are sampled simultaneously and continuously for spectral bands 0.340 to 1.273 μm , plus one of the six bands on the filter wheel (1.55–2.30 μm) (Gatebe and King, 2016). The filter wheel can either cycle through all six spectral bands at a prescribed interval (usually changing filter every fifth scan line), or lock onto any one of the six spectral bands (King et al., 1986). The CAR scan mirror rotates 360° in a plane perpendicular to the direction of flight and the data are collected through a 190° aperture that allows observations of the earth-atmosphere scene around the starboard horizon from local zenith to nadir. In this study, we report results of data analysis from channels between 0.340 μm and 1.273 μm . The UV bands are analyzed in spite of significant errors, which could impact our results especially for the cloud top cases. The filter wheel channels were not consistently stable during measurements and therefore are excluded from our analysis.

Radiometric calibration is performed at Goddard Space Flight Center (GSFC) prior to and just after the field experiment, at intervals of about two months (Gatebe et al., 2007). In order to determine a suitable calibration for a given flight during the experimental campaign, we assume a linear change between pre- and post-flight calibration and as a function of only the number of flights flown during the field campaign. We note that the calibration ratios post-flight/pre-flight are normally about 0.9 for 340 λ 1273 nm. We use the spectral calibration of the CAR that defines the bandpass functions and central wavelengths conducted for each experiment (cf. Gatebe et al., 2007; Gatebe et al., 2003).

Geometric correction is applied to remove image distortions through algorithms developed for processing CAR Level 1 data that allows pixels to be matched to their actual scan angle by use of airplane roll, pitch, and scan line pixel corresponding to the horizon (matched to a scan angle of 90°) that is easily identified on a scan line by the contrast between sky and surface especially on a clear day. A plot of sky radiance as a function of azimuthal angle helps in identifying asymmetry due to errors in the geometrical correction. It should be noted here that the documented error of the differential global position system (Trimble TANS/vector) is $\pm 0.5^\circ$.

The data files contain geographical coordinates of the observation site, local time, solar zenith angle θ_0 , azimuth angle relative to the Sun ϕ , altitude of the observation, 182 values of radiance in reflectance units I_i in viewing zenith angles θ_i in ranges from zenith to nadir (0–180°) at 1° intervals. At every altitude level of observation several scans with the same

viewing geometry were registered that provide estimation of occasional errors and accomplishing regularization of result.

The data from CAR are available online at NASA Goddard Earth Sciences Data and Information Services Center (GES DISC): <https://disc.gsfc.nasa.gov/information/news/5a5e1d7c8de99badff092774/cloud-absorption-radiometer-car-data-product-release>.

2.2 Marine Stratocumulus Offshore of Namibia and California

The marine stratocumulus clouds off the western coast of southern Africa are layered clouds, associated with subtropical anticyclonic subsidence and cold water upwelling along the Benguela current (Hansen, 1991). These clouds are characterized by a well-defined cloud top height corresponding to a strong boundary layer inversion. They are impacted by both natural (sulfur from phytoplankton DMS production and decay processes) and anthropogenic sources (industrial and fire emission products) that can influence the clouds microphysical and optical properties, an effect that is not quantitatively well understood (cf. Lawrence, 1993). On 13 September 2000, the CAR on board the University of Washington Convair CV-580 research aircraft obtained measurements over these marine stratocumulus offshore of Namibia at two locations, A (21.63°S, 13.62°E) and B (20.66°S, 13.13°E) as shown in Figure 1c. The solar incident angles (θ_0 , ϕ_0) was about (37.14°, 51.13°) at A and (28.47°, 34.27°) at B. The flight altitude ranged from 799 m to 1178 m above mean sea level over point A (see also Fig. 2a, Section a) and 1005 m to 1381 m above mean sea level over point B. Cloud top and base were found to be at levels of about 700 m and 400 m above mean sea level, respectively. The CV-580 flew below, inside and above these clouds as the CAR obtained data through a 190° field of view of the Earth-atmosphere scene around the starboard horizon at selected wavelengths in the UV, visible, and near-infrared (cf. Gatebe et al., 2003). These measurements were taken as a part of the Southern African Regional Science Initiative (SAFARI 2000) campaign (Swap et al., 2002).

The marine stratocumulus clouds off the western coast of California are similar to those offshore of Namibia: layered, but associated with subtropical anticyclonic subsidence and cold water upwelling along the Californian coastline. On 22 June 2008, the CAR onboard the NASA P-3B research aircraft obtained measurements of marine stratocumulus offshore of California at latitude 37.3–36.9°N and 122.2–122.6°W. Cloud base and top were 60 m and 1060 m above mean sea level, respectively. The P-3B flew below, inside and above these clouds as the CAR obtained good data over these marine stratocumulus clouds. These measurements were taken as a part of the Arctic Research of the Composition of the Troposphere from Aircraft and Satellites (ARCTAS 2008) experiment in California (Jacob et al., 2010; Gatebe et al., 2012).

Figure 1(a–b) shows the aircraft ground tracks and satellite images of the cloud environment for the two airborne campaigns during the CAR measurements. Figures 1(c–d) show the flight altitude, which are further illustrated in Figure 2(a–b) from observations made either above or below or inside clouds during the SAFARI-2000 and ARCTAS campaigns.

2.3 Observational uncertainties

During the flight the aircraft position is determined by roll, pitch and yaw (King, 1987). Even with a stable flight, these angles are influenced by random variations, which in turn can lead to random errors in radiance measurements as a function of viewing directions. We need to at least average several scans (4 to 10) at each aircraft altitude in order to obtain an estimate of the observational errors (cf. Melnikova and Vasilyev, 2004).

Averaging the radiances for every viewing zenith angle over all scans (from 4 to 10) at a given altitude provides a standard deviation, where the deviations of the intensity in every scan are compared with the mean square deviation (MSD) of the intensity and then

calculated according to the formulae $MSD = \frac{\sqrt{\sum (I_{i,aver} - I_i)^2}}{I_{i,aver}}$. Intensities with the same

viewing zenith angle are averaged over 4 to 10 scans. Since all the scans are registered within a short time interval (less than a minute), one can therefore assume that the solar zenith and azimuth angles are the same for all the scans and therefore the relative azimuth angle is constant for a straight flight path. And the intensity is to be the same if the aircraft position is not varied. The MSD is taken as an absolute error and it is plotted on the ordinate axis, which is used here to define the random measurement error shown in Fig. 3. Figure 3 shows the observational relative errors of radiance versus the viewing zenith angle for the reflected radiance above the cloud (a), transmitted radiance under the cloud (b), SAFARI 2000 cases (solid lines) and ARCTAS cases (dotted lines) for three spectral channels: 340 nm, 682 nm and 1273 nm. Similar results can be demonstrated for relative errors of downward (D) and upward (U) radiance inside the cloud.

3. Retrieval Methods

3.1 Asymptotic Theory

Our retrieval method is based on asymptotic theory for thick atmospheres, and the presence of a diffusion domain within the clouds. Asymptotic theory provides a rigorous solution to the equation of radiative transfer in optically thick layers. Here, we briefly review the radiative transfer theory and lead to the asymptotic theory as applied in this study.

The radiative transfer equation describes the change in observed intensity in frequency, ν , given a medium of varying optical depth relative to the original radiation source, represented by a source function $S()$ can expressed in the form of a first order differential equation (cf. Chandrasekhar, 1960):

$$\mu \frac{dI(\tau; \mu, \varphi)}{d\tau} = I(\tau; \mu, \varphi) - S(\tau; \mu, \varphi) \quad (1)$$

where $I(\tau, \mu, \varphi)$ represents the total radiant intensity (direct and diffuse) at an arbitrary level defined by an optical depth τ in a unit solid angle along a direction (μ, φ) , μ is the cosine of the emergent direction, φ is the azimuth angle of emergent direction from a reference plane, and $S(\tau, \mu, \varphi)$ is the source function, which represents augmentation of radiation in a medium

characterized by scattering and emission and represents several processes such as single scattering of the direct solar radiation, multiple scattering of the diffuse intensity, and emission by the media. Thus, the source function can be expressed mathematically as:

$$S(\tau; \mu, \varphi) = \frac{\omega}{4\pi} \int_0^{2\pi} \int_{-1}^1 p(\mu, \varphi; \mu', \varphi') I(\tau, \mu', \varphi') d\mu' d\varphi' + S'(\tau; \mu, \varphi) \quad (2)$$

where ω is the single scattering albedo, the ratio of scattering to extinction coefficient, $p()$ is the phase function and describes a single scattering event from (μ', φ') to (μ, φ) . S' represents radiation arising from internal or external sources of radiation.

If the total radiance is separated into direct and diffuse radiation fields, Eq. (1) and (2) can be reduced to the integro-differential equation for the diffuse radiance in the form:

$$\begin{aligned} \mu \frac{dI_d(\tau; \mu, \varphi)}{d\tau} &= I_d(\tau; \mu, \varphi) - \frac{\omega_0}{4\pi} \int_0^{2\pi} \int_{-1}^1 p(\mu, \varphi; \mu', \varphi') I_d(\tau; \mu', \varphi') d\mu' d\varphi' \quad (3) \\ &- \frac{\omega_0}{4\pi} F_0 p(\mu, \varphi; -\mu_0, \varphi_0) \exp\left(-\tau/\mu_0\right) + S'(\tau; \mu, \varphi) \end{aligned}$$

Using an upper and lower boundary conditions defined by:

$$I^-(0, \mu, \varphi) = \delta(\mu - \mu_0) \delta(\varphi - \varphi_0) \pi F_0 \quad (4a)$$

$$I^+(\tau, \mu, \varphi) = \frac{1}{\pi} \int_0^{2\pi} \int_0^1 A_g(\mu, \varphi; \mu', \varphi') I^-(\tau, -\mu', \varphi') d\mu' d\varphi' \quad (4b)$$

where the delta function $\delta()$ is zero everywhere except at the origin and $A_g(\mu, \varphi; \mu', \varphi')$ is the bidirectional reflectance distribution function, which depends on the incidence (μ', φ') and reflectance (μ, φ) directions. The simplest methods for finding a solution include Trapezium and Simpson rules, while the more advanced techniques such as Gaussian quadrature rule are suitable for integration of complex functions and considered better for numerical integration. Other solution methods and techniques are described in Chandrasekhar's book (Chandrasekhar, 1960). The discrete-ordinate method (DISORT), spherical-harmonic method, adding-doubling method, and Monte Carlo are now the main computation methods used and are reviewed in a book by Lenoble (1985). However, the solution of these methods becomes increasingly difficult as optical thickness increases. Consequently, three classes of radiative transfer approximations that include asymptotic theory for thick layers, two-stream approximations, and single scattering have been used to derive approximate expressions for

the quantities characterizing the radiation field such as the plane albedo, total transmission and fractional absorption of a layer. These approximations differ substantially in both their assumptions and accuracies as a function the optical thickness and solar zenith angle (King and Harshvardhan, 1986).

Note that, if the surface is black (e.g., over the ocean at near-infrared wavelengths outside of sunglint), $A_g(\mu, \phi; \mu', \phi') = 0$. For $A_g(\mu, \phi; \mu', \phi') = \text{constant}$, the surface reflectance is described by Lambert's law and for a surface dominated by specular reflection, e.g., a quiet flat ocean, $A_g(\mu, \phi; \mu', \phi') = \delta(\mu - \mu') \delta(\phi - \phi') A_f$, where A_f is given by the Fresnel's law. In reality A_g is generally a complex function of both incidence and reflectance directions and is not well known for various kinds of surface boundaries and sometimes is estimated by empirical equations.

