Unobserved Heterogeneity of Frailty in the Analysis of Socioeconomic Differences in Health and Mortality

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Abstract The concepts of unobserved frailty and selection have been extensively analyzed with respect to phenomena like mortality deceleration at old ages and mortality convergence or cross overs between populations (for example American black and white populations, men and women). Despite the long-time observation of converging mortality risks in differential socioeconomic mortality research, the interest in the connection between frailty, selection, and health and mortality inequalities over a life course approach has increased only recently. This overview of the literature summarizes the main concepts of unobserved frailty and socioeconomic differences in mortality and how frailty and selection relate to these differences at old ages. It then reviews the evidence coming from the existing studies. Although the number of studies is still limited, the body of research on unobserved frailty and socioeconomic inequalities in health and mortality in a life course approach is growing. The results, however, are contradictory, and whether selection plays a major role in shaping the observed socioeconomic mortality patterns at old ages is still controversial.

Keywords Frailty · Heterogeneity · Socioeconomic inequality · Mortality · Health

1 Introduction

The notion of unobserved heterogeneity of frailty, from which the literature about frailty models has sprouted, was introduced in demography by Vaupel et al. (1979). The concept pointed out the fact that all populations have a higher degree of

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heterogeneity than the one explained by observed covariates, because individuals differ one from another for many other unobserved characteristics called *frailty*. Even when the investigator has at his disposal a very rich data set with individual level information (a situation that, however, seldom occurs), there will always be a component of heterogeneity that is unobserved. Frailty, in fact, is a general concept that does not distinguish between acquired weakness, lifestyle factors, environmental risks, and innate biological frailty. It combines in a single measure all the factors that operate to increase or decrease a given individual's mortality risk, regardless of the source of heterogeneity (Manton et al. 1981).

Besides affecting the individual's mortality risk, the presence of unobserved heterogeneity of frailty influences mortality at the population level, contributing to shaping the observed dynamics. A population, in fact, is a mixture of several individuals with unobserved features. Some individuals are frailer and some are more robust. As the cohort ages, frailer individuals exit the population earlier than more robust ones, gradually selecting the surviving population toward robustness. This implies that the observed dynamic at the population level does not necessarily correspond to the dynamic experienced at the individual level. For example, at old ages the observed population risk of death decelerates and eventually levels off. This phenomenon is known as mortality deceleration. However, this does not mean that the individuals, as they age, see their probability of death increasing less quickly. To address the problem of unobserved heterogeneity and to improve the fit of mortality models to populations, especially at very old ages, frailty models have been suggested. These allow for individual, hidden differences to be taken into account, explain cross over and deceleration at old ages (Vaupel and Yashin 1983, 1985), correct biased estimates of regression coefficients in proportional hazard regression (Chamberlain 1985) and to separate biological effects from compositional ones in studies about aging (Manton et al. 1986).

The gradual selection and compositional change caused by unobserved heterogeneity have been used to explain phenomena like decreasing relative mortality differences at old ages between men and women (Manton et al. 1995; Kohler and Kohler 2000) and the American black and white populations, whose age trajectories not only converge, but produce a cross over (Hambright 1969; Kitagawa and Hauser 1973). Although the validity of this pattern has been questioned and the cross over attributed to data problems and inaccuracy of information at old ages (Coale and Kisker 1986; Preston et al. 1996), studies conducted on more accurate subpopulation data have also reported this phenomenon (Wing et al. 1985), testifying for its consistence. Selective effects of high early mortality experienced by the disadvantaged group would leave more robust individuals at old ages, causing convergence with the hazard of the lower mortality group that is subjected to weaker selection (Manton and Stallard 1981).

These concepts also find relevance in social epidemiology, where narrowing differences in mortality and health by socioeconomic status are often observed with age (Huisman et al. 2003, 2004, 2005; Merlo et al. 2003; Enroth et al. 2013; Martelin 1994; Shkolnikov et al. 2008; Jaffe et al. 2008; Martínez et al. 2009; Hoffmann 2005, 2011b). According to the unobserved frailty framework, the convergence would be an artifact of selection at the population level and would not

necessarily mean that the single individuals experience a reduction of the advantage/disadvantage as they age.

