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Two-Phase Displacements In Microchannels of Triangular Cross-Section

Yafei Liu^{#1}, Andrew Hansen^{#1}, Erica Block^{1,3}, Norman R. Morrow¹, Jeff Squier², and John Oakey^{1,*}

¹Department of Chemical Engineering, University of Wyoming, Laramie, WY 82070, USA

²Department of Physics, Colorado School of Mines, Golden, CO 80401, USA ³Present address: School of Microengineering, École Polytechnique Fédérale de Lausanne Lausanne, Switzerland

These authors contributed equally to this work.

Abstract

Varying microfluidic channel cross-sectional geometry can dramatically alter fluid flow behavior, particularly for capillary-driven flow. Most fabrication techniques, however, are planar and therefore incapable of providing depth-dependent variations in width. We introduce an ultrafast laser ablation technique that enables the fabrication of microchannels with arbitrary triangular cross sectional geometry. Triangular channels were fabricated with widths ranging from 45 to 116 μm and aspect ratios between 0.7 to 1.9. This experimental platform was utilized to observe two-phase flow and evaluate the capillary pressures required to initiate flow within triangular capillaries. Applying Mayer, Stowe and Princen (MS-P) theory, critical drainage capillary pressures were predicted for varying cross sections and compared to experimental observations. Results indicate the capability to predict capillary pressures inside triangular channels with perfectly water wet surfaces, providing the first instance of experimental validation of the theory for arbitrary triangular cross sections. This work was extended to intermediate wet conditions, which provides an insight into the prediction of capillary pressure under more realistic conditions. The fabrication techniques and validation of predictive frameworks presented here provide an approach to microfluidic experimental design that will impact a wide range of fundamental and applied technology areas.

Graphical Abstract

*To whom correspondence should be addressed. joakey@uwyo.edu.

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Introduction

Innovations in microscale fabrication technologies are consistently providing the ability to create increasingly complex microfluidic networks with superior microscale fluid handling control. Such microchannel networks, for instance, have been applied as molecular and cellular analysis platforms[1], materials synthesis processing systems[2,3] and physical models to represent complex porous media[4,5]. The relatively recent introduction of soft lithography for the rapid and precise replication of complex microfluidic systems in poly(dimethylsiloxane) (PDMS) [6,7], followed by chemical modifications of surface chemistry and functionalization[8–10] have enabled the production of devices for precise fluid handling in laboratories without the need for a clean room. Most patterns from which PDMS devices are molded, however, are still typically produced by planar photolithography, which yields patterns of uniform width throughout their cross sectional depth. The reliance upon photolithographic patterning has limited the application of geometrically-mediated hydrodynamic principles and interfacial mechanisms to microfluidic platform design and development. While fabricating microfluidic networks with non-uniform depth and arbitrary cross-sectional geometry is challenging or cumbersome[4], the potential utility for such structures is broad.

Many microfluidic applications could significantly benefit from a facile fabrication approach that provides arbitrary cross-sectional control. For instance, centrifugal microfluidics utilize capillary pressure control for fluid dispensing, controlled mixing, retention and separations within the devices[11]. Similarly, capillary pumping exploits surface tension-driven forces[12,13] and is dictated by fluid meniscus movement via capillary pressure, while resistance to flow dictates flow rates. Variations in the curvature of the inscribed meniscus in triangular and other polygonal channels, for instance, can produce differential capillary pressures; which is strongly correlated with meniscus curvature and the shape of the cross-sectional area[14,15]. As illustrated in Figure 1, threshold drainage capillary pressure varies with the cross sectional shape and, accordingly, wetting phase saturation decreases when the capillary pressure increases (calculation details provided in Results and Discussion section). Accurate predictions of meniscus curvature configuration are critical for understanding the processes like displacement and imbibition in porous media. Additionally, the ability to fabricate microchannels of arbitrary cross section can provide increased versatility for passive devices, thus enabling the development of low cost diagnostic tools[16–20].

Previous efforts to create fluid conduits with variations in cross sectional geometry have met with mixed success. Early attempts to create non-rectangular cross sections include bead-

packs or capillary tube bundles[21–23], which are useful replica geometries but limited to simple networks. Defining structure height via gray scale photolithography[24,25] provides a measure of control over microfluidic channel depth, but has low resolution and produces unreliable results. Isotropic wet etching[26] can be used to create trenches with semi-cylindrical cross sections, while etching with KOH[27] can be used to create sloped side walls with fixed angles[28]. Both of these approaches are limited by the range of aspect ratios that may be created.