The retrieval approach taken in this study relies on asymptotic theory for thick atmospheres, and the presence of a diffusion domain within the clouds. According to King et al. (1990), in the diffusion domain region of an optically thick cloud, the diffuse radiation field is azimuthally independent of the solar zenith angle and assumes an asymptotic form characterized by simple properties such as similarity parameter and single scattering albedo $\omega = 1$ (conservative case), the expressions for reflection function ($R(\tau_c; \mu, \mu_0, \varphi)$) and transmission function ($T(\tau_c; \mu, \mu_0, \varphi)$) can be defined by (e.g. van de Hulst, 1980; King, 1987):

$$R(\tau_c; \mu, \mu_0, \varphi) = R_\infty(\tau_c; \mu, \mu_0, \varphi) - \frac{K(\mu)K(\mu_0)}{3(1-g)(\tau_c + 2q_0)} \quad (5)$$

$$T(\tau_c; \mu, \mu_0, \varphi) = \frac{4K(\mu)K(\mu_0)}{3(1-g)(\tau_c + 2q_0)} \quad (6)$$

where $R_\infty(\tau_c; \mu, \mu_0, \varphi)$ is the reflection function of a semi-infinite cloud layer, which can be obtained by solving the Ambartsumian's nonlinear integral equation (cf. Mishchenko et al., 1999; Zhao and Sun, 2010), $K(\mu)$ the escape function and q_0 the extrapolation length for conservative scattering.

For the non-conservative case ($\omega < 1$):

$$R(\tau_c; \mu, \mu_0, \varphi) = R_\infty(\tau_c; \mu, \mu_0, \varphi) - \frac{ml}{1 - l^2 e^{-2k\tau_c}} K(\mu)K(\mu_0) e^{-2k\tau_c} \quad (7)$$

$$T(\tau_c; \mu, \mu_0, \varphi) = R_\infty(\tau_c; \mu, \mu_0, \varphi) - \frac{ml}{1 - l^2 e^{-2k\tau_c}} K(\mu) K(\mu_0) e^{-2k\tau_c} \quad (8)$$

where k is the diffusion exponent representing the attenuation of radiation in the diffusion domain, and m and l are constants which are governed by the single scattering albedo and asymmetry factor (King 1987).

The functions and constants in equations (5–8), e.g. the escape function $K(\mu_0)$ as well as the asymptotic constants q_0 and l can be obtained using asymptotic fitting method suggested by van de Hulst (1968) whereby computational results from the doubling method are fit to known general forms of the asymptotic equations. Nakajima and King (1992) derived matrix equation for computing the functions and constants that appeared in the asymptotic expressions by using discrete ordinates method of radiative transfer. Zhao and Sun (2010) used an iterative method to solve the Ambartsumian's nonlinear integral equation, suggested by Mishchenko et al. (1999) by expanding the phase function in a Legendre polynomial series. They obtained diffusion pattern and the diffusion exponent by solving a characteristic equation with a combination of the normalization condition of diffusion pattern and the so-called Sobolev-van de Hulst relation.

In the following sections, we will describe our method of obtaining the diffusion pattern and the diffusion exponent.

3.2 Algorithm Description

Most of the earlier retrieval approaches assumed significant limits for desired parameters: semi-infinite optical thickness, single scattering albedo equaling unity (non-absorbing scattering), absence of spectral dependence of the optical thickness, etc., and only one optical parameter could be retrieved (either optical thickness or single scattering albedo) (cf. King, 1987; King et al., 1990; Rozanov and Kokhanovsky, 2004). Here, the method described in (Melnikova and Vasilyev, 2004; Melnikova and Mikhailov, 1994; Melnikova et al., 2000b) is used for cloud optical parameters retrieval. It does not put such restrictions and makes it possible to retrieve two or even three optical parameters (optical thickness, τ_0 , single scattering albedo, ω , and phase function asymmetry parameter, g , for every wavelength independently, and has already been applied to satellite data and ground-based observations of intensity and aircraft flux measurements of reflected and transmitted solar radiation. In this paper, to solve the problem, we used measurements at the top and the bottom of the cloud, as well as measurements taken within the cloud. Different expressions have been derived for every case (Melnikova et al., 2009). The algorithm of the problem solution is described in (Genya et al., 2010). The following optical parameters are determined directly from measurements: $s^2 = (1 - \omega) / [3(1 - g)]$ – a parameter that describes absorption in the cloud, $\tau' = 3(1 - g) \tau_0$ – triple scaled optical thickness defining scattering of the radiation, and the phase function asymmetry parameter g . Note that the parameter s^2 is convenient in our consideration because it is used in the expansions of the asymptotic constant and functions. Subject to the geometrical thickness z of the whole cloud or

sublayer thickness between measurements levels z_i it is easy to change to volume absorption $\kappa = s^2 \tau' / z$ and extinction coefficient $\alpha = \tau_0 / z$.

Equations for computing the escape function, diffusion pattern, diffusion exponent, and the reflection function of a semi-infinite atmosphere are given in the next section, and their full derivations are shown in the Appendix.

Figure 4 shows spectral radiance (870 nm and 1273 nm) as a function of CAR scan angle (where 0° corresponds to zenith direction and 180° corresponds to nadir direction) for scattered radiation measured at different flight altitude ranging from 372 m to 1178 m above mean sea level, offshore of Namibia during SAFARI 2000 experiment. The cloud top and base were found to be at 800 m and 400 m above mean sea level, respectively, so the first two curves (Alt_372m and Alt_393m) show observations taken below the cloud, and the last two lines (Alt_804m and Alt_1178m) were taken above the cloud. The rest of the lines represent observations inside the cloud at different distances from the cloud edge. The peak intensity seen coincides with the solar incident angle of about 37° . The curves corresponding to levels 372 m, 393 m, and 404 m point to diffusion domain (scan angles: $0-90^\circ$) because there is no solar aureole peak. Clearly, measurements inside the cloud, especially those that were obtained too close to the top (altitude levels from 639 m to 1178 m) do not satisfy the diffuse domain conditions as shown in Figure 4 as per the maximums on the solar aureole. King et al. (1990) describe the criteria that can be used to determine whether an individual scan of the CAR inside an optically thick water cloud pertains to the diffusion domain (cf. Gatebe et al., 2012).

Even though the diffusion conditions were not met in most cases, we attempted to retrieve the optical parameters from measurements inside the cloud to test the performance of our approach in non-diffusion conditions. Processing data was accomplished for the viewing directions close to the nadir and zenith, which turned out to give a positive solution for most sub-layers between the measurement levels. The break of the diffusion domain condition leads to greater processing errors, which are accounted for through regularization of the solution. The averaging and estimation of the standard deviation is accomplished using a minimum of 6 scans at every level. The special procedure is analogous to regularization that permits taking into account an average error at every height, which is then applied to vertical profiles to help with smoothing. We should note that observations taken further away from the cloud top or cloud edge could provide more reasonable results with a higher accuracy.

3.3. Method of processing experimental data

3.3.1. Observation above the cloud—Multiangular data of reflected radiation at cloud top gives intensity ρ_j in relative units of the incident solar flux in viewing zenith angles $\theta_j = \arccos \mu_j$, the ratio of two differences $[\rho_\infty(\mu_1, \mu_0, \varphi) - \rho_1] / [\rho_\infty(\mu_2, \mu_0, \varphi) - \rho_2]$ is considered, where ρ_1 and ρ_2 are the observed intensities in two viewing zenith angles $\arccos \mu_1$ and $\arccos \mu_2$, and $\rho_\infty(\mu_i, \mu_0, \varphi)$ – reflection coefficient of semi-infinite conservative atmosphere ($\tau_0 = \infty$, $\omega_0 = 1$) which can be calculated using the approximation by Melnikova and Vasilyev (2004). Derivations of these methods are shown in the Appendix. Further details are also available in Melnikova et al. (2000a). After algebraic transformations,

expressions for the parameter s^2 (Melnikova et al., 2000b) and triple scaled optical thickness τ' (King, 1987) are given by:

$$s^2 = \frac{[\rho_\infty(\varphi, \mu_1, \mu_0) - \rho_1]K_0(\mu_2) - [\rho_\infty(\varphi, \mu_2, \mu_0) - \rho_2]K_0(\mu_1)}{[\rho_\infty(\varphi, \mu_2, \mu_0) - \rho_2]K_0(\mu_1) \left(\frac{K_2(\mu_1)}{K_0(\mu_1)} - \frac{K_2(\mu_2)}{K_0(\mu_2)} \right) + \frac{a_2(\mu_0) K_0(\mu_1)a_2(\mu_2) - K_0(\mu_2)a_2(\mu_1)}{12q'} \frac{1}{[\rho_\infty(\varphi, \mu_2, \mu_0) - \rho_2]K_0(\mu_1)}} \quad (9)$$

$$\tau' = (2s)^{-1} \ln \left\{ \frac{m\bar{l}K(\mu_i)K(\mu_0)}{\rho_\infty(\varphi, \mu_i, \mu_0) - \rho_i} + \bar{l}\bar{l} \right\} \quad (10)$$

where $q' = 0.714$. m and l are determined by the optical properties of the considered cloud and are calculated by formulas in (Melnikova and Vasilyev, 2004) after determining parameter s . The overbar over the value $\bar{l}$ indicates that the function accounts for the ground albedo A . Subscript i implies that determination of τ' is possible for both viewing zenith angles in the pair. $K(\mu_i)$, $K_0(\mu_i)$ and $K_2(\mu_i)$ are escape functions, and the coefficients of their expansions over the small parameter s ; $a_2(\mu_i)$ is the second coefficient in the expansion of the plane albedo function defined as follows:

$$K_0(\mu) = 0.797\mu + 0.442 \quad (11)$$

$$K_2(\mu) = 5/3n_2(\mu^2 + 0.1)$$

$$a_2(\mu) = 3K_0(\mu) \left(\frac{3}{1+g} (1.271\mu - 0.9) + 4q' \right)$$

The form of these functions is known and their values for fixed solar and viewing zenith angles can be found in tables or calculated using the approximation by Melnikova and Vasilyev (2004).

3.3.2 Observation under the cloud.—Similar expressions for a case of multiangular data of intensity σ_j of transmitted solar radiation in relative units of the incident solar flux (transmission coefficient) under the cloud are obtained in Melnikova et al. (2000b) and summarized below:

$$s^2 = \left[\frac{\sigma_1 \bar{K}_0(\mu_2)}{\sigma_2 \bar{K}_0(\mu_1)} - 1 \right] \frac{1}{\frac{\bar{K}_2(\mu_1)}{\bar{K}_0(\mu_1)} - \frac{\bar{K}_2(\mu_2)}{\bar{K}_0(\mu_2)}}, \quad (12)$$

$$\tau' = s^{-1} \ln \left[\frac{\sqrt{4\sigma(\tau, \mu_j, \mu_0)^2 \bar{l}\bar{l} + m^2 \bar{K}(\mu_j)^2 K(\mu_0)^2} + m\bar{K}(\mu_j)K(\mu_0)}{2\sigma(\tau, \mu_j, \mu_0)} \right],$$

The overbar over the function $\bar{K}(\mu)$ and the value $\bar{l}$ indicates that the function accounts for the ground albedo A :

$$\bar{l} = l - \frac{A m n^2}{1 - A a^\infty}, \quad \bar{K}(\mu) = K(\mu) + \frac{A a(\mu) n}{1 - A a^\infty}$$

Details of derivations are in the Melnikova et al., (2000b).

3.3.3 Observation inside the cloud—Below are the formulas for determining the optical parameters from measurements inside the cloud. Note that the expressions for parameters retrieval are different for the cloud sublayer adjacent to the top, base and away from the boundaries. Denoting $I_i^\downarrow = I(\mu)$, $I_i^\uparrow = I(-\mu)$ and $J_i = I_i^\downarrow - I_i^\uparrow$. There are n cloud sublayers between observational levels, and subscript i points to the number of the observational level within a cloud layer. The derivation of formulas for processing observation inside the cloud is presented in the Appendix.

