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Estimation of Supercapacitor Charge Capacity Bounds Considering Charge Redistribution

Hengzhao Yang [Member, IEEE]

Department of Electrical Engineering at California State University, Long Beach, CA 90840, USA

Abstract

This paper investigates the impact of charge redistribution on supercapacitor charge capacity by experimentally estimating the charge capacity bounds. An analysis of a physics-based RC ladder circuit model for supercapacitors reveals the theoretical bounds and provides guidelines for designing experiments to estimate the bounds. The upper bound corresponds to a long time constant voltage charging process and the lower bound is established using a charging process with the largest possible current. Bounds of two types of supercapacitor charge capacity are estimated: the total charge stored in the supercapacitor that can be released during multiple discharging processes and the utilized charge delivered during one discharging action. The relationship between the utilized charge capacity and the discharge current is examined and different patterns are observed depending on the supercapacitor state of charge (SOC). For a fully charged supercapacitor, the utilized capacity increases when the discharge current decreases. For a supercapacitor partially charged by a relatively large current, the utilized capacity is dependent on both the discharge current and the supercapacitor operation voltage range. The difference between the bounds is significant for both types of charge capacity. These observations provide guidelines for optimizing the supercapacitor charging and discharging policies for different applications.

Index Terms

Supercapacitor; charge capacity; charge redistribution; state of charge; discharge time

I. Introduction

Supercapacitor-based energy storage systems have been employed by a variety of applications including electric and hybrid vehicles, smart grid, wireless sensor nodes, and biomedical devices. To better utilize this energy storage technology, one of the key research and development issues is to accurately and quickly estimate the supercapacitor state in terms of state of charge (SOC), state of energy (SOE), or state of health (SOH). Although the supercapacitor terminal voltage is a natural indicator of its state, accurate estimation of the state is still challenging because the supercapacitor capacitance and equivalent series resistance (ESR) are affected by multiple factors such as its terminal voltage [1], ambient temperature [2], and aging condition [3] in a complex manner. Numerous frameworks have been proposed to identify supercapacitor parameters and estimate supercapacitor states, which include offline methods based on electrochemical impedance spectroscopy (EIS) [4] or waveform relaxation [5] as well as online approaches utilizing Kalman filters [6], [7], recursive least squares [8], [9], or Lyapunov-based adaptation law [10].

On the other hand, modeling and characterization of supercapacitors based on its physics have also been investigated. Supercapacitors are usually constructed using porous carbon electrodes [11]. The porous electrode theory [12] suggests that an electrode can be modeled as an RC transmission line [13]. To reduce the model complexity, different versions of simplified equivalent circuit models [14]–[17] have been proposed. In addition, various models have been developed to investigate a particular aspect of the supercapacitor behavior such as self-discharge [18]–[20], charge redistribution [21]–[23], voltage dependency of capacitance [21], [24], [25], as well as power and energy capabilities [26]–[29].

This paper estimates the supercapacitor charge capacity bounds based on its physics and takes into account charge redistribution, which is a relaxation process originated from the porous structure of the electrodes. In addition to modeling and characterization [21]–[23], various aspects of this process have been investigated such as the terminal voltage behavior [30]–[33], energy loss [30], [34], and impact on power management in wireless sensor networks [30], [35]–[37]. This paper studies the impact of charge redistribution on the amount of charge delivered by a supercapacitor during one or multiple discharging processes. Specifically, this paper first examines an RC ladder circuit model and reveals the theoretical bounds of the supercapacitor charge capacity, which also provides guidelines for properly designing the experiments. Following this analysis, four sets of experiments are designed to determine the experiment settings that can be used to accurately estimate the bounds. Based on these settings, the bounds of the total and utilized charge capacity are estimated for four 100 F supercapacitor samples at the rated voltage of 2.7 V. Results show that the difference between the upper and lower bounds is significant for all samples and therefore experimentally demonstrate the impact of charge redistribution on supercapacitor charge capacity.

The remainder of this paper is organized as follows. Section II analyzes an RC ladder circuit model to illustrate the impact of charge redistribution on supercapacitor charge capacity. Section III introduces an experimental approach to estimate the supercapacitor charge capacity bounds. Section IV evaluates the bounds of the total charge capacity delivered during multiple discharging cycles. Section V examines the bounds of the charge capacity utilized during one discharging process. Section VI concludes this paper.

II. Impact of Charge Redistribution on Supercapacitor Charge Capacity

To illustrate the impact of charge redistribution on supercapacitor charge capacity, an RC ladder circuit model for supercapacitors is analyzed, as shown in Fig. 1. This model captures the distributed nature of supercapacitor capacitance and resistance. Because of the pore effect and the transmission line characteristic, there is a potential distribution down the electrode pores when the electrode is charged or discharged [38], or equivalently, the pores at or near the surface of the electrode are accessed first followed by the pores at the bottom of the electrode. This behavior can be described by the RC ladder circuit model. The supercapacitor terminal voltage is denoted as V_T , which is a measurable parameter. In fact, V_T equals the voltage across the first RC branch composed of R_1 and C_1 . During a charging or discharging process, the source or load is applied to the supercapacitor terminals and the capacitor of each RC branch is accessed through a series connection of all resistors from the

supercapacitor terminals to the branch in question. The time constant of each RC branch can be written as

$$\tau_i = C_i \sum_{k=1}^i R_k, \quad (1)$$

and the porous electrode theory gives that

$$\tau_1 < \tau_2 < \cdots < \tau_n. \quad (2)$$

During a charging process, the supercapacitor terminal voltage is monitored and used as a condition to terminate the process. At a given terminal voltage, the amount of charge stored in the supercapacitor may vary significantly depending on the SOC of each RC branch capacitor. Specifically, the lower bound of the charge capacity corresponds to the case in which only the first branch capacitor is charged to establish the terminal voltage while the upper bound is obtained when all branch capacitors are charged. This analysis provides guidelines for properly designing experiments to estimate the supercapacitor charge capacity bounds, which are elaborated in Section III.

III. An Experimental Approach for Estimating Supercapacitor Charge Capacity Bounds

A. Experiment Design

The analysis in Section II reveals the theoretical bounds of the supercapacitor charge capacity. To accurately estimate the bounds, an experiment needs to meet two requirements: (1) the charging process is carefully designed so that the minimum or maximum possible charge is injected into the supercapacitor to establish the lower or upper bound, and (2) the discharging phase is sufficiently long to guarantee that the stored charge is completely extracted from the supercapacitor. To meet the first requirement, the following charging conditions are used. For the upper bound, all RC branch capacitors within the supercapacitor should be fully charged, which can be realized by charging the supercapacitor using a constant voltage source for a sufficiently long time. For the lower bound, only the first branch capacitor should be charged to establish the terminal voltage and the charge time should be minimized to avoid charge redistribution, which implies a charging process using the largest possible current specified in the supercapacitor datasheet. As for the second requirement, multiple discharging-redistribution cycles are performed to exhaust the charge stored in the supercapacitor. To determine the proper experiment settings, this paper examines four sets of experiments using the supercapacitor samples listed in Table I. The rated capacitance is 100 F and the rate voltage is 2.7 V for all samples.