Multiphase flow behavior also depends heavily upon channel cross-section[29–31]. Continuity of the wetting phase through connecting corner flows simulates numerous phenomena that occur in a naturally occurring porous media. With a proper experimental platform, these phenomena can be accurately reproduced, observed, and quantified in a microfluidic device[14,32,33]. The implications of more sophisticated microfluidic devices for multiphase flow are particularly relevant to problems associated with immiscible displacements during enhanced oil recovery operations such as waterflooding and surfactant flooding[34–38]. In these applications, glass microfluidic systems, commonly referred to as micromodels[39], are designed to provide two-dimensional topology of interconnected pores as a simplistic representation of oil reservoir rock for the study of multiphase flow. Micromodels have been traditionally produced through an isotropic etching process, producing trough-like cross sections without clearly defined walls and floors[34,40,41]. More contemporary microfluidic platforms are generally created using reactive ion etching or photolithography, which produce rectangular cross sections with well-defined corners[42,43]. All of the microfabrication methods discussed here are effectively two dimensional in nature: the depth created is uniform throughout the extent of a given channel within a microfluidic system. Many applications of microfluidics could benefit greatly from additional cross sectional control as well as simple channel depth control[28].

Recent advances in femtosecond pulse laser micro-machining[44–47] have enabled the fabrication and replication[48] of high aspect ratio and intricately patterned microstructures that can be utilized directly or replicated to create devices. Femtosecond pulsed laser ablated structures provide the same workflow simplicity as a device master that has led to soft lithography becoming a popular laboratory technique[7]. We demonstrate that femtosecond pulsed laser ablation is an effective tool to reliably produce channels with well-controlled triangular cross sections, which can be subsequently replicated to produce elastomeric microfluidic devices. Microchannels of triangular cross section are of potential significance in a wide variety of microfluidics-related fields. We demonstrate the application of replicated triangular channels as functional microfluidic devices, allowing entry capillary pressures to be experimentally measured and used, for the first time, to validate the MS-P theory for triangular tubes.

Fabrication and Experimental Section

Machining Triangular Channels in Glass

Triangular channel fabrication was performed using a 10 kHz repetition rate, Yb:CaF₂ Chirped pulse amplification system, which produces ~1.3 W average power[44] using a standard focusing geometry. At focus this system produces 200 fs pulses. With this

technique, triangular channels were micro-machined on both Gold Seal 25.4mm×76.2mm×1mm glass microscope slides and Esco Optics, 50.8mm diameter × 1.59mm thick, Commercial Quality Flat Circular Pyrex® Windows (#P720063). A CNC milling stage is used to raster the glass through the stationary focus, thus forming a three dimensional pattern in the substrate. Stage motion is directed by converting AutoCAD files to stage coordinates using G-code commands. Vapor debris is expelled using a laser fume extraction system (BOFA, AD Oracle IQ). Plasma mediated ablation, as utilized here is reviewed by Philips et al[49]. The projection of the axial profile of the laser along the interior channel walls results, as a function of ablation depth, in the production of triangular channels as illustrated in Figure 2A&B. Single channels cut by this technique exhibit dimensions common to those used as microfluidic channels with depths ranging from 5 to 150 μm and widths from 15 to 110 μm.

PDMS Microfluidic Device Fabrication from Glass Masters

Femtosecond pulsed laser machined glass channels were employed as masters to create PDMS channels, necessitating a two step replica molding procedure[50]. An alternative PDMS (Silastic 7–4860, Dow Corning) was used for the initial replication step. Owing to this elastomer's high tensile strength, high fidelity, intact replicas can be produced. The glass master was first coated with (Tridecafluoro-1,1,2,2-Tetrahydrocetyl) Trichlorosilane (trichlorosilane) (Gelest, Inc) by vacuum deposition to prevent silicone adhesion to or within glass features[51]. The silastic was then fully cured in an oven for 24 hours at 100 °C. The cured Silastic positive replica of the triangular channels was used as an intermediate to conduct a second soft lithographic step before final device fabrication[50]. Silastic replicas were again treated with trichlorosilane prior to the application of Sylgard (Sylgard 184, Dow Corning) to avoid silicone adhesion. Upon curing the Sylgard was carefully removed from the silastic mold and trimmed prior to bonding to a thin PDMS film-coated glass microscope slide. Sylgard devices and PDMS-coated glass slide were exposed to an oxygen plasma (Harrick Plasma Cleaner, PDC-001) at 475 mTorr pure oxygen and 10.2 W for 90 seconds. After the plasma treatment, the two device halves were placed in contact and cured in an oven at 100° for 10 minutes to form a secure covalent bond.