For the upper sub-layer adjacent to the cloud top (Melnikova et al., 2009):

$$s^2 = \frac{(\rho_\infty - \rho)^2 - 4J_1^2 J K_0^2(\mu) / 9\mu^2}{16K_0^2(\mu)K_0^2(\mu_0) + (\rho_\infty - \rho)^2 \left[2\frac{K_2(\mu)}{K_0(\mu)} + \frac{48q'K_0^2(\mu)K_0^2(\mu_0)}{(\rho_\infty - \rho)} - \frac{a_2(\mu_0)a_2(\mu)}{6q'(\rho_\infty - \rho)} - \frac{(I_1^\uparrow + I_1^\downarrow)^2}{(I_1^\uparrow - I_1^\downarrow)^2} \right]} \quad (13)$$

$$\tau'_1 = \frac{1}{2s} \ln \left\{ l \left[\frac{I_1^\downarrow(1 - 6\mu s + 18\mu^2 s^2) - I_1^\uparrow}{I_1^\downarrow - (1 - 6\mu s + 18\mu^2 s^2)I_1^\uparrow} \right] \frac{(\rho_\infty - \rho) + 4K_0(\mu_0)K_0(\mu)s + \frac{a_2(\mu_0)a_2(\mu)}{12q'}s^2}{(\rho_\infty - \rho) - 4K_0(\mu_0)K_0(\mu)s + \frac{a_2(\mu_0)a_2(\mu)}{12q'}s^2} \right\}$$

For the internal cloud sub-layers:

$$s^2 = \frac{(J_i^2 - J_{i-1}^2)J_i^2}{36\mu^2[J_{i-1}^2 I_i^\downarrow I_i^\uparrow - J_i^2 I_{i-1}^\downarrow I_{i-1}^\uparrow]} \quad (14)$$

$$\tau'_i - \tau'_{i-1} = \frac{1}{2s} \ln \left[\frac{J_{i-1} - 6I_{i-1}^\downarrow \mu s(1 - 3\mu s)}{J_{i-1} + 6I_{i-1}^\uparrow \mu s(1 - 3\mu s)} \right] + \frac{1}{2s} \ln \left[\frac{J_i + 6I_i^\uparrow \mu s(1 - 3\mu s)}{J_i - 6I_i^\downarrow \mu s(1 - 3\mu s)} \right]$$

For the lower sub-layer adjacent to the cloud base:

$$\begin{aligned}
s^2 = & \frac{9\mu^2\sigma^2/K_0^2(\mu)J_{n-1}^2 - 1}{\frac{2}{\bar{K}_0(\mu)} \left[K_2(\mu) + A \frac{a_2(\mu) + n_2 + 24\bar{K}_0(\mu)\bar{q}'}{1-A} + \frac{12q'A^2}{[1-A]^2} \right] - 9q' - 9\mu^2 \frac{(I_{n-1}^\downarrow + I_{n-1}^\uparrow)^2}{J_{n-1}^2}} \quad (15) \\
\tau'_n - \tau'_{n-1} = & \frac{1}{2S} \ln \left[\frac{2}{3\mu\sigma} \frac{I_{n-1}^\downarrow - I_{n-1}^\uparrow (1 - 6\mu s + 18\mu^2 s^2)}{1 - 3\mu s + 9\mu^2 s^2} \right] \\
& + \ln \left[K_0(\mu)(1 - 3q's) + K_2(\mu)s^2 + \frac{A}{1-A} (1 - 3q's + n_2 s^2)(1 - 4K_0(\mu)s + a_2(\mu)s^2) \right]
\end{aligned}$$

Observations at every level within the cloud are used for retrieving optical parameters of the layer between levels for all chosen viewing zenith angles. Then obtained values are averaged to help smooth out observational and retrieval errors.

3.4 The case of conservative scattering

The single scattering albedo is equal to unit and the parameter s^2 is zero. The optical thickness of the sublayer adjacent to cloud top is expressed with formulas that are derived from known relations of conservative cases (e.g. King et al., 1990):

$$3(1-g)\tau_1 = \frac{4K_0(\mu)K_0(\mu_0)}{\rho_\infty - \rho} - \frac{3\mu(I_1^\downarrow + I_1^\uparrow)}{(I_1^\downarrow - I_1^\uparrow)} - 3q' \quad (16)$$

For the scaled optical thickness of sublayers inside the cloud the following formula is used:

$$3(1-g)(\tau_i - \tau_{i-1}) = 3\mu \left[\frac{(I_{i-1}^\downarrow + I_{i-1}^\uparrow)}{(I_{i-1}^\downarrow - I_{i-1}^\uparrow)} - \frac{(I_i^\downarrow + I_i^\uparrow)}{(I_i^\downarrow - I_i^\uparrow)} \right] \quad (17)$$

And the following formula is used for the scaled optical thickness of the sublayer adjacent to the cloud base:

$$3(1-g)\tau_{n-1} = \frac{4\bar{K}_0(\mu)K_0(\mu_0)}{\sigma} - 3\mu \frac{(I_{n-1}^\downarrow + I_{n-1}^\uparrow)}{(I_{n-1}^\downarrow - I_{n-1}^\uparrow)} - 3q' \quad (18)$$

Details of derivations are in Appendix.

3.5 The ground albedo retrieval

It should be noted that a difficulty arises when the ground albedo A is obtained from measurements. The ground albedo is determined in terms of fluxes; in our case, observations only include the radiation intensity. The relation $A = I(48^\circ)/I(132^\circ)$, is used here to determine the ground albedo from observations as discussed elsewhere (e.g. Melnikova and Vasilyev, 2004; Varotsos et al, 2014) with a Lambertian surface assumption. It uses observations obtained under the cloud in two viewing zenith angles 48° and 132° . The mean value of albedo is calculated from all the scans at a selected altitude 370 m, where the MSD, dA, of the ground albedo is estimated and presented in Table 2 in different spectral channels for both experiments. It is seen that the standard deviations are quite small.

3.6 Phase function asymmetry parameter.

We also retrieved the phase function asymmetry parameter. Let us note the retrieved asymmetry parameter as G in order to differentiate it from g from the Mie calculation for cloud droplets. There is the following expression proposed in (Melnikova and Vasilyev 2004; Melnikova and Mikhailov, 2001):

$$G = \left[\frac{\sigma_1 \bar{K}_0(\mu_2)}{\sigma_2 \bar{K}_0(\mu_1)} - 1 \right] / 5s^2(\mu_1 - \mu_2) \quad (19)$$

The parameter s^2 might be obtained from observations inside the lower sublayer, and then substituted in the expression (19) for retrieving the asymmetry parameter G . Using data from many viewing directions allows the retrieval, and the regularization provides a stable solution.

3.7 Regularization of the solution

Therefore, as already mentioned above, random variations in the angles that define an aircraft flight affect measurement uncertainty, which could lead to the angular dependence of errors in the observed intensity. The heterogeneity of the real cloud contributes also to considerable uncertainty. The estimation of these errors is provided by several scans registered at every observational level. Fig. 3 shows the mean square deviation of the intensity, obtained by averaging over the scans measured above the cloud top (cf. Fig. 3a) at the altitude of 800 m, under the cloud (cf. Fig. 3b) at the altitude of 370 m in 3 spectral channels, and within the cloud at the level 670 m (Fig. 3c) and 200 m (Fig 3d) in 6 spectral channels. In Figs 3a and 3b the spectral channels are indicated in the legend. The observational errors were used in the regularization procedure together with the retrieval errors. This involves the use of a procedure developed by Melnikova & Vasilyev (2009):

$$x = \frac{\sum_j x_j \frac{1}{(\Delta x_j)^2}}{\sum_j \frac{1}{(\Delta x_j)^2}}, \quad (20)$$

where, x_j is the retrieved parameter for j^{th} viewing zenith angle (or J^{th} pair of angles); x_j is the sum of measurement errors and processing or computation errors. They are calculated using expressions for indirect errors and which give the uncertainty of desired values. Processing errors are obtained from the formulas (9) – (18) for retrieving the required parameter (τ' , s^2 and g). The analysis of the inversion formulas for the errors caused by the proposed procedure of data processing is described in Melnikova & Vasilyev (2004) and Melnikova et al. (2009). For example, the case of observation under the cloud gives the following expression for parameter s^2 uncertainty:

$$\frac{\Delta s}{s} = \frac{\Delta \sigma \bar{K}_0(\mu_1) + \sigma \Delta \bar{K}_0}{\sigma_1 \bar{K}_0(\mu_2) - \sigma_2 \bar{K}_0(\mu_1)} + \frac{\Delta \sigma}{2\sigma} + \frac{\Delta \bar{K}_0}{2\bar{K}_0(\mu)} + \frac{2\mu \Delta \mu}{\left(\frac{\bar{K}_2(\mu_1)}{\bar{K}_0(\mu_1)} - \frac{\bar{K}_2(\mu_2)}{\bar{K}_0(\mu_2)} \right)^2} \quad (21)$$

and for the scaled optical thickness:

$$\frac{\Delta \tau'}{\tau'} = \frac{\Delta s}{s} + \frac{1}{\tau'} \left[\frac{\Delta \sigma}{\sigma} + 3 \frac{\Delta s}{s} + 2 \frac{\Delta K}{K} - \frac{\frac{2\sigma(\mu_{1,2})^2}{\bar{l} \bar{m} \bar{K}(\mu_{1,2})^K(\mu_0)} \left(4 \frac{\Delta \sigma}{\sigma} + 3 \frac{\Delta s}{s} + 2 \frac{\Delta K}{K} \right)}{2\bar{l} \sqrt{\frac{2\sigma(\mu_{1,2})^2}{\bar{l} \bar{m} \bar{K}(\mu_{1,2})^K(\mu_0)} + 1} \left(\sqrt{\frac{2\sigma(\mu_{1,2})^2}{\bar{l} \bar{m} \bar{K}(\mu_{1,2})^K(\mu_0)} + 1} + 1 \right)} \right] \quad (22)$$

σ/σ – is the observational uncertainty. Similar expressions are derived for all other cases of observations that were used for the retrieval. Uncertainties calculated with these formulas are used for the regularization of the results in the processing algorithm. This is a typical inversion problem and results in a solution that demonstrates instability, which is clearly seen in Fig 5. It shows no regularized results of retrieved optical thickness and parameter s^2 versus viewing zenith angles. It is seen that the optical thickness in some cases, obtained from measurements under the cloud (Fig. 5a, base, SAFARI) and above cloud (Fig 5c, 340 and 470 nm) practically does not depend on the viewing zenith angles (as it should be, ideally).

The regularization in this example accounts for the dependence of observational errors in viewing directions and cloud irregularities, and gives the value of parameter $s^2 = 0.001568$ (base) and 0.001469 (top), in the time when simple averaging without accounting for the errors dependence on the viewing zenith angles, brings the value to 0.002004 (base) and 0.001327 (top). The differences are 22% and 10%, respectively. The negative values at certain viewing directions are possible because it is the ill-posed inverse problem. Just by this reason the regularization procedure is needed. Here we put no restriction to the parameter s^2 (even the demand of a positive value) as is usually done in many cases of regularization of inverse problem solution. It is possible to put this additional restriction, but the result appears positive without it.

4. Results and discussions

4.1 Optical parameters from top and base observations

The optical parameters (ground albedo A , parameter s^2 and scaled optical thickness τ') for the whole cloud is obtained with the use of the above formulas from observations at the cloud base (B) at a height of 370 m, and at the cloud top (T) at the height 800 m. The ground albedo is presented in Table 2. Results similar to Table 2 and Figure 4 were also reported by King et al. (1990).

The approach for obtaining the parameter s^2 needs observation from two viewing angles from both above (T) and below (B) the cloud. The selected pair of radiation intensities in two viewing zenith angles in the Eqs. (1) and (3) has to be successively sampled.

Measurements data are analyzed for a range of zenith (transmitted radiation: 0-60°) and nadir (reflected radiation; 120-180°) viewing zenith angles. Viewing zenith angles very close to the horizon were not considered due to increased uncertainties in the plane parallel layer model.

During processing, we considered only the viewing angles that meet the proximity values of the optical thickness determined in the approximation of conservative scattering (up to 0.5%) and the difference of cosines of viewing angles in a pair more than 0.2 for diminishing the processing error according to corresponding formulas, because the close values in nearby directions lead to small differences in the numerator and to increased errors of the retrieval.

Next, the obtained values of the required parameters are averaged over all considered pairs with a weight equal to the inverse value of the error for the regularization of solution. The processing algorithm provides an assessment of the standard deviations of the results. After retrieving parameter s^2 , the scaled optical thickness is obtained with a similar procedure for regularization. Results are presented in Table 3.

The sum of optical thicknesses retrieved from observations between the levels inside the cloud, shown in the row noted as SAFARI $\Sigma\tau_i$ of Table 3 are closer to values retrieved from observations at the cloud base. Results obtained from ARCTAS top and base observations are close to each other. In the case of SAFARI data the results differ especially in the UV 340 nm channel. It seems that errors of top observations were higher or it might be explained by the impact of stray light scattering within the instrument because of strong registered signal above cloud, which diminishes the received signal.