B. Estimation of Upper Bound

To estimate the upper bound, two sets of experiments are designed. Fig. 2(a) shows the measured terminal voltage for supercapacitor sample 3 during a 10 A experiment of the first set. During this experiment, the supercapacitor is first charged by a constant voltage source of 2.7 V for 12 hours (Fig. 2(a) only shows the last 0.5 hours of this charging phase), then discharged by a 10 A constant current source to 0.01 V. After that, the discharge current is disconnected and the supercapacitor experiences charge redistribution, which results in an increase in the terminal voltage. Once the terminal voltage increase rate is less than 0.01 V per 5 minutes, the charge redistribution process is considered complete and the discharge current is applied again. This discharging-redistribution cycle is repeated 10 times to extract the charge stored in the supercapacitor to the maximum extent possible. The charge voltage of 2.7 V is the rated voltage of the supercapacitor. In fact, this paper estimates the charge capacity bounds for all supercapacitor samples at the rated voltage. The charge time of 12 hours is selected based on the results of the second set of experiments. The discharge current of 10 A is determined based on the supercapacitor datasheet. The discharging termination voltage of 0.01 V and redistribution termination condition of 0.01 V per 5 minutes are determined based on the capabilities of the supercapacitor tester, which is an automated Maccor Model 4304 testing system. During the other two experiments of this set, the discharge current is 5 and 1 A, respectively, and the other settings remain unchanged. This experiment set is designed to verify the assumption that if the supercapacitor has been fully charged, then the charge capacity estimated using different discharge currents should be approximately equal, which is indeed the case as shown below.

The impact of charge redistribution on the supercapacitor terminal voltage change is shown in Fig. 2(a). In the first discharging-redistribution cycle, the supercapacitor is first discharged from 2.6998 to 0.0098 V in 26.12 s. After that, as shown in Fig. 2(b), the voltage increases from 0.0098 (point A) to 0.0797 V (point B) due to ESR within 0.01 s, which is the time resolution of the supercapacitor tester. The voltage finally reaches 0.3191 V after 30 minutes of charge redistribution and triggers the second discharging action. Obviously, the voltage boost due to ESR and charge redistribution is the largest (from 0.0098 to 0.3191 V) in the first cycle and the smallest (from 0.0098 to 0.1059 V) in the last (tenth) cycle. It should be noted that the supercapacitor terminal voltage is 0.1059 V after 10 cycles, which is mainly contributed by ESR. In fact, in the last cycle, the voltage first increases from 0.0098 to 0.0813 V due to ESR and then to 0.1059 V due to charge redistribution. This analysis explains the voltage profiles during three experiments with different discharge currents, as shown in Fig. 3. A smaller discharge current corresponds to a smaller final voltage: 0.0629 V for 5 A and 0.0282 V for 1 A. This is because a smaller discharge current results in a smaller voltage boost due to ESR. After 10 cycles, the charge redistribution process can be considered complete and the final voltage is mainly determined by the ESR voltage.

Assuming that the charge stored in the supercapacitor has been completely extracted after 10 discharging-redistribution cycles, the charge capacity is estimated as

$$Q = \int I dt, \quad (3)$$

where I is the discharge current and t is the discharge time. The estimated capacity is 294.3, 293.4, and 295.1 C for the 10, 5, and 1 A experiments, respectively. These three values are reasonably close and the assumption is verified.

The second experiment set is designed to examine if the charging phase is sufficiently long to ensure that the supercapacitor has been fully charged and therefore the upper bound of the charge capacity has been reached. Three experiments are performed using supercapacitor sample 2. The supercapacitor is first charged by a constant voltage source of 2.7 V for 3, 12, and 48 hours, respectively, and then undergoes 10 discharging redistribution cycles. The discharge current is fixed at 10 A during three experiments. The charge times are selected based on the typical charge time used in self-discharge experiments, which is normally 1 hour [18]–[20]. It is usually assumed that the supercapacitor has been fully charged after 1 hour of constant voltage charging. To guarantee that the supercapacitor has been fully charged, this paper examines longer charge times. The estimated capacity is 328.2, 326.0, and 322.8 C for the 3, 12, and 48 hour experiments, respectively. The results are reasonably close and 12 hours is used as the charge time in the first set of experiments. In summary, these two experiment sets show that the upper bound of the charge capacity corresponds to a long time constant voltage charging process.

C. Estimation of Lower Bound

Another two sets of experiments are designed to estimate the lower bound. As analyzed above, the lower bound corresponds to a charging process using the largest possible current. To verify this analysis, the third experiment set is designed for supercapacitor sample 1. To ensure that the charge injected into the supercapacitor during the charging phase can be accurately estimated using the discharging-redistribution cycles, ideally the supercapacitor should be completely discharged before the charging phase is initiated. In practice, the supercapacitor is discharged by a constant voltage source of 0.01 V for 12 hours to approximate the ideal initial condition. Then it is charged by a constant current source of 10, 5, and 1 A to 2.7 V, respectively. After that, it undergoes 10 discharging-redistribution cycles and the discharge current is fixed at 10 A during three experiments. The estimated capacity is 250.5, 260.0, and 271.4 C for the 10, 5, and 1 A experiments, respectively. The results are consistent with the theoretical analysis: a larger charge current corresponds to a smaller amount of charge injected into the supercapacitor. Other than the discharging phase, the charging process can also be used to estimate the charge capacity. Based on the charge current and time, the estimated capacity is 267.0, 276.2, and 295.05 C for the 10, 5, and 1 A experiments, respectively. These results also verify the theoretical analysis. It should be noted that the capacity estimated using the charging process is greater than the one using the discharging process. The difference is significant: 16.5, 16.2, and 23.65 C for the 10, 5, and 1 A experiments, respectively. Several factors may contribute to this difference: charging/discharging efficiency of the supercapacitor tester, charge redistribution duration, residual

charge, and voltage change due to ESR. Comparisons of these two estimates of the charge capacity are further elaborated in Section IV.

The fourth experiment set is designed to examine if the supercapacitor has been fully discharged before the constant current charging phase is initiated. Three experiments are conducted using supercapacitor sample 4. The supercapacitor is first discharged by a constant voltage source of 0.01 V for 3, 12, and 48 hours, respectively. After this conditioning phase, it is charged by a constant current source of 10 A to 2.7 V and then undergoes 10 discharging-redistribution cycles. The capacity estimated using the discharging-redistribution cycles is 242.4, 243.9, and 245.7 C for the 3, 12, and 48 hour experiments, respectively. If the charging phase is used, the estimated capacity is 251.5, 253.6, and 256.8 C, respectively. Both methods give reasonably close results and 12 hours is selected as the conditioning time for consistency. Once again, noticeable differences are observed between the results using these two methods: 9.1, 9.7, and 11.1 C, respectively. In summary, these two experiment sets verify that the lower bound of the charge capacity corresponds to a charging process using the largest possible current.

IV. Bounds of Total Charge Capacity

It should be noted that the charge capacity estimated in Section III is the amount of “total” charge stored in the supercapacitor, which is the overall available charge that can be extracted during multiple discharging processes. Due to charge redistribution, the total charge cannot be fully released during one discharging cycle. Therefore, it is necessary to differentiate these two types of charge capacity. This section examines the total charge capacity and Section V investigates the “utilized” charge capacity, which is the amount of charge delivered during one discharging process.