Experimental methods for entry pressure measurement

Two sets of displacement experiments were conducted to investigate the entry capillary pressures under two wetting conditions and the validity of MS-P theory in predicting those capillary pressures.

1) Water wet surface—Wettability of PDMS devices varies greatly following different treatments. Poly(ethylene glycol) (PEG) (average mol wt 200, Sigma Aldrich) was injected to the triangular channels immediately after the plasma treatment[52] and as illustrated in Figure 3 PDMS exhibits hydrophilicity after the PEG treatment. Subsequently, channels were rinsed with an excess of deionized water to remove residual PEG solution and establish the initial water saturation. Capillary pressures were determined by displacing deionized water (wetting phase) in the channels with air (non-wetting phase). The water-air interface was observed in bright field through the bottom of the device as it sat upon an inverted optical microscope (Olympus IX51). Air was introduced to the device via a microbore

Tygon® tubing (United States Plastic Corp.) and regulated by LabVIEW software. Air pressure was automatically increased in 0.001 psi increments until it began to displace water within the channel at the “breakthrough pressure” or “entry pressure” as indicated in Figure 4A. Figure 4A also shows a series of time lapse images taken after breakthrough, during the advancement of the air phase’s main terminal meniscus.

2) Intermediate wet surface—Bonded PDMS device can recover from the plasma treatment[53,54] to its intrinsic intermediate (slightly hydrophobic) wettability (Figure 3) by resting in a 150°C oven for at least 72 hours. Before conducting experiments, PDMS devices were partially submerged in deionized water in a petri dish for one hour to fully saturate the PDMS matrix with deionized water[53]. Capillary pressures were determined by displacing air in the channels with deionized water. Water pressures were controlled by a variable height water column created by partially filling a burette, which was connected to the device via Tygon tubing (Figure 5A). The burette was raised to increase the hydrostatic pressure until water began to displace air within the device at the “breakthrough pressure”. The right part of Figure 5A shows a series of time lapse images taken after the breakthrough, during the water main terminal meniscus advancement.

Each experiment was conducted in at least two separate but identical sets of channels. Three measurements were recorded in each of the channels in each device.

Device Characterization

To predict theoretical capillary pressures within channels used in flow studies, obtaining accurate measurements of channel dimensions is essential. Channels were characterized using an LEXT OLS4000 optical profilometer (Olympus) and critical dimensions for each channel in the PDMS were measured (Figure 2C) at three locations equally spaced along the channel length. Dimensions of the channels in each PDMS replica may vary slightly due to variability introduced by PDMS replication.

The Wilhelmy plate method was used to anticipate the contact angle of water within PDMS channels following bonding. Thin, uniform samples of PDMS were prepared by pouring an approximately 2 mm thick layer of uncured PDMS on a polished silicone wafer. The wafer was vacuumed and allowed to sit at room temperature for two hours upon a level surface. The wafer was then carefully transferred to a 70 °C oven and allowed to cure. PDMS films were cut into 25mm wide and 20 mm long rectangles, which were suspended within an oxygen plasma under bonding conditions mentioned before. Two different treatments on PDMS samples were conducted prior to the contact angle measurement to achieve different wettability as described in the preceding sections. Contact angle measurements were made in deionized water using the Wilhelmy plate method as adapted for a Krüss K100 tensiometer. As shown in Figure 3, the advancing contact angles for native PDMS and PEG treated PDMS are $94.6^{\circ} \pm 7.1^{\circ}$ and $21.7^{\circ} \pm 7.2^{\circ}$ respectively. The receding contact angles for native PDMS and PEG treated PDMS are $61.9^{\circ} \pm 5.2^{\circ}$ and $0.8^{\circ} \pm 0.9^{\circ}$ respectively. The value of the advancing contact angle of the native PDMS was used to calculate the breakthrough pressure by water in an intermediate wet system. The value of the receding contact angle of the PEG treated PDMS was used to calculate the breakthrough pressure by air in a water wet

system. Of note is the relatively large standard deviation of measured contact angles, particularly for native PDMS. Microscopic surface roughness contributes to the hysteresis and standard deviation, as does wetting heterogeneity. Variation in contact angles for surfaces cut from the same sample implies that native PDMS exhibits significant surface energy variation.