To obtain the single scattering albedo (SSA), ω , the optical thickness, τ and extinction coefficient, σ the spectral values of the asymmetry parameter, g were taken according to (Stephens, 1979; Lobanova et al, 2010).

The extinction and absorption coefficients are presented in Table 4 for both CAR experiments retrieved from cloud top and base observations.

Fig. 6a demonstrates spectral values of the water surface albedo obtained with the above described procedure from CAR observations at levels 370 m (SAFARI) and 50 m (ARCTAS) and albedo spectral dependences directly calculated from the ratio of measured upward and downward fluxes at 300 m above sea surface (Varotsos et al., 2014). The comparison of cloud single scattering co-albedo (1-SSA) and optical thickness retrieved from the CAR data and Russian observations of spectral fluxes above sea surface is shown in the Fig. 6b. SAFARI 2000 results are represented as blue symbols: rhombus – for observations under the cloud base, triangles – for observations above the cloud top. ARCTAS results are represented by red symbols. Spectral fluxes have been measured at the top and base of the cloud above the Atlantic Ocean in the range of Global Atlantic Tropical Experiment – GATE (1974), Azov Sea (1972) and Black Sea (1971) in the range of Complete Atmospheric Energetics Experiment - CAENEX (Kondratyev, 1972; Kondratyev et al., 1977). The radiative divergence shown in Fig. 6c is calculated as the second difference from flux observations, and applying the Eddington approach on the base of retrieved optical parameters from the CAR experiments. These results might be a kind of validation of retrieval results (cf. Gatebe et al., 2014).

4.2. Vertical profiles of optical parameters inside cloud

During the SAFARI-2000 campaign, the CAR measurements inside the cloud were taken within 10 m of cloud top (aircraft altitude levels from 600 m to 800 m) and do not completely satisfy the diffusion domain conditions. But we considered it as a good case to test our approach. We attempted to retrieve the optical parameters from measurements at all available levels inside the cloud for view zenith angles within 10° from nadir and zenith directions (and 10° away from the solar aureole), which turned out to be successful for most sub-layers between the observational levels. The averaging and estimation of the standard deviation is accomplished using over 6 scans at every level. Results appeared negative at the two levels nearest to the top and were omitted. The positive solution was obtained at all the lower levels. It is important to point out that the errors of this retrieval increase because the diffusion domain conditions are not satisfied.

The special procedure, analogous to regularization, while taking into account the average errors at every height, is applied to vertical profiles for smoothing. Figures 7 (a) and (b) show vertical profiles of the volume absorption and scattering coefficients retrieved from CAR observation taken on September 13, 2000.

Finally, the optical thickness retrieved in all sublayers was summed and compared with the result obtained for the whole layer. The summed values are presented in Table 3, raw SAFARI $\Sigma\tau_i$. The good agreement with retrievals from cloud base and cloud top shows consistency in our approach. Scattering and absorption coefficients locally in thin layers from 685 m to 750 m are high enough and thus, these high values might point to local cloud heterogeneity. Their variations are similar in all the spectral channels, which again point to consistency in our approach and might point to the cloud vertical heterogeneity. The scattering coefficient values $300 - 500 \text{ km}^{-1}$ are reliable for a tropical cloud (values of droplet is about $10 \text{ }\mu\text{m}$, concentration is about 100 cm^{-3} , which would provide a scattering coefficient of about 300 km^{-1}).

There are two maximum peaks of the absorption coefficient at heights 647 m and 685 m, and the lower one coincides with the scattering coefficient maximum. It is necessary to take into account that observations were taken too close to the cloud top, which seems to push the asymptotic formulas region of applicability (diffuse domain) and leads to larger retrieval errors. However, these retrieval results indicate that the positive continuous solution would not exist if the region of applicability is completely violated as described by King et al. (1990) (cf. Gatebe et al., 2012). Here we gain by obtaining the result, but with increased uncertainty. It is important to note that the approach appears effective even for conditions that does not meet diffusion domain criteria and might be recommended for other similar observational data.

Observations inside cloud from the ARCTAS-2008 experiment has been accomplished in the lower part of the cloud (50-258 m) in 10 meters intervals. Retrieval results are demonstrated in the Figures 7 (c) and (d), where one can see considerable irregularity with respect to height within the cloud especially for absorption coefficient. Maximal values of coefficients are similar to values obtained in SAFARI-2000 cases.

4.3 Phase function asymmetry parameter

Values of the parameter s^2 , obtained in the lower sublayer from ARCTAS-2008 cloud cases were used for estimation of the phase function asymmetry parameter G using the expression (19). In processing the data, we used 85 viewing directions, which give many pairs for estimating the parameter G . The observational errors together with retrieval errors of the parameter s^2 were accounted for using regularization.

The SSA and optical thickness are re-retrieved using new values of parameter G in every spectral channel for the whole cloud. Results are presented in Table 5. Mean square deviation is calculated over all considered viewing direction pairs that were used to retrieve the uncertainty. Theoretical values of the parameter g (Stephens, 1979; Lobanova et al., 2010) that was used for the above data processing are also shown in Table 5 for comparison. It is seen that the asymmetry parameter values are lower than calculated from Mie scattering theory. We speculate that this might be due to the influence of molecules and atmospheric aerosols, which regulate the scattering process in cloud together with droplets.

Other optical parameters change compared with values retrieved from observation at the cloud base in Table 2 because of the parameter G difference from values of g . Mean square deviation appears stable over the wavelength channels for all the retrieved parameters.

4.4. Radiative calculation on the basis of retrieved optical parameters

We also considered 2 cloud optical models named here as SAFARI 1 and ARCTAS 1, which contain the retrieved optical thickness, SSA, and ground albedo, based on retrievals below the cloud which were found to be more smooth and stable. The asymmetry parameter was taken from theoretical calculation by Stephens (1979) and Lobanova (2010). And also models SAFARI 2 and ARCTAS 2 corresponding to all retrieved optical parameters including asymmetry parameter (Table 6).

The reflectivity and net flux at the cloud top, transmissivity at the cloud base and radiative divergence are calculated using the Delta-Eddington method (Joseph et al., 1976).

Two-stream methods provide an acceptable accuracy to calculate the radiative fluxes and divergence, which is of particular interest in the study of the radiation regime of the atmosphere (King and Harshvardhan, 1986). Since the Delta-Eddington method and its application has been published, we will not repeat the set of basic formulas that were used in this study.

Fluxes of the solar radiation (irradiances) reflected $F^{\uparrow}$ at the cloud top $\tau=0$ and transmitted $F^{\downarrow}$ to the cloud base are calculated in relative units of the incident solar flux $F_0\mu_0$ for solar angles $\arccos\mu_0=30^\circ$ and 26° , together with the radiative divergence D in the cloud layer (absorption). After calculation of radiation characteristics, the values obtained should be multiplied by the cosine of the solar zenith angle and values of the incident solar flux at the top of the atmosphere at the corresponding wavelength.

It should be noted that different values of the solar flux in the UV region are given in publications because of the high spectral variability and different spectral intervals for averaging. Here, we used the values from (Makarova et al., 1991). It is necessary to calculate net fluxes at the top, base and within a layer in order to estimate the cloud forcing.

For calculating radiative cloud forcing the radiative characteristics for a clear atmosphere with similar aerosols are needed. It was shown in (Ginzburg et al., 2016) that the clear and cloudy atmospheric layers containing same aerosols are distinguished only by scattering optical thickness of a cloud and have the same absorption optical thickness. Values of the absorption optical thickness obtained here for clouds point to the model Aerosol 2 from (Ginzburg et al., 2016).

Thus, the absorption coefficient (Table 6) is used for clear optical model and the scattering optical thickness for clear atmosphere is taken from (Ginzburg et al., 2016). The optical models taken for our calculations are presented in Table 6. Values of the aerosol optical thickness assumed here for SAFARI 1 model are consistent with values observed during the SAFARI-2000 Field Campaign (Magi et al., 2008), which also points to reliability of the above obtained results. The radiative characteristics for clouds and clear layers are presented in Table 7 in relative units. Values appear in ranges of observation from (Dong et al., 2006; Bouniol et al., 2012).

4.5 The radiative forcing

We should point out that the radiative forcing analyzed here is local and instantaneous value differs from the classical notion corresponding to global and time averaged values. We determined values of the direct (instantaneous for solar zenith angle $\arccos\mu_0$) cloud forcing at the top f_T , base f_B , and within f_A the cloud layer using the following approach:

$$\begin{aligned}
 f_T &= \left[(1 - F^\uparrow)_{cloud} - (1 - F^\uparrow)_{clear} \right] F_0^\downarrow \mu_0 \\
 f_B &= \left[(1 - A) F_{cloud}^\downarrow - (1 - A) F_{clear}^\downarrow \right] F_0^\downarrow \mu_0 \\
 f_A &= [D_{cloud} - D_{clear}] F_0^\downarrow \mu_0
 \end{aligned} \quad (23)$$

Calculated spectral values of forcings at the cloud top, base and within cloud are presented in Table 8 in relative units of the incident flux together with values integrated over wavelength in energetic units. It is seen that practically all forcing are negative. The models with Mie calculated phase function parameter g lead to smaller values of forcings than models containing retrieved values. The radiative forcing at the top and base is sufficiently high, but it is cloud instantaneous forcing at noon and in low latitude.

The comparison of obtained values with hourly mean shortwave radiative forcing from (Dong et al., 2006) and monthly mean values from (Bouniol et al., 2012) at the surface points to a similar order of magnitude.

5. Summary and Conclusions

This study shows the advantages of using an analytical approach for solving the inverse problem. The method does not have additional restrictions and links to required parameters directly. It does not unbound desired parameters values and wavelength dependence and, hence, it allows results to be closer to the real nature. The phase function asymmetry parameter is first retrieved from observations. The method for ground albedo obtained from intensity data appears effective. The observational and retrieval errors are accounted for by regularization, which allows us to obtain a more stable and smooth solution. The small distance between observational levels inside cloud allows retrieving the vertical structure of the cloud optical parameters, but increases retrieval errors. High values of the scattering and absorption coefficients in local scale are consistent for all spectral channels at the same level and might point to strong cloud heterogeneity.

The applied asymptotic formulas work well in the diffuse domain. We also found that our approach works for cases violating the diffusion domain conditions, but retrieval uncertainties are higher, which should be included in regularization procedure.

The obtained cloud optical parameters allow the construction of optical models for radiation calculation of radiative characteristics: reflectivity, transmissivity, radiative divergence, forcing, and heating rate for climate modeling applications. Although it is hard to validate our results, the retrieved radiative characteristics are consistent with results obtained in other similar studies. The optical model, constructed with retrieved parameters, provides radiative characteristics consistent with direct observations. It might be considered as adequate for the retrieval.

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8.: Appendix

Derivation of formulas for inside cloud layer for parameter s^2 and τ'

The model of a slab of homogeneous atmospheric layer can be represented schematically as:

Fig. A1.

The model of a homogeneous atmospheric slab layer

The model of slab homogeneous atmospheric layer is considered. The solar incident flux is F_0 .

The radiation field is different above, inside and under the cloud layer. Thus, the cases of the sublayer adjusted to the top, to the base and remote from borders are analyzed separately.

