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A Multiphysics Model for Radiofrequency Activation of Soft Hydrated Tissues

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Abstract

A multi-physics model has been developed to investigate the effects of cellular level mechanisms on the electro-thermo-mechanical response of hydrated soft tissues with radiofrequency (RF) activation. A micromechanical model generates an equation of state (EOS) that provides the additional pressure arising from evaporation of intra- and extracellular water as well as temperature to the continuum level thermo-mechanical model. A level set method is used to capture the interfacial evolution of tissue damage with the level set evolution equation derived from the second law of thermodynamics, which is consistent with Griffith's fracture evolution criterion. The discretized equations are solved simultaneously using a Krylov subspace based iterative solver (GMRES) with block preconditioning that effectively deflates the spectrum of the system matrix, resulting in exponential convergence of the Arnoldi iterations. Example problems, including experimental validation, illustrate the computational accuracy and efficiency of the technique.

Keywords

Multi-physics modeling; Electrosurgery; Level set method; Equation-of-state; Preconditioning

1. Introduction

Instruments that utilize radiofrequency (RF) alternating current in the range of 300Hz-5 MHz, are increasingly being utilized to perform a variety of surgical and other therapeutic procedures [1]. RF has the advantage of dissecting tissue with simultaneous coagulation, leading to hemostasis. The frequency range precludes neuromuscular stimulation and therefore entails no risk of electrocution [2]. However, to avoid significant tissue damage and other adverse events during surgical procedures as well as to develop more effective devices and procedures, there is need to develop models that account for the effects of the biophysics of RF electricity on the thermo-mechanical response of soft tissues.

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When RF electrical energy is concentrated in a very small area of the tissue the resulting high current density increases intra- and extracellular temperature due to oscillations of large protein ion in the cytosol of the cells. This leads to vaporization of water from the cells, protein denaturation, and various tissue effects including coagulation, desiccation or dehydration, and carbonization [3]. The efficacy of electrosurgical techniques depends on being able to dissect tissue locally while minimizing inadvertent thermal damage of neighboring tissue to prevent adverse surgical outcomes. The dissection or cutting mechanism is associated with continuous delivery of currents into tissue governed by Joule's law [4]. Previous studies have investigated the mechanism of explosive vaporization for creating dissections [5–7]. The “cut” mode, with its continuous delivery of current, creates higher tissue temperatures in a shorter time, leading to boiling of intra- and extracellular water and consequently cause the cell membrane to rupture, release, and evaporation of the cellular water resulting in tissue dehydration and desiccation. Therefore, the tissue is precisely dissected with surrounding coagulated tissue walls. The explosive vaporization may also result in the fragmentation of the tissue in its proximity and during the collapse of cellular membrane, a fast water transport may form causing severe tissue damage far away from the initial energy deposition site [8].

Although clinical application of electrosurgical instrument has been well documented, the effects of cellular level micromechanical mechanisms on the thermomechanical response of RF activated soft tissues have not been adequately modeled, the understanding of underlying cellular mechanism of electrosurgical cutting and tissue damaging remain limited. Therefore, it is imperative to develop such understanding for predicting the clinical outcome of these procedures. The purpose of this work is to develop a multi-physics computational model to study the effects of micromechanical mechanisms, due to Joule heating, on the thermomechanical response of RF activated soft tissue.

Several models have been proposed to study the effects of RF electrical energy on the thermomechanical response of soft tissue. Dodde et al. [9] have conducted a thermal-electric finite element analysis on RF activated soft tissue, neither tissue deformation nor dissection is considered in this study. Kuroda et al. [10] proposed an electrosurgical cutting simulation system with consideration of structural change due to thermal stress criteria. Lu et al. [11] proposed an electrosurgical cutting method using isotherm to determine tissue cutting. Pennes bioheat equation [12] is used to predict heat transfer in biological tissue in all these studies. Other references have shown the usefulness of this equation [13,14]. However, at higher temperatures during electrosurgical dissection, Pennes bioheat equation does not adequately capture all the physical processes affecting the tissue temperature field. These processes include evaporation of water, transport of water vapor and tissue damage [15]. Therefore, these previous models, which disregard the latent heat of water-to-vapor phase transition, predict monotonic rise in temperature beyond the boiling temperature, which is not observed experimentally. Also, cellular level micromechanical effects are critical in determining the depth and spread of thermal damage, for which satisfactory models have yet to be developed.

In this paper, we propose a multi-physics model to study the effects of RF activation on the thermo-mechanical response of hydrated soft tissue. A thermodynamically consistent

continuum model for the soft tissue is presented within the framework of small deformation linear thermoelasticity. The micromechanical model based equation of state (EOS) describes the thermodynamic states such as temperature, and pressure arising from evaporation of intra- and extracellular water by absorbing heat due to Joule heating. The thermodynamics of tissue micromechanical solid-water-vapor system is motivated by a micro-explosion-based physical model for the laser ablation process [16], which was developed assuming a spherical cavity embedded in a non-absorbing elastic medium. In [17] a similar cavitation based model is proposed to study effects of ultrasonically activated soft tissue. In our method, a tissue damage model is employed to describe tissue dissection due to cellular wall rupture, with effective thermal and mechanical quantities as well as incremental effective heat generation at a macroscopic spatial point obtained from the cellular model.

Another aspect of RF activation that we are interested in is tissue dissection. Tissue division using a scalpel has been modeled in [18]. Modeling of electrosurgical dissection is complicated as the cut must follow the motion of the electrosurgical tool, but must also evolve in width and depth. The actual depth and extent of tissue injury is dependent upon a number of factors including power density, electrode size and shape, and nature of the tissue being dissected. Techniques ranging from local remeshing [19–21] to the extended finite element methods [22–24] have been developed to model topological changes. In [21], an online remeshing technique has been proposed, where an adaptive framework has been used to accelerate computations. In [19,20], a dynamic meshing framework is proposed for fracture simulation, which allows a subsequent coarsening once areas are no longer likely to undergo topology changes. The extended finite element methods (XFEM) are a class of generalized finite element techniques that have been developed to represent discontinuities in finite element computations, without the need for remeshing, by directly representing the discontinuity using enrichment functions within the partition of unity framework. In [24], a shifted enrichment function based XFEM cutting method is presented for real-time surgery simulation. However, during electrosurgery, the cut evolves in width and depth. Within the XFEM framework, the discontinuities are modeled independent of the finite element mesh, unable to keep track of the evolution of these discontinuities. In [11], a dual-mesh dynamic triangulation algorithm was presented for electrosurgical tissue dissection based on isotherms. However, this technique requires a very fine mesh in order to resolve the cut and as the cut progresses, new elements are created increasing the computational cost. To overcome these problems in existing methods, we present a level set-based technique to reduce meshing and re-meshing complexity.

Level set methods have been shown to be very effective for modeling topological changes without re-meshing [25]. Physics-based modeling of the velocity field to drive the evolution of the level set interface has been investigated in fracture mechanics and Stefan problems [26,27], in which the level set velocities were derived from shape derivatives and jump conditions between different phases, respectively. In this paper, a level set method is used to capture the interfacial evolution of tissue damage. An important aspect of this work is the derivation of the level set evolution equation from the second law of thermodynamics. The formulation is motivated by the damage evolution problem discussed in [28], and is shown to be consistent with Griffith's fracture evolution criterion [29].