Unobserved heterogeneity has another important implication. In duration dependence models (i.e. survival analysis models), its presence poses problems for estimating the hazard function and the effect of observed covariates on the survival chances. Because of unobserved heterogeneity, in fact, the population undergoes a constant compositional change that, if neglected, can lead to biased estimates of the hazard rate and of the regression coefficients for the covariates (Aalen 1994, 1988), even if the omitted source of variation is uncorrelated with the predictors included in the model, because, at every age, the death rate is computed based on a population at risk whose composition is constantly changing (Trussell and Rodriguez 1990; Rodriguez 1994).

The investigation of these dynamics has gone on and new approaches have been developed, for example in the bayesian framework (Cai 2010). Moreover, the general umbrella of heterogeneity today considers various other sources of bias it is important to be aware of, like disparate types of attrition in panel studies (Zajacova and Burgard 2013), cohort specific confounding factors that can determine the age patterns of group mortality (Masters 2012) as well as the association between various health indicators and health (Willson et al. 2007). Recently, attention has been paid again to the fact that societal changes in economic structures over time can lead to changing distributions of covariates and their associations (Chen et al. 2013). As already pointed out by Riley (1973, 1987), a single life course does not exist, but rather cohorts experience distinct life courses embedded in their specific historical context.

In the last decade, the interest in the connection between unobserved heterogeneity and socioeconomic differences in health and mortality has increased. Most of the debate is focused on whether selection processes play a role in the life course pattern of socioeconomic differentials, although the number of studies directly addressing this question is still limited.

The aim of this paper is to (1) offer a brief theoretical summary of the main concepts of unobserved frailty, selection processes, socioeconomic differences in health and mortality (with a particular focus on old ages) and how these three things relate to each other; (2) offer a summary of the evidence coming from the existing studies that analyze the connection between frailty, selection and old age socioeconomic differences in health and mortality.

2 Socioeconomic Differences in Mortality

The relation between socioeconomic status, health and mortality conditions has been known and analyzed for a long time (Antonovsky 1967). Several studies in different countries consistently find health and mortality disadvantage of the lower social classes (Mackenbach et al. 2006, 2008; Vallin 1980; Black et al. 1980; Martikainen et al. 2009, 2001; Huisman et al. 2004; Elo et al. 2006; Koskinen et al. 2007; Braveman et al. 2010). A growing body of research has confirmed the existence of a substantial gradient between social groups, independently on the

indicator used as proxy of the socioeconomic status: income (Martikainen et al. 2009), occupational status (Martikainen et al. 2001), education (Elo and Preston 1996), house value as a measure of accumulated wealth (Connolly et al. 2009), housing tenure or a combination of these (Huisman et al. 2004; Elo et al. 2006; Koskinen et al. 2007; Dalstra et al. 2006).

The explanations of how socioeconomic status can influence health and mortality outcomes are diverse. Generally, they emphasize the causal direction from socioeconomic to health inequality, recalling behavioral factors, material factors, and psychosocial factors. Among the first explanations, the most important role seems to be played by cigarette smoking (Mackenbach et al. 2004; Blakely and Wilson 2005; Stringhini et al. 2010) and dietary and exercise habits (Smith and Brunner 1997; Lindström et al. 2001; Stringhini et al. 2010). Material factors include, among others, occupational health risks (Schrijvers et al. 1998; Rahkonen et al. 2006), access to medical care (Mackenbach et al. 1989; Doorslaer et al. 2006), and housing conditions (Krieger and Higgins 2002). Psychosocial factors, instead, relate to stressful situations like feelings of deprivation, financial problems, and employment insecurity (Wilkinson 1997; Aldabe et al. 2011). A systematization of the proximate determinants of socioeconomic inequality in health and mortality can be found in Hoffmann (2008).

Other scholars suggest putting more emphasis on the sociological approach to explain the persisting association between socioeconomic status and health. They point out the importance of contextualizing individuals risk factors and specific mechanisms: Social conditions become the “fundamental causes” of diseases, because they mediate the access to the resources that affect health outcomes through these specific risk factors and mechanisms, and when these are changed or eliminated, the disparities would endure by finding new pathways (Link and Phelan 1995; Phelan et al. 2004).