1. The sublayer adjusted to the top ($0, \tau_1$)

Formulas for the intensity escaping from the cloud at the top $\rho(\varphi, \mu, \mu_0)$ and intensity at the first observational level τ_1 are used:

$$\rho(\varphi, \mu, \mu_0) = \rho^\infty(\varphi, \mu, \mu_0) - \frac{K(\mu)K(\mu_0)m\bar{l}e^{-2k\tau_0}}{1 - \bar{l}le^{-2k\tau_0}} \quad (A1)$$

Where $K(\mu)$ is the escape function and $\rho^\infty(\varphi, \mu, \mu_0)$ is the reflection function for the semiinfinite atmosphere

$$\frac{I(\tau_1, -\mu, \mu_0)}{I(\tau_1, \mu, \mu_0)} = b(\tau_1, \mu) = \frac{b^\infty(\mu) - \bar{l}e^{-2k(\tau_0 - \tau_1)}}{1 - b^\infty(\mu)\bar{l}e^{-2k(\tau_0 - \tau_1)}} \quad (A2)$$

$$J(\tau_1) = I(\tau_1, \mu, \mu_0) - I(\tau_1, -\mu, \mu_0) = \frac{6\mu s K(\mu_0)e^{-k\tau_1}}{1 - \bar{l}le^{-2k\tau_0}} \left[1 + \bar{l}e^{-2k(\tau_0 - \tau_1)} \right] \quad (A3)$$

It follows from Eq. (A1):

$$\frac{e^{2k\tau_0}}{\bar{l}} = \frac{mK(\mu)K(\mu_0) + \bar{l}[\rho^\infty(\varphi, \mu, \mu_0) - \rho(\varphi, \mu, \mu_0)]}{[\rho^\infty(\varphi, \mu, \mu_0) - \rho(\varphi, \mu, \mu_0)]} \quad (A4)$$

The expression at the level τ_1 follows from Eq. (2)

$$\bar{l}e^{-2k\tau_0}e^{2k\tau_1} = \frac{b^\infty(\mu) - b(\mu, \tau_1)}{1 - b^\infty(\mu)b(\mu, \tau_1)} \quad (A5)$$

The value $\bar{l}$ accounts for the ground albedo influence defined by: $\bar{l} = l - \frac{Amn^2}{1 - Aa^\infty}$

The intermediate expression is derived after substituting Eqs. (A4) and (A5) to the Eq.(A3) and dropping the arguments of functions ρ^∞ , σ , b^∞ and b

$$J(\tau_1) = \frac{3\mu}{4} \frac{(mK(\mu)K(\mu_0) + N[\rho^\infty - \rho])}{K(\mu)} \left[1 + \frac{b^\infty - b}{1 - b^\infty b} \right] e^{-k\tau_1} \quad (\text{A6})$$

Both parts of the Eq (A6) are squared, the expression for $e^{-k\tau_1}$ is accounted for and the equation is obtained after cancelling the common factors:

$$K^2(\mu)J(\tau_1)^2 = \frac{9\mu^2}{16} (mK(\mu)K(\mu) + l[\rho^\infty - \rho])[\rho^\infty - \rho] \frac{1}{b^\infty - b} \frac{(1 + b^\infty)^2 (1 - b)^2}{(1 - b^\infty b)} \quad (\text{A7})$$

The expansions over the small parameter s for asymptotic constants and functions are substituted to the Eq(A7):

$$\begin{aligned} m &= 8s \left[1 + s \left(6 - 7.5g + \frac{3.6}{1 + g} \right) \right] l = 1 - 6q's + 18q'^2 s^2 \quad (\text{A8}) \\ a^\infty &= 1 - 4s + 12q's^2 \\ b^\infty &= 1 - 6\mu s + 18\mu^2 s^2 \end{aligned}$$

$$\begin{aligned} \rho^\infty &= \rho_0 - 4K_0(\mu_0)K_0(\mu)s + \frac{a_2(\mu)a_2(\mu_0)}{12q'} s^2 \quad (\text{A9}) \\ K(\mu) &= K_0(\mu)(1 - 3q's) + K_2(\mu)s^2 \\ a(\mu) &= 1 - 4K_0(\mu)s + a_2(\mu)s^2 \\ a_2(\mu) &= 3K_0(\mu) \left(\frac{3}{1 + g} (1.271\mu - 0.9) + 4q' \right) = 3K_0(\mu)f(\mu) + 12q'K_0(\mu) \\ K_2(\mu) &= 1.667 \left(1.6 + 3g - \frac{2}{1 + g} \right) (\mu^2 + 0.1) \end{aligned}$$

Multiply the polynomials while keeping the items with power less or equal to 2 and cancel common factors in both parts of the equation:

$$\begin{aligned}
& \frac{16}{9\mu^2} J(\tau_1)^2 K_0^2(\mu) = \\
& \left[8K_0(\mu_0)K_0(\mu)s \left(1 - 6q's + 9q'^2s^2 + s^2 \left(\frac{K_2(\mu_0)}{K_0(\mu_0)} + \frac{K_2(\mu)}{K_0(\mu)} \right) \right) \left((\rho_0 - \rho) - 4K_0(\mu_0)K_0(\mu)s + \frac{a_2(\mu_0)a_2(\mu)}{12q'}s^2 \right) + \right. \\
& \left. (1 - 6q's + 18q'^2s^2) \left((\rho_0 - \rho)^2 + 16K_0^2(\mu_0)K_0^2(\mu)s^2 - 8K_0(\mu_0)K_0(\mu)(\rho_0 - \rho)s + \frac{a_2(\mu_0)a_2(\mu)}{6q'}(\rho_0 - \rho)s^2 \right) \right] \times \\
& \left(1 + 6q's - 18q'^2s^2 - 2\frac{K_2(\mu)}{K_0(\mu)} + 36q's^2 \right) \frac{1}{b^\infty - b} \frac{(1 + b^\infty)^2(1 - b)^2}{(1 - b^\infty b)} \\
& = \left[(\rho_0 - \rho)^2 - 16K_0^2(\mu_0)K_0^2(\mu)s^2 - 48q'K_0(\mu_0)K_0(\mu)(\rho_0 - \rho)s^2 + \frac{a_2(\mu_0)a_2(\mu)}{6q'}(\rho_0 - \rho)s^2 - 2\frac{K_2(\mu)}{K_0(\mu)}(\rho_0 - \rho)^2s^2 \right] \times \\
& 4 \left(1 + 9\mu^2s^2 \left(1 - \frac{4b_1}{(1 - b_1)^2} \right) \right) \\
&)
\end{aligned}$$

Accounting the equality following from the definition $b(\mu, \tau)$: $1 - \frac{4b}{(1 - b)^2} = 1 - \frac{4I^\uparrow I^\downarrow}{(I^\uparrow - I^\downarrow)^2}$, The linear equation with respect to s^2 is obtained:

$$\begin{aligned}
& \frac{4J_1(\mu)^2 K_0^2(\mu)}{9\mu^2} = \\
& \left[(\rho_0 - \rho)^2 - 16K_0^2(\mu_0)K_0^2(\mu)s^2 + (\rho_0 - \rho)^2 \left(\frac{a_2(\mu_0)a_2(\mu)}{6q'(\rho_0 - \rho)} - \frac{48q'K_0(\mu_0)K_0(\mu)}{(\rho_0 - \rho)} - 2\frac{K_2}{K_0} + 9\mu^2 \right. \right. \\
& \left. \left. + \frac{36b_1\mu^2}{(1-b_1)^2} \right) s^2 \right] = \\
& \left[(\rho_0 - \rho)^2 - 16K_0^2(\mu_0)K_0^2(\mu)s^2 + (\rho_0 - \rho)^2 \left(\frac{a_2(\mu_0)a_2(\mu)}{6q'(\rho_0 - \rho)} - \frac{48q'K_0(\mu_0)K_0(\mu)}{(\rho_0 - \rho)} - 2\frac{K_2}{K_0} \right. \right. \\
& \left. \left. + 9\mu^2 \left(1 + \frac{4I_1^\uparrow I_1^\downarrow}{(I_1^\downarrow - I_1^\uparrow)^2} \right) \right) s^2 \right] \\
& \quad \quad \quad (A10)
\end{aligned}$$

Thus, the solution s^2 of the Eq.(A10) is given as:

$$\begin{aligned}
s^2 = & \frac{(\rho_0 - \rho)^2 - \frac{4J_1^2 K_0^2(\mu)}{9\mu^2}}{16K_0^2(\mu)K_0^2(\mu_0) + (\rho_0 - \rho)^2 \left[2\frac{K_2(\mu)}{K_0(\mu)} + \frac{48q'K_0(\mu_0)K_0(\mu)}{(\rho_0 - \rho)} - \frac{a_2(\mu_0)a_2(\mu)}{6q'(\rho_0 - \rho)} - 9\mu^2 \frac{(I_1^\uparrow + I_1^\downarrow)^2}{(I_1^\downarrow - I_1^\uparrow)^2} \right]} \\
& \quad \quad \quad (A11)
\end{aligned}$$

Optical thickness of the upper sublayer τ_1 is determined from Eqs. (A4) and (A5)

$$e^{2k\tau_1} = \frac{b^\infty - b(\tau_1)}{1 - b^\infty b(\tau_1)} \frac{mK(\mu)K(\mu_0) + l(\rho^\infty - \rho)}{(\rho^\infty - \rho)} = \frac{b^\infty I^\downarrow - I^\uparrow}{I^\downarrow - b^\infty I^\uparrow} \left[\frac{mK(\mu)K(\mu_0)}{(\rho^\infty - \rho)} + l \right] \quad (A12)$$

Then Eq. (A12) together with expansions (A8) and (A9) gives:

$$\begin{aligned}
e^{2k\tau_1} = & \frac{b^\infty - b(\tau_1)}{1 - b^\infty b(\tau_1)} \frac{mK(\mu)K(\mu_0) + l(\rho^\infty - \rho)}{(\rho^\infty - \rho)} = \\
& \frac{I_1^\downarrow(1 - 6\mu s + 18\mu^2 s^2) - I_1^\uparrow}{I_1^\downarrow - (1 - 6\mu s + 18\mu^2 s^2)I_1^\uparrow} \left[\frac{8s[K_0(\mu)K_0(\mu_0)(1 - 6q's) + (K_2(\mu) + K_2(\mu_0))s^2]}{\rho_0 - \rho - 4K_0(\mu)K_0(\mu_0)s + \frac{1}{12q'}a_2(\mu)a_2(\mu_0)s^2} + 1 - 6q's + 9q'^2 s^2 \right]
\end{aligned}$$

And finally, the solution for the optical thickness of the upper sublayer is given by:

$$\tau' = 3\tau_1(1 - g) = \frac{1}{2s} \ln \left[l \frac{I_1^{\downarrow}(1 - 6\mu s + 18\mu^2 s^2) - I_1^{\uparrow}}{I_1^{\downarrow} - (1 - 6\mu s + 18\mu^2 s^2)I_1^{\uparrow}} \right] \quad (\text{A13})$$

$$\left[\frac{(\rho_0^{\infty} - \rho) + 4K_0(\mu_0)K_0(\mu)s + \frac{a_2(\mu_0)a_2(\mu)}{12q'}s^2}{(\rho_0^{\infty} - \rho) - 4K_0(\mu_0)K_0(\mu)s + \frac{a_2(\mu_0)a_2(\mu)}{12q'}s^2} \right]$$

The sublayer inside of a cloud remote from borders (τ_{i-1} , τ_i)

Derivation of needed formulas is based on Eqs (3) and (5) for 2 levels τ_{i-1} and τ_i . Consider the ratio of differences of intensities measured in 2 directions, defined with zenith viewing angles $\arccos \mu$ and $\arccos(-\mu)$ at 2 levels τ_{i-1} and τ_i :

$$c = \frac{J(\tau_i)}{J(\tau_{i-1})} = \frac{1 + \bar{l}e^{-2k(\tau_0 - \tau_i)}}{1 + \bar{l}e^{-2k(\tau_0 - \tau_{i-1})}} \frac{e^{-k\tau_i}}{e^{-k\tau_{i-1}}} = \frac{1 + \bar{l}e^{-2k(\tau_0 - \tau_i)}}{1 + \bar{l}e^{-2k(\tau_0 - \tau_{i-1})}} e^{-k(\tau_i - \tau_{i-1})} \quad (\text{A14})$$

And taking into account that:

$$\bar{l}e^{-2k(\tau_0 - \tau_i)} = \frac{b^{\infty}(\mu) - b(\mu, \tau_i)}{1 - b^{\infty}(\mu)b(\mu, \tau_i)}$$

$$c^2 = \frac{(1 - b_i)(1 + b^{\infty})^2(1 - b^{\infty}b_{i-1})}{(1 - b_{i-1})(1 + b^{\infty})^2(1 - b^{\infty}b_i)} e^{-k(\tau_i - \tau_{i-1})}$$

$$e^{2k(\tau_i - \tau_{i-1})} = \frac{e^{-2k(\tau_0 - \tau_i)}}{e^{-2k(\tau_0 - \tau_{i-1})}} = \frac{b^{\infty} - b_i}{1 - b^{\infty}b_i} \frac{1 - b^{\infty}b_{i-1}}{b^{\infty} - b_{i-1}} \quad (\text{A15})$$

the intermediate expression is derived:

$$c^2 = \frac{(1-b_i)^2(1-b^\infty b_{i-1})^2}{(1-b_{i-1})^2(1-b^\infty b_i)^2} \frac{b^\infty - b_{i-1}}{1-b^\infty b_{i-1}} \frac{1-b^\infty b_i}{b^\infty - b_i} = \frac{(1-b_i)^2(1-b^\infty b_{i-1})}{(1-b_{i-1})^2(1-b^\infty b_i)} \frac{b^\infty - b_{i-1}}{b^\infty - b_i} \quad (\text{A16})$$

