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Fourier-Domain Shift Matching: A Robust Time-of-flight Approach for Shear Wave Speed Estimation

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Abstract

Our primary objective of this work was to design and test a new time-of-flight (TOF) method that allows measurements of shear wave speed (SWS) following impulsive excitation in soft tissues. Particularly, under the assumption of the local plane shear wave, this work named Fourier-Domain shift matching (FDSM) method, estimates SWS by aligning a series of shear waveforms either temporally or spatially using a solution space deduced by characteristic curves of the well-known one-dimensional wave equation. The proposed SWS estimation method was tested using computer-simulated data, and tissue-mimicking phantom and *ex vivo* tissue experiments. Its performance was then compared with three other known TOF methods: lateral time-to-peak method with robust random sampling consensus (RANSAC) fitting method, Radon-Sum Transformation method, and a modified cross-correlation method. Hereafter, these three TOF methods are referred to as the TTP-RANSAC, Radon-Sum and X-corr methods, respectively. Additionally, an adapted form of the 2D Fourier Transform (2D-FT) based method in which the (group) SWS was approximated by averaging phase SWS values was considered for comparison.

Based on data evaluated, we found that the overall performance of the above-mentioned temporal implementation of the proposed FDSM method was most similar to the established Radon-Sum method (correlation = 0.99, scale factor = 1.03 and mean difference = 0.07 m/s) and the 2D-FT (correlation = 0.98, scale factor = 1.00 and mean difference = 0.10 m/s) at high signal quality. However, results obtained from the 2D-FT method diverged (correlation = 0.201) from these of the proposed temporal implementation in the presence of diminished signal quality, whereas the agreement between the Radon-Sum approach and the proposed temporal implementation largely remained the same (correlation = 0.98).

Index Terms

Ultrasound; shear wave elastography; shear wave speed; elastography

I. Introduction

Pathological evolution of many diseases such as cancer, cardiovascular diseases and fibrotic changes in livers is typically accompanied by gradual changes in a spectrum of tissue viscoelastic properties. In the last decade, significant research efforts have been devoted to ultrasound-based elastographic techniques [1], [2], [3] that can be used to non-invasively quantify viscoelastic properties of soft tissues *in vivo*. Among them, shear wave elastography (SWE) [3] characterizes tissue elasticity mainly based on quantification of propagating shear wave speed (SWS) inside the tissue of interest. This is done by first inducing shear waves using mechanical excitation and then estimating the propagating SWS through a sequence of time-resolved (particle) displacement or velocity curves. Hence, tissue elasticity can be inferred from a material model relating the elasticity to SWS. The SWE technology is of interest in a number of clinical applications, including staging of liver fibrosis [4], estimations of muscle loading [5], estimation of tissue hyperelastic parameters [6], [7] and assessment of the risk of pre-term birth [8].

A breadth of techniques for SWS estimation has been proposed in the ultrasound SWE literature. Most SWS estimation algorithms for SWE driven by impulsive excitation such as point-SWE and Supersonic Shear Imaging [9] can be categorized as time-of-flight (TOF) methods [10], [11], [12], [13], [14]. Before we can utilize most of these TOF methods, wave arrival time values must be extracted from the time-resolved displacement/velocity data that were ultrasonically measured. Using these wave arrival time values as the input, McLaughlin and Renzi [10] proposed a method of estimating the SWS by inversely solving the Eikonal Equation. Palmeri et al. [11] have attempted to fit these arrival time values as a linear function of the measurement positions along the shear wave propagation direction. Thus, the slope of the above-mentioned linear function becomes the estimated SWS. Their method is known as the lateral time-to-peak (TTP) method.