A. Upper Bound of Total Charge Capacity

To estimate the upper bound of the total charge capacity, a set of seven experiments is performed for all supercapacitor samples. The supercapacitor is first charged by a constant voltage source of 2.7 V for 12 hours and then undergoes 10 discharging-redistribution cycles. Seven discharge currents are used: 10, 5, 1, 0.5, 0.1, 0.05, and 0.01 A. Note that in addition to the three currents (10, 5, and 1 A) examined in Section III, another four (0.5, 0.1, 0.05, and 0.01 A) are included. With this extended range, the impact of discharge current on the utilized charge capacity of supercapacitors can be investigated, as elaborated in Section V.

Denoting the upper bound calculated from one of the seven experiments as Q_{max_i} , their mean ($\overline{Q_{max}}$) is taken as the upper bound estimate:

$$\overline{Q_{max}} = \frac{1}{7} \sum_{i=1}^7 Q_{max_i}. \quad (4)$$

The standard deviation (s_{max}) and coefficient of variation (cv_{max}) are estimated as

$$s_{max} = \sqrt{\frac{1}{6} \sum_{i=1}^7 (Q_{max_i} - \overline{Q_{max_i}})^2}, \quad (5)$$

$$cv_{max} = \frac{s_{max}}{\overline{Q_{max}}}. \quad (6)$$

Take supercapacitor sample 2 for instance. The Q_{max_i} results are shown in Fig. 4 and the statistics are listed in Table II, both denoted as dataset M1.

To examine the data repeatability, the seven experiments are repeated once. The results are denoted as dataset M2 in Fig. 4 and Table II. The difference between the two estimates of $\overline{Q_{max}}$ is 13.02 C, which is approximately 4% of the M1 estimate (320.54 C). This difference may be originated from various factors such as the residual charge [39] stored in the supercapacitor. The impact of residual charge needs to be minimized to improve the data repeatability. To this end, the original experiment procedures used for datasets M1 and M2 are modified. Fig. 5(a) shows a modified 10 A experiment for supercapacitor sample 2. Two modifications are introduced. First, an initial conditioning phase is added before the constant voltage charging process to minimize the effect of residual charge. Second, the duration of the constant voltage charging phase is reduced from 12 to 3 hours to shorten the overall experiment time while ensuring that the supercapacitor can still be fully charged. The 10 discharging-redistribution cycles remain unchanged. The initial conditioning phase is composed of 10 cycles. During each cycle, the supercapacitor is first charged from 0.001 to 2.7 V by a constant current source of 1 A and then undergoes charge redistribution for 5 minutes. After that, it is discharged to 0.001 V by a constant current source of 1 A. Fig. 5(b) shows the supercapacitor voltage during the first, second, and last (tenth) cycles. Two sets of experiments are performed using the modified procedures, which are denoted as M3 and M4 in Fig. 4 and Table II.

To illustrate the impact of residual charge, the charge time and end voltage of the charge redistribution phase during each conditioning cycle are extracted, as shown in Fig. 6(a) and Fig. 6(b), respectively. Among the seven experiments of M3, four are shown in Fig. 6 for better readability. It can be observed that the charge time decreases and the charge redistribution end voltage increases when the cycle index increases. Both metrics tend to stabilize after 10 conditioning cycles. By adding this initial conditioning phase before the constant voltage charging process, the supercapacitor is tuned to be in a relatively stable state during each of the seven experiments, which improves the data repeatability.

The results for M3 and M4 are also included in Fig. 4 and Table II. The difference between the two estimates of $\overline{Q_{max}}$ is 3.01 C, or 1% of the M3 estimate (300.91 C). Compared to M1 and M2, the data repeatability is improved. Based on this result, the average of M3 and M4 is used to estimate the upper bound, which is denoted as M5 in Fig. 4 and Table II. In fact,

the modified experiment procedures are also performed using the other three supercapacitor samples and the data repeatability is improved as well. For instance, Fig. 7 shows the results for sample 4. Dataset S1 uses the original procedures while S2 and S3 use the modified ones. Dataset S4 is the average of S2 and S3, which is used to estimate the upper bound for this device.

The estimated upper bounds for all samples are listed in Table III. It should be noted that the results are based on the average of the two datasets using the modified experiment procedures (e.g., M5 in Table II for sample 2 and S4 in Fig. 7 for sample 4). For all samples, the coefficient of variation is acceptably small (i.e., less than 1.3%) and a relatively accurate estimate of the total charge capacity upper bound is obtained.

To understand the origin of the data dispersion, Fig. 8 shows the charge delivered during each of the 10 discharging-redistribution cycles for four experiments of M3 for supercapacitor sample 2. It can be observed that the first cycle releases the majority of the total charge stored in the supercapacitor for all experiments. The delivered charge decreases when the cycle index increases. While this pattern applies to all experiments with different discharge currents, the overall charge accumulated during the 10 cycles shows variation. For the 10 A experiment, the charge delivered during the first and last cycles is 252.4 and 0.7 C, respectively. The total charge released during the 10 cycles is 293.7 C. For the 0.01 A experiment, the three quantities are 300.2039, 0.2253, and 303.1481 C. When seven experiments are used to estimate the total charge capacity, a certain degree of data dispersion appears. To improve the data dispersion, it may help if the number of discharging-redistribution cycles is selected based on the discharge current, which is fixed at 10 for all currents in this paper. On the other hand, it can also be observed that the charge extracted from the last cycle is already a fairly small percentage of that from the first cycle. For the 10 and 0.01 A experiments, it is 0.28% and 0.08%, respectively, which suggests that the amount of unextracted charge is reasonably small. Therefore, the upper bound estimated using the approach described in this section gives a good estimate of the actual value.

B. Lower Bound of Total Charge Capacity

Similar to the upper bound case, the lower bound is first estimated using the original experiment procedures described in Section III. Specifically, the supercapacitor is first discharged by a constant voltage source of 0.01 V for 12 hours during each of the seven experiments. After that, it is charged by a 10 A constant current source to 2.7 V and then undergoes 10 discharging-redistribution cycles. The same set of discharge currents is used: 10, 5, 1, 0.5, 0.1, 0.05, and 0.01 A. For the lower bound, two estimates are calculated from each experiment: Q_{min_j} is estimated using the discharging-redistribution cycles and $Q_{min_i}^*$ is estimated using the charging phase. The means ($\overline{Q_{min}}$ and $\overline{Q_{min}^*}$), standard deviations (s_{min} and s_{min}^*), and coefficients of variation (cv_{min} and cv_{min}^*) are determined accordingly. This dataset is denoted as N1. For example, Fig. 9 shows the results and Table IV lists the statistics for supercapacitor sample 2.

To minimize the effect of residual charge, the original experiment procedures are modified and the initial conditioning phase used in the upper bound experiments is also included in the lower bound experiments. The modified experiments are repeated twice for all supercapacitor samples, which are denoted as datasets N2 and N3. The average of N2 and N3 is denoted as N4, which is used to estimate the lower bound. The results and statistics for sample 2 are shown in Fig. 9 and Table IV. It can be observed that the data repeatability is improved. For both $\overline{Q_{min}}$ and $\overline{Q_{min}^*}$, the difference between the N2 and N3 estimates is small: 0.16 C for the former and 0.20 C for the latter. Additionally, cv_{min}^* is much smaller than cv_{min} for all datasets. This is because the Q_{min} calculation process involves 10 discharging-redistribution cycles, as illustrated in Fig. 8 for the upper bound case while Q_{min}^* only utilizes a single constant current charging phase. The statistics for all supercapacitor samples are listed in Table III. Similar to sample 2, the results are based on dataset N4, which is the average of N2 and N3 using the modified experiment procedures.

