Spectral extrapolation for ultra-wide band radio frequency super-resolution tumor localization in the breast

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Abstract The use of ultra-wide band (UWB) radio frequency (RF) as an alternative to x-ray mammography for the detection and localization of breast tumors has been an area of focused research over the last decade. Unlike x-rays, UWB RF is non-ionizing and poses no risk of inducing cancer in examined patients. However, the reduction in operating frequency of UWB RF compared to x-rays results in much poorer localization when using classical space-time adaptive signal processing. This work investigates the synergistic use of a temporal signal spectrum extrapolation technique and contemporary basis pursuit de-noising (BPDN) beamforming to achieve medically relevant tumor localization accuracy within a heterogeneous simulated breast volume. If the beamforming process is viewed as a matched filtering operation, the spectrum extrapolation technique artificially increases the bandwidth of the received signal to afford much sharper correlations and hence contributing to increased localization accuracy. BPDN also contributes to increased localization accuracy by employing L_1 -norm constraints which shows marked clutter suppression effects in this context.

Keywords Compressive sensing · Breast tumor localization · UWB · Spectral extrapolation

1 Introduction

Since the very late 1990s ultra-wide band (UWB) radio frequency (RF) has been an investigated alternative technology for the early detection and localization of tumors in the breast [1]. The primary motivation for UWB RF investigations in this application stems from the fact that energy propagated at UWB RF frequencies is non-ionizing and, in contrast to x-ray mammography, eliminates the possibility of inducing cancer within the patient. Research such as [2] has shown that cancerous and non-cancerous breast tissues exhibit greatly different dielectric properties which makes the RF backscatter from the tumor discernable. Henceforth from this observation the major research efforts over the last decade has been aimed at obtaining tumor localization performance commensurate with x-ray mammography at significantly lower UWB RF operating frequencies [3–10].

Recently several works have examined tumor localization performance in the breast using the most recent super-resolution techniques such as orthogonal matching pursuit (OMP) [9] and compressive sensing [10]. In both [9] and [10] excellent localization performance was demonstrated. However, either clutter was not included or the number of tumors was assumed known a priori. In the work presented here the number of tumors is not assumed to be known a priori and clutter is included for added realism. The clinical relevance of the proposed approach stems from the fact that both the number of tumors is not known a-priori and clutter due to inhomogeneities in real breasts [1] can be significant. Furthermore the proposed approach herein considers both practical engineering implementation considerations and super-resolution medical needs.

The main attributes of the proposed approach are; (a) a multiple-input multiple-output (MIMO) radar configuration

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which employs a 200 MHz signal bandwidth operating at 6 GHz, (b) a spectral extrapolation technique to artificially quadruple the signal bandwidth to 800 MHz, and (c) a super-resolution L_1 -norm basis pursuit de-noising (BPDN) beamformer.

From a practical implementation and engineering point of view, a 200 MHz signal bandwidth is at the upper limit of common commercial digitization hardware and the selection of a super-resolution BPDN beamformer achieves (at least in the simulation presented) medically relevant tumor localization (e.g. 2mm accuracy) [1]. The artificial quadrupling of the signal bandwidth offered by a spectral extrapolation technique achieves a signal bandwidth incapable of digitization with common commercial digitization hardware.

This work makes the following simplifying assumptions to demonstrate a proof-of-concept. First, signal backscatter from the breast skin surface may be either neglected due to advances in RF sensing technology [9] or effectively eliminated with techniques such as that in [1]. Second, a reasonably flat received signal magnitude spectrum with linear phase can be generated and received by UWB RF transmitters and receivers. And third, signal-to-noise ratio is appreciably high and the signal-to-clutter ratio presents the dominant challenge.

This paper is organized in the following way. Section II presents the problem formulation supported in the context of a UWB RF signal model and the proposed approach consisting of spectral extrapolation and BPDN beamforming. Section IV offers a simulated experiment with two tumors in a cluttered environment. Section V discusses extensions of this work and Section VI presents a few concluding remarks. Section VII provides a list of references.

