Analytic simulator and image generator of multiple-scattering Compton camera for prompt gamma ray imaging

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Received: 20 August 2018 / Revised: 28 August 2018 / Accepted: 30 August 2018 / Published online: 10 September 2018
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Abstract

For prompt gamma ray imaging for biomedical applications and environmental radiation monitoring, we propose herein a multiple-scattering Compton camera (MSCC). MSCC consists of three or more semiconductor layers with good energy resolution, and has potential for simultaneous detection and differentiation of multiple radio-isotopes based on the measured energies, as well as three-dimensional (3D) imaging of the radio-isotope distribution. In this study, we developed an analytic simulator and a 3D image generator for a MSCC, including the physical models of the radiation source emission and detection processes that can be utilized for geometry and performance prediction prior to the construction of a real system. The analytic simulator for a MSCC records coincidence detections of successive interactions in multiple detector layers. In the successive interaction processes, the emission direction of the incident gamma ray, the scattering angle, and the changed traveling path after the Compton scattering interaction in each detector, were determined by a conical surface uniform random number generator (RNG), and by a Klein–Nishina RNG. The 3D image generator has two functions: the recovery of the initial source energy spectrum and the 3D spatial distribution of the source. We evaluated the analytic simulator and image generator with two different energetic point radiation sources (Cs-137 and Co-60) and with an MSCC comprising three detector layers. The recovered initial energies of the incident radiations were well differentiated from the generated MSCC events. Correspondingly, we could obtain a multi-tracer image that combined the two differentiated images. The developed analytic simulator in this study emulated the randomness of the detection process of a multiple-scattering Compton camera, including the inherent degradation factors of the detectors, such as the limited spatial and energy resolutions. The Doppler-broadening effect owing to the momentum distribution of electrons in Compton scattering was not considered in the detection process because most interested isotopes for biomedical and environmental applications have high energies that are less sensitive to Doppler broadening. The analytic simulator and image generator for MSCC can be utilized to determine the optimal geometrical parameters, such as the distances between detectors and detector size, thus affecting the imaging performance of the Compton camera prior to the development of a real system.

Keywords Multiple-scattering Compton camera · Prompt gamma imaging · Analytic simulator · Image generator

1 Introduction

In order to recover an emission energy spectrum and generate a source image with an unknown radiation distribution for biomedical applications and environmental radiation monitoring, we consider herein the use of a multi-

scattering Compton camera (MSCC) which could detect simultaneously and differentiate multi-energetic gamma rays [1–3]. An MSCC consists of three or more semiconductor detector layers with good energy resolution that measures gamma rays by coincidence detection over multiple detector layers, and recovers the energy spectrum and the spatial distribution of the prompt gamma rays from the measured data. One interesting potential feature of MSCC is its capability for simultaneous, multiple radioisotope detection. Motomura et al. [4] obtained separate gamma images of three radiotracers of I-131 (364.5 keV), Sr-85 (514 keV), and Zn-65 (1115.5 keV), based on in vivo

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imaging of a living mouse using their Compton camera. Another potential feature is the spectroscopic ability by coincidence detection through three or more detector layers without any prior knowledge on the gamma energies that will be detected. Another study showed the recovered energy spectrum of the secondary prompt gamma rays emitted during ion beam therapy using three-stacked detectors [5].

Onsite environmental radiation monitoring should be one possible application of MSCC given the abilities of multi-tracer imaging and spectroscopy. The integrated environmental radiation monitoring network (IERNet) [6] managed by the Korea institute of nuclear safety (KINS) is a public service for the environmental radiation monitoring over inland and nearby oceanic regions of Korea. In particular, the periodic oceanic radiation monitoring is conducted every 3 months to investigate the anomalous changes of radiation in the sea around the Korean peninsula by sea water samples of 60 kg at 21 representative oceanic spots, and by analyzing the radioactivities in the laboratory. The current periodic monitoring process is quite time-consuming for daily, real-time observations of problematic situations in the ocean. In order to understand changing situations in a more efficient manner, identify problems, and take appropriate and correct actions, onsite radiation detection is essential. When remotely operated machines are equipped with the MSCC, it can be used for daily onsite environmental radiation monitoring, especially after nuclear power plant accidents.