Consistent with the observation in Section III, the lower bound of the total charge capacity estimated using the 10 discharging-redistribution cycles is smaller than the one using the charging process for all supercapacitor samples, as shown in Table III. This is because the charging process determines the maximum possible charge injected into the supercapacitor and the discharging-redistribution phase only extracts a relatively large percentage of the stored charge. To further interpret this observation, Fig. 10 shows the difference between the two estimates of the lower bound for sample 2. Take the dataset N4 for example. The minimum and maximum difference is 0.43 C (1 A experiment) and 7.95 C (10 A experiment), respectively. The mean of the seven experiments is 2.99 C, which is approximately 1.2% of the estimated lower bound (247.70 C for $\overline{Q_{min}}$ or 250.69 C for $\overline{Q_{min}^*}$). This difference measures the amount of unextracted charge after the extended discharging-redistribution process that lasts from approximately 6,000 s (10 A experiment) to 28,000 s (0.01 experiment).

C. Comparisons of Total Charge Capacity Bounds

To quantify the difference between the upper and lower bounds of the total charge capacity, the following parameters are defined:

$$\Delta_Q = \overline{Q_{max}} - \overline{Q_{min}}, \quad (7)$$

$$\Delta_Q^* = \overline{Q_{max}} - \overline{Q_{min}^*}, \quad (8)$$

$$r_Q = \frac{\overline{Q_{max}}}{\overline{Q_{min}}}, \quad (9)$$

$$r_Q^* = \frac{\overline{Q_{max}}}{\overline{Q_{min}^*}}. \quad (10)$$

The results are listed in Table V and it is clear that the difference between the upper and lower bounds is significant for all samples. The maximum and minimum difference is observed for samples 2 and 1, respectively. For the lower bound, $\overline{Q_{min}^*}$ is greater than $\overline{Q_{min}}$ for all samples and it gives conservative results when it comes to evaluating the difference between the bounds. In terms of Δ_Q^* , the maximum and minimum difference is 48.72 and 33.06 C, respectively. In terms of r_Q^* , the maximum and minimum ratio is 1.194 and 1.137, respectively. Therefore, even for sample 1 with the smallest difference between the upper and lower bounds of the total charge capacity, at least 33.06 C or 13.7% more charge is stored in the supercapacitor at the rated voltage if it is fully charged using a long time constant voltage charging process compared to the case in which it is charged by the largest possible current during a relatively short period of time.

V. Bounds of Utilized Charge Capacity

While it is important to estimate the total charge stored in the supercapacitor, estimation of the actual charge delivered during one discharging process is also an interesting problem. This section investigates the bounds of the charge released during one discharging action and examines the impact of discharge current on the utilized charge capacity. The experiments performed to estimate the total charge capacity bounds are also used for estimating the utilized charge capacity bounds. Take sample 2 for instance. Dataset M5 in Fig. 4 is used for the upper bound case and dataset N4 in Fig. 9 is used for the lower bound case.

A. Upper Bound of Utilized Charge Capacity

The utilized charge capacity is defined as the amount of charge delivered during the first of the 10 discharging-redistribution cycles. Take the 10 A experiment of dataset M3 for sample 2 for instance. As shown in Fig. 8, the utilized charge capacity is 252.4 C if the termination voltage (0.01 V) of the discharging process is taken as the cutoff voltage. In practice, a supercapacitor usually operates between its rated voltage and half of that. Therefore, half of the rated voltage is set as another cutoff voltage, which is 1.35 V for all samples examined in this paper. The utilized charge capacity for this cutoff voltage is 123.4 C. As shown in Table V, the difference between the upper and lower bounds of the total charge capacity is the largest for sample 2. Therefore, this section presents the numerical results for this sample in Table VI and shows the results for all samples in graphs. Table VI lists the utilized charge capacity bounds associated with the two cutoff voltages for sample 2. The subscript “max” means that a quantity is associated with the upper bound. The subscripts “full” and “half” denote the cutoff voltages of 0.01 and 1.35 V, respectively. Note that U_{max_full} and U_{max_half} are 250.55 and 122.50 C in Table VI for the 10 A experiment. They are different from 252.4 and 123.4 C because Table VI uses dataset M5 instead of M3 to be consistent with the total

charge capacity case. Results for all samples are plotted in Fig. 11 with Fig. 11(a) for the cutoff voltage of 0.01 V and Fig. 11(b) for 1.35 V.

As shown in Table VI and Fig. 11, for both cutoff voltages, the upper bound of the utilized charge capacity increases when the discharge current decreases for all samples. Take sample 2 for instance. If the cutoff voltage is 0.01 V, U_{max_full} increases from the minimum of 250.55 C at 10 A to the maximum of 299.41 C at 0.01 A resulting in a difference of 48.86 C. In fact, the impact of discharge current on the charge delivered during a discharging process is consistent with Peukert's law [40], an empirical law originally developed to relate the lead-acid battery capacity to the discharge current. For supercapacitors, the relationship between the utilized charge capacity and the discharge current can be explained using the RC ladder circuit model shown in Fig. 1. Since the supercapacitor is charged by a constant voltage source for a long time, it can be assumed that all RC branch capacitors have been fully charged. When a discharge current is applied to bring the supercapacitor terminal voltage down to the cutoff voltage, the charge stored in the RC branches is extracted progressively from the first branch with the smallest time constant to the last branch with the largest time constant. Obviously, a smaller discharge current requires a longer time to reduce the supercapacitor terminal voltage (mainly the voltage across the first branch capacitor) to the cutoff voltage. As the discharge time extends, more charge stored in the slow branches with large time constants is extracted. In the meantime, the charge stored in the slow branches redistributes to the first branch, which decelerates the drop in the terminal voltage and prolongs the discharge time. Therefore, the amount of charge extracted from the supercapacitor is larger during the extended discharging phase if a smaller discharge current is applied.

To verify the above analysis, a generic 5-branch RC ladder circuit model for 100 F supercapacitors shown in Fig. 12 is simulated using LTspice. The component values of the five RC branches are selected to generate time constants of $\tau_1 = 1.05$, $\tau_2 = 10$, $\tau_3 = 100$, $\tau_4 = 1,000$, and $\tau_5 = 10,000$ s because the actual discharge time of the supercapacitor samples ranges between 10 (lower bound case, 1.35 V cutoff voltage, and 10 A discharge current) and 30,000 s (upper bound case, 0.01 V cutoff voltage, and 0.01 A discharge current). Therefore, the model is tuned to simulate the supercapacitor behavior at various time scales. The total capacitance of the five branch capacitors is 100 F. The first branch capacitance is 70% of the total capacitance because this is the major branch. The capacitances are 16, 8, 4, and 2 F for the remaining four branches with a scale factor of 0.5 based on the fact that a slower branch makes a smaller contribution to the total capacitance. As for resistors, the first branch resistance is based on the typical ESR values specified in the supercapacitor sample datasheets. The other four resistances are calculated based on the time constants and capacitances using (1). It should be noted that this model is conceived to explain the impact of charge redistribution on the utilized charge capacity. The model component values are assumed with certain arbitrariness and they are not determined by characterizing any of the four supercapacitor samples because it is experimentally challenging to identify the 10 component values in an effective manner.