It is easy to see that the calculation of wave arrival time is important to both methods [10], [11]. The original lateral TTP algorithm [11] used the maximum displacement as the surrogate of the shear wave arrival. However, even with the presence of a moderate amount of noise in the ultrasonically measured displacement-time data, picking individual peaks may lead to outliers among estimates of the wave arrival time which cannot be simply compensated by a least square line fitting. Noise sources in the time-resolved displacement data include physiological motion, low-quality displacement estimates, wave interference due to spatial inhomogeneities in the medium and some system-dependent factors [15]. An improvement over the original TTP algorithm stemmed from that fact that the random sample consensus (RANSAC) methodology was used to iteratively remove outliers among these estimated wave arrival time values during the above-mentioned slope fitting process; this modified method [13] was known as the TTP-RANSAC method. However, while the TTP-RANSAC method improved the robustness for SWS estimation, two potential limitations were identified [12]. First, the TTP-RANSAC method is slow because a large number of trial solutions (e.g. 5000) are required. Second, the accuracy of the SWS estimation could be dependent on a pre-determined critical threshold that is used to distinguish between inlier and outlier data during the slope fitting, thereby hampering the usefulness of the TTP-RANSAC method.

Rouze et al. [12] took a different approach. They used the Radon-Sum Transformation to calculate all possible trajectories of the wave; their method is hereafter referred to as the Radon-Sum method. In fact, graphically, the Radon-Sum method actually selects a trajectory through which the individual (displacement) peaks can be connected using a line for a given wave propagation period.

More recently, Zhao et al. used cross-correlation to identify the wave arrival by tracking relative motion among a set of displacement-time curves in their modified TOF method [14]. Of note, the use of cross-correlation to detect the wave arrival, to our knowledge, was first suggested by McLaughlin and Renzi [10].

Fourier-based SWS estimation methods such as shear wave spectroscopy [16] and 2D Fourier Transform (2D-FT) based methods [17], [18], [19], [20] have also been proposed. These approaches leverage the Fourier Transform to resolve relatively high-quality shear wave data into frequency dependent phase SWS, in contrast to the previously described TOF methods which are targeted toward robust estimation of single group SWS. However, a method leveraging the shape information contained in the Fourier Transform to robustly estimate the group SWS has yet to be described.

In this study, we propose a new alternate Fourier-domain method for estimating the group SWS by utilizing the full shape of the waveform. Particularly, the proposed method leverages knowledge of the one-dimensional wave equation and the time-shift property of the Fourier Transform. Because the proposed method utilizes the shift properties of the Fourier Transform, we have coined it Fourier-domain shift matching (FDSM). Toward this end, our objective is twofold. First, we intend to demonstrate the feasibility of the proposed FDSM method. Second, we want to compare and contrast the proposed FDSM method to three other current TOF methods of SWS estimation, namely, the TTP-RANSAC method [13], the Radon-Sum method [12] and modified TOF method based on the cross-correlation [14] as well as a 2D-FT based approach that was modified from [19]. It is important to note that, in the original 2D-FT method [19], only phase SWS values were obtained and here we approximated the group SWS by averaging phase SWS values (see Section III-B3). In this feasibility study, these comparisons and evaluations were conducted using computer-simulated SWS data [21], a commercial elastography phantom (Model 049, CIRS, VA) and *ex vivo* tissues. To date, such an algorithmic comparison has not been published.

II. Theory

When a shear wave propagates through an elastic medium, the shear wave induces a transient displacement field $u(x, t)$, where x and t are a spatial coordinate corresponding to the shear wave propagation direction (*i.e.* lateral to the impulsive excitation source) and time, respectively. Assuming the medium is at least locally homogeneous and the shear wave can be approximated by a plane shear wave, the following 1-dimensional wave equation holds as follows [22],

$$\frac{\partial^2 u(x, t)}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u(x, t)}{\partial t^2} \quad (1)$$

where c is the SWS. It can be shown by the method of characteristic curves, that the general solution of Eqn. (1) is the following [22]:

$$u(x, t) = h(x + ct) + g(x - ct) \quad (2)$$

where $h(x + ct)$ and $g(x - ct)$ are two arbitrary waveforms propagating away from the impulsive excitation source in the negative and positive directions, respectively. Without losing generality, in this study, only the wave propagating along the positive direction is considered. In the proposed FDSM method, the entire shape of the propagating shear waveform is $g(x - ct)$. If the wave attenuation is not considered in the elastic media, the shape of the forward-propagating waveform at one time-point is a shifted version of itself at a preceding or proceeding time-point. Fig. 1 displays an illustrative example of such a wave behavior. Applying the Fourier transform to the spatial dimension, x on $g(x - ct)$, and taking advantage of the time-shift property of the Fourier transform, it is found that

$$\mathcal{F}_x \{g(x - ct)\} = e^{-2\pi j f_x ct} \mathcal{F}_x \{g(x)\} = e^{-2\pi j f_x ct} G(f_x) \quad (3)$$

where $\mathcal{F}_x\{\bullet\}$ denotes a Fourier transform of an arbitrary function and $G(f_x)$ is a complex function representing Fourier transform of $g(x)$ in the spatial frequency domain f_x . In Eqn. (3), $j = \sqrt{-1}$. Thus, as a result of the applying the Fourier Transform (FT) over x , the shifting of the waveform is converted into multiplication with a complex exponential. Here we propose first multiplying Eqn.(3) by another complex exponential $e^{2\pi j f_x \theta t}$, where θ is a dummy variable, and then integrating over some finite time-domain T . $t = 0$ has been set as the center of this time domain T for mathematical convenience. This results in the following relationship:

$$\int_{-T/2}^{T/2} e^{2\pi j f_x (\theta - c)t} G(f_x) dt = G(f_x) \frac{\sin(\pi f_x T(\theta - c))}{\pi f_x (\theta - c)} \quad (4)$$

Eqn. (4) suggests that all shear waveforms $g(x - ct)$ can be first aligned and then summed together in the time domain t to achieve a maximum value if θ is equal to the SWS c . Of note, the integration for a continuous signal in Eqn. (4) is equivalent to a summation for a discrete signal for practical purposes.

In the case where a temporal FT would be preferable to the spatial one in Eqn. (3), the development of Eqn. (2) can be adjusted such that the SWS c is applied to t rather than x , *i.e.* a FT applied to the temporal waveform and integrated over a spatial domain, and the same

SWS c can be estimated. See Appendix A for a more detailed development. Such an adjustment modifies Eqn. (2) into its reflected form

$$u(x, t) = h\left(t + \frac{1}{c}x\right) + g\left(t - \frac{1}{c}x\right) \quad (5)$$

Following the same process that was used to arrive at Eqn. (4), we arrive at the following relation:

$$\int_{-X/2}^{X/2} e^{2\pi j f_t \left(\theta - \frac{1}{c}\right)x} G(f_t) dx = G(f_t) \frac{\sin\left(\pi f_t X \left(\theta - \frac{1}{c}\right)\right)}{\pi f_t \left(\theta - \frac{1}{c}\right)} \quad (6)$$

where $G(f_t)$ is a complex function representing FT of $g(x)$ in the frequency domain f_t . Similarly, Eqn. (6) also suggests that all shear waveforms $g(x - ct)$ can be first aligned and then summed together over the spatial domain x to achieve a maximum value if θ is equal to $\frac{1}{c}$

III. Methods and Materials

A. Implementation

The mathematical insight described in Section II was the basis for the development of the proposed FDSM method. In this study, given a sequence of ultrasound radiofrequency echo data, $u(x, t)$ was estimated using a speckle tracking algorithm (e.g. [24], [25]). Using a (discretely sampled) displacement field $u(x, t)$ (see Fig. 1) as the starting point, both Eqns. (4) and (6) were separately implemented and will be referred to respectively as FDSM-spatial and FDSM-temporal from here on.

1) FDSM-spatial Implementation—As shown in Fig. 2, the FDSM-spatial implementation consists of