Results and Discussion

Theory

Theories for entry (drainage) capillary pressures originate with models to predict entry pressures for arrays of spheres[22]. Essentially the same theory was independently developed and presented by Princen[21,55,56] subsequently for capillary rise between two rods and for uniform tubes formed by rods. Princen noted that the theory was an approximation for capillary rise because the effect of gravity on meniscus shape was not explicitly accounted for. Mason and Morrow identified the connection between the Mayer and Stowe approximation for spheres and the solution presented by Princen for uniform tubes and referred to the conserved approach for calculating interface curvatures as the MS-P method[57]. The MS-P method can be applied to predict and examine multiphase flows such as oil drainage and imbibition process in reservoirs. Most model porous media flow experiments are conducted within planar pore geometries as a result of fabrication limitations that prevent more accurate pore level representations from being produced.

Strongly water wet condition

Theoretical relationships for capillary pressures and wetting phase saturations during imbibition and drainage in triangular cross sections of arbitrary geometry were presented by Mason and Morrow[14]. In solving the MS-P equation for wetted triangular pores, a shape factor G was introduced and defined by[14]:

$$G = \frac{A}{P^2} \quad (1)$$

where P is a triangular tube's cross sectional perimeter and A is its area. The radius of the arc menisci of displacement that run along the corners of the invaded length of the triangular pore, as shown in Figure 4B, is defined as r_d and can be calculated for a given cross sectional geometry by directly adopting Equation 2 from [14]. Because PEGylated PDMS surfaces, prepared as described here, display a 0.8° receding contact angle according to the Wilhelmy plate measurement, they can therefore be described as a perfectly water wet system.

$$\frac{P}{r_d} = \frac{1}{2G} + \left(\frac{\pi}{G}\right)^{1/2} \quad (2)$$

The capillary displacement pressure, p_c , was calculated by Equation 17 from Ma et al[15] and adapted for an air-water system:

$$\frac{p_c}{\sigma_{aw}} = \frac{L_{aw} + L_{as} \cos \theta}{A_{eff}} \quad (3)$$

where A_{eff} is the channel cross sectional area occupied by the non-wetting phase (air), σ_{aw} is air-water interfacial tension (0.0728 N/m), L_{aw} is the length of the air-water interface, L_{as} is the length of the capillary wall contacting air, and θ is the receding contact angle of water. L_{aw} and L_{as} can be calculated geometrically as a function of r_d . L_{aw} is shown in blue in Figure 4B and is equal to the perimeter of a circle formed by all three arc menisci as illustrated in blue dashed line. L_{as} was calculated by the following equation where β_1 , β_2 , and β_3 are the experimentally measured vertex angles of the triangular tube.

$$L_{as} = P - 2 \frac{r_d}{\tan(\beta_1/2)} - 2 \frac{r_d}{\tan(\beta_2/2)} - 2 \frac{r_d}{\tan(\beta_3/2)} \quad (4)$$

A_{eff} is comprised of one circle, three rectangles and one similar triangle, which was calculated as follows:

$$A_{eff} = \pi r_d^2 + L_{as} r_d + A \left(\frac{L_{as}}{P} \right)^2 \quad (5)$$

Besides computing the entry pressure, water saturations can be predicted based upon the geometry of the triangular cross-section and the varying arc meniscus radius r ([14] and Equation 21).

$$S_w = \left(\frac{r}{P} \right)^2 \left(\frac{1}{4G} - \pi \right) G^{-1} \quad (6)$$

By substituting r with r_d , the critical water saturation upon air entry may be calculated, and is shown to be dependent on the shape of the triangular cross-section. As the capillary pressure in the channel increases (arc meniscus curvature increases), water saturation will decrease.

Theoretical capillary pressures were calculated for each channel and were compared with experimental data in Figure 4C. The experimentally observed breakthrough pressures almost overlap with the theoretical values calculated for each geometry by the MS-P method; which validates the capability of the MS-P theory in predicting capillary pressures of wetted triangular channels with arbitrary cross-section. The only relatively large discrepancy between theoretical values and experimental data is when A_{eff}/P_{eff} is small (~ 9) which is attributed to increasingly sensitive experimental measurements when the entry pressure is high. Nonetheless, a quantitatively accurate prediction of entry capillary pressures in wetted

triangular channels with non-ideally smooth surfaces was obtained and can be employed to estimate capillary pressures if direct measurements are not feasible. Conversely, by measuring the entry pressure of an arbitrary fluid into a wetted triangular channel of known cross sectional geometry, the contact angle may be calculated via Equation 3 when direct contact angle measurements are not practical. This is particularly beneficial if the amount of the liquid of interest is limited as this approach requires less than 200 μ L of fluid to run such an experiment.