The simulation setup and results are as follows. The initial voltages of the five branch capacitors are set to be 2.7 V. In addition to the seven discharge currents used in

experiments, another three are simulated: 50, 0.005, and 0.001 A. Fig. 13(a) shows the simulated upper bound of the utilized charge capacity for the two cutoff voltages of 0.01 and 1.35 V. The relationships between the utilized charge capacity and the discharge current for both cutoff voltages are consistent with the experiment results shown in Fig. 11. Fig. 13(b) shows the simulated supercapacitor terminal voltage V_T and five branch capacitor voltages $V_1 - V_5$ when the discharge current is 1 A. It can be observed that V_T almost overlaps with V_1 . Charge is mainly extracted from C_1 and its voltage drops rapidly. Because of the increasingly larger time constant of the corresponding branch, the drop in $V_2 - V_5$ becomes smaller at a particular point of time compared to the drop in V_1 . Given the voltage differences between $V_2 - V_5$ and V_1 , charge is unidirectionally redistributed from $C_2 - C_5$ to C_1 during the entire discharging process, which decelerates the drop in V_1 . When the discharge current is smaller, these effects are more significant and therefore more charge is extracted from the supercapacitor.

B. Lower Bound of Utilized Charge Capacity

The lower bound of the utilized charge capacity is estimated using the experiments performed for estimating the lower bound of the total charge capacity. Table VI lists the results for sample 2. The subscript “min” means that a quantity is associated with the lower bound. Fig. 14 plots the results for all samples. These results show that the relationship between the utilized charge capacity and the discharge current is similar for all samples. However, the relationship is different from that for the upper bound case in which the utilized charge capacity increases when the discharge current decreases. In addition, the relationship is different for the two cutoff voltages. When the cutoff voltage is 0.01 V, the utilized capacity is relatively flat for the current range between 0.01 and 1 A while it decreases noticeably when the current increases from 1 to 10 A, as shown in Fig. 14(a). Specifically, for samples 1 and 2, the utilized capacity increases when the current decreases from 1 to 0.01 A although the change is minimal. The numerical results for sample 2 are listed in Table VI. For sample 1, the utilized capacity is 235.47 C at 1 A and 236.53 C at 0.01 A. For samples 3 and 4, the utilized capacity first increases when the current decreases from 1 to 0.1 A and then decreases when the current decreases from 0.1 to 0.01 A. Again, the change is minimal. Numerically, the utilized capacity of sample 3 peaks at 233.15 C at 0.1 A and decreases to 232.17 C at 0.01 A. For sample 4, the values are 234.66 C at 0.1 A and 232.90 C at 0.01 A. Therefore, considering the relatively minor changes in the utilized capacity in the flat region, it can be assumed that the utilized capacity for the 0.01 V cutoff voltage remains constant when the discharge current is small to medium (0.01–1 A) and drops when the discharge current increases from 1 to 10 A. For the cutoff voltage of 1.35 V, the relationship shown in Fig. 14(b) introduces a new feature. While the utilized capacity decreases significantly when the discharge current increases from 1 to 10 A, it also drops dramatically when the discharge current decreases from 1 to 0.01 A. In other words, the utilized capacity peaks at 1 A and decreases noticeably when the discharge current deviates from this particular value.

The dispersion of the utilized charge capacity lower bound for both cutoff voltages can be quantified using the metrics defined in Section IV. Take sample 2 for instance. For the cutoff voltage of 0.01 V, the mean, standard deviation, and coefficient of variation calculated using

the U_{min_full} data listed in Table VI is 240.33 C, 4.83 C, and 0.0201, respectively. For the 1.35 V cutoff voltage, they are 108.64 C, 4.36 C, and 0.0401. Compared to the statistics of the total charge capacity lower bound listed in Table III, the standard deviation of the full (4.83 C) and half (4.36 C) utilized charge capacity lower bound is approximately 54 and 48 times larger if $s_{min}^* = 0.09$ C is used, respectively. For this comparison, s_{min}^* is more suitable than s_{min} because both the utilized capacity (U_{min_full} and U_{min_half}) and the total capacity ($\overline{Q_{min}^*}$) are estimated using a single discharging or charging process while the total capacity ($\overline{Q_{min}}$) associated with s_{min} utilizes 10 discharging-redistribution cycles and therefore overestimates the data dispersion. The differences are also significant when the coefficient of variation is considered: 0.0201 vs. 0.0004 for the 0.01 V cutoff voltage case and 0.0401 vs. 0.0004 for 1.35 V. Therefore, the lower bound of the utilized charge capacity is dependent on the discharge current for both cutoff voltages.

The relationship between the utilized charge capacity and the discharge current for both cutoff voltages can also be explained using the model shown in Fig. 12. To simulate the lower bound case, the initial voltages of the five branch capacitors are set to be 0 V. A constant current source of 10 A is applied and the supercapacitor terminal voltage reaches 2.7 V at 20.3 s. Then the charging current source is removed and the discharging current source is connected. Fig. 15(a) shows the simulated lower bound of the utilized charge capacity. Again, the relationships between the utilized charge capacity and the discharge current are consistent with the experiment results shown in Fig. 14 for both cutoff voltages. For the 0.01 V case, the utilized charge capacity increases quickly from 124.79 to 192.09 C when the discharge current decreases from 50 to 1 A and increases slowly to 201.69 C at 0.001 A. For the 1.35 V case, the utilized charge capacity peaks at 75.88 C at 1 A and decreases to 30.69 C at 50 A and 67.70 C at 0.001 A.

Fig. 15(b) shows the simulated supercapacitor terminal and branch capacitor voltages when the discharge current is 1 A. As expected, the charging process using a relatively large current mainly charges the first branch capacitor and the slow branch capacitors are not fully charged. In fact, the five branch capacitor voltages $V_1 - V_5$ are 2.550, 1.471, 0.116, 0.001 and 0.000 V at the end of the charging process ($t = 20.3$ s). When the discharge current is applied, charge is first extracted from C_1 . Although V_1 drops continuously, it is still greater than $V_2 - V_5$ during certain periods of time and therefore charge is transferred from C_1 to $C_2 - C_5$. Take V_3 for instance. It increases from 0.116 ($t = 20.3$ s) to 1.067 V ($t = 120.102$ s) after the charging current source is removed because during this period of time charge is redistributed from C_1 and C_2 to C_3 . As the discharging process continues, C_3 is accessed by the discharge current and its voltage drops. As long as V_3 is greater than V_1 and V_2 , a portion of charge flows back from C_3 to C_1 and C_2 . Therefore, different from the upper bound case shown in Fig. 13(b), charge redistribution in the lower bound case is bidirectional during the entire discharging process: charge redistributes from the fast branches to the slow branches during the early stage and the charge transfer direction is reversed during the late stage. This explains why the pattern of the utilized charge capacity is different for the two cutoff voltages when the discharge current is small to medium (0.01–1 A), as shown in Fig. 14 and Fig. 15(b). Specifically, for the cutoff voltage of 0.01 V, the

discharge time is almost twice of that for the 1.35 V case. Although part of the charge redistributes from the fast branches to the slow branches during the early stage, this portion of charge flows back to the fast branches during the late stage. Therefore, all branches are almost fully discharged and the total charge extracted from the supercapacitor is approximately equal. On the other hand, when the cutoff voltage is 1.35 V, the late stage is not considered and there is no time for the charge redistributed to the slow branches during the early stage to be extracted. This effect is more significant when a smaller discharge current is applied and therefore the utilized charge capacity drops when the discharge current decreases. When the discharge current is relatively large (1–10 A), the relationship between the utilized charge capacity and the discharge current is similar for both cutoff voltages, which is also similar to that for the upper bound case. Therefore, the explanation for the upper bound case also applies.