Crack growth in soft tissues such as arterial tissues has been considered to be driven by energy-based failure criterion such as the critical fracture energy (Griffith-type critical energy release rate) [30,31]. A comparison of various failure criteria, such as energy-based, Tsai–Wu, Hill, and principal stress criteria, for the crack propagation in arterial tissues subjected to uniaxial and planar biaxial deformations has shown that the energy-based criterion reflects stable and physically realistic crack growth, particularly in complex three-dimensional geometries [31]. Further, Luo et al. [32] proposed a computational model for tear propagation in fiber-reinforced soft tissue subjected to internal pressure, which is based on Griffith's energy minimization principle. Similar to the crack phase field models [30,31], the proposed level set approach to the interfacial evolution of tissue damage is based on a Griffith-type critical energy release rate.

Finally, the issue of computational efficiency in the solution of the discretized set of equations is important for multiphysics problems. Current best practices for the efficient solution of the such problem are based almost exclusively on multigrid methods and preconditioned Krylov subspace methods [33]. Design of efficient application specific monolithic multiphysics multigrid solvers or preconditioners is challenging [34]. Here, we show that a block right preconditioner can effectively deflate the spectrum of the system matrix leading to exponential convergence of the Arnoldi iterations [35].

The paper is organized as follows. The continuum scale electro-thermo-mechanical model for the RF activated soft tissue along with the micromechanical model based EOS and level set method based interfacial evolution of tissue damage are presented in Section 2. The numerical solution approach of the governing equations is presented in Section 3. Example problems, including experimental validation, are solved in Section 4 demonstrating the accuracy and computational efficiency of the solution process. Conclusions and future scope of work are presented in Section 5.

2. Mathematical formulation

The balance laws are presented in Section 2.1 followed by the level set-based tissue dissection methods in Section 2.2. Constitutive laws and the derivation of level set evolution equation from the second law of thermodynamics are presented in Section 2.3. The temperature evolution equation is presented in Section 2.4 followed by the micromechanical model in Section 2.5.

2.1 Balance equations

Let $\Omega \in \mathbb{R}^3$ be an open bounded domain with external boundary Γ as shown in Fig. 1, where $\Omega_d \subset \Omega$ denotes the damaged zone from where tissue is removed due to dissection, $\Omega_h = \Omega \setminus \Omega_d$ denotes the healthy region, and $\Gamma_i = \Omega_h \cap \Omega_d$ the interface of Ω_d and Ω_h .

The electric potential (V), temperature (T) and displacement field ($\mathbf{u}$) must satisfy the following local forms of the balance laws for all $\mathbf{x} \in \Omega$ and $t \in (0, \tau)$.

Electric charge conservation—The electromagnetic effects are treated by solving Maxwell's equations [12] for the electric field distribution in the tissue. At typical operating

frequencies of electrosurgical instruments, the characteristic wavelengths (λ_{wave}) of the electromagnetic waves are much larger than the characteristic dimensions of the tissue (L), i.e.,

$$L \ll \lambda_{wave} \Rightarrow Lf \ll \frac{1}{\sqrt{\epsilon\mu}} \quad (1)$$

where f is the frequency of the electromagnetic wave, ϵ and μ are the permittivity and permeability of the tissue, respectively. This justifies a quasi-static assumption of the Maxwell's equations [36]. The boundary value problem for charge conservation is

Find $V: \bar{\Omega} \rightarrow \mathbb{R}$ such that

$$\nabla \cdot \mathbf{J} = 0 \text{ in } \Omega \quad (2)$$

subject to

$$\mathbf{n} \cdot \mathbf{J} = j_n \text{ on } \Gamma_j$$

$$V = \bar{V} \text{ on } \Gamma_V$$

with boundary $\Gamma = \Gamma_j \cup \Gamma_V$ and $\Gamma_j \cap \Gamma_V = \emptyset$ and $\bar{\Omega} = \Omega \cup \Gamma$, where V denotes the electric potential, $\mathbf{J}$ is the current density, j_n is the current density normal to the Neumann boundary Γ_j , $\mathbf{n}$ is the unit outward normal to Γ_j , and $\bar{V}$ is the prescribed electric potential on the Dirichlet boundary Γ_V .

At the frequencies used in electrosurgical procedures, capacitive losses are negligible and heat generation can be considered to be mainly resistive [37]. The current density can then be described by Ohm's law as

$$\mathbf{J} = \sigma \mathbf{E} \quad (3)$$

where σ is the electrical conductivity and $\mathbf{E}$ is the electric field given by

$$\mathbf{E} = -\nabla V \quad (4)$$

The heat generated (q_f) due to radiofrequency electrical excitation is

$$q_f = \mathbf{J} \cdot \mathbf{E} \quad (5)$$

Linear momentum balance—The deformation of soft tissue due to pore pressure induced by evaporation of water under Joule heating is obtained by solving the local linear momentum balance equation. In electrosurgery, the time scales over which temperature changes occur are orders of magnitude larger than the time scales of the elastic waves [38]. Hence, tissue deformation is described by the quasi-static momentum balance equation. The boundary value problem in elastostatics can be given as

Find $\mathbf{u} : \bar{\Omega} \rightarrow \mathbb{R}$ such that

$$\nabla \cdot \boldsymbol{\sigma} + \mathbf{b} = 0 \text{ on } \Omega \quad (6)$$

subject to

$$\boldsymbol{\sigma} \cdot \mathbf{n} = \bar{\mathbf{t}} \text{ on } \Gamma_t$$

$$\mathbf{u} = \bar{\mathbf{u}} \text{ on } \Gamma_u$$

with boundary $\Gamma = \Gamma_t \cup \Gamma_u$ and $\Gamma_t \cap \Gamma_u = \emptyset$, and $\bar{\Omega} = \Omega \cup \Gamma$, where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\mathbf{b}$ is the body force, $\bar{\mathbf{t}}$ is the surface traction on the Neumann boundary Γ_t , $\bar{\mathbf{u}}$ is the prescribed displacement on the Dirichlet boundary Γ_u , $\mathbf{n}$ is the outward normal to Γ_t .

Energy balance equation—The local energy balance equation is given by

$$\rho \dot{e} = \boldsymbol{\sigma} : \dot{\mathbf{e}} + q - \nabla \cdot \mathbf{q} \text{ on } \Omega \times]0, \tau[\quad (7)$$

where ρ is the effective density of tissue as described in Section 2.5, e is the internal energy per unit mass, $\mathbf{e}$ is the strain measure, q is the effective heat generation per unit volume as described in Section 2.5, and $\mathbf{q}$ is the heat flux vector. The temperature evolution equation for soft tissue subjected to Joule heating is derived in Section 2.4.