Differences in health and mortality related to socioeconomic status vary with age. Absolute differences increase at old ages, due to the fact that the overall mortality level grows exponentially, and thus even a small difference in the rate of increase leads to wider and wider absolute differences. On the contrary, relative differences tend to decrease with age. Evidence has been found in both morbidity (Huisman et al. 2003; Merlo et al. 2003; Enroth et al. 2013) and mortality (Martelin 1994; Merlo et al. 2003; Huisman et al. 2004, 2005; Shkolnikov et al. 2008; Jaffe et al. 2008; Martínez et al. 2009; Hoffmann 2005, 2011b), with few exceptions (Olausson 1991; Huisman et al. 2004; Rostad et al. 2009).

Different theoretical frameworks can currently be found in the literature addressing the age pattern of socioeconomic differences in mortality.

The *cumulative disadvantage* hypothesis states that the effect of socioeconomic status on health and mortality increases with age, creating pathways of disadvantage that accumulate over the life course and widen inequalities (Ross and Wu 1996; Beckett et al. 2002). Suggested ways of accumulation are related to the time lag between the experience of some deleterious event and its effects on health (Lauderdale 2001), increased vulnerability at old ages that makes differential exposure more dangerous for lower social classes (as they are supposed to be disadvantaged in several risk factors like unhealthy work conditions, high levels of

stress, and economic deprivation House et al. 1994). On an empirical basis, however, evidence supporting the arguments of the *cumulative disadvantage* hypothesis are limited (Dupre 2007).

The *age as leveler* framework states that health and mortality inequalities decrease at old ages. The weakening association between socioeconomic status and mortality at old ages is seen as a result of factors like governmental support for the elderly (House et al. 1994; Decker and Rapaport 2002; Dor et al. 2006), general vulnerability at old ages (Elo and Preston 1996; Liang et al. 2002) and disengagement from systems that contribute to social stratification like the labor market (Marmot and Shipley 1996). These factors would contribute to level off differences between individuals, that, consequently, become more equally susceptible as they age.

Finally, a third explanation for the convergence by social classes at old ages is represented by mortality selection. As frailer individuals die faster than more robust ones the population is gradually selected toward robustness, thus causing the convergence of the high mortality group toward the low mortality group (Vaupel and Yashin 1985). Not only higher frailty, but exposure to higher mortality act through the same mechanism. Lower social groups suffer from higher early mortality (van Raalte et al. 2011; Zarulli et al. 2012), which is a stronger selective force. This leads to more compositional change as the population ages, leaving a more robust surviving population at old ages compared with upper social groups. This would decrease the association between social class and health with age.

3 Unobserved Heterogeneity of Frailty

Every individual has a specific level of frailty, z , that defines the individual risk of death in a context of proportional hazard models as:

$$\mu(x, z) = z\mu(x) \quad (1)$$

where $\mu(x)$ is the baseline risk and z acts multiplicatively on it.

A population comprises many individuals, with different unobserved frailties, that all contribute to the population dynamic. At any age x , what is observed is the mean mortality rate at that age, $\bar{\mu}(x)$, for the survivors of each frailty. Since frailty cannot be negative, this can be expressed as:

$$\bar{\mu}(x) = \int_0^{\infty} z\mu(x)f_x(z)dz \quad (2)$$

where $f_x(z)$ is the probability density function of frailty at age x among the survivors. $\mu(x)$ can be taken out of the integral and what is left inside is the mean frailty among survival at age x :

$$\bar{\mu}(x) = \mu(x)\bar{z}(x) \quad (3)$$

Equation 3 shows that individual and population hazards are different. In particular, the individual hazard $\mu(x)$ increases at a faster pace than the population hazard $\bar{\mu}(x)$, because the average frailty of the population, $\bar{z}(x)$, decreases with age. On a log scale, when age-specific death rates are plotted against age, the points follow a straight line, but they slow down at older ages, causing a mortality deceleration. As the frailest ones die out faster, the population hazard decelerates. Eventually, the population hazard would converge to the level of the lowest frailty. In particular, because $\bar{z}(x)$ decreases with age, the population hazard grows less rapidly than the individual one. The difference between the two becomes bigger and bigger at advanced age, as the selection process has had enough time to select the more robust individuals. The result, at the population level, is a mixture model of many individual curves, which reflects the frailty compositional change over age and causes the mortality deceleration because the surviving population comprises less frail individuals (Caselli et al. 2000; Vaupel 2010).