C. Comparisons of Utilized Charge Capacity Bounds

With the utilized charge capacity bounds, a comparative study is conducted to illustrate the impact of charge redistribution on the charge released during a single discharging process. The differences between the bounds are quantified as follows:

$$\Delta_{U_full} = U_{max_full} - U_{min_full}, \quad (11)$$

$$r_{U_full} = \frac{U_{max_full}}{U_{min_full}}, \quad (12)$$

$$\Delta_{U_half} = U_{max_half} - U_{min_half}, \quad (13)$$

$$r_{U_half} = \frac{U_{max_half}}{U_{min_half}}. \quad (14)$$

The numerical results for sample 2 are summarized in Table VI. For all samples at the two cutoff voltages, Fig. 16 shows the bound differences and Fig. 17 shows the bound ratios. Three major observations can be made based on these results.

First, the charge capacity utilized during a discharging process is dependent on the supercapacitor state and the difference between the bounds is significant. For all samples at the two cutoff voltages, the charge capacity delivered after a long time constant voltage charging process is greater than that after a short time constant current charging process regardless of the discharge current magnitude. Take sample 2 for instance. In terms of the difference between the upper and lower bounds, U_{full} ranges between 19.60 C (at 10 A) and 55.16 C (at 0.01 A) for the cutoff voltage of 0.01 V. Or equivalently, 8.5% (at 10 A) or

22.6% (at 0.01 A) more charge can be extracted from the supercapacitor, which also means that the discharge time of the supercapacitor is extended accordingly. Consider the maximum difference when the discharge current is 0.01 A. The discharge time is 29,941 and 24,425 s, respectively, which results in a difference of 5,516 s or 22.6% of the lower bound. Therefore, the optimistic and conservative runtimes of the supercapacitor can be estimated based on the utilized charge capacity bounds.

Second, the difference between the utilized charge capacity bounds (or equivalently, the discharge time bounds) is dependent on the discharge current. The difference is larger for a smaller discharge current for all samples. The maximum and minimum difference is reached at 0.01 and 10 A, respectively. Referring to the RC ladder circuit model, the fundamental reason is that the slow branches can be accessed during the extended discharging process when a small discharge current is applied. In the meantime, the charge redistribution effect is more significant when the discharge current is smaller, as elaborated for both the upper and lower bound cases using the 5-branch model.

Third, the difference between the utilized charge capacity bounds is mainly contributed by the upper half of the supercapacitor operation voltage range (i.e., 1.35–2.7 V). As shown in Fig. 16, the difference associated with the 1.35 V cutoff voltage is a significant percentage of that for the 0.01 V case at a given discharge current. To understand this observation, Fig. 18 shows the ratio of the half to full utilized charge capacity for both the upper and lower bounds. The ratios are defined as

$$\eta_{U_max} = \frac{U_{max_half}}{U_{max_full}}, \quad (15)$$

$$\eta_{U_min} = \frac{U_{min_half}}{U_{min_full}}. \quad (16)$$

As shown in Fig. 18(a), η_{U_max} is greater than 0.5 for all discharge currents except 10 A, which means that more charge is extracted from the upper half of the supercapacitor operation voltage range (1.35–2.7 V) compared to that from the lower half (0–1.35 V) when the supercapacitor is fully charged. On the other hand, Fig. 18(b) shows that η_{U_min} is less than 0.5 for all discharge currents when the supercapacitor is only partially charged. When the difference between the utilized charge capacity in these two scenarios is considered, the gap between η_{U_max} and η_{U_min} results in the fact that the 1.35–2.7 V range makes a greater contribution. It can also be observed that the two ratios for sample 2 are the smallest for both the lower and upper bound cases. Coupled with the magnitude of the two bounds, it is possible that both the half and full utilized charge capacity of sample 2 are greater than those of the other three samples. This is the case for the upper bound, as shown in Fig. 11. It is also possible that the full utilized charge capacity of sample 2 is greater than those of the other three samples while the half is not, as shown in Fig. 14 for the lower bound case.

Although this paper only examines the supercapacitor charge capacity bounds at the rated voltage, the results clearly demonstrate the significant impact of charge redistribution on supercapacitor charge capacity. It should be noted that all experiments are performed at room temperature and the supercapacitor samples are relatively new. Based on the methodology presented in this paper, a comprehensive study is being conducted to incorporate the effects of various factors including supercapacitor terminal voltage, ambient temperature, and aging condition on supercapacitor charge capacity, which could lead to a framework useful to many supercapacitor-based applications.

VI. Conclusion

This paper demonstrates the effects of the charge redistribution characteristic on supercapacitor charge capacity by experimentally estimating the bounds of the total and utilized charge capacity. Based on an RC ladder circuit model for supercapacitors, the impact of charge redistribution on supercapacitor charge capacity is analyzed and experiment design guidelines are developed. The upper bound requires that the supercapacitor is fully charged, which can be realized by a long time constant voltage charging process. The lower bound is reached if the supercapacitor is partially charged, i.e., only the first RC branch with the smallest time constant is used to establish the supercapacitor terminal voltage, which corresponds to a charging process with the largest possible current. The experiment results for four supercapacitor samples show that the difference between the upper and lower bounds of the total charge capacity is significant.

Moreover, the impact of discharge current on the charge capacity utilized during a discharging process is investigated. Different relationships between the utilized charge capacity and the discharge current are observed depending on the supercapacitor SOC. For a fully charged supercapacitor, the utilized charge capacity increases when the discharge current decreases. If a supercapacitor is partially charged by a relatively large current, the relationship is dependent on both the discharge current and the supercapacitor operation voltage range. When the supercapacitor is fully discharged to a zero voltage, the utilized charge capacity remains almost constant if the current is small to medium and decreases as the current increases if the current is relatively large. When the supercapacitor is discharged to half of its rated voltage, the utilized charge capacity peaks at a medium current and drops if the current either increases or decreases. These different patterns are explained using the RC ladder circuit model and the impact of charge redistribution is illustrated. The bounds of the utilized charge capacity are compared and the differences are also significant. Therefore, based on the results for the total and utilized charge capacity bounds, it is concluded that the charge redistribution process should be taken into account when it comes to accurately estimating the supercapacitor charge capacity.