2.2 A level set approach to tissue dissection

Topology change of the tissue due to thermal damage is modeled using a level set approach. In the level set method [25], an evolving interface $\Gamma_A(t)$ in Ω is represented as the zero level set of a smooth function $\phi(\mathbf{x}, t)$, i.e. $\Gamma_A(t) = \{\mathbf{x} | \phi(\mathbf{x}, t) = 0\}$. In the domain Ω , we parameterize the damaged zone $\Omega_d \subset \Omega$ (Fig. 1) by means of the level set function $\phi(\mathbf{x}, t)$ such that

$$\phi(\mathbf{x}, t) = \begin{cases} < 0 & \mathbf{x} \in \Omega_h \\ > 0 & \mathbf{x} \in \Omega_d \\ = 0 & \mathbf{x} \in \Gamma_i(t) \end{cases} \quad (8)$$

The level set function is evolved using the following equation

$$\frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi = 0, \text{ on } \Omega \times]0, \tau[\quad (9)$$

$$\phi(\mathbf{x}, 0) = \phi_0(\mathbf{x}) \text{ on } \mathbf{x} \in \Omega$$

where $\mathbf{v}$ is the interface velocity and $\mathbf{v} \cdot \nabla \phi = v_N |\nabla \phi|$, where v_N is the normal component of the interface velocity and ϕ_0 is the initial condition. The interface velocity may depend on the position, time, geometry of the interface, and the physics of radiofrequency (RF) induced tissue damage. In this paper, we derive the level set evolution equation from the second law of thermodynamics in the next Section 2.3.

2.3 Constitutive relations and level set evolution equation

We assume a general form of the free energy functional

$$\psi = \hat{\psi}(\boldsymbol{\epsilon}, T, \phi) \quad (10)$$

then the rate of change of free energy can be expressed as

$$\dot{\psi} = \frac{\partial \psi}{\partial \boldsymbol{\epsilon}} : \dot{\boldsymbol{\epsilon}} + \frac{\partial \psi}{\partial T} \dot{T} + \frac{\partial \psi}{\partial \phi} \dot{\phi} \quad (11)$$

The Clausius-Duhem inequality [39] $\rho \dot{\psi} + \rho \eta \dot{T} - \boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}} \geq 0$ becomes

$$(\boldsymbol{\sigma} - \rho \frac{\partial \psi}{\partial \boldsymbol{\epsilon}}) : \dot{\boldsymbol{\epsilon}} - \rho(\eta + \frac{\partial \psi}{\partial T}) \dot{T} - \rho \frac{\partial \psi}{\partial \phi} \dot{\phi} \geq 0 \quad (12)$$

where η denotes the specific entropy per unit mass.

The above inequality holds for all admissible deformation and temperature fields, leading to the following restrictions on the stress tensor, entropy and remainder of the dissipation inequality

$$\boldsymbol{\sigma} = \rho \frac{\partial \psi}{\partial \boldsymbol{\epsilon}}, \eta = - \frac{\partial \psi}{\partial T}, \rho \frac{\partial \psi}{\partial \phi} \dot{\phi} \leq 0 \quad (13)$$

The Cauchy stress tensor can be split into hydrostatic pressure (p) and deviatoric stress ($\boldsymbol{\tau}$) as

$$\boldsymbol{\sigma} = -p \mathbf{I} + \boldsymbol{\tau} \quad (14)$$

where p is obtained from the micromechanical based EOS (Section 2.5) and $\boldsymbol{\tau} = \boldsymbol{\sigma} - \frac{1}{3}tr(\boldsymbol{\sigma})\mathbf{I}$.

The tissue damage is assumed to be dissipative in nature, hence

$$\beta \dot{\phi} = -\rho \frac{\partial \psi}{\partial \phi} \quad (15)$$

where $\beta > 0$ is a kinetic coefficient.

We postulate the following form of the Helmholtz free energy of the soft tissue with damage

$$\rho \psi = [1 - H(\phi)]\psi_e + H(\phi)\kappa \quad (16)$$

where κ is a damage parameter, ψ_e defines the elastic strain energy density of the soft tissue, and $H(\phi)$ defines a Heaviside function, with

$$H(\phi) = \begin{cases} 0, & \phi < 0 \\ 1, & \phi \geq 0 \end{cases} \quad (17)$$

The transition from healthy to damaged state occurs when the release of elastic strain energy exceeds the parameter κ .

Eqs. (15) and (16) then leads to the level set evolution equation

$$\beta \dot{\phi} = (\psi_e - \kappa)\delta(\phi) \quad (18)$$

where δ is the Dirac delta function. A standard rescaling can be made by replacing $\delta(\phi)/\beta$ by $|\nabla \phi|$ [40], resulting in the level set evolution equation

$$\dot{\phi} = (\psi_e - \kappa)|\nabla \phi| \quad (19)$$

with the interface velocity v_N as the difference between the elastic strain energy and the damage parameter ' κ '. This formulation is consistent with Griffith's energy minimization criterion for crack evolution if ' κ ' is interpreted as the fracture toughness of the tissue [29]. The parameter ' κ ' can be measured experimentally.

2.4 Temperature evolution

Introducing the Legendre transform of the internal energy, i.e. $\psi = e - \eta T$ into the energy balance Eq. (7) gives the following entropy evolution equation

$$\rho \dot{\eta} T = -\rho \dot{\psi} - \rho \eta \dot{T} + \boldsymbol{\sigma} : \dot{\mathbf{e}} + q - \nabla \cdot \mathbf{q} \quad (20)$$

Substituting Eq. (11) for the rate of change of free energy along with constitutive relations (Eq. (13)) and the level set rate law (Eq. (15)) into Eq. (20) yields

$$\rho \dot{\eta} T = \beta \dot{\phi}^2 + q - \nabla \cdot \mathbf{q} \quad (21)$$

Further, using Gibbs' relation $\eta = -\psi / T$, the rate of change of entropy takes the following form

$$\dot{\eta} = -\frac{\partial^2 \psi}{\partial T \partial \boldsymbol{\epsilon}} : \dot{\boldsymbol{\epsilon}} - \frac{\partial^2 \psi}{\partial T^2} \dot{T} - \frac{\partial^2 \psi}{\partial T \partial \phi} \dot{\phi} \quad (22)$$

Using the definition of the specific heat at constant deformation

$$c = \frac{\partial e}{\partial T} = \frac{\partial}{\partial T}(\psi + \eta T) = \frac{\partial \psi}{\partial T} + \eta + T \frac{\partial \eta}{\partial T} = -T \frac{\partial^2 \psi}{\partial T^2} \quad (23)$$

and Grüneisen tensor at constant temperature

$$\boldsymbol{\gamma} = \frac{1}{c} \frac{\partial \eta}{\partial \boldsymbol{\epsilon}} = -\frac{1}{c} \frac{\partial^2 \psi}{\partial T \partial \boldsymbol{\epsilon}} \quad (24)$$

the rate of change of entropy in Eq. (22) leads to the temperature evolution equation as

$$\rho c \dot{T} = -\rho c T \boldsymbol{\gamma} : \dot{\boldsymbol{\epsilon}} + q - \nabla \cdot \mathbf{q} + \beta \dot{\phi}^2 + \rho T \frac{\partial^2 \psi}{\partial T \partial \phi} \dot{\phi} \quad (25)$$