2 Methods

2.1 Problem formulation

This work explores the use of a MIMO RF radar configuration consisting of N transmitters, M receivers, and L active RF pulses transmitted from each transmitter. The expected received signal spectrum at the m^{th} receiver from the n^{th} transmitter is modeled as a linear superposition,

$$X_{nm}(\omega) = \sum_{s=1}^S \sum_{k=1}^W \alpha_{snm}(\omega_k) \exp\left(j\omega_k \left(\frac{r_{snm}}{c} + (n-1)T_d\right)\right). \quad (1)$$

In Equation (1), s is a scatterer index, k is a temporal discrete-Fourier transform (DFT) frequency component, r_{snm} is the straight-line travel distance from the n^{th}

transmitter to the s^{th} scatterer and on to the m^{th} receiver, α_{snm} is the back-scattered signal amplitude traveling along path r_{snm} , and T_d is a known fixed signal transmission delay from the n^{th} transmitter used to allow simple temporal separation of transmitter pulses received at each receiver (i.e. time-division multiplexing).

In Equation (1), the known quantities are assumed to be the number of transmitters and their locations, the number of receivers and their locations, the transmitted spectral components ω_k , and T_d . The number of scatterers comprised of tumors and other breast inhomogeneities (clutter) and their locations is assumed unknown. Additionally, the number of strong scatterers due to cancerous tumors is assumed to be small in medically relevant cases.

According to earlier research [1, 2] one expects cancerous breast tumors to exhibit much larger backscatter. That is, one expects $\alpha_{snm}(\omega)$ to be large when reflected from a tumor and considerably smaller when reflected from other inhomogeneous scatterers in the breast. The authors of [1] noted that when operating in the RF spectrum between 3 GHz to 10 GHz cancerous scatterers exhibited backscatter that was approximately 5 times larger than non-cancerous scatterers.

In the following, details of the proposed approach comprised of a spectral extrapolation technique and the use of an L_1 -norm BPDN beamformer are presented.

Coherent signal processing techniques always chooses a reference signal with which to compare signal phase differences. If one chooses a reference DFT component of the received signal spectrum to be ω_1 and a reference transmitter and receiver to be $m = n = 1$ (arbitrarily) then Equation (1) may be re-written as

$$Y_{nm}(\omega) = \sum_{s=1}^S \sum_{k=1}^W \alpha_{snm}(\omega_k) \exp\left(j(\omega_1 + (k-1)\Delta\omega) \left(\frac{r_{s11} + \Delta r_{snm}}{c} + (n-1)T_d\right)\right) \quad (2)$$

where $\Delta\omega$ is a known DFT component separation in Hz and Δr_{snm} is the difference in straight-line travel distance from the n^{th} transmitter to the s^{th} scatterer and on to the m^{th} receiver compared to the reference straight-line travel distance.

Equation (2) shows two relationships when a scatterer (s), transmitter (n), and receiver (m) are specified. First, DFT phase components of the received signal are linearly related to $\Delta\omega$ and second, by choosing a reference transmitter and receiver, DFT phase differences are related by Δr_{snm} . The linear relationship to $\Delta\omega$ motivates the spectral extrapolation technique discussed next. Both the aforementioned relationships allow creating parameterized beamforming steering vectors.

2.2 Spectral extrapolation

In the context of spatial discrimination of wireless telephone users using a small uniform linear RF sensor array (ULA), the authors of [11] showed that the relationship of the element-to-element sampled signal of the linear array can be represented by a P^{th} order auto-regressive (AR) model where P was the number of wireless telephone users. Using the AR coefficients the authors artificially extrapolated the physical aperture of the ULA to offer improved spatial resolution. In this work, the concept is used in the temporal domain.

If Equation (2) is compared to the context of the ULA in [11] then one can make the association that adjacent DFT components of the received signal spectrum can be similarly estimated with an P^{th} order AR process by

$$Y_{nmk} = \sum_{p=1}^P a_{pnm} Y_{nm(k-p)} \quad (3)$$

where a_{pnm} is the p th AR coefficient of the received signal spectrum at the m^{th} receiver from the n^{th} transmitter.

expected must be chosen in the appropriate context (e.g. 10).

2.3 L_1 -norm BPDN

Basis-Pursuit De-Noising (BPDN) [13] is a method for determining a set of basis vectors that most closely fits observed data while allowing one to trade-off parameterized data fitting for the number of basis vectors chosen using a regularization parameter. In this work, Equation (2) is exploited to form a set of parameterized basis vectors describing the expected DFT phase response from many possible tumor locations. Three components are required to employ BPDN; a matrix of basis vectors, and observation vector, and a value for the regulation parameter.