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References

1. Rafik F, Gualous H, Gallay R, Crausaz A, Berthon A. Frequency, thermal and voltage supercapacitor characterization and modeling. *Journal of Power Sources*. 2007; 165(2):928–934.
2. Reichbach N, Mellincovsky M, Peretz MM, Kuperman A. Long-term wide-temperature supercapacitor ragone plot based on manufacturer datasheet. *IEEE Transactions on Energy Conversion*. 2016; 31(1):404–406.
3. Ktz R, Ruch P, Cericola D. Aging and failure mode of electrochemical double layer capacitors during accelerated constant load tests. *Journal of Power Sources*. 2010; 195(3):923–928.
4. Marracci M, Tellini B, Catelani M, Ciani L. Ultracapacitor degradation state diagnosis via electrochemical impedance spectroscopy. *IEEE Transactions on Instrumentation and Measurement*. 2015; 64(7):1916–1921.
5. Moayedi S, Cingoz F, Davoudi A. Accelerated simulation of high-fidelity models of supercapacitors using waveform relaxation techniques. *IEEE Transactions on Power Electronics*. 2013; 28(11):4903–4909.
6. Nadeau A, Hassanalieregh M, Sharma G, Soyata T. Energy awareness for supercapacitors using Kalman filter state-of-charge tracking. *Journal of Power Sources*. 2015; 296:383–391.
7. Zhang L, Wang Z, Sun F, Dorrell DG. Online parameter identification of ultracapacitor models using the extended kalman filter. *Energies*. 2014; 7(5):3204–3217.
8. Reichbach N, Kuperman A. Recursive-least-squares-based real-time estimation of supercapacitor parameters. *IEEE Transactions on Energy Conversion*. 2016; 31(2):810–812.
9. Eddahech A, Ayadi M, Briat O, Vinassa J-M. Online parameter identification for real-time supercapacitor performance estimation in automotive applications. *International Journal of Electrical Power & Energy Systems*. 2013; 51:162–167.
10. Chaoui H, Gualous H. Online lifetime estimation of supercapacitors. *IEEE Transactions on Power Electronics*. 2017; 32(9):7199–7206.
11. Pandolfo A, Hollenkamp A. Carbon properties and their role in supercapacitors. *Journal of Power Sources*. 2006; 157(1):11–27.
12. de Levie R. On porous electrodes in electrolyte solutions: I. capacitance effects. *Electrochimica Acta*. 1963; 8(10):751–780.
13. Yaniv M, Soffer A. The transient behavior of an ideally polarized porous carbon electrode at constant charging current. *Journal of The Electrochemical Society*. 1976; 123(4):506–511.
14. Belhachemi F, Rael S, Davat B. A physical based model of power electric double-layer supercapacitors. *Conference Record of the 2000 IEEE Industry Applications Conference*; 2000. 3069–3076.
15. Nelms RM, Cahela DR, Tatarchuk BJ. Modeling double-layer capacitor behavior using ladder circuits. *IEEE Transactions on Aerospace and Electronic Systems*. 2003; 39(2):430–438.
16. Faranda R. A new parameters identification procedure for simplified double layer capacitor two-branch model. *Electric Power Systems Research*. 2010; 80(4):363–371.
17. Buller S, Karden E, Kok D, Doncker RWD. Modeling the dynamic behavior of supercapacitors using impedance spectroscopy. *IEEE Transactions on Industry Applications*. 2002; 38(6):1622–1626.
18. Diab Y, Venet P, Gualous H, Rojat G. Self-discharge characterization and modeling of electrochemical capacitor used for power electronics applications. *IEEE Transactions on Power Electronics*. 2009; 24:510–517.
19. Zhang Y, Yang H. Modeling and characterization of supercapacitors for wireless sensor network applications. *Journal of Power Sources*. 2011; 196(8):4128–4135.
20. Yang H, Zhang Y. Self-discharge analysis and characterization of supercapacitors for environmentally powered wireless sensor network applications. *Journal of Power Sources*. 2011; 196(20):8866–8873.
21. Zubieta L, Bonert R. Characterization of double-layer capacitors for power electronics applications. *IEEE Transactions on Industry Applications*. 2000; 36(1):199–205.

22. Graydon JW, Panjehshahi M, Kirk DW. Charge redistribution and ionic mobility in the micropores of supercapacitors. *Journal of Power Sources*. 2014; 245:822–829.
23. Sedlakova V, Sikula J, Majzner J, Sedlak P, Kuparowitz T, Buegler B, Vasina P. Supercapacitor equivalent electrical circuit model based on charges redistribution by diffusion. *Journal of Power Sources*. 2015; 286:58–65.
24. Szewczyk A, Sikula J, Sedlakova V, Majzner J, Sedlak P, Kuparowitz T. Voltage dependence of supercapacitor capacitance. *Metrology and Measurement Systems*. 2016; 23(3):403–411.
25. Schonberger J, GmbH P. Modeling a supercapacitor using PLECS. [Online]. Available: https://plexim.com/files/plecs_supercap.pdf
26. Spyker R, Nelms R. Analysis of double-layer capacitors supplying constant power loads. *IEEE Transactions on Aerospace and Electronic Systems*. 2000; 36(4):1439–1443.
27. Mellincovsky M, Kuperman A, Lerman C, Gadelovits S, Aharon I, Reichbach N, Geula G, Nakash R. Performance and limitations of a constant power-fed supercapacitor. *IEEE Transactions on Energy Conversion*. 2014; 29(2):445–452.
28. Yang H, Zhang Y. Estimation of supercapacitor energy using a linear capacitance for applications in wireless sensor networks. *Journal of Power Sources*. 2015; 275:498–505.
29. Yang H, Zhang Y. Characterization of supercapacitor models for analyzing supercapacitors connected to constant power elements. *Journal of Power Sources*. 2016; 312:165–171.
30. Yang H, Zhang Y. Analysis of supercapacitor energy loss for power management in environmentally powered wireless sensor nodes. *IEEE Transactions on Power Electronics*. 2013; 28(11):5391–5403.
31. Yang H, Zhang Y. A study of supercapacitor charge redistribution for applications in environmentally powered wireless sensor nodes. *Journal of Power Sources*. 2015; 273:223–236.
32. Yang H. Analysis of supercapacitor charge redistribution through constant power experiments. *Proceedings of The 2017 IEEE Power & Energy Society General Meeting (PESGM 2017)*; 2017. p. in press
33. Yang H. Bounds of supercapacitor open-circuit voltage change after constant power experiments. *Proceedings of The 10th Electrical Energy Storage Applications and Technologies (EESAT 2017)*; 2017. p. in press
34. Chai R, Zhang Y. A practical supercapacitor model for power management in wireless sensor nodes. *IEEE Transactions on Power Electronics*. 2015; 30(12):6720–6730.
35. Yang H, Zhang Y. A task scheduling algorithm based on supercapacitor charge redistribution and energy harvesting for wireless sensor nodes. *Journal of Energy Storage*. 2016; 6:186–194.
36. Yang H, Zhang Y. Power management in supercapacitor-based wireless sensor nodes. *Supercapacitor Design and Applications*. 2016:165–179.
37. Ju Q, Zhang Y. Charge redistribution-aware power management for supercapacitor-operated wireless sensor networks. *IEEE Sensors Journal*. 2016; 16(7):2046–2054.
38. Black J, Andreas HA. Effects of charge redistribution on self-discharge of electrochemical capacitors. *Electrochimica Acta*. 2009; 54(13):3568–3574.
39. Torregrossa D, Bahramipanah M, Namor E, Cherkaoui R, Paolone M. Improvement of dynamic modeling of supercapacitor by residual charge effect estimation. *IEEE Transactions on Industrial Electronics*. 2014; 61(3):1345–1354.
40. Doerffel D, Sharkh SA. A critical review of using the peukert equation for determining the remaining capacity of lead-acid and lithium-ion batteries. *Journal of Power Sources*. 2006; 155(2):395–400.