Intermediate wet condition

Previous discussion demonstrates the ability of MS-P theory to accurately predict capillary entry pressures when the system is perfectly water-wet. However, more realistic porous media surrogate systems possess a finite contact angle. PDMS devices recovered from the excited state created by plasma bonding are usually slightly hydrophobic, as illustrated by the measured advancing contact angle of around 94°. Therefore, capillary pressure predictions within microchannels of intermediate wettability were performed using bonded PDMS device. During the displacement of water, air resides the corners. Figure 5B schematically illustrates the theoretical arc menisci configuration in the corners based on theoretical predictions from Ma, et al[15].

a) Direct curvature measurements—The radius of the arc meniscus (r_d) is considered to be equal to the thickness of the air in the corners ($l_{measured}$) which was directly measured during experiments as shown in Figure 5A. Therefore L_{ws} , L_{aw} and A_{eff} were calculated by the following equations based upon the proposed configuration of the cross section as sketched in Figure 5B.

$$L_{ws} = P - 3 \times 2r_d \quad (7)$$

$$L_{aw} = \pi r_d \quad (8)$$

$$A_{eff} = A - \frac{\pi}{2} r_d^2 \quad (9)$$

b) Predicted curvature—MS-P theory provides a relationship that allows the radius of the arc meniscus ([15] and Equation 19) to be calculated. Therefore, r_d can be expressed as follows based on the geometry and the configuration of the arc meniscus (Figure 5B):

$$r_d = \frac{A_{eff}}{P_{eff}} = \frac{A - \frac{\pi}{2} r_d^2}{P - 6r_d + \pi r_d} \quad (10)$$

Subsequently, by rearranging Equation 10, r_d can be expressed as a function of the perimeter and cross sectional area of a triangle as follows:

$$r_d = \frac{-P + \sqrt{P^2 + (6\pi - 24)A}}{3\pi - 12} \cong \frac{P(1 - \sqrt{1 - 5.2G})}{2.6} \quad (11)$$

The relationship between entry pressure and air saturation, S_a , at r is given by:

$$S_a = \left(\frac{r_d}{P}\right)^2 \frac{\pi}{2G} \quad (12)$$

Prediction of the air saturation can only be made at r_d because the curvature will be unknown as capillary pressure increases. Equation 7–9 were applied to calculate L_{ws} , L_{aw} and A_{eff} . In this case, the equation to calculate the capillary pressure was derived by equating the work of a small displacement dx to the surface free energy change by the following equation

$$p_c A_{eff} dx = (\sigma_{aw} L_{aw} + \sigma_{ws} L_{ws} - \sigma_{as} L_{ws}) dx \quad (13)$$

Where σ_{aw} is the interfacial tension between air and water, σ_{ws} is the interfacial tension between water and solid, and σ_{as} is the interfacial tension between air and solid. The relationship between air-solid, air-water and water-solid interfacial tension is expressed by Young's equation.

$$\sigma_{as} = \sigma_{ws} + \sigma_{aw} \cos\theta \quad (14)$$

Therefore by combining Equation 13 and 14, the expression for capillary pressure is given by:

$$\frac{p_c}{\sigma_{aw}} = \frac{L_{aw} - L_{ws} \cos\theta}{A_{eff}} \quad (15)$$

The experimental data and calculated theoretical capillary pressures for intermediate wet channels are shown in Figure 5C. While predicted capillary pressures are slightly smaller than experimentally measured entry pressures however in good agreement indicating that cross sectional geometry effects are accurately captured by the MS-P model. Discrepancies in the magnitude between experimental results and theoretical values can be attributed to both the heterogeneity of the PDMS surface chemistry and surface roughness since the MS-P method assumes smooth surfaces and does not account for irregular surface chemistry and roughness. Roughness of channel surfaces can also add resistance to the entering fluid through wetting phase retention by interfacial pinning at asperity peaks and edges, which

may contribute to higher capillary pressure. The discrepancy can also be attributed to the fact that the contact angle measured from the Wilhelmy plate method might not accurately represent the actual contact angle within channels during displacement experiments. A final point of note is that arc meniscus curvature evaluated by direct experimental observation and prediction are in excellent agreement, thus justifying the use of this approach to predict arc meniscus radii under intermediate wet conditions. The relatively good agreement between theory and experimental data in intermediate wet channels illustrates that the classical MS-P method derived for perfectly wet triangular tubes can be extended to finite contact angles. This application of the MS-P method may also provide a valuable estimate of the effective contact angles in channel, when they are expected to be different from those measured by the Wilhelmy plate method or other direct approaches.