Heterogeneity of frailty can take different shapes, not necessarily proportional to each other. Even with different configurations of frailty, the differential selection of the individual at higher risks can produce patterns of population mortality very different from the mortality of its individuals (Duchateau and Janssen 2008).

The relation between population and individual hazard is complicated by the fact that frailty, by definition, is unobservable. Therefore, a distribution must be assumed for frailty, which is usually assumed to follow a gamma distribution.

Gamma is a convenient distribution because it is positive (and frailty can not be negative) and has a very flexible shape (this is convenient because we do not know what the frailty distributions looks like), and its Laplace transform allows us to express the population survival function as a gamma mixture of individual survival functions.

The Laplace transform of a function $f(t)$ is a function $F(s)$ defined as

$$F(s) = \mathcal{L}f(t) = \int_0^{\infty} e^{-st} f(t) dt \quad (4)$$

If $f(x)$ is the frailty distribution $f(z)$ and s is the cumulative baseline hazard $M(x)$, the population survival turns out to be the Laplace transform of the frailty distribution evaluated at the cumulative baseline hazard:

$$\bar{S}(x) = F(s) = \mathcal{L}f(z) = \int_0^{\infty} e^{-M(x)z} f(z) dz \quad (5)$$

The probability density function of a gamma distribution with shape and scale parameters α and β is:

$$f(z) = z^{\alpha-1} e^{-\beta z} \beta^{\alpha} / \Gamma(\alpha) \quad (6)$$

Its mean and variance are $E(z) = \frac{\alpha}{\beta}$, $var(z) = \frac{\alpha}{\beta^2}$ and its Laplace transform is $\left(\frac{\beta}{\beta+s}\right)^{\alpha}$. Therefore,

$$\bar{S}(x) = \left(\frac{\beta}{\beta + M(x)} \right)^\alpha \quad (7)$$

By standardizing the frailty distribution to mean 1 and variance σ^2 , so that $\alpha = \beta = \frac{1}{\sigma^2}$, the model becomes identifiable.

It follows that, under gamma-distributed frailty, the population survival is:

$$\bar{S}(x) = \frac{1}{(1 + \sigma^2 M(x))^{\frac{1}{\sigma^2}}} \quad (8)$$

and the population hazard is:

$$\bar{\mu}(x) = \frac{d}{dx} - \ln(\bar{S}(x)) = \frac{\mu(x)}{1 + \sigma^2 M(x)} \quad (9)$$

where $\mu(x)$ is the standard individual (or baseline) hazard, $M(x)$ is the cumulative baseline hazard and σ^2 is the variance of frailty at the initial age.

Other distributions have been suggested for modeling unobserved heterogeneity of frailty (Hougaard 1986). Popular distributions are the inverse-Gaussian (Hougaard 1984) and the log-normal (McGilchrist and Aisbett 1991). However, Missov and Finkelstein (2011), showed that, given the observed mortality pattern at old ages (that decelerates and, at very old ages, tends to reach a plateau Gampe 2010), the inverse-Gaussian and the log-normal distributions are not admissible, because they do not lead to a plateau in the mortality rate, while the gamma distribution is among the admissible distributions.

The assumption of a gamma-distributed frailty yields useful mathematical results that are helpful for the development of the estimation procedures. In particular, frailty among the survivors is also gamma-distributed, but now with parameters α and $\beta + M(x)$; frailty among those who die is also gamma-distributed with parameters $\alpha + 1$ and $\beta + M(x)$.