The first term on the right-hand side of Eq. (25) contributes to the structural heating, whereas the last two terms correspond to tissue damage. Neglecting structural heating and dissipation due to tissue damage compared to Joule heating (Eq. (5)) and using the specific form of Helmholtz free energy in Eq. (16), we arrive at the following form of the temperature evolution equation

$$\rho c \dot{T} = q - \nabla \cdot \mathbf{q} \quad (26)$$

The corresponding *initial boundary value problem* is

Find $T : \bar{\Omega} \times [0, \tau] \rightarrow \mathbb{R}$ such that

$$\rho c \frac{\partial T}{\partial t} + \nabla \cdot \mathbf{q} = q \text{ on } \Omega \times \left] 0, \tau \right[\quad (27)$$

subject to

$$-\mathbf{q} \cdot \mathbf{n} = q_n \text{ on } \Gamma_q \times]0, \tau[$$

$$T = \bar{T} \text{ on } \Gamma_T \times]0, \tau[$$

$$T(\mathbf{x}, 0) = T_0(\mathbf{x}) \mathbf{x} \in \Omega$$

with the boundary $\Gamma = \Gamma_q \cup \Gamma_T$ and $\Gamma_q \cap \Gamma_T = \emptyset$, and $\bar{\Omega} = \Omega \cup \Gamma$, where $\mathbf{q}$ is the heat flux vector, q is the effective heat generation per unit volume, q_n is the component of heat flux normal to the Neumann boundary Γ_q , $\mathbf{n}$ is the positive unit normal to Γ_q , $\bar{T}$ is the prescribed temperature at the Dirichlet boundary Γ_T , T_0 is the initial temperature, and ρ and c are the effective density and effective specific heat capacity of the tissue, respectively. The heat flux is defined in terms of temperature gradient using generalized Fourier's law as

$$\mathbf{q} = -k \nabla T \quad (28)$$

where k is the effective thermal conductivity of the tissue.

The effective parameters (ρ , c and k) along with effective heat generation (q) obtained from the micromechanical model are described in Section 2.5.

2.5 Micromechanical model

Following the work of Majaron et al.[16] and Rahul and De [17], we postulate that the tissue water is trapped in microscopic pores in a non-absorbing elastic medium. Initially, the pore of volume V_0 is filled with water at ambient temperature T_a and atmospheric pressure p_0 . As the water absorbs the heat input from Joule heating, its temperature increases until the boiling point T_b is reached. Further heating converts a fraction (x) of the water mass into vapor and causes the pores to expand under added pressure. The current volume V of the pore can be calculated by adding the volume of the water (V_w) and vapor (V_v) as

$$V = V_0 \left[\left(\frac{rT}{p} \rho_w - 1 \right) x + 1 \right] \quad (29)$$

where r is the specific gas constant for water vapor, ρ_w denotes the density of liquid water at current pore pressure p and temperature T . The water vapor is assumed to follow the ideal gas equation of state. The above Eq. (29) is derived using mass conservation $\rho_w V_0 = \rho_w V_w + \rho_v V_v$, where ρ_v is the density of water vapor, related to the pore pressure through the ideal gas law $p = \rho_v r T$ and vapor content, i.e. mass fraction of water converted to vapor, $x = \rho_v V_v / \rho_w V_0$. For the ideal gas approximation of vapor much below the critical temperature,

an explicit relation can be obtained from the Clausius-Clapeyron equation that relates the equilibrium vapor pressure p to temperature T as [41]

$$p = p_0 \exp \left[-\frac{\lambda}{r} \left(\frac{1}{T} - \frac{1}{T_b} \right) \right] \quad (30)$$

where λ denotes the average specific enthalpy of evaporation in the interval $[T_b, T]$.

The current volume of the pore can also be approximated from the condition of mechanical equilibrium for a single spherical cavity in an infinite elastic medium as [42]

$$V = V_0 \left[1 + \frac{1+\nu}{2E} (p - p_0) \right]^3 \quad (31)$$

where E is the young's modulus of the tissue and ν is the Poisson's ratio.

Eqs. (29) and (31) lead to an EOS that relates the pore pressure p and temperature T with the vapor content x as,

$$x = \left[\left\{ 1 + \frac{1+\nu}{2E} (p - p_0) \right\}^3 - 1 \right] \left(\frac{rT}{p} \rho_w - 1 \right)^{-1} \quad (32)$$

For the quasi-static expansion of the micro pore, the energy balance follows from the first law of thermodynamics, where the change of specific internal energy of the closed system over a time interval dt can be given as $de = \delta q_f - p dv$. The last term of the energy balance corresponds to the specific work done by the pore system on the surrounding tissue. In terms of specific enthalpy, the energy balance can be given as $dh = \delta q_f + v dp$. For the pore system the energy balance takes the following incremental form

$$\frac{q_f}{\rho} \Delta t = m[\lambda \Delta x + x(c_v + r)\Delta T + (1-x)c_w \Delta T] - \frac{1}{\rho} \Delta p + (1-m)c_s \Delta T \quad (33)$$

where m is the water content, ρ_s is the density of the solid part of the tissue, and c_s , c_w and c_v , respectively, denotes the specific heat capacity of solid, water, and vapor part of the tissue. T denotes incremental change in temperature and x denotes incremental change in water content. In Eq. (33), the first three terms describe the enthalpy changes in additional evaporation, and heating of vapor and water to the new equilibrium temperature, respectively. The last term accounts for eventual conductive thermalization of the solid tissue components.

Effective quantities—The effective density, specific heat capacity and thermal conductivity of the tissue at $\mathbf{x} \in \Omega$ is computed using vapor content x as

$$\rho = wx\rho_v + w(1-x)\rho_w + (1-w)\rho_s, \quad (34)$$

$$c = wxc_v + w(1-x)c_w + (1-w)c_s, \quad (35)$$

$$k = wxk_v + w(1-x)k_w + (1-w)k_s \quad (36)$$

where the subscripts v , w , and s represent properties corresponding to the vapor, water, and solid phase, respectively.

Effective heat generation—The effective heat generation at a spatial point $\mathbf{x} \in \Omega$ is then given by

$$q = \rho c \frac{\Delta T}{\Delta t} \quad (37)$$

Remark—The pore pressure p at a spatial point $\mathbf{x} \in \Omega$ contributes to the hydrostatic pressure in the tissue.

3. Discretized equations and numerical solution approach

The weak forms of the electric charge conservation, linear momentum balance equation, and temperature evolution equation are presented below followed by numerical solution approach for the level set evolution equation.

Electric charge conservation

Let $V_v = \{w(\mathbf{x}) \mid \mathbf{x} \in \bar{\Omega}, w(\mathbf{x}) \in H^1(\Omega), w(\mathbf{x}) = 0 \text{ on } \Gamma_v\}$ be the space of weighing functions, where H^1 is the first order Hilbert space. Let $\zeta_v = \{V(\mathbf{x}) \mid \mathbf{x} \in \bar{\Omega}, V(\mathbf{x}) \in H^1(\Omega), V(\mathbf{x}) = 0 \text{ on } \Gamma_v\}$ be the space of trial solutions. The weak form of the electric charge conservation law (Eq. (2)) is given as

Find $V \in \zeta_v$, such that for all $w \in V_v$

$$\int_{\Omega} \nabla w \cdot \mathbf{J} d\Omega = \int_{\Gamma_j} w j_n d\Gamma, \quad (38)$$

where the current density $\mathbf{J}$ is described by Ohm's law (Eq. (3)).