In this study, we developed an analytic simulator and an image generator for a MSCC. A preliminary simulation study is essential to work prior to the development of a real MSCC. In many biomedical applications, the simulation becomes a useful tool to validate the systematic performances and to compare with the real experimental results [7–9]. Through the simulation study, we could investigate the physical factors affecting the system performance and optimize the geometrical parameters, such as the distance between detectors, the size of the detector, as well as the image reconstruction parameters. Additionally, the simulator provides guidelines on how the developed system is verified. It is possible to conduct a simulation with various sources and detector factors and to confirm the recovered energy spectrum and the 3D spatial source images.

2 Materials and methods

2.1 Detection process and image generation of multiple-scattering Compton camera

MSCC is a feasible detection system when we have no prior knowledge of the energies and of the spatial distribution of prompt gamma rays. The MSCC consists of three

or more detector layers operated in the coincidence detection mode, and records the successive interactions of gamma rays in multiple detector layers. It is assumed that the most frequent interaction in the detectors is Compton scattering. The energy spectrum and spatial distribution of the prompt gamma emission are recovered from the MSCC measurements. For example, in Fig. 1, for each coincidence detection, the n -th detector records the interaction position ($\mathbf{P}_n$) and the lost energy (ΔE_n) after Compton scattering interactions. The recorded positions and the lost energies determine the emission spectrum ($\mathbf{E}_0$) and the spatial distribution of the gamma rays.

The incidence gamma energy (E_{n-1}) onto the n -th detector could be computed by summing two energies, the leaving (E_n) and the lost (ΔE_n) energies, after Compton scattering. The lost energy is directly measured by the detector and the leaving energy is determined with the scattering angle (ω_n). At the n -th detector, the scattering angle is

$$\cos(\omega_n) = 1 - m_e c^2 \left(\frac{1}{E_n} - \frac{1}{E_n + \Delta E_n} \right) = \frac{(\mathbf{P}_{n-1} - \mathbf{P}_n) \cdot (\mathbf{P}_n - \mathbf{P}_{n+1})}{\|\mathbf{P}_{n-1} - \mathbf{P}_n\| \cdot \|\mathbf{P}_n - \mathbf{P}_{n+1}\|}, \quad (1)$$

where m_e is the electron rest mass and c is the speed of light. The scattering angle is defined based on the energetic or geometrical relationships of the successive measurements. While the first scattering angle, ω_1 , was calculated based on the energetic relationship, the other scattering angles at all the remaining detectors were computed by the geometrical relationship of the three successive positions in (1). Additionally, each incidence energy, (E_{n-1}), was computed as

$$E_{n-1} = \frac{1}{2} \left(\Delta E_n \pm \sqrt{\Delta E_n^2 + \frac{4m_e c^2 \Delta E_n}{1 - \cos(\omega_n)}} \right). \quad (2)$$

Through the repetitive computation of (1) and (2) with the last to the first detector measurements recorded as a

Fig. 1 Detection and imaging principle of the multiple-scattering Compton camera

coincident event, we can recover the initial emission energy (E_0). Furthermore, the line of $\overline{\mathbf{P}_1\mathbf{P}_2}$ and ω_1 become the axis and the half angle of a cone (e.g., the red-colored cone in Fig. 1). The spatial emission distribution is then reconstructed using reverse conical surface integrals of the measured coincidence events.

2.2 Analytic simulator of multiple-scattering Compton camera

The analytic simulator for a MSCC is based on two models of source emission and successive coincident detections to determine the emission direction, the scattering angle, and the changed traveling direction, after a Compton scattering interaction in each detector.

2.2.1 Emission model of radiation sources

The emission model of polyenergetic gamma sources generates randomly the emission directions of gamma rays from the emulated radioactive decays.

Based on the Cartesian coordinate system shown in Fig. 2, the intensities of gamma rays are expressed by a spectroscopic source distribution according to the specific emission position, $\mathbf{P}_0(x_0, y_0, z_0)$, and the initial energy E_0 in the field-of-view (FOV). Herein, the emission direction was limited by the angle between the emission vector and the detector surface to reduce redundant computation. For example, the angle of 90° corresponds to the half-unit gray-colored sphere in Fig. 2. Within a limited unit sphere, emission unit vectors $\mathbf{e}(e_x, e_y, e_z)$ were randomly generated by a uniform random number generator (RNG).

$$\mathbf{e}(e_x, e_y, e_z) = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta). \quad (3)$$

The uniform RNG generates polar angles, θ , and azimuthal angles, φ , randomly and the emission unit vectors are then computed.