This implies that at age x , the average frailty among survivors is somewhat lower than frailty among those who die and shows how the selection process operates eliminating the high-risk individuals:

$$\bar{z}(x)_{\text{surv}} = \frac{1}{1 + \sigma^2 M(x)} \quad (10)$$

$$\bar{z}(x)_{\text{dead}} = \frac{1 + \sigma^2}{1 + \sigma^2 M(x)} \quad (11)$$

It also implies that the variance decreases over age and the population becomes more homogeneous in absolute terms:

$$\text{var}z(x)_{\text{surv}} = \frac{\sigma^2}{(1 + \sigma^2 M(x))^2} \quad (12)$$

$$\text{var}z(x)_{\text{dead}} = \frac{\sigma^2(1 + \sigma^2)}{(1 + \sigma^2 M(x))^2} \quad (13)$$

But how does the selection hypothesis apply to the convergence of the mortality age trajectories of different groups, for example two socioeconomic groups?

The mechanism is based on different paces of selection: as lower social groups are likely to suffer from higher early mortality (van Raalte et al. 2011; Zarulli et al. 2012), they are selected at a faster rate than the other groups. Therefore, at the same (old) age, what is left in these groups is a more selected population in terms of robustness. The difference in the rates of mortality selection causes the mortality curves to converge and gives the impression that the effect of the covariate declines with age.

Let $\mu_l(x)$ be the individual hazard for the low-education group and $\mu_h(x)$ the individual hazard for the high-education group. From Eq. 3 it follows that the ratio of the two educational population hazards is

$$\frac{\bar{\mu}_l(x)}{\bar{\mu}_h(x)} = \frac{\mu_l(x)\bar{z}_l(x)}{\mu_h(x)\bar{z}_h(x)} \quad (14)$$

This ratio declines because, although the ratio of the two individual hazards remains constant over age, the two groups undergo different paces of selection. In particular, the low-education group is selected at a faster pace and its average frailty declines also faster (the numerator of the right-hand side of Eq. 14 decreases at a faster pace than the denominator) (Zajacova et al. 2009).

More formally, considering that in the framework of the frailty models the mean frailty at the initial age is assumed 1, at such age the left-hand side of Eq. 14 is equal to the right-hand side. This implies that, at the initial age, the ratio between the two educational groups is the same both at the individual and at the population level. As it is showed in Aalen et al. (2008a)[250], let r be this ratio at the initial age 0, the hazard for the low-education group can be expressed in terms of the hazard of the high-education group: $\mu_l(0) = r\mu(0)_h$, where $r > 1$. It follows that, as age goes by, the ratio between the two population hazards is

$$\frac{\bar{\mu}_l(x)}{\bar{\mu}_h(x)} = r \frac{1 + \sigma^2 M_h(x)}{1 + \sigma^2 r M_h(x)} \quad (15)$$

Equation 15 shows how the ratio between the two groups is r only at the initial age (when the cumulative hazard M is 0) and decreases as the age increases, because r is multiplied by a decreasing quantity.

It is important to remark that no causal direction is implied in the mechanisms connecting frailty and health susceptibility with socioeconomic status. Because of their higher mortality, the lower socioeconomic classes are considered to be frailer than the others. However, this does not imply that the higher frailty attributed to them determines their lower social condition. It may be the opposite as well: Given that exposure to detrimental conditions (working related risks, worse housing

conditions, bad health behaviors like smoking and so on) is socioeconomically patterned, lower socioeconomic groups can be considered frailer.

When dealing with life time data in duration dependence models, the presence of unobserved heterogeneity poses problems for calculating the effect of the covariates on the survival chances. Not taking this component into account, in fact, biases the parameter estimates of both the risk function and the regression coefficients. Contrary to linear regression models, the estimates are biased even if the omitted source of variation is uncorrelated with the predictors included in the model (Gail et al. 1984; Trussell and Rodriguez 1990; Rodriguez 1994). This is caused by the most distinctive feature of survival analysis, the time-dependent hazard. Selection, due to the unobserved (i.e., omitted) covariates, acts constantly on the risk set by gradually eliminating the frailest individuals (Chamberlain 1985; Aalen et al. 2008a, b).

When one important factor is ignored, the relative risk of the covariates on the survival times is underestimated. This can lead to loss of power in studies of treatment comparison (Schumacher et al. 1987; Schmoor and Schumacher 1997). More generally, when only one covariate is left in the model, whatever the number of omitted variables are, the effect of that covariate on survival is underestimated; when several covariates remain in the model, underestimation still holds, at least up to some reasonably long time (Bretagnolle and Huber-Carol 1988). Other econometric and biometric studies provide further substantial evidence for the bias introduced by ignoring the presence of hidden variability (Lin et al. 1998; Betensky et al. 2002) and show that the amount of bias depends on the hidden variability present in the population. The amount of the bias also depends on the extent of heterogeneity: The higher the heterogeneity of frailty, the bigger the bias (Henderson and Oman 1999).