Assuming a finite element discretization of the electric potential as $V(\mathbf{x}) = \sum_I N_I(\mathbf{x}) V_I$, where $N_I(\mathbf{x})$ is the shape function of node I and V_I is the nodal electric potential, the discrete form of (Eq. (38)) is given as

$$\mathbf{K}_V \mathbf{V} = \mathbf{F}_V, \text{ with } \mathbf{K}_V = \int_{\Omega} \mathbf{B}^T \sigma \mathbf{B} d\Omega, \text{ and } \mathbf{F}_V = \int_{\Gamma_j} \mathbf{N}^T j_n d\Gamma \quad (39)$$

where $\mathbf{N}(\mathbf{x})$ is the vector of shape functions, and $\mathbf{B} = \nabla \mathbf{N}(\mathbf{x})$.

Energy balance

Let $V_t = \{w(\mathbf{x}) \mid \mathbf{x} \in \bar{\Omega}, w(\mathbf{x}) \in H^1(\Omega), w(\mathbf{x}) = 0 \text{ on } \Gamma_T\}$ be the space of weighing functions and $\zeta_t = \{T(\mathbf{x}, t) \mid \mathbf{x} \in \bar{\Omega}, T(\mathbf{x}, t) \in H^1(\Omega), T(\mathbf{x}, t) = \bar{T}(\mathbf{x}, t) \text{ on } \Gamma_T\}$ be the space of trial solutions. The weak form of the heat equation (Eq. (27)) may be stated as [43]

Find $T(t) \in \zeta_t$, $t \in [0, \tau]$ such that for all $w \in V_t$

$$\int_{\Omega} w \rho c \frac{\partial T}{\partial t} d\Omega + \int_{\Omega} \nabla w \cdot \mathbf{q} d\Omega = \int_{\Omega} w q d\Omega + \int_{\Gamma_q} w q_n d\Gamma \quad (40)$$

Assuming a finite element discretization of the temperature field as $T(\mathbf{x}, t) = \sum_I N_I(\mathbf{x}) T_I(t)$, where T_I is the nodal temperature at node I , the discrete form of Eq. (40) is given as

$$\mathbf{M}_T \dot{\mathbf{T}} + \mathbf{K}_T \mathbf{T} = \mathbf{F}_T \quad (41)$$

with $\mathbf{M}_T = \int_{\Omega} \rho c \mathbf{N}^T \mathbf{N} d\Omega$, $\mathbf{K}_T = \int_{\Omega} k \mathbf{B}^T \mathbf{B} d\Omega$, and $\mathbf{F}_T = \int_{\Omega} \mathbf{N}^T q d\Omega - \int_{\Gamma_q} \mathbf{N}^T q_n d\Gamma$.

The time integration of Eq. (41) is performed using a midpoint rule. The resulting set of equations to be solved at time step t is

$$(\mathbf{M}_T + \frac{1}{2} \Delta t \mathbf{K}_T) \mathbf{T}^{t+\Delta t} = (\mathbf{M}_T - \frac{1}{2} \Delta t \mathbf{K}_T) \mathbf{T}^t + \Delta t \mathbf{F}_T^{t+\frac{1}{2}\Delta t} \quad (42)$$

Linear momentum balance

Let $V_u = \{\mathbf{w}(\mathbf{x}) \mid \mathbf{x} \in \bar{\Omega}, \mathbf{w}(\mathbf{x}) \in H^1(\Omega), \mathbf{w}(\mathbf{x}) = 0 \text{ on } \Gamma_u\}$ be the space of weighing functions and $\zeta_u = \{\mathbf{u}(\mathbf{x}) \mid \mathbf{x} \in \bar{\Omega}, \mathbf{u}(\mathbf{x}) \in H^1(\Omega), \mathbf{u}(\mathbf{x}) = \bar{\mathbf{u}}(\mathbf{x}) \text{ on } \Gamma_u\}$ the space of trial solutions. The weak form of the linear momentum balance (Eq. (6)) is given as

Find $\mathbf{u} \in \zeta_u$, such that for all $w \in V_u$

$$\int_{\Omega} \nabla \mathbf{w} \cdot \boldsymbol{\sigma} d\Omega = \int_{\Omega} \mathbf{w} \cdot \mathbf{b} d\Omega + \int_{\Gamma_t} \mathbf{w} \cdot \bar{\mathbf{t}} d\Gamma \quad (43)$$

Assuming a finite element discretization of the displacement field as $\mathbf{u}(\mathbf{x}) = \sum_I N_I(\mathbf{x}) \mathbf{u}_I$, where $\mathbf{u}_I$ is the nodal displacement vector at node I , the discrete form of Eq. (43) is given as

$$\mathbf{K}_u \mathbf{U} = \mathbf{F}_u \quad (44)$$

with $\mathbf{K}_u = \int_{\Omega} \mathbf{B}^T \mathbf{D} \mathbf{B} d\Omega$, $\mathbf{F}_u = \int_{\Omega} \mathbf{N}^T \mathbf{b} d\Omega + \int_{\Gamma_t} \mathbf{N}^T \bar{\mathbf{t}} d\Gamma$, and $\mathbf{D}$ is the elasticity tensor in Voigt notation.

Level set evolution

The level set equation (Eq. (19)) is solved on triangulated domains using an explicit positive coefficient scheme proposed by Barth and Sethian [44]. Let $\chi = \{T_1, T_2, \dots, T_n\}$ denote a triangulation set in $\mathbb{R}^d$ such that $\chi = \cup T_j$, $T_i \cap T_j = \emptyset$, for $i \neq j$. The solution on χ is approximated using a standard piecewise linear finite element subspace, i.e. $\phi(\mathbf{x}) = N_I(\mathbf{x}) \phi_I$. The numerical approximation of the level set Eq. (19) can be given as

$$\phi_j^{t+\Delta t} = \phi_j^t + \Delta t \frac{\sum_l^n \alpha_j^l \int_{T_l} (\psi_e - \kappa) |\nabla \phi| d\Omega}{\sum_l^n \alpha_j^l \text{meas}(T_l)} \quad (45)$$

where α_j^l are bounded, positive weighting coefficients computed on simplex T_l and $\text{meas}(T_l)$ corresponds to the volume of simplex T_l . For detail algorithmic description of the explicit positive coefficient scheme we refer to [44].

Solution of the algebraic set of equations within each time step

The resulting linear algebraic set of equations for the electric charge conservation (Eq. (39)), linear momentum balance equation (Eq. (44)), and temperature evolution equation (Eq. (41)) are solved simultaneously as

$$\mathbf{F}^t = \begin{Bmatrix} \mathbf{F}_V \\ \bar{\mathbf{F}}_T \\ \mathbf{F}_u \end{Bmatrix} = \begin{bmatrix} \mathbf{K}_V & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{K}}_T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_u \end{bmatrix} \begin{Bmatrix} \mathbf{V} \\ \mathbf{T} \\ \mathbf{U} \end{Bmatrix} \equiv \mathbf{K}^t \mathbf{X}^t \quad (46)$$

where $\bar{\mathbf{F}}_T = (\mathbf{M}_T - \frac{1}{2} \Delta t \mathbf{K}_T) \mathbf{T}^t + \Delta t \mathbf{F}_T^{t+\frac{1}{2}\Delta t}$ and $\bar{\mathbf{K}}_T = (\mathbf{M}_T + \frac{1}{2} \Delta t \mathbf{K}_T)$.

The system of equations is solved using a GMRES iterative solver [45] with an efficient block preconditioning technique.