Then Eq (A8) for the value b^∞ is substituted, after deleting common factors, we obtain the following expression:

$$c^2 = \frac{(1-b_i)^2(1-b^\infty b_{i-1})}{(1-b_{i-1})^2(1-b^\infty b_i)} \frac{b^\infty - b_{i-1}}{b^\infty - b_i}$$

Then Eq (A8) for b^∞ is accounted for and polynomials are multiplied and keeping items proportional to s^2 , and after deleting common items a new expression is obtained

$$\begin{aligned} c^2 &= \frac{J_i^2}{J_{i-1}^2} = \frac{1 - 6\mu s + 18\mu^2 s^2 - \frac{36b_i \mu^2 s^2}{(1-b_i)^2}}{1 - 6\mu s + 18\mu^2 s^2 - \frac{36b_{i-1} \mu^2 s^2}{(1-b_{i-1})^2}} \\ &= \left[1 - 6\mu s + 18\mu^2 s^2 - \frac{36b_i \mu^2 s^2}{(1-b_i)^2} \right] \left[1 + 6\mu s - 18\mu^2 s^2 + \frac{36b_{i-1} \mu^2 s^2}{(1-b_{i-1})^2} + 36\mu^2 s^2 \right] \\ &= 1 - 6\mu s + 18\mu^2 s^2 - \frac{36b_i \mu^2 s^2}{(1-b_i)^2} + 6\mu s - 18\mu^2 s^2 + \frac{36b_{i-1} \mu^2 s^2}{(1-b_{i-1})^2} + 36\mu^2 s^2 - 36\mu^2 s^2 \\ &= 1 + 36\mu^2 s^2 \left[\frac{b_{i-1}}{(1-b_{i-1})^2} - \frac{b_i}{(1-b_i)^2} \right] = 1 + 36\mu^2 s^2 \frac{[b_{i-1}(1-b_i)^2 - b_i(1-b_{i-1})^2]}{(1-b_{i-1})^2(1-b_i)^2} \\ &= 1 + 36\mu^2 s^2 \frac{I_{i-1}^\downarrow I_{i-1}^\uparrow J_i^2 - I_i^\downarrow I_i^\uparrow J_{i-1}^2}{J_{i-1}^2 J_i^2}, \end{aligned}$$

$$J_i^2 J_i^2 = J_i^2 J_{i-1}^2 + 36\mu^2 s^2 [I_{i-1}^\downarrow I_{i-1}^\uparrow J_i^2 - I_i^\downarrow I_i^\uparrow J_{i-1}^2] \quad (\text{A17})$$

From definitions of values c and b_i , the result is derived:

$$s^2 = \frac{(J^2(\tau_i) - J^2(\tau_{i-1}))J^2(\tau_i)}{36\eta^2 [J^2(\tau_{i-1})I(\eta, \tau_i)I(-\eta, \tau_i) - J^2(\tau_i)I(\eta, \tau_{i-1})I(-\eta, \tau_{i-1})]} \quad (\text{A18})$$

After finding the logarithm of the Eq. (A15) the expression for the optical thickness between levels $i-1$ and i is following:

$$e^{2k(\tau_i - \tau_{i-1})} = \frac{e^{-2k(\tau_0 - \tau_i)}}{e^{-2k(\tau_0 - \tau_{i-1})}} = \frac{b^\infty - b_i}{1 - b^\infty b_i} \frac{1 - b^\infty b_{i-1}}{b^\infty - b_{i-1}} \quad (\text{A19})$$

$$\tau'_i - \tau'_{i-1} = 3(\tau_i - \tau_{i-1})(1 - g) = \frac{1}{2s} \left\{ \ln \left[\frac{b^\infty I_i^\downarrow - I_i^\uparrow}{I_i^\downarrow - b^\infty I_i^\uparrow} \right] + \ln \left[\frac{I_{i-1}^\downarrow - b^\infty I_{i-1}^\uparrow}{b^\infty I_{i-1}^\downarrow - I_{i-1}^\uparrow} \right] \right\} \quad (\text{A20})$$

or in other form:

$$\tau'_i - \tau'_{i-1} = 3(\tau_i - \tau_{i-1})(1 - g) = \frac{1}{2s} \ln \left[\frac{J(\tau_{i-1}) - 6I(\mu, \tau_{i-1})\mu s(1 - 3\mu s)}{J(\tau_{i-1}) + 6I(-\mu, \tau_{i-1})\mu s(1 - 3\mu s)} \right] + \frac{1}{2s} \ln \left[\frac{J(\tau_i) + 6I(-\mu, \tau_i)\mu s(1 - 3\mu s)}{J(\tau_i) - 6I(\mu, \tau_i)\mu s(1 - 3\mu s)} \right]$$

The sublayer adjacent to the cloud base (τ_{N-1} , τ_N)

The formula for transferred radiation is applied

$$\sigma(\tau_N, \mu, \mu_0) = \frac{\bar{K}(\mu)\bar{K}(\mu_0)m e^{-k\tau_0}}{1 - \bar{l}e^{-2k\tau_0}} = \frac{8s\bar{K}(\mu)K(\mu_0)e^{-k\tau_0}}{1 - \bar{l}e^{-2k\tau_0}} \quad (\text{A21})$$

Assume that $m = 8s$

There is also the expression:

$$J(\tau_{N-1}) = I(\tau_{N-1}, \mu, \mu_0) - I(\tau_{N-1}, -\mu, \mu_0) = \frac{6\mu s K(\mu_0) e^{-k\tau_{N-1}}}{1 - \bar{l}e^{-2k\tau_0}} \left[1 + \bar{l}e^{-2k(\tau_0 - \tau_{N-1})} \right]$$

then

$$\frac{6\mu s K(\mu_0)}{1 - \bar{l}e^{-2k\tau_0}} = \frac{J(\tau_{N-1}) e^{k\tau_{N-1}}}{\left[1 + \bar{l}e^{-2k(\tau_0 - \tau_{N-1})} \right]}$$

The following expression is derived:

$$\sigma(\tau_0, \mu, \mu_0) = \frac{4}{3\mu} \bar{K}(\mu) e^{-k\tau_0} e^{k\tau_{N-1}} \frac{J(\tau_{N-1})}{\left[1 + \bar{l} e^{-2k(\tau_0 - \tau_{N-1})}\right]} = \frac{4}{3\mu} \bar{K}(\mu) \frac{J(\tau_{N-1}) e^{-k(\tau_0 - \tau_{N-1})}}{\left[1 + \bar{l} e^{-2k(\tau_0 - \tau_{N-1})}\right]}$$

The Eq. (A5) is taken for the level τ_{N-1} and noting as $b(\mu, \tau_{N-1}) = b_{N-1}$:

$$\bar{l} e^{-2k(\tau_0 - \tau_{N-1})} = \frac{b^\infty(\mu) - b(\mu, \tau_{N-1})}{1 - b^\infty(\mu) b(\mu, \tau_{N-1})}$$

$$1 + \bar{l} e^{-2k(\tau_0 - \tau_{N-1})} = \frac{1 - b^\infty b_{N-1} + b^\infty - b_{N-1}}{1 - b_{N-1} b^\infty} = \frac{(1 - b_{N-1})(1 + b^\infty)}{1 - b_{N-1} b^\infty}$$

The intermediate expression is defined by:

$$\frac{4}{3\mu} \bar{K}(\mu) J(\tau_{N-1}) = \sigma \frac{(1 - b_{N-1})(1 + b^\infty)}{(1 - b_{N-1} b^\infty)} e^{k(\tau_0 - \tau_{N-1})} \quad (\text{A22})$$

Both parts are squared:

$$\frac{16}{9\mu^2} \bar{K}^2(\mu) J^2(\tau_{N-1}) = \sigma^2(\tau_0) \frac{(1 - b_{N-1})^2 (1 + b^\infty)^2}{(1 - b_{N-1} b^\infty)^2} e^{2k(\tau_0 - \tau_{N-1})}$$

Taking into consideration the relation:

$$e^{2k(\tau_0 - \tau_{N-1})} = \bar{l} \frac{1 - b^\infty b_{N-1}}{b^\infty - b_{N-1}}, \quad (\text{A23})$$

then the following expression is obtained

$$\frac{16J_{N-1}^2 \bar{K}^2(\mu)}{\bar{l}} = 9\mu^2 \sigma^2 \frac{(1 - b_{N-1})^2 (1 + b^\infty)^2}{(1 - b_{N-1} b^\infty)^2 (b^\infty - b)} \quad (\text{A24})$$

Take into account the surface albedo influence as:

$$\bar{K}(\mu) = K(\mu) + \frac{Aa(\mu)n}{1 - Aa^\infty} \bar{l} = l - \frac{Amn^2}{(1 - Aa^\infty)},$$

$$\bar{K}_0(\mu) = K_0(\mu) + \frac{A}{1 - A}$$

$$\bar{q}' = q' + \frac{4}{3} \frac{A}{1 - A} \bar{n} = \frac{n}{1 - Aa^\infty} \bar{n}_1 = -3 \frac{\bar{q}'}{1 - A} \bar{n}_2 = \frac{1}{1 - A} \left[n_2 + 12 \frac{A(\bar{q}' + q')}{1 - A} \right]$$

$$\bar{K}(\mu) = K(\mu) + \frac{Aa(\mu)n}{(1 - Aa^\infty)} = \bar{K}_0(\mu) - 3\bar{q}'\bar{K}_0(\mu)s + \left(K_2(\mu) + \frac{12A\bar{q}'\bar{K}_0(\mu)}{1 - A} + \frac{12A^2q'}{(1 - A)^2} + \frac{A(a_2(\mu) + n_2)}{1 - A} \right) s^2$$

$$\bar{K}^2(\mu) = \bar{K}_0^2(\mu) \left\{ 1 - 6\bar{q}'s + 9\bar{q}'^2s^2 + \frac{2}{\bar{K}_0} \left[K_2 + \frac{24A\bar{q}'\bar{K}_0(\mu)}{1 - A} + \frac{12A^2\bar{q}'}{(1 - A)^2} + \frac{A(a_2(\mu) + n_2)}{1 - A} \right] s^2 \right\}$$

$$\bar{l} = l - \frac{Amn^2}{(1 - Aa^\infty)} = \frac{l(1 - Aa^\infty) - Amn^2}{(1 - Aa^\infty)} = 1 - 6\bar{q}'s + 18\bar{q}'^2s^2$$

The result of multiplying and dividing the left part of the Eq. (A24) with the expression $\frac{\bar{K}^2}{\bar{l}}$ and keeping items proportional to s and s^2 :

$$\frac{\bar{K}^2}{\bar{l}} = \bar{K}_0^2 \left\{ 1 - 9\bar{q}'^2s^2 + \frac{2s^2}{\bar{K}_0} \left[K_2 + \frac{24\bar{q}'\bar{K}_0A}{1 - A} + \frac{A}{1 - A} \left(a_2(\mu) + n_2 + \frac{12q'A}{1 - A} \right) \right] \right\} \quad (A25)$$