Given an estimate of a scatterer range from the reference transmitter and receiver, $\hat{r}_{s11}$, one can exploit Equation (2) (with now Q total DFT components due to extrapolation) to form a very large steering vector involving Q DFT components, N transmitters, and M receivers. The final steering vector $\mathbf{a}(\hat{r}_{s11})$ shown in Equation (4) is $QNM \times 1$.

$$\mathbf{a}(\hat{r}_{s11}) = \begin{bmatrix} \exp\left(-j\omega_1 \left(\frac{\hat{r}_{s11}}{c}\right)\right) \\ \exp\left(-j(\omega_1 + \Delta\omega) \left(\frac{\hat{r}_{s11}}{c}\right)\right) \\ \dots \\ \exp\left(-j(\omega_1 + (W-1)\Delta\omega) \left(\frac{\hat{r}_{s11}}{c}\right)\right) \\ \exp\left(-j(\omega_1) \left(\frac{\hat{r}_{s11} + \Delta r_{s12}}{c}\right)\right) \\ \exp\left(-j(\omega_1 + \Delta\omega) \left(\frac{\hat{r}_{s11} + \Delta r_{s12}}{c}\right)\right) \\ \dots \\ \exp\left(-j(\omega_1 + (W-1)\Delta\omega) \left(\frac{\hat{r}_{s11} + \Delta r_{s12}}{c}\right)\right) \\ \dots \\ \exp\left(-j(\omega_1 + (W-1)\Delta\omega) \left(\frac{\hat{r}_{s11} + \Delta r_{s1M}}{c}\right)\right) \\ \dots \\ \exp\left(-j(\omega_1 + (W-1)\Delta\omega) \left(\frac{\hat{r}_{s11} + \Delta r_{sNM}}{c}\right) + (N-1)T_d\right) \end{bmatrix} \quad (4)$$

Equation (3) can be iterated to artificially extrapolate the received signal spectrum by estimating non-existing DFT components increasing the number of DFT components from W to Q . The P AR coefficients may simply be estimated with a least-squares algorithm [12] using any set of P measured DFT components of the received signal. A set of P coefficients must be estimated for each received signal combination resulting in NMP total coefficients. Since P is unknown (S being the true value), a larger value than

If the entire breast region is gridded in three dimensions each grid point can be used to compute a different hypothesized $\hat{r}_{s11}$. The collection of hypothesized $\hat{r}_{s11}$ parameterizing different steering vectors are collected to form the basis for the observed (and extrapolated) received signal DFT component data. In this work the basis matrix $\mathbf{A}(\mathbf{r})$ has the form,

$$\mathbf{A}(\mathbf{r}) = [\mathbf{a}(\hat{r}_1), \mathbf{a}(\hat{r}_2), \mathbf{a}(\hat{r}_3) \dots \mathbf{a}(\hat{r}_J)] \quad (5)$$

where each vector $\mathbf{a}(\hat{r}_i)$ $i = 1:J$ is of the form of Equation (4) and is parameterized by the straight-line travel distance from the n th transmitter to the i th hypothesized scatterer location (grid volume) and on to the m th receiver. $\hat{r}_i$ has replaced $\hat{r}_{s11}$ to simplify notation.

The extrapolated DFT component data of the received signal from each receiver due to each transmitter is stacked into one large $QNM \times 1$ observation vector $\mathbf{y}$. Recall that a transmission delay is imposed to allow separation of received transmitter signals at each receiver. The selection of the regularization parameter is deferred to the next section.

Given Equation (5) and the observation vector $\mathbf{y}$, the BPDN problem is posed as

$$\min \|\mathbf{y} - A(\mathbf{r})\mathbf{x}\|_2 \text{ s.t. } \lambda < \|\mathbf{x}\|_1 \quad (6)$$

where λ is the regularization parameter and $\mathbf{x}$ is a $J \times 1$ vector with non-zero entries only where columns of $A(\mathbf{r})$ correlate strongly with $\mathbf{y}$. In this context vector $\mathbf{x}$ will contain the magnitude of the contribution of the hypothesized tumor responses associated with the columns in $A(\mathbf{r})$ to the received signal $\mathbf{y}$. One expects a higher correlation when a column of $A(\mathbf{r})$ corresponds to the location of a true tumor. As λ is decreased fewer columns of $A(\mathbf{r})$ are selected which enables empirical suppression of clutter.

The solution to Equation (6) is usually solved with now publically available mathematical routines such as [14].