The frailty approach in the analysis of life time data allows one to account for the presence of unobserved variability, which has an impact on the survival chances, by introducing a random effect for the frailty term that is multiplicative on the hazard function. There are two broad classes of frailty models: the univariate frailty models and the multivariate frailty models.

The first class deals with univariate, independent, and identically distributed survival data, for example the age at death of unrelated individuals. In the traditional proportional hazards regression setting, the hazard depends on the baseline hazard, assumed to be unique and common to all individuals, and on the observed covariate profiles. In addition to them, in the frailty model, the hazard depends also on the frailty, introduced in the model as a random effect term that modulates the baseline hazard multiplicatively. In this way, the variability of the data is split into two parts, one accounted for by measurable and observed risk factors and one resulting from the hidden frailty, which aims to account for omitted covariates and to describe the non-proportionality of the population hazards (for example when the hazards of two subgroups converge at old ages) (Yashin et al. 2001).

The second class of models are used with multivariate survival data, where independence between event times cannot be assumed. This is the case, for example, of recurrent events for the same individual (occurrence of a disease) or the age at death of related individuals, like twins or parents and children. These event

times are clustered, and each cluster (the family, the pair of twins, the individual) is assumed to have a common hidden frailty that is responsible for creating dependence between the event times within the cluster. In this setting, the assumption is that the event times are independent, conditional on the latent frailty variable.

Two models have been developed in the multivariate setting. The shared frailty model assumes that individuals in a cluster share the same frailty. This is used for strongly related individuals, like monozygotic twins, or repeated measurements for the same individual. However, the assumption of an identical frailty can be too strong and restrictive in several situations (for a detailed discussion on these limitations see, for example, Xue and Brookmeyer 1996; Wienke 2010). To avoid the limitations, the correlated frailty models have been developed. The individuals in each cluster are assigned to have separate frailties, in the form of random variables that are assumed to be associated and to have a joint distribution. The correlation between frailties is a much less strict assumption than shared frailty and it does not restrict the correlation to one type. For example, the correlation could also be negative, inducing a negative association between survival times in the cluster, while in most cases, the shared frailty model only induces a positive association within the cluster.

4 Frailty Models and Differential Mortality

In the last decade, the interest in the connection between unobserved heterogeneity and socioeconomic differences in health and mortality has increased. The number of studies directly addressing this question or using frailty models in the estimation of health and mortality differentials, however, is still limited. The main problem that has so far posed limits to this quest was the lack of availability of individual, longitudinal data, long enough to allow the investigation of the impact of unobserved heterogeneity of frailty and selection on the observed socioeconomic patterns of health and mortality. Fortunately, such datasets have recently become more and more available, thanks to the availability of long term surveys or registry based administrative data.

One study focused on the potential underestimation of the covariates produced by neglecting the frailty component and found that, in the majority of the estimated models, the educational difference in mortality obtained by survival analysis models without frailty was narrower than when frailty was included in the models (Zarulli et al. 2013).

However, most of the debate is focused on the role of unobserved heterogeneity and selection on the observed converging hazards by socioeconomic groups at old ages. The central question is whether the observed pattern of gradual narrowing health inequalities is due to unobserved frailty and selection (i.e., attenuation bias), which would act through compositional changes in the surviving population, or to other mechanisms like differential aging processes in different groups or changes in the impact of socioeconomic status over life. Empirical evidence, even though growing, is still little, and the results so far are fragmentary and not consistent.

Some findings support the idea that different socioeconomic groups have different aging processes (with the upper social groups being able to postpone health decline up to older ages than the lower classes) (Beckett 2000; House et al. 1994). Others lend support to the idea that the social context and its effect on health change with age (so that factors like retirement and social security slow down the health deterioration for those in the lower classes) (Herd 2006).