Taking into account Eq. (A8) for $b^\mathbb{X}$, we obtained the right part of the Eq.(A24):

$$\begin{aligned}
& \frac{(1-b_{N-1})^2(1+b^\infty)^2}{(1-b_{N-1}b^\infty)(b^\infty-b)} = \frac{(1-b_{N-1})^2(1+1-6\mu s+18\mu^2s^2)^2}{(1-b_{N-1}+6b_{N-1}\mu s-18b_{N-1}\mu^2s^2)(1-b_{N-1}-6\mu s+18\mu^2s^2)} \\
& = \frac{4(1+9\mu^2s^2-6\mu s+18\mu^2s^2)}{(1-b_{N-1})^2\left(1+\frac{6b_{N-1}\mu}{1-b_{N-1}}s-\frac{18b_{N-1}\mu^2}{1-b_{N-1}}s^2\right)\left(1-\frac{6\mu}{1-b_{N-1}}s+\frac{18\mu^2}{1-b_{N-1}}s^2\right)} = \\
& \frac{4(1-b_{N-1})^2}{(1-b_{N-1})^2} \frac{1+9\mu^2s^2-6\mu s+18\mu^2s^2}{1-6\mu s+18\mu^2s^2-36\frac{b_{N-1}\mu^2s^2}{(1-b_{N-1})^2}} \\
& = 4(1+9\mu^2s^2-6\mu s+18\mu^2s^2)\left(1+6\mu s-18\mu^2s^2+36\frac{b_{N-1}\mu^2s^2}{(1-b_{N-1})^2}+36\mu^2s^2\right) \\
& = 4\left[1+9\mu^2s^2-36\mu^2s^2+36\frac{b_{N-1}\mu^2s^2}{(1-b_{N-1})^2}+36\mu^2s^2\right] \\
& = 4\left(1+9\mu^2s^2\left(1+\frac{4b_{N-1}}{(1-b_{N-1})^2}\right)\right)
\end{aligned}$$

(A26)

Finally, the right part is derived:

$$\begin{aligned}
& 9\mu^2\sigma^2(\tau_0)\left(1+9\mu^2s^2\left(1+\frac{4b_{N-1}}{(1-b_{N-1})^2}\right)\right) = 9\mu^2\sigma^2(\tau_0)\left(1+9\mu^2s^2\left(1+\frac{(1+b_{N-1})^2}{(1-b_{N-1})^2}\right)\right) \quad (A27) \\
& = 9\mu^2\sigma^2(\tau_0)\left(1+9\mu^2s^2\frac{(I_{N-1}^\downarrow+I_{N-1}^\uparrow)}{J_{N-1}^2}\right)
\end{aligned}$$

Eqs (A25 and (A27) lead to the linear equation for desired s^2 and the solution is defined by:

$$s^2 = \frac{9\mu^2\sigma^2 - 4\bar{K}_0^2(\mu)J_{N-1}^2}{4\bar{K}_0^2J_{N-1}^2 \left[\frac{2}{\bar{K}_0} \left[\bar{K}_2 + \frac{A}{1-A} (a_2(\mu) + n_2 + 24\bar{q}'\bar{K}_0 + \frac{12q'A}{1-A}) \right] - 9\bar{q}'^2 \right] - 36\mu^2\bar{K}_0^2(I_{N-1}^\downarrow + I_{N-1}^\uparrow)^2} \quad (\text{A28})$$

After finding the logarithm of the Eq. (A23) the expression for the optical thickness between levels $N-1$ and N is defined by:

$$\tau'_N - \tau'_{N-1} = 3(\tau_N - \tau_{N-1})(1-g) = \frac{1}{2s} \ln \left[\frac{2}{3\mu\sigma} \frac{I_{N-1}^\downarrow - I_{N-1}^\uparrow (1 - 6\mu s + 18\mu^2 s^2)}{1 - 3\mu s + 9\mu^2 s^2} \right] + \ln \left[K_0(\mu)(1 - 3q's) + K_2(\mu)s^2 + \frac{A}{1-A} (1 - 3q's + n_2 s^2) (1 - 4K_0(\mu)s + a_2(\mu)s^2) \right] \quad (\text{A29})$$

or

$$\tau_N - \tau_{N-1} = \frac{1}{2s} \ln \left(\frac{1 - b^\infty b_{N-1}}{b^\infty - b_{N-1}} \right) = \frac{1}{2s} \ln \left(\frac{I_{N-1}^\downarrow - b^\infty I_{N-1}^\uparrow}{b^\infty I_{N-1}^\downarrow - I_{N-1}^\uparrow} \right),$$

Conservative scattering

The optical thickness of the sublayer close to the cloud top:

$$3(1-g)\tau_0 = \frac{4K_0(\mu_0)K_0(\mu)}{\rho_0(\mu, \mu_0) - \rho} - \left(6q' + \frac{4A}{1-A} \right), \quad (\text{A30})$$

$$I(\tau, \mu) = \frac{K_0(\mu_0) \{ (1-A)[3(1-g)(\tau_0 - \tau) + 3q' + 3\mu] + 4A \}}{(1-A)[3(1-g)\tau_0 + 6q'] + 4A} \quad (\text{A31})$$

The following chain of transformations is done through modification of the well known expressions (A30) and (A31):

$$3(1-g)\tau_0 - 3(1-g)\tau_1 = \frac{I(\tau_1, \mu)3(1-g)\tau_0 + I(\tau_1, \mu)6q' + I(\tau_1, \mu)\frac{4A}{1-A}}{K_0(\mu_0)} - 3(q' + \mu) - \frac{4A}{1-A},$$

$$3(1-g)\tau_1 = \frac{4K_0(\mu_0)K_0(\mu)}{\rho_0(\mu, \mu_0) - \rho} - \left(6q' + \frac{4A}{1-A}\right) - I(\tau_1, \mu)\frac{4K_0(\mu)}{\rho_0(\mu, \mu_0) - \rho} + \frac{I(\tau_1, \mu)\left(6q' + \frac{4A}{1-A}\right) - I(\tau_1, \mu)\left(6q' + \frac{4A}{1-A}\right)}{K_0(\mu_0)} + 3(q' + \mu) + \frac{4A}{1-A}.$$

The result for τ_1 follows as:

$$3(1-g)\tau_1 = \frac{4K_0(\mu)(K_0(\mu_0) - I_1^\downarrow)}{(\rho_0 - \rho)} - 3(q' + \mu) \quad (\text{A32})$$

Let us consider the sum and difference of downward and upward intensities:

$I_{i-1}^\downarrow = I(\tau_{i-1}, \mu)$ and $I_{i-1}^\uparrow = I(\tau_{i-1}, -\mu)$ at two inner levels τ_{i-1} and τ_i :

$$I_{i-1}^\downarrow + I_{i-1}^\uparrow = \frac{K_0(\mu_0)\left\{[6(1-g)(\tau_0 - \tau) + 6q'] + \frac{8A}{(1-A)}\right\}}{3(1-g)\tau_0 + 6q' + \frac{4A}{(1-A)}} \quad (\text{A33})$$

and

$$I_{i-1}^\downarrow - I_{i-1}^\uparrow = \frac{6\mu K_0(\mu_0)}{3(1-g)\tau_0 + 6q' + \frac{4A}{(1-A)}} \quad (\text{A34})$$

The ratio of the sum and difference is equal to:

$$\frac{I_{i-1}^\downarrow + I_{i-1}^\uparrow}{I_{i-1}^\downarrow - I_{i-1}^\uparrow} = \frac{3(1-g)(\tau_0 - \tau_{i-1}) + 3q' + 4\frac{A}{1-A}}{3\mu} \quad (\text{A35})$$

$$\frac{I_i^\downarrow + I_i^\uparrow}{I_i^\downarrow - I_i^\uparrow} = \frac{3(1-g)(\tau_0 - \tau_i) + 3q' + 4\frac{A}{1-A}}{3\mu}$$

The scaled optical thickness of the inner sublayer is derived from Eqs. (A35):

$$3(1-g)(\tau_i - \tau_{i-1}) = 3\mu \left[\frac{(I_{i-1}^{\downarrow} + I_{i-1}^{\uparrow})}{(I_{i-1}^{\downarrow} - I_{i-1}^{\uparrow})} - \frac{(I_i^{\downarrow} + I_i^{\uparrow})}{(I_i^{\downarrow} - I_i^{\uparrow})} \right] \quad (\text{A36})$$

The optical thickness of the sublayer close to the cloud base is obtained using the Eq. (A31) for the level τ_{N-1} and the known relation for the total optical thickness $\tau_N = \tau_0$:

$$3(1-g)\tau_0 = \frac{4K_0(\mu_0)\bar{K}_0(\mu)}{\sigma(\tau_0, \mu, \mu_0)} - \left(6q' + \frac{4A}{(1-A)} \right), \quad (\text{A37})$$

$$3(1-g)\tau_N - 3(1-g)\tau_{N-1} = \frac{I_{N-1}^{3(1-g)\tau_N + I_{N-1}^{6q' + I_{N-1} \frac{4A}{1-A}}}{K_0(\mu_0)} - 3(q' + \mu) - \frac{4A}{1-A},$$

The resulting formula after substituting the expression (A37) for $\tau_N = \tau_0$ is as follows:

$$3(1-g)(\tau_N - \tau_{N-1}) = \frac{4I_{N-1}\bar{K}_0(\mu)}{\sigma} - 3(q' + \mu) - \frac{4A}{(1-A)} \quad (\text{A38})$$

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Fig. 1a.
Flight Track (in red): Atlantic Ocean off Namibian Coastline. The background image is from MODIS/Terra during the CAR flight.

Fig. 1b.
Flight Track (in red): Pacific Ocean off California Coastline. The background image is from MODIS/Aqua during the CAR flight.

Fig. 1c.

Flight track GPS height during SAFARI-2000. The aircraft obtained measurements over the marine stratocumulus offshore of Namibia at two locations, A (21.63°S , 13.62°E) and B (20.66°S , 13.13°E).

Fig. 1d.
Flight track GPS height during ARCTAS. The aircraft obtained measurements of marine stratocumulus offshore of California at X, latitude 37.3–36.9°N and 122.2–122.6°W.

Fig. 2a:

Aircraft flight profile during SAFARI-2000 cloud measurements overlaid on CAR quick-look image (constructed from bands at 1.04, 0.87, and 0.47 μm); section a – above cloud, Section b, below clouds, Section c inside cloud with the CAR scanning in the principal plane, and Section d, inside cloud with the CAR scanning away from the principal plane. The measurements shown in each sub-panel are from different clouds, which were combined to form one image.

Fig. 2b:
Aircraft flight profile during ARCTAS cloud measurements overlaid on CAR quicklook image (constructed from bands at $1.04\ \mu\text{m}$, $0.87\ \mu\text{m}$, and $0.47\ \mu\text{m}$); Section a – above cloud, and Section b, below clouds.

Fig. 3.

Dependence of observational random errors on viewing zenith angle above the cloud (a), under the cloud (b) (Up and Down) during SAFARI (solid lines) and ARCTAS (dotted lines) experiments for three spectral channels 340, 682 and 1273 nm. Relative observational uncertainties refer to error divided by the value itself.

Fig. 4. Spectral radiance (870 nm and 1273 nm) as a function of CAR scan angle (where 0° corresponds to zenith direction and 180° corresponds to nadir direction) for scattered radiation measured at different flight altitude ranging from 372 m to 1178 m above mean sea level, offshore of Namibia during SAFARI 2000 experiment.

Fig. 5.
Unregularized results retrieved from SAFARI-2000 experiment (a,b) and ARCTAS-2008 (c,d)

Fig. 6.

Optical parameters retrieved and radiative divergence calculated for NASA SAFARI-2000 (blue symbols), NASA ARCTAS-2008 (red symbols) and obtained from hemispherical fluxes observations above sea surface (black lines): a) ground albedo; b) single scattering coalbedo (1-SSA); c) optical thickness; d) radiative divergence (in relative units)

Fig. 7.

Vertical profiles of the volume scattering coefficient (a, c); volume absorption coefficient (b, d) from airborne observation SAFARI-2000 (a, b) and ARCTAS-2008 (c, d)

Table 1.

Cloud Absorption Radiometer specifications

Platform	the University of Washington CV-580 (SAFARI 2000), NASA P-3B (ARCTAS 2008) aircrafts
Ground speed	80 m s ⁻¹ (nominal)
Total field of view	190°
Instantaneous field of view	17.5 mrad (1°)
Imaging modes	4 (zenith, BRDF, starboard, nadir)
Pixels per scan line	382
Scan rate	1.67 scan lines per second (100 rpm)
Spectral channels (μm; bandwidth (FWHM))	14 (8 continuously sampled and last six in filter wheel: 0.340(0.009), 0.381(0.006), 0.472(0.021), 0.682(0.022), 0.870(0.022), 1.036(0.022), 1.219(0.022), 1.273(0.023), 1.556(0.032), 1.656(0.045), 1.737(0.040), 2.103(0.044), 2.205(0.042), 2.302(0.043))
Output channels	9 channels at 16 bits
Data rate	61.85 MB hr ⁻¹
Instrument mass	49 kg
Radiometric calibration	Laboratory integrating sphere measurements before and after a research mission

Table 2.