3 Results

This section demonstrates the tumor localization performance of the proposed approach by simulation. Localization results are presented using a single simulated data set. The localization performance of the proposed approach is expected to be comparable to those of [9] and [10] in clutter-free and known single-tumor scenarios. Since this work considers the inclusion of clutter and an unknown number of tumors, the localization performance of the proposed approach is compared to the popular minimum variance distortionless response (MVDR) beamformer [12]. Though many conventional adaptive beamformers exist [15], beamformers such as MVDR and MUSIC [15] continue to be highly popular in sonar and radio frequency (RF) applications.

A three-dimensional breast volume is simulated in the Cartesian coordinate system with a maximum support of 150 cm in each dimension. The spectral response at each receiver due to each transmitter is simulated with a center

Fig. 1 **a** MVDR x - y plane with extrapolation. **b** BPDN x - y plane without extrapolation. **c** MVDR x - z plane with extrapolation. **d** BPDN x - z plane without extrapolation. **e** MVDR y - z plane with extrapolation. **f** BPDN y - z plane without extrapolation

Fig. 2 **a** BPDN x - y plane with extrapolation. **b** BPDN x - z plane with extrapolation. **c** BPDN y - z plane with extrapolation

frequency of 6 GHz, pulse bandwidth of 200 MHz, and DFT component separation of 20 MHz (10 DFT bins). Three transmitters and eight receivers are placed equidistantly over the surface of the breast volume. Two random point-scattering tumors are placed within the volume with thirty clutter sources throughout the breast volume. Permittivity values of $\epsilon_t = 50$ and $\epsilon_c = 10$ are used in simulating tumor and clutter scattering responses respectively. The breast volume is gridded into 2 mm grid steps in each dimension. Each grid volume is used to form a hypothetical tumor location comprising a single column of $A(\mathbf{r})$. When spectrum extrapolation is performed, the response of the received signal is extrapolated from 200 MHz to 800 MHz keeping the same DFT component separation of 20 MHz

(40 DFT bins) and using an AR order of $P = 10$ in Equation (3) and DFT components $Y_{nmP-9}:Y_{nmP}$. An optimal value for λ is unknown to the authors in this context, however a wide range of values of λ produced nearly identical results using BPDN which is consistent with behavior seen in other contexts [16]. The results of this section are presented using a value of $\lambda = 35$. Lastly, though SNR is not expected to be a limiting condition in this application, a small amount of complex white Gaussian noise was added to the simulated received signal spectrum to add non-uniformity.

Fig. 1a–f show each plane of the breast volume using extrapolation for MVDR and without using extrapolation for BPDN (MVDR without extrapolation is quite poor and does not provide a valuable comparator). The yellow diamonds are shown above the true tumor locations. MVDR is able to identify the presence of the two tumors in two dimensions but localization accuracy is inadequate for medical purposes (well in excess of 2mm). Fig. 1e showed that MVDR was unable to separate the two tumors in the y - z dimension. Fig. 1a, c, e of the MVDR results show several sizeable responses resulting from clutter included in the simulation. In comparison, Fig. 1b, d, and f show markedly better localization using BPDN without the use of extrapolation but it is still not able to localize the correct tumors within 2mm of their true location in all dimensions and exhibits additional spurious peaks due to the presence of clutter.

Fig. 2a–c show BPDN with spectral extrapolation. In each figure the peaks of the spectrum are located within 1 grid point of the true tumor locations placing the localization accuracy under 2mm. Additionally, all of the previously present spurious clutter responses have been eliminated.

4 Discussion

This work explored the concept of spectral extrapolation for UWB applications of breast tumor localization. Leveraging prior work using extrapolation in a spatial context, this work extrapolated the received signal spectrum temporally to bandwidths far greater than could be sampled with currently available digitization hardware. The resulting localization accuracy examined with a cluttered breast simulation yielded medically relevant localization accuracy of approximately 2mm. In closing, one extension is discussed in the following subsection as well as the effect of the AR model order selection.

4.1 Non-ideal received signal response spectrums

As implemented in this work, the proposed method is limited by near-ideal amplitude-rectangular and linear

phase received signal spectrums. This is a limitation not experienced by conventional approaches such as MVDR. Real received signals will be distorted by both the breast medium and signal filtering. To remove this limitation in the case where the spectrum is significantly shaped, the received spectrum could be pre-processed to flatten it (perhaps by one of many windowing options). Enhancing the realism of the received signal spectrums will be a continuation of this work.

4.2 AR order

In the simulations presented in the prior section the true AR order would have been $P = S = 2$. The effect of the AR order was empirically observed to only adversely impact localization performance when it was less than the true AR order ($P < S$).

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