A deflation based block preconditioner

We present a right preconditioner for the current time step ' t ' for the system in Eq. (46) along the lines proposed in [35]. We use information generated from the GMRES process in previous time steps $j \in \{1, 2, \dots, t-1\}$ to develop an effective block preconditioner for the current time step ' t ':

Definition 3.1—Let the system matrix at time step ' j ', i.e. $\mathbf{K}^j \in \mathbb{R}^{n \times n}$, $j \in \{1, 2, \dots, t-1\}$, permit the factorization $(\mathbf{V}_{m_j+1}^j \mathbf{Q}_{m_j}^j)^T \mathbf{K}^j \mathbf{V}_{m_j+1}^j = \mathbf{R}_{m_j}^j$. Then for the nonsingular $\mathbf{R}_{m_j}^j$, we define the block preconditioner

$$(\mathbf{M}^t)^{-1} = \mathbf{I}_n + \sum_j \mathbf{V}_{m_j+1}^j \left\{ \left(\alpha_j \left(\mathbf{R}_{m_j}^j \right)^{-1} \left(\mathbf{Q}_{m_j}^j \right)^T ; \left(\mathbf{Q}_{m_j+1}^j \mathbf{e}_{m_j+1} \right)^T \right) - \mathbf{I}_{m_j+1} \right\} \left(\mathbf{V}_{m_j+1}^j \right)^T, \quad (47)$$

where ' m_j ' is the number of GMRES iterations for convergence in time step ' j ', $\mathbf{V}_{m_j+1}^j$ is the Krylov matrix, $\mathbf{R}_{m_j}^j$ and $\mathbf{Q}_{m_j+1}^j$ are the QR factors of the corresponding upper Hessenberg matrix $\bar{\mathbf{H}}_{m_j}^j \in \mathbb{R}^{(m_j+1) \times m_j}$, $\alpha_j \in \mathbb{R}$, $\alpha_j \neq 0$, and $\mathbf{I}_{m_j+1} \in \mathbb{R}^{(m_j+1) \times (m_j+1)}$ is an identity matrix.

The block preconditioner (Eq. (47)) requires storage and use of the Krylov matrix $(\mathbf{V}_{m_j+1}^j)$ and the upper Hessenberg matrix $(\bar{\mathbf{H}}_{m_j}^j)$ from previous time steps $j \in \{1, 2, \dots, t-1\}$ in current time step ' t ', where ' m_j ' is the number of GMRES iterations required for convergence in time step ' j '. The parameter α_j is defined as

$$\alpha_j = \left\{ \left\| \mathbf{K}^j \mathbf{v}_{m_j+1}^j \right\|_2 \mid j \in \{1, 2, \dots, t-1\} \right\}, \quad (48)$$

where $\mathbf{v}_{m_j+1}^j$ is the last computed Arnoldi vector in the time step ' j '. In section 4, we show that the action of the preconditioner (Eq. (47)) deflates the spectrum of the preconditioned matrix by clustering eigenvalues around α_j .

Remark 3.1—The action of preconditioner $(\mathbf{M}^t)^{-1}$ on an Arnoldi vector requires $O(\sum_j (4n(m_j+1)))$ flops for $m_j \ll n$, $\forall j \in \{1, 2, \dots, t-1\}$. The block preconditioner requires $O\left(\sum_j (m_j+1)\left(\frac{m_j}{2} + n + 1\right)\right)$ locations for the storage of the Krylov matrices $\mathbf{V}_{m_j+1}^j$, upper

triangular matrices $\mathbf{R}_{m_j}^j$, and ' m_j+1 ' independent elements of Given's rotation matrices required to form $\mathbf{Q}_{m_j}$

The algorithmic flow of the numerical solution approach is presented in Fig. 2.

4. Numerical examples

The effects of radiofrequency (RF) electric current on the thermo-mechanical response of liver tissue are investigated using the multi-physics model in conjunction with the underlying micromechanical model providing the EOS. The model parameters that have been assumed are summarized in Table 1.

For the given model parameters, the time scales for temperature changes ($\rho c L^2/k$) and elastic waves ($L\sqrt{\rho/\mu}$) [38] are 3.6 and 10^{-4} , respectively, justifying the quasi-static assumption for the linear momentum balance holds in Section 2.1.

A finite element computational model has been developed to simulate two scenarios: (i) an active electrode placed at a fixed point of the tissue plane as shown in Fig. 3 and (ii) the electrode is moved along an arbitrary path. The computational model is a $0.2 \times 0.1 \times 0.1 \text{ m}^3$ rectangular tissue block. The active electrode, which is ball shaped with a radius of 1.17 mm, is placed at a fixed point (0.1, 0.05, 0.1 m) of the tissue plane for scenario (i). The applied current density of the active electrode is 0.0572 A/mm^2 for 50 W, 0.0444 A/mm^2 for 25 W, and 0.0271 A/mm^2 for 10 W power setting. The bottom plane (CDHG) is placed on the dispersive electrode.

Table 2 specifies the boundary condition for planes marked by letters A–H in Fig. 3. In most commercially available electrosurgical units (ESU), the output variable that can be adjusted is power. This power setting, in conjunction with the output power vs. tissue impedance characteristics of the ESU, allows the surgeon to control the current input. The tissue is assumed to be room temperature (288.5 K) and the micromechanical tissue pores are initially filled with water at room temperature and atmospheric pressure (101,325 Pa). The simulation is conducted on an Intel Core 4 Xeon(R) CPU and NVIDIA QUADRO K5000 graphics processor. The FEniCS library is used for automated, efficient solution of differential equations associated with the finite element analysis of the coupled multi-physics problem [49]. The computational domain is discretized using tri-linear tetrahedral elements with one Gauss point per element. Implementations of solvers are extensively based on the Portable Extensible Toolkit for Scientific computations (PETSc) library [45]. A GPU-based fast parallel solver for the level set equations [50] is incorporated into our framework to allow computationally efficient tracking of the changes in the tissue topology due to vaporization. The thermodynamic states (p , T) at a macroscale integration point are obtained from the micromechanical model by solving the local energy balance equation (Eq. (33)) along with the EOS (Eq. (32)) numerically using Newton's method in parallel using CUDA.

In the following sections, we first present an example of electrosurgery using fixed electrode and the corresponding model validation in Section 4.1, followed by a moving electrode in Section 4.2.

4.1. Fixed electrode

We present simulation results with a fixed electrode, a condition known as thermal ablation, and validate the temperature evolution results with the experimental data for *ex vivo* porcine liver [15]. Power setting is one of the major factors that has a significant effect on the tissue response [1]. Therefore, for validation purpose, we compare the simulation results with experimental data under various power settings (10, 25, 50 W). We examine the efficacy of our multi-physics model in capturing the radial distribution of the temperature field from the tip of the electrode at four different instants of time as presented by the dashed lines in Fig. 4a, 4c and 4e. The time evolution of the temperature field at four different radial distances from the electrode are shown by the dashed lines in Fig. 4b, 4d and 4f. The solid lines represent the mean of the experimental data and the error bars represent the standard deviation. The temperature close to the electrode increases rapidly, but decays fast with radial distance from it. The simulations are seen to capture the general trends in the data with an error of less than 11%, which is within the standard deviation of the experimental data points.