When it comes to testing the selection hypothesis, some studies found that selection causes the convergence of the hazards at old ages, while others did not find evidence that selection plays a key role in such a converging pattern.

Among the studies that found empirical evidence for a role of selection, Ferraro and Farmer (1996), while analyzing a 15-year-long longitudinal survey, found that selective mortality biased downward the racial and gender differentials in the USA for two of the four health measures considered. In the study by McMunn et al. (2009), selective mortality appears to explain the decline in heart diseases inequalities at old ages. In Dupre (2007) convergence by educational groups in disease prevalence was observed, but the rates of incidence and mortality were greater for the lowly educated groups than for highly educated ones. This suggests that educational differences in incidence and mortality resulted in the convergence of prevalence, reflecting the bias caused by selection. Other studies focused specifically on mortality by various types of income groups (individual income, household income, income of the spouse and deprivation area index) and found a strong and significant presence of unobserved heterogeneity, even though the attenuation bias that resulted is not particularly big (Congdon 1994; Frijters et al. 2011; Kalwij et al. 2013; Kalwij 2014).

Among the studies that did not find empirical evidence for a role of selection, Lynch (2003) found that, once the potential cohort effects were controlled for,¹ the impact of education on health strengthened and not weakened with age, as predicted by the selection mechanism. However, when evaluating the correlation between mortality and education over age, one must be aware that in addition to cohort effects, period effects must also be considered. In particular, factors such as the cardiovascular revolution and the subsequent mortality reduction at old ages or the general increase of the average level of education, might represent period confounders on the way mortality can be affected by education (Lauderdale 2001). Herd (2006), while following an individual cohort over time from the Health and Retirement Study (1992–2002), found that selection did not fully explain the shrinking educational disparities in functional health at old ages. It must be pointed out that the results could have been affected by some data limitations such as an inadequate follow-up, which did not allow the following of individuals into their mid-to-late 70s and 80s. That the mortality selection bias is unlikely to be a major explanatory factor was found also by Kim and Durden (2007), whose results indicate that age-related patterns of health trajectories may differ according to the used dimensions of socioeconomic and health status.

¹ In the investigation of decreasing health inequalities at older ages, one should be aware that these inequalities might be displaying cohort effects and not just age effects as showed by Willson et al. (2007) and Masters (2012).

Finally, another type of selection that has been investigated is sample selection in survey data, that is, the possibility that the non respondents might be a selected population. In particular, their characteristics may be related to poor health, especially at old ages (Goldman et al. 1995). This is a particularly important point, as most of the studies are carried out with survey data. With this respect, Beckett (2000), while investigating the role of sample selection due to loss of follow-up in panel data, found no evidence that this is responsible for the convergence in education differentials in health conditions or functional impairment. On the contrary, Zajacova and Burgard (2013) found evidence that sample selection and other disparate types of attrition can cause the observed convergence of hazards at old ages.

5 Conclusion

The present paper focused on selection due to unobserved heterogeneity of frailty in the analysis of socioeconomic differences in health and mortality at old ages.

It is important to discuss this issue because (1) the framework of selection due to hidden frailty offers an explanation for the often observed pattern of reduction of the relative differences in health and mortality by socioeconomic status at old ages; (2) neglecting the presence of the unobserved frailty component might lead to biased estimates of the hazard rate and the covariates in duration dependence models.

With respect to the first point, there is a lot of empirical evidence showing that socioeconomic differentials in health and mortality narrow down at old ages (Huisman et al. 2003, 2004, 2005; Merlo et al. 2003; Enroth et al. 2013; Martelin 1994; Shkolnikov et al. 2008; Jaffe et al. 2008; Martínez et al. 2009; Hoffmann 2005, 2011b). The selection hypothesis explains this pattern as a result of the gradual selection of the most robust individuals and a compositional change of the population toward robustness as the frail ones die out earlier. Different socioeconomic groups would therefore undergo differential paces of selection, which in turn would lead to the convergence of the group specific mortality risk as age increases, thus showing a weakening association between socioeconomic status and mortality (Zajacova et al. 2009; Aalen et al. 2008a).