Conclusions

We have introduced femtosecond pulsed laser micromachining as a tool to directly write microchannels of triangular cross section within glass substrates. Using this fabrication method, with replication in poly(dimethylsiloxane) (PDMS) via soft lithography enabled by a high tensile strength silicone elastomer intermediate, we have produced a series of triangular microfluidic channels that were used to validate Mayer, Stowe and Princen (MS-P) theory that predicts the capillary displacement pressure within triangular channels. The MS-P model has been hypothesized to be applicable to triangular tubes with corner imperfections of arbitrary cross sectional geometry, and has been widely utilized by network models to predict pore pressures without experimental validation. [30,58] Here, experimentally observed breakthrough pressures in triangular channels are shown to be consistent with MS-P predicted capillary pressures, confirming the applicability of the theory to arbitrary triangular cross sections under perfect wet conditions. Strong correlation between experimentally observed and theoretically predicted capillary pressures also verified that triangular channels are well-defined and uniform over a range of cross sectional dimensions. The application of MS-P theory has been extended to the intermediate wet condition that has been previously studied by computational methods[31]. Our work provides an approach to estimate the entry capillary pressure as well as the effective contact angles under a range of wetting conditions. These results have important implications for microfluidic technologies, specifically related to the preparation of two dimensional pore network micromodels for investigation of two-phase flow mechanisms[59,60]. While the MS-P model is independent of geometry, its application to arrays of spheres becomes increasingly inaccurate with increase in contact angle but remains exact for tubes of uniform cross-section. This further indicates that triangular channels fabricated here possessed uniform cross-section throughout their length and suggest that this fabrication approach can be used to investigate the limitations of the MS-P method to nonuniform pore geometries with increasing contact angle.

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Nomenclature

A	area
G	shape factor
L	contact length
P	perimeter
p_c	capillary pressure
r_d	radius of the arc meniscus of displacement
S	saturation
β	inner angle of a triangle
θ	contact angle
σ	interfacial tension

Subscripts

a	air
aw	air-water
as	air-solid
eff	effective
w	water
ws	water-solid

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Figure 1. Illustration of the threshold drainage pressure in triangular channels with various cross-sectional shape. Wetting phase saturation decreases as the drainage curvature (capillary pressure) increases from the threshold drainage curvature.

Figure 2.

(A) Schematic of femtosecond pulsed laser cutting used to produce triangular channels. Blue rectangle represents the glass substrate and the yellow dot represents the laser. The sequence of the images illustrates the process of laser cutting on the glass and ultimately results in a triangular cross section. (B) Image of a representative triangular cross section. (C) Microscopy images of the triangular cross sections observed by slicing through the PDMS channel cross section. The table below summarizes the dimensions of the triangular channels measured by the profilometer.

Figure 3. Contact angle measurements of both native and treated PDMS samples by Wilhelmy plate method. (A) Dynamic contact angle is plotted versus the immersion depth of the PDMS sample into water (B) Average advancing and receding contact angle values are reported.

Figure 4.

(A) A series of images showing the displacement progression of the wetting phase water (lighter) by the non-wetting phase air (darker). The schematic on the left demonstrates that air was controlled and monitored by a pressure gauge. (B) A schematic of a triangular cross section with arc menisci at strongly water wet condition. (C) “Entry pressure” data measured during the experiment compared with theoretical values calculated by MS-P theory for triangular channels varying shape.

Figure 5.

(A) Series of images showing the displacement progression of air (darker) by water (lighter). The schematic on the left demonstrates that water was controlled by varying the height of the water column. Black bars drawn on the third picture indicate direct measurements, which correspond to the radius of arc menisci with intermediate wet surfaces. (B) A schematic of a triangular cross section with arc menisci at the intermediate wet condition. (C) “Entry pressure” data measured during the experiment compared with theoretical values calculated by MS-P theory for triangular channels varying shape.