Fig. 5 shows that without the micromechanics-based model the temperature continues to rise with increasing simulation time, whereas the micromechanical model allows the temperature to level off in a manner that is observed experimentally [15]. From Fig. 6, we observe that for the 25 and 50 W power settings, the RF heat produced is sufficient for the tissue to reach boiling temperature within 1 s, which is not the case with 10 W power setting. Therefore, the lower power setting needs longer time to reach the coagulation temperature, which is consistent with experimental observations [1].

The effect of applied power of the electrosurgical unit (ESU) on the thermo-mechanical response of the tissue is investigated at 10 W, 25 W, and 50 W power settings. From Fig. 6, we can see that as the temperature reaches its boiling point (Fig. 6b), water starts to evaporate (Fig. 6c), and then the pressure starts to increase due to evaporation of water trapped in the interstitials (Fig. 6a). Increased pressure results in volume expansion in the RF activated part of the tissue, which may lead to mechanical damage. Beyond the boiling temperature, the temperature levels off as the tissue undergoes protein denaturation leading to the formation of a coagulum. We observe that increasing the power setting of the electrode from 25W to 50 W, results in faster temperature rise leading to rapid evaporation of intra- and extracellular water. Therefore, higher applied power of ESU leads to early protein denaturation and coagulation of tissue.

Next, we study the computational efficiency of our numerical solution approach. The results are presented at third time step for 39560 number of degrees of freedom (NDOF). We begin with examining the effectiveness of various preconditioners by comparing spectrum of system matrix with that of preconditioned matrix. We test the well-known block Jacobi preconditioner [45] and an algebraic multigrid (AMG) based preconditioner (HYPRE boomerAMG, with a V cycle type, Jacobi relaxation smoother, and maximum 5 levels in a

hierarchy) [51] along with the block-preconditioner defined in Eq. (47). From Fig. 7, we see that the deflation based block-preconditioner clusters eigenvalues of system matrix more effectively around parameters α_j , defined in Eq. (48), resulting in a smaller spectral radius. Fig. 8 shows the effects of eigenvalue clustering on the convergence of the preconditioned GMRES iterations, in the L^2 norm of the residual. We see that the block-preconditioned iterations result in exponential convergence as shown in [35] when compared to other two preconditioners.

In Fig. 9, we compare the average computational cost within a single time step for the three preconditioned iterations with increasing NDOF. Consistent with the convergence behavior, the deflation based block-preconditioner outperforms the other two with linear increase in computational cost with increasing NDOF.

To solve the level set equation efficiently, a fast parallel strategy [50], which combines the narrowband scheme [52] for the tissue damage interface, with a domain decomposition technique is adopted into our framework to allow computationally efficient tracking of the changes in the tissue topology. In Fig. 10, the performance comparison of solving the level set equation using CPU and Graphics Processing Unit (GPU) is conducted with increasing NDOF, when h-type refinement is performed. While the computational cost for both the approaches are comparable at lower NDOF, the GPU based parallel strategy is computationally more efficient with increasing NDOF with larger variance.

The distribution of the electric potential, current density, temperature distribution, and displacement field for the fixed electrode scenario are presented in Fig. 11. The contour plots correspond to 0.4875 s, 1.0938 s, and 1.5313 s, respectively, for the tissue activated at 50W power setting. The active electrode is located at the reference coordinate (0.1, 0.05, 0.1 m). From the current density distribution in Fig. 11b and temperature distribution in Fig. 11c, we observe that the current density concentrates at the tip of the active electrode which results in intense Joule heating of the tissue. The current density is significantly lower on the dispersive electrode (bottom of the tissue block) as the current spreads over a much larger area. From the temperature distribution in Fig. 11c and the displacement fields in Fig. 11d, we clearly see that the electrosurgical tool not only increases the temperature of the tissue but also inflates it due to evaporation of intra- and extracellular water, causing damage. The damaged zone evolves in width and depth as shown in Fig. 11d.

4.2. Moving electrode

In this example, the active electrode is moved along a curvilinear path at speeds of 5.28 mm/s, 8.82 mm/s, and 12.35 mm/s. The contour plots in Fig. 12 are taken when the active electrode goes through reference point (0.0579, 0.0551, 0.1 m), (0.1085, 0.0597, 0.1 m), and (0.1558, 0.0350, 0.1 m) at the speed of 5.28 mm/s, respectively, for the tissue at 50 W power setting.

The contour plots in Fig. 13 are time evolution at a cross section going through the reference point (0.1085, 0.0597, 0.1 m) and normal to the electrode movement direction, at speeds of 5.28 mm/s, 8.82 mm/s, and 12.35 mm/s from bottom to top. Speed of the electrode movement is one of the factors that determines the actual width and depth of tissue ablation

and collateral tissue damage. From Fig. 13, we observe that the slower the speed of the active electrode, the more energy delivered to the tissue, which results in wider and deeper cut.

Along the line that goes through the reference point (0.1085, 0.0597, 0.1 m) and normal to the electrode movement direction, we take some reference points to show the variance of pressure and vapor content for the three different speeds (Fig. 14). We observe that with increase in speed of the electrode, the zone of the tissue with increased pressure and vapor content decreases. Consequently, this results in less collateral damage of the tissue, which is consistent with what is observed in literature [1].

Next, we examine the computational efficiency of our numerical solution approach for the moving electrode scenario. The convergence of the GMRES iterations in L^2 norm of the residual for the choice of preconditioners, that is, block Jacobi, AMG, and deflation based block-preconditioner is shown in Fig. 15. As in the case of fixed electrode scenario, it is clear that the deflation based block preconditioning has a significant effect on reducing the number of GMRES iterations from a total of 27 and 35 iterations, respectively, for the AMG and block Jacobi preconditioner to just 2 iterations. This is reflected in the much lower average computational cost per time step for the deflation based preconditioner in comparison to other two preconditioning strategies, see Fig. 16. The convergence plots are presented at third time step for 39560 NDOF.

5. Conclusions

A multi-physics model that accounts for the electro-thermo-mechanical response of radiofrequency (RF) activated hydrated soft tissues is presented. A thermodynamically consistent continuum model for the soft tissue is presented within the framework of small deformation linear thermoelasticity. A micromechanical model based equation-of-state describes the thermodynamic states due to evaporation of tissue water by absorbing heat due to Joule heating. A thermodynamically consistent and computationally efficient level set approach is presented to track the topology changes due to electrosurgical cutting. The level set evolution equation that is derived from the second law of thermodynamics leads to physically realistic interface velocity. A GMRES solver with deflation based block-preconditioner is shown to solve the system of equations of the coupled multiphysics problem efficiently with exponential convergence. Model validation is performed by comparing the experimental temperature distributions measured from electrosurgical ablation testing on *ex vivo* porcine liver at three different power settings. The higher power settings yield faster temperature rise leading to rapid evaporation of tissue water and, hence, resulting in early protein denaturation and coagulation of the tissue, consistent with experimental observations. The model is shown to successfully capture the characteristics of RF activated soft tissue temperature and damage interface evolution.