Ground albedo, retrieved from airborne observations under cloud of diffused solar radiance from SAFARI-2000 and ARCTAS-2008 campaigns

λ , nm	340	381	472	681	870	1035	1219	1273
A, SAFARI	0.066	0.067	0.050	0.046	0.043	0.042	0.040	0.044
dA, SAFARI	0.002	0.0004	0.0002	0.0002	0.0002	0.0002	0.0001	0.0002
A ARCTAS	0.072	0.078	0.066	0.045	0.040	0.037	0.038	0.044
dA, ARCTAS	0.003	0.007	0.002	0.004	0.006	0.006	0.007	0.007

Table 3.

Optical thickness and single scattering albedo (SSA) from airborne observation

λ, nm		340	381	472	682	870	1035	1219	1273
τ	SAFARI (B)	20.9	15.4	14.5	10.9	9.2	9.3	10.4	10.1
	SAFARI (T)	10.0	13.9	15.2	13.3	15.2	12.3	12.1	12.7
	SAFARI $\Sigma\tau_i$	18.0	15.7	15.2	14.4	8.5	10.0	9.1	9.6
	ARCTAS (B)	13.0	12.4	10.8	12.2	10.0	7.9	9.7	10.3
	ARCTAS (T)	14.0	19.4	14.7	14.2	13.3	14.2	14.6	11.8
$MSD\tau$	SAFARI	0.002	0.003	0.003	0.003	0.003	0.004	0.003	0.0001
	ARCTAS	0.0003	0.0007	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
ω	SAFARI (B)	0.9967	0.9965	0.9965	0.9961	0.9958	0.9955	0.9962	0.9962
	SAFARI (T)	0.9981	0.9971	0.9970	0.9946	0.9967	0.9997	0.9960	0.9958
	ARCTAS (B)	0.9935	0.9962	0.9942	0.9949	0.9945	0.9933	0.9936	0.9935
	ARCTAS (T)	0.9973	0.9973	0.9984	0.9978	0.9973	0.9970	0.9973	0.9977
$MSD\omega_I$	SAFARI	0.001	0.001	0.002	0.004	0.004	0.009	0.006	0.008
	ARCTAS	0.0010	0.0068	0.0006	0.0004	0.0027	0.0005	0.0210	0.0535

Table 4.

Extinction and absorption coefficients, 1/km

λ, nm		340	381	472	682	870	1035	1219	1273
σ, km^{-1}	SAFARI (B)	52.2	38.2	36.2	27.2	23.1	23.4	30.9	26.8
	SAFARI (T)	25.0	34.7	38.0	33.1	38.0	30.2	30.2	31.7
	ARCTAS (B)	12.4	11.8	10.3	11.5	9.6	7.6	9.7	9.8
	ARCTAS (T)	11.4	18.2	14.0	13.5	12.7	13.5	13.9	11.2
κ, km^{-1}	SAFARI (B)	0.171	0.138	0.125	0.108	0.095	0.105	0.108	0.105
	SAFARI (T)	0.026	0.069	0.114	0.253	0.150	0.012	0.176	0.177
	ARCTAS (B)	0.080	0.044	0.060	0.058	0.053	0.049	0.060	0.064
	ARCTAS (T)	0.028	0.073	0.034	0.055	0.068	0.060	0.051	0.046

Table 5.

Phase function parameter and other optical parameters retrieved taking into account retrieved values of G from SAFARI-2000 and ARCTAS-2008 experiments

	$\lambda \text{ nm}$	340	381	472	682	870	1035	1219	1273
	$g \text{ theor}$	0.854	0.846	0.852	0.842	0.816	0.834	0.811	0.821
SAFARI	G	0.666	0.671	0.601	0.625	0.679	0.626	0.676	0.713
	MSD/g	0.0002	0.0008	0.0003	0.0004	0.0007	0.0007	0.0007	0.0004
	τ	6.7	6.4	4.7	4.0	5.7	3.9	5.0	6.8
	MSD/τ	0.002	0.003	0.002	0.003	0.002	0.003	0.004	0.003
	ω	0.9729	0.9829	0.9766	0.9760	0.9800	0.9727	0.9664	0.9861
	MSD/ω	0.108	0.104	0.114	0.116	0.108	0.096	0.111	0.109
ARCTAS	G	0.541	0.684	0.550	0.454	0.569	0.483	0.496	0.551
	MSD/g	0.0001	0.0005	0.0046	0.0017	0.0033	0.0029	0.0040	0.0031
	τ	5.57	5.86	4.47	3.48	3.40	3.34	3.82	5.20
	MSD/τ	0.007	0.011	0.005	0.015	0.016	0.015	0.014	0.014
	ω	0.9868	0.9831	0.9804	0.9691	0.9703	0.9738	0.9753	0.9843
	MSD/ω	0.152	0.161	0.114	0.113	0.127	0.118	0.124	0.119

Table 6.

Optical models of cloud and clear layers for ARCTAS and SAFARI conditions

λ , nm	ARCTAS 1						ARCTAS 2					
	cloud			clear			cloud			clear		
	ω	τ_{ext}	τ_{abs}	τ_{scat}	$\bar{\tau}_{\Sigma}$	ω	ω	τ_{ext}	τ_{abs}	τ_{scat}	$\bar{\tau}_{\Sigma}$	ω
340	0.9967	13.5	0.111	1.06	1.167	0.9049	0.9852	6.0	0.166	1.2	1.367	0.8778
381	0.9973	10.4	0.082	0.9	0.982	0.9165	0.9845	5.8	0.166	0.97	1.136	0.8538
472	0.9984	8.1	0.048	0.4	0.448	0.8929	0.9831	5.0	0.187	0.6	0.787	0.7624
682	0.9978	6.9	0.060	0.28	0.34	0.8235	0.9866	4.8	0.147	0.34	0.487	0.6982
870	0.9973	5.5	0.069	0.22	0.289	0.7612	0.9835	4.0	0.181	0.28	0.461	0.6074
1035	0.9973	5.0	0.073	0.21	0.283	0.7421	0.9728	3.7	0.299	0.28	0.559	0.5009
1219	0.9973	4.5	0.068	0.20	0.268	0.7463	0.9776	3.5	0.241	0.25	0.491	0.5092
1273	0.9977	4.0	0.061	0.20	0.261	0.7660	0.9813	4.0	0.197	0.25	0.447	0.5593
λ , nm	SAFARI 1						SAFARI 2					
	cloud			clear			Cloud			clear		
	ω	τ_{ext}	τ_{abs}	τ_{scat}	$\bar{\tau}_{\Sigma}$	ω	ω	τ_{ext}	τ_{abs}	τ_{scat}	$\bar{\tau}_{\Sigma}$	ω
340	0.9983	20.6	0.069	1.06	1.125	0.9387	0.9846	16.0	0.246	1.2	1.446	0.8299
381	0.9987	15.6	0.0424	0.9	0.942	0.9554	0.9852	15.2	0.225	0.97	1.195	0.8117
472	0.9985	14.4	0.0318	0.4	0.432	0.9259	0.9857	13.0	0.186	0.6	0.786	0.7634
682	0.9986	10.9	0.0382	0.28	0.318	0.8805	0.9866	12.7	0.170	0.34	0.510	0.6667
870	0.9985	9.2	0.0359	0.22	0.256	0.8594	0.9891	12.6	0.137	0.28	0.417	0.6715
1035	0.9983	9.3	0.0423	0.21	0.252	0.8333	0.9883	12.5	0.147	0.28	0.427	0.6557
1219	0.9982	10.3	0.0445	0.20	0.245	0.8163	0.9870	12.6	0.164	0.25	0.414	0.6039
1273	0.9981	10.1	0.0467	0.20	0.247	0.8097	0.9850	12.0	0.180	0.25	0.430	0.5814

Table 7.

Radiative characteristics calculated from ARCTAS and SAFARI experiments

λ , nm	ARCTAS 1						ARCTAS 2					
	cloud			Clear			cloud			clear		
	$F^\uparrow$	$F^\downarrow$	D	$F^\uparrow$	$F^\downarrow$	D	$F^\uparrow$	$F^\downarrow$	D	$F^\uparrow$	$F^\downarrow$	D
340	0.500	0.447	0.085	0.337	0.544	0.159	0.537	0.355	0.134	0.334	0.471	0.229
381	0.464	0.523	0.054	0.323	0.605	0.119	0.442	0.419	0.171	0.298	0.520	0.091
472	0.401	0.616	0.024	0.206	0.781	0.065	0.469	0.406	0.152	0.212	0.595	0.232
682	0.369	0.632	0.027	0.150	0.811	0.076	0.438	0.403	0.177	0.148	0.708	0.176
870	0.350	0.650	0.026	0.124	0.825	0.084	0.380	0.475	0.164	0.121	0.700	0.207
1035	0.304	0.699	0.023	0.117	0.825	0.088	0.428	0.442	0.146	0.103	0.623	0.297
1219	0.311	0.695	0.020	0.115	0.834	0.082	0.450	0.407	0.159	0.101	0.663	0.262
1273	0.279	0.739	0.015	0.120	0.842	0.075	0.506	0.366	0.145	0.110	0.699	0.222
λ , nm	SAFARI 1						SAFARI 2					
	cloud			clear			cloud			clear		
	$F^\uparrow$	$F^\downarrow$	D	$F^\uparrow$	$F^\downarrow$	D	$F^\uparrow$	$F^\downarrow$	D	$F^\uparrow$	$F^\downarrow$	D
340	0.630	0.321	0.070	0.364	0.569	0.104	0.451	0.286	0.288	0.306	0.406	0.315
381	0.591	0.395	0.040	0.348	0.631	0.064	0.475	0.362	0.187	0.278	0.466	0.288
472	0.567	0.425	0.029	0.208	0.788	0.044	0.426	0.412	0.183	0.213	0.584	0.233
682	0.511	0.482	0.029	0.161	0.827	0.050	0.351	0.556	0.119	0.147	0.680	0.205
870	0.507	0.488	0.027	0.136	0.854	0.047	0.478	0.346	0.191	0.134	0.731	0.167
1035	0.478	0.505	0.039	0.130	0.852	0.054	0.416	0.426	0.175	0.132	0.722	0.177
1219	0.540	0.442	0.036	0.124	0.853	0.057	0.416	0.337	0.261	0.117	0.718	0.194
1273	0.517	0.468	0.092	0.126	0.851	0.060	0.478	0.382	0.157	0.116	0.704	0.211

Table 8.

Radiative direct cloud forcing at the cloud top, base and in the layer: spectral values in relative units and integral values, Wm^{-2} (last row)

λ , nm	ARCTAS 1			ARCTAS 2			SAFARI 1			SAFARI 2		
	f_T	f_B	f_A	f_T	f_B	f_A	f_T	f_B	f_A	f_T	f_B	f_A
340	-0.163	-0.093	-0.074	-0.203	-0.116	-0.095	-0.266	-0.248	-0.034	-0.145	-0.112	-0.027
381	-0.141	-0.082	-0.065	-0.144	-0.101	0.080	-0.241	-0.233	-0.024	-0.197	-0.104	-0.101
472	-0.195	-0.165	-0.041	-0.257	-0.189	-0.080	-0.359	-0.363	-0.015	-0.196	-0.172	-0.050
682	-0.219	-0.179	-0.049	-0.290	-0.305	-0.001	-0.350	-0.345	-0.021	-0.204	-0.124	-0.086
870	-0.226	-0.175	-0.058	-0.259	-0.225	-0.043	-0.371	-0.366	-0.020	-0.344	-0.385	0.024
1035	-0.196	-0.126	-0.065	-0.325	-0.181	-0.151	-0.465	-0.347	-0.015	-0.284	-0.296	-0.002
1219	-0.159	-0.139	-0.062	-0.349	-0.411	-0.103	-0.416	-0.411	-0.021	-0.299	-0.381	0.067
1273	-0.217	-0.103	-0.060	-0.396	-0.256	-0.077	-0.391	-0.383	0.032	-0.362	-0.322	-0.054
$\int d\lambda \text{ Wm}^{-2}$	-191	-158	-55	-157	-228	-84	-348	-313	-20	-257	-217	25