Biography

Hengzhao Yang (M'13) received the Ph.D. degree in electrical and computer engineering from the Georgia Institute of Technology in 2013, the M.S. degree in microelectronics and solid-state electronics from the Shanghai Institute of Microsystem and Information Technology, Chinese Academy of Sciences in 2008, and the B.S. degree in optoelectronics from Chongqing University in 2005.

He has been an Assistant Professor at California State University, Long Beach since 2016. He was a Postdoctoral Fellow at the Georgia Institute of Technology from 2013 to 2015 and a Visiting Assistant Professor at Miami University from 2015 to 2016. His current research interests include energy storage devices and systems for a variety of applications including wireless sensor networks, smart grid, biomedical devices, cyber-physical systems, and the Internet of Things.

Fig. 1.
An RC ladder circuit model for supercapacitors.

Fig. 2. Measured terminal voltage during a 10 A experiment for supercapacitor sample 3. (a) Overview. (b) Voltage increase due to ESR.

Fig. 3.
Measured terminal voltage during three experiments for supercapacitor sample 3.

Fig. 4.
Upper bound of total charge capacity for supercapacitor sample 2.

Fig. 5. Measured terminal voltage during a 10 A experiment for supercapacitor sample 2 using modified experiment procedure. (a) Overview. (b) Initial conditioning phase.

Fig. 6. Impact of residual charge on supercapacitor behavior for sample 2. (a) Charge time. (b) Charge redistribution end voltage.

Fig. 7.
Upper bound of total charge capacity for supercapacitor sample 4.

Fig. 8.
Charge delivered during each discharging-redistribution cycle for supercapacitor sample 2.

Fig. 9. Lower bound of total charge capacity for supercapacitor sample 2. (a) Estimate using discharging-redistribution phase. (b) Estimate using charging phase.

Fig. 10. Difference between two estimates of total charge capacity lower bound for supercapacitor sample 2.

Fig. 11.

Upper bound of utilized charge capacity for all supercapacitor samples. (a) Cutoff voltage is 0.01 V. (b) Cutoff voltage is 1.35 V.

Fig. 12.
A generic 5-branch RC ladder circuit model.

Fig. 13. Simulation results for upper bound case. (a) Upper bound of utilized charge capacity. (b) Supercapacitor terminal and branch capacitor voltages for 1 A current simulation.

Fig. 14.

Lower bound of utilized charge capacity for all supercapacitor samples. (a) Cutoff voltage is 0.01 V. (b) Cutoff voltage is 1.35 V.

Fig. 15. Simulation results for lower bound case. (a) Lower bound of utilized charge capacity. (b) Supercapacitor terminal and branch capacitor voltages for 1 A current simulation.

Fig. 16.

Difference between upper and lower bounds of utilized charge capacity for all supercapacitor samples. (a) Cutoff voltage is 0.01 V. (b) Cutoff voltage is 1.35 V.

Fig. 17.

Ratio of upper to lower bounds of utilized charge capacity for all supercapacitor samples. (a) Cutoff voltage is 0.01 V. (b) Cutoff voltage is 1.35 V.

Fig. 18. Ratio of half to full utilized charge capacity for all supercapacitor samples. (a) Upper bound. (b) Lower bound.

TABLE I

Supercapacitor Samples.

Sample	1	2	3	4
Manufacturer	Maxwell	AVX	Eaton	Maxwell
Model	BCAP0100	SCCV60B107MRB	HV1860-2R7107-R	BCAP0100
Capacitance (F)	100	100	100	100
Voltage (V)	2.7	2.7	2.7	2.7

TABLE II

Statistics of Total Charge Capacity Upper Bound for Supercapacitor Sample 2.

Dataset	M1	M2	M3	M4	M5
$\overline{Q}_{max} \text{ (C)}$	320.54	307.52	300.91	297.90	299.41
$s_{max} \text{ (C)}$	2.64	3.87	3.60	3.77	3.62
cV_{max}	0.0082	0.0126	0.0120	0.0126	0.0121

TABLE III

Statistics of Total Charge Capacity Bounds for All Supercapacitor Samples.

Sample	1	2	3	4
$\overline{Q}_{max}$ (C)	273.73	299.41	272.78	274.02
s_{max} (C)	3.44	3.62	2.63	3.22
cv_{max}	0.0126	0.0121	0.0096	0.0118
$\overline{Q}_{min}$ (C)	237.76	247.70	236.13	235.94
s_{min} (C)	2.82	2.55	2.25	3.01
cv_{min}	0.0119	0.0103	0.0095	0.0127
$\overline{Q}_{min}^*$ (C)	240.67	250.69	239.11	239.39
s_{min}^* (C)	0.07	0.09	0.16	0.04
cv_{min}^*	0.0003	0.0004	0.0007	0.0002

TABLE IV

Statistics of Total Charge Capacity Lower Bound for Supercapacitor Sample 2.

Dataset	N1	N2	N3	N4
$\overline{Q}_{min}$ (C)	257.90	247.78	247.62	247.70
s_{min} (C)	4.33	2.53	2.58	2.55
cv_{min}	0.0168	0.0102	0.0104	0.0103
$\overline{Q}_{min}^*$ (C)	267.11	250.79	250.59	250.69
s_{min}^* (C)	0.81	0.20	0.07	0.09
cv_{min}^*	0.0030	0.0008	0.0003	0.0004

TABLE V

Comparisons of Total CHARGE Capacity Bounds for All Supercapacitor Samples.

Sample	1	2	3	4
$\overline{Q}_{max}$ (C)	273.73	299.41	272.78	274.02
$\overline{Q}_{min}$ (C)	237.76	247.70	236.13	235.94
$\overline{Q}_{min}^*$ (C)	240.67	250.69	239.11	239.39
Q (C)	35.97	51.71	36.65	38.08
ΔQ^* (C)	33.06	48.72	33.68	34.62
r_Q	1.151	1.209	1.155	1.161
r_Q^*	1.137	1.194	1.141	1.145

TABLE VI

Comparisons of Utilized Charge Capacity Bounds for Supercapacitor Sample 2.

Current (A)	10	5	1	0.5	0.1	0.05	0.01
$U_{max_full}(C)$	250.55	258.93	272.01	276.37	287.09	291.80	299.41
$U_{min_full}(C)$	230.95	236.60	241.74	242.36	243.16	243.28	244.25
$U_{full}(C)$	19.60	22.33	30.27	34.01	43.93	48.52	55.16
rU_{full}	1.085	1.094	1.125	1.140	1.181	1.199	1.226
$U_{max_half}(C)$	122.50	129.80	138.49	140.74	145.45	147.25	150.17
$U_{min_half}(C)$	107.10	111.82	113.42	112.34	108.52	106.35	100.90
$U_{half}(C)$	15.40	17.98	25.07	28.40	36.93	40.90	49.27
rU_{half}	1.144	1.161	1.221	1.253	1.340	1.385	1.488