2.2.2 Detection model of multiple-scattering Compton camera (MSCC)

The detection model has three substeps to generate (1) an intersection point of the incident direction vector and the detector plane, (2) a scattering angle related to Compton scattering interaction, and (3) the exiting direction vector changed after Compton scattering at each detector. The coincident detection of an MSCC event is completed by repeating the three substeps of detection model at every detector layer.

The first substep was used to determine the intersection point of the extended line along the incident direction vector ($\mathbf{u}$) and the corresponding detector plane. The line and plane equations are

$$\mathbf{P}_n = \mathbf{P}_{n-1} + t\mathbf{u}_n \text{ and } \mathbf{N}_n \cdot (\mathbf{O}_n - \mathbf{P}_n) = 0. \quad (4)$$

In (4), $\mathbf{N}_n$ and $\mathbf{O}_n$ are the normal and origin vectors of n -th detector layer, respectively. On the first detector, $\mathbf{u}_n$ becomes the emission unit vector, $\mathbf{e}$, as shown in Fig. 2. In order to determine the intersection point $\mathbf{P}_n$ in (4), the length, t , was calculated as

$$t = \frac{(\mathbf{N}_n \cdot \mathbf{O}_n) - (\mathbf{N}_n \cdot \mathbf{P}_{n-1})}{\mathbf{N}_n \cdot \mathbf{u}_n}. \quad (5)$$

Assuming that primary type of interaction of the incident gamma ray in the detector is Compton scattering, the second substep determines a scattering angle (ω_n). Given an incidence energy (E_n), the angular probability follows the Klein–Nishina (KN) probability density function (pdf) [10] as

$$p(\omega) = \pi r_0 \sin(\omega) \left[\frac{m_e c^2}{E_n [1 - \cos(\omega)] + m_e c^2} \right]^2 \left[\frac{m_e c^2}{E_n [1 - \cos(\omega)] + m_e c^2} + \frac{E_n [1 - \cos(\omega)] + m_e c^2}{m_e c^2} - \sin^2(\omega) \right]. \quad (6)$$

In (6), $m_e c^2$ is 511 keV based on the rest mass of the electron, and r_0 is classical electron radius. We propose the

Fig. 2 Detection process in an analytical simulator of MSCC

use of a KN-RNG based on the cumulative distribution of the KN pdf in (6). In the cumulative KN pdf, we could estimate the scattering angle from the given cumulative probability. For KN-RNG, we built a lookup table of cumulative probabilities corresponding to the sampled scattering angles between 0° and 180° . In KN-RNG, we did not consider the back-scattered events by limiting the range of scattering angles between 0° and 180° . First, the KN-RNG chooses one cumulative probability using uniform RNG, and the corresponding angle is then computed linearly from two-nearest sampled scattering angles in the lookup table. Figure 3 shows the lookup tables of the cumulative pdf for KN and compares a generated histogram from the KN-RNG and the calculated KN pdf at 100 keV and 20 MeV. As shown in Fig. 3, the KN-RNG histogram matched the shape of the KN pdf when considerably huge angles are generated. After choosing a scattering angle ω_n through KN-RNG, the lost energy (ΔE_n) in the detector is generated as in (7), and is recorded with the detection position $\mathbf{P}_n$ as a Compton event. In (7), the detector energy resolution (E_{fwhm}) affected the uncertainty of the deposited energy in the detector through a Gaussian RNG, in accordance with $N(\mu, \sigma)$.

$$\Delta E_n \sim N\left(\frac{E_{n-1}^2[1 - \cos(\omega_n)]}{E_{n-1}[1 - \cos(\omega_n)] + m_e c^2}, \frac{E_{fwhm}}{2.355}\right). \quad (7)$$

Finally, the leaving direction vector is generated by a uniform conical RNG after the Compton scattering interaction with a scattering angle ω_n . The possible leaving direction vectors, $\{\mathbf{u}_{n+1}\}$ form a conical surface with an apex $\mathbf{P}_n$ (interaction point in n -th detector), an axis $\mathbf{u}_n$ (incident direction vector), and a half angle ω_n , are shown in Fig. 4.

In order to generate a leaving direction vector $\mathbf{u}_{n+1}$ of gamma rays in the n -th detector, the conical RNG chooses randomly a vector $\mathbf{c}$ on the surface of a reference cone according to a uniform pdf and then computes the changed direction vector $\mathbf{u}_{n+1}$ through a rotation transformation $\mathbf{R}$, as shown in Fig. 5.