The age as leveler hypothesis argues that this happens because factors such as health policies, pensions and governmental support to the elderly make individuals more equal as they grow older and older (House et al. 1994; Decker and Rapaport 2002; Dor et al. 2006; Elo and Preston 1996; Liang et al. 2002; Marmot and Shipley 1996). This hypothesis has been opposed by the cumulative disadvantage hypothesis, which states that the socioeconomic advantage/disadvantage increases with age through accumulating pathways (Ross and Wu 1996; Beckett et al. 2002; Lauderdale 2001; House et al. 1994), even though the empirical evidence for this hypothesis is limited.

The age as leveler hypothesis and the cumulative disadvantage hypothesis have often been seen as contrasting and mutually exclusive. However, they could be reconciled on the grounds of the selection hypothesis. What is observed at the population level is the result of the interaction between the individual dynamics that

create the final aggregate population dynamic. While at the population level the disadvantage of lower social classes is observed to diminish at old ages, the selection hypothesis makes it possible to still have this pattern, without assuming that the individuals experience the same reduction at the individual level over their life course.

With respect to the potential estimation bias, the statistical theory predicts that, in duration dependence models, neglecting the presence of unobserved heterogeneity can bias the estimates of the hazard function and the effect of the covariates on the survival chances. This bias occurs even if the omitted source of variation is uncorrelated with the predictors included in the model and has been shown to be toward zero (Gail et al. 1984; Trussell and Rodriguez 1990; Rodriguez 1994; Chamberlain 1985; Aalen et al. 2008a, b; Schumacher et al. 1987; Schmoor and Schumacher 1997; Bretagnolle and Huber-Carol 1988; Lin et al. 1998; Betensky et al. 2002; Henderson and Oman 1999). Therefore, when analyzing socioeconomic differences in health and mortality using standard survival analysis models, ignoring this component can lead to an underestimation of the health and mortality differentials. Frailty models have been developed to account for this problem, but yet in differential mortality research they are seldom used. This is quite surprising given the strong statistical evidence showing that ignoring unobserved characteristics can lead to an underestimation of the relative risks between groups.

The number of studies directly addressing the connection between this source of variation and socioeconomic differences in health and mortality is still limited, although in the last decade the interest has increased and the body of research is growing. Many of the existing studies have focused either on narrowing differences in health or mortality, while fewer have analyzed both dimensions. Results are so far contradictory. Some studies found evidence that selection processes are responsible for the converging hazards at old ages by socioeconomic groups (Ferraro and Farmer 1996; McMunn et al. 2009; Dupre 2007; Congdon 1994; Frijters et al. 2011; Kalwij et al. 2013; Kalwij 2014). Other studies found that selection plays no significant role in this often observed pattern (Lynch 2003; Lauderdale 2001; Herd 2006; Kim and Durden 2007). In general, more studies failed to detect a significant role of selection when the focus of the analysis was on health. The majority of the studies focusing on mortality, on the contrary, found a key role of selection in explaining the narrowing gap between different socioeconomic groups.

These conflicting findings produce a lively debate. Some authors support the age as a leveler hypothesis, while others affirm that this hypothesis does not adequately describe the real patterns, especially at the individual level. More recently, studies have found stable mortality differentials across ages but narrowing differentials when health deteriorates. This suggests that, instead of age, health status might act as leveler (Hoffmann 2005, 2011a). On the other hand, even if it is reasonable to assume that increasing age is related to worsening health, for analytical purposes, these two dimensions should be kept separate (Kelley-Moore 2010): While age increases for everyone, health can decline in varied ways between different social groups and between individuals. Moreover, the declining pattern can depend on the type of health indicator used. All these factors undoubtedly contribute to make the

picture somehow blurry and the identification of significant selection effects more difficult.

This paper has demonstrated how intricate the question of how and why the socioeconomic health and mortality gap tends to reduce at old ages is. Biological and social, individual, and collective factors are all likely to play an important role. Moreover, the picture is complicated by the action of unobserved factors and dynamics that cannot be ignored in order to eliminate or reduce the bias in the results. More empirical evidence is needed to be able to disentangle the various components of this complex phenomenon. Until recently, one of the major obstacles to this kind of investigations was the lack of appropriate data but, fortunately, future research has at its disposal increasingly available longitudinal datasets.

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