There are several limitations of the model. The rapid transport of tissue fluid, which may be an important determinant of tissue damage far away from the initial energy deposition site, has not been considered in our model. Further, the model does not account for the frictional contact between the electrode and the tissue at the interface, where tissue may undergo large

deformation due to stiction with coagulated tissue. The model may be expanded to include fluid transport, electrode-tissue interaction, and finite deformation kinematics in future work. Experiments need to be designed to calibrate the damage parameter. More complex damage models including thermal and mechanical damage mechanisms may be developed based on experimental characterization. The monolithic multiphysics solver can be combined with reduced modeling to solve problems with even larger degrees of freedom.

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Fig. 1.

Computational domain, where solid and dotted boundary segments indicate representative Dirichlet and Neumann boundaries for mechanical, electrical and thermal problems, respectively. Each type of boundary for the three problems does not necessarily overlap.

Algorithm 1 Multiscale multiphysics coupling framework: Outline of the algorithm for proposed framework

```

1: Initialization:
2: Set  $t \leftarrow 0$ , initialize  $T_0, \phi_0, p_0$ 
3: while  $t < \tau$  do
4:   Set  $t \leftarrow t + \Delta t$ 
5:   for each macroscale Gauss point:
6:     if  $T > T_b$  then
7:       Solve micromechanical model:
7:        $x = f(p, T)$  (equation of state)
7:        $dh = \delta q_f + v dp$  (energy balance)
7:        $p = p_0 \exp[-\frac{\lambda}{r}(\frac{1}{T} - \frac{1}{T_b})]$  (Clausius-Claperron equation)
8:       return effective properties  $\rho_{eff}, c_{eff}, k_{eff}, q_{eff}$  and hydrostatic pressure  $p$ 
9:     else
10:      return initial values for effective properties and hydrostatic pressure
11:    end if
12:  end for
13:  Solve coupled systems of equations:
13:   $\mathbf{K}^t \mathbf{X}^t = \mathbf{F}^t$ 
14:  Solve level set equation:
14:  
$$\phi_j^{t+\Delta t} = \phi_j^t + \Delta t \frac{\sum_l^n \alpha_j^l \int_{T_l} (\psi_\varepsilon - \kappa) |\nabla \phi| d\Omega}{\sum_l^n \alpha_j^l meas(T_l)}$$

15: end while

```

Fig. 2.
Overview of the simulation.

Fig. 3.
Schematic of the 3D FEM model. Red dot indicates active electrode position.

Fig. 4. Comparison of mean axis-symmetric experimental radial temperature evolution from [15] to simulation results for (a–b) 10W, (c–d) 25W and (e–f) 50 W power settings.

Fig. 5. Comparison of temperature evolution with time at the reference coordinate (0.1, 0.05, 0.1 m) with and without micromechanical model for 50 W power setting.

Fig. 6. Evolution of (a) pressure, (b) temperature and (c) vapor content with time at the reference coordinate (0.1, 0.05, 0.1 m).

Fig. 7.

Spectrum of the (a) non-preconditioned, (b) block Jacobi preconditioned, (c) HYPRE BoomerAMG preconditioned, and (d) deflation based block-preconditioned matrices.

Fig. 8.

Comparison of GMRES convergence with the block Jacobi (BJ), AMG, and deflation based block-preconditioned (BDF) iterations.

Fig. 9.

Plots of the CPU time (s) versus NDOF when an h-refinement is performed. Here, 'BJ', 'BDF', and 'AMG' represent, respectively, the block Jacobi preconditioner, deflation based block-preconditioner, and boomerAMG preconditioner.

Fig. 10.
Performance comparison of the solution of level set evolution equation using CPU and GPU.

Fig. 11.

Contour plots are shown for 50 W power setting with a fixed active electrode: (a) electric potential, (b) current density, (c) temperature (with topology change) and (d) von Mises stress (stress fields are cross sections along the $x=0.1$ m axis) at 0.4875 s, 1.0938 s, and 1.5313 s.

Fig. 12.

Contour plots are shown for 50 W power setting with a moving active electrode: (a) electric potential, (b) current density, (c) temperature (with topology change) at a speed of 5.28 mm/s.

Fig. 13.

Time evolution at a cross section going through the reference point (0.1085, 0.0597, 0.1 m) and normal to the electrode movement direction, at speeds of 5.28 mm/s, 8.82 mm/s, and 12.35 mm/s from bottom to top.

Fig. 14.

Comparison of (a) pressure and (b) vapor content corresponding to the three electrode speeds along radial distance from integration point (0.1085, 0.0597, 0.1 m) normal to electrode movement direction.

Fig. 15.

Comparison of GMRES convergence of moving electrode scenario with the block Jacobi (BJ), AMG, and deflation based (BDF) preconditioned iterations.

Fig. 16.

Plots of the CPU time (s) versus NDOF for moving electrode scenario, when an h-refinement is performed. Here, 'BJ', 'BDF', and 'AMG' represent block Jacobi preconditioner, deflation based block-preconditioner, and AMG preconditioner, respectively.

Table 1

Electrical, thermal and mechanical properties of liver tissue

Electrical conductivity (σ)	0.33 S/m
Thermal conductivity (k_s)	0.512 W/ (m·K)
Thermal conductivity (k_v)	0.03 W/ (m·K)
Thermal conductivity (k_w)	0.6 W/ (m·K)
Convection coefficient (h)	25W/(m ² ·K)
Young's modulus (E)	0.27 MPa
Poisson's ratio (ν)	0.4
Damage parameter (κ)	0.7×10^5 J/m ³
Average density (ρ_s)	1.08×10^3 kg m ⁻³
Water content (w)	70%
Vapor density (ρ_v)	1.0×10^3 kg m ⁻³
Water density (ρ_w)	0.994×10^3 kg m ⁻³
Specific heat capacity of solid part (c_s)	1.7×10^3 J kg ⁻¹ K ⁻¹
Specific heat capacity of water part (c_w)	4.187×10^3 J kg ⁻¹ K ⁻¹
Specific heat capacity of vapor part (c_v)	1.996×10^3 J kg ⁻¹ K ⁻¹
Specific gas constant for water vapor (r)	4.620×10^2 J kg ⁻¹ K ⁻¹
Gabriel [46], Kemper et al. [47] and Hegemann [48]	

Table 2

FEM boundary conditions as marked in Fig. 3

Plane	Electrical BC	Thermal BC	Mechanical BC
ABCD	Insulated $\mathbf{J} \cdot \mathbf{n} = 0$	$-\mathbf{q} \cdot \mathbf{n} + h(T - T_a) = 0$	
ABFE	Insulated $\mathbf{J} \cdot \mathbf{n} = 0$	$-\mathbf{q} \cdot \mathbf{n} + h(T - T_a) = 0$	
EFGH	Insulated $\mathbf{J} \cdot \mathbf{n} = 0$	$-\mathbf{q} \cdot \mathbf{n} + h(T - T_a) = 0$	
CDHG	Grounded $V = 0$	$-\mathbf{q} \cdot \mathbf{n} + h(T - T_a) = 0$	$u = 0$
ADHE	Insulated $\mathbf{J} \cdot \mathbf{n} = 0$	$-\mathbf{q} \cdot \mathbf{n} + h(T - T_a) = 0$	
BCGF	Insulated $\mathbf{J} \cdot \mathbf{n} = 0$	$-\mathbf{q} \cdot \mathbf{n} + h(T - T_a) = 0$	
Input	$\mathbf{J} \cdot \mathbf{n} = j_0$		