The reference cone is defined with an axis $\mathbf{r}(0, 0, 1)$ and a half-angle ω_n , and a new direction vector $\mathbf{u}_{n+1} (= \mathbf{R}\mathbf{c})$ is determined with a rotation matrix, as in (8).

$$\mathbf{R} = \mathbf{I} + [\mathbf{r} \times \mathbf{u}_n]_{\times} + [\mathbf{r} \times \mathbf{u}_n]_{\times}^2 \frac{1 - (\mathbf{r} \cdot \mathbf{u}_n)}{|\mathbf{r} \times \mathbf{u}_n|^2} \quad (8)$$

In (8), the cross-product matrix of a vector $\mathbf{v}$ is

$$[\mathbf{v}]_{\times} = \begin{bmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{bmatrix}. \quad (9)$$

The three substeps of the detection model are repeated by the number of detectors to generate the Compton events.

When the interaction position in the last detector is determined, a valid Compton event of successive interactions is recorded through multiple detectors in accordance with $\{\mathbf{P}_1, \mathbf{P}_2, \dots, \mathbf{P}_n, \Delta E_1, \Delta E_2, \dots, \Delta E_n\}$.

2.3 Image generator of multiple-scattering Compton camera

The 3D image generator has two functions: the recovery of the initial source energy spectrum, and the mapping of the 3D spatial source distribution from the measured Compton data. The source energy spectrum is derived with the successive detection positions $\{\mathbf{P}_1, \mathbf{P}_2, \dots, \mathbf{P}_n\}$ and energies $\{\Delta E_1, \Delta E_2, \dots, \Delta E_n\}$ using (1) and (2). Additionally, the 3D image is recovered by simple backprojection based on the conical surface integrals of all measured Compton events over the entire image space. Each measured cone is defined with the detection position $\mathbf{P}_1$ (apex) and the scattering angle ω_1 (half-angle of a cone) in the first detector, as shown in Fig. 6. To complete the conical surface integral over the image space, the conical surface is sampled uniformly into N lines starting from $\mathbf{P}_1$, as shown by the red dotted lines in Fig. 8. Furthermore, the intersecting lengths between the sampled lines and image voxels are computed by the ray-tracing method [11], and are used as weights for backprojection. Each voxel value is determined by the weighted sum of all measured Compton data.

3 Results

An analytical simulator and a 3D image generator of MSCC were developed for prompt gamma imaging. Figure 7 shows the graphic user interface (GUI) of the simulator and image generator. The upper part of the GUI is for the simulator, where we could set the detailed conditions, such as the detector positions and sizes for simulation. Furthermore, we could confirm the simulation results on the right upper part. The lower part of the panel shows the features used to recover the source energy spectrum and 3D spatial source distribution. After the reconstruction process of the loaded measurement is completed, we could see the recovered energy spectrum and the selected image plane.

As shown in Fig. 8, we performed a simulation with two points sources, namely, Cs-137 (661 keV) and Co-60 (1172 keV), placed at two different distances, namely, at 5 and 50 cm from the MSCC, which can measure the successive interactions using the three detectors. The size of each detector is $20 \times 20 \times 5 \text{ mm}^3$ and the distance between the two detectors is 3 cm. The detector segment size is 1.25 mm. The distance between Cs-137 and Co-60 on the same x-y plane was 2 cm. The simulator emitted

Fig. 3 Cumulative probability function (left), Klein–Nishina random number generator (middle), and Klein–Nishina probability density function (right) at 100 keV (c) and at 20 MeV (d–f)

Fig. 4 Changed direction model of leaving gamma rays from the detector after Compton scattering interactions

Fig. 5 Uniform conical surface random number generator

5×10^6 gamma rays from each point source and generated 4557 and 570 Compton events from two point sources at 5 and 50 cm, respectively. The larger the distance from the

MSCC is, the smaller the obtained sensitivity is. The generated simulation data was recovered in the source energy spectrum and the 3D source distribution image.

Figure 9 shows the recovered energy spectra of Cs-137 and Co-60 located at 5 (a) and 50 (b) cm from the MSCC. Comparison of the energy spectra at 5 cm, shows that the uncertainty of the peak energy at 50 cm is increased. The image voxel and the volume sizes are 1 mm^3 and $60 \times 60 \times 60 \text{ mm}^3$. In Fig. 10, we could see the separate Co-60 and Cs-137 images and the colored fused image of the two point sources. Owing to the limited detector segment size, the spatial image resolution was deteriorated as the distance from MSCC is increased.

Fig. 6 Ray-tracing model for conical surface integral using simple backprojection

Fig. 7 Graphic user interface (GUI) for analytical simulator and image generator of MSCC

Fig. 8 MSCC simulation of Cs-137 (662 keV) and Co-60 (1172 keV) point sources located at 5 and 50 cm from the MSCC

Fig. 9 Recovered source energy spectra at distances of **a** 5 and **b** 50 cm from the MSCC

Fig. 10 Reconstructed separate gamma images of Co-60 and Cs-137 colored fused images, and scatter plots of two point sources at distances of **a** 5 and **b** 50 cm from the MSCC

4 Discussion

In this study, we developed an analytical simulator and an image generator for an MSCC. The MSCC is capable of the simultaneous detection of prompt multi-isotopes without any prior knowledge of radiation sources, and can provide a source energy spectrum and a 3D image of the spatial distribution of radiation. There are some advantages in monitoring the therapeutic effects and irradiation accuracy to target organs if the secondary radiation arising during radiation therapy could be detected. Additionally, MSCC

can be utilized for spectroscopic measurements of radiation pollution on the ground and the sea after radiation accidents, such as the Fukushima Daiichi nuclear disaster in March 2011. Owing to the portability because of its compact size, and the spectroscopic measurement ability, there are some added applications associated with Compton cameras that are mounted on remotely operated machines, e.g., an unmanned helicopter [12, 13].

The developed analytic simulator in this study emulated the randomness of the detection process of an MSCC, including the inherent degradation factors of detectors,

such as the limited spatial resolution of the detector segmentation, and the energy resolution. However, the detection process did not consider Doppler broadening effects that arise from Compton scattering interactions. Thus, there will be a larger simulation uncertainty for low-energy radio-isotopes. However, Doppler broadening is not a dominant factor for the deterioration of the measurement performance for detecting high-energy gamma rays. The current image generator based on simple backprojection could be updated with more robust image reconstruction techniques and with more sophisticated system models, e.g., ordered subset expectation maximization [14], prior knowledge [15], or resolution recovery technique [16].

Using the simulator and image generator, preliminary studies can be performed prior to the development of the MSCC. Correspondingly, preliminary studies can be performed to determine the optimal geometrical parameters, such as the distances between detectors or detector sizes, thus affecting the imaging performance of the Compton camera prior to the development of a real system. The resolution degradation factors, such as the limited energy resolutions, Doppler broadening effect, and the segment size of the detectors [17], can be tested with simulation data, and the proper detector properties could then be determined.

5 Conclusions

For effective prompt gamma imaging for biomedical and daily environmental radiation monitoring, we proposed an onsite prompt gamma monitoring system using an MSCC. In instances where unknown energy spectra of radiation emitting sources exist, the Compton camera is necessary to construct the multiple detectors that require coincidence detection. The MSCC can provide a source energy spectrum and a three-dimensional image of spatial source distribution from the measurements. In this study, we developed an analytical simulator and an image generator, which were applicable in investigating the various types of MSCCs with different detection factors. The analytic simulator generated the Compton events through the following successive models: (1) source model for the emission of incident radiation toward a detector using a uniform RNG, (2) detection model of Compton scattering interaction on each detector by inducing an interaction position and energy, scattering angle (Klien–Nishina RNG), and changed traveling direction (conical surface-based uniform RNG) of the incident radiation in a detector. The measured Compton event was recorded with detection positions and energies $\{\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3, \Delta E_1, \Delta E_2\}$, and was used to induce the initial energy spectrum of the incident radiation on the MSCC and the 3D images of multiple radio-isotopes. The

simulator accounted for the effects of the limited detector segment size and the energy resolution, but not Doppler broadening. Prompt environmental radiation monitoring is one possible application of MSCC because most of the isotopes of interest, such as Cs-137 have high energies that are less sensitive to Doppler broadening. For a verification of the developed analytic simulator and image generator, the Compton events were generated from 5×10^6 emissions of Cs-137 (662 keV) and correspondingly emissions from Co-60 (1172 keV). In the simulation of the Cs-137 and Co-60 point sources, two different gamma rays were well differentiated, thus providing the energy spectrum and 3D images. Through the simulation, we can formulate the guidelines on how to verify the real system or how to determine the optimal parameters of the fast and robust image reconstruction algorithms.

Acknowledgements This work was supported by research Grants from Korea Institute of Ocean Science and Technology (PE99672).

Compliance with ethical standards

Conflict of interest The author declares that there is no conflict of interest.

Ethical approval It is declared that no human or animal has been used at any stage during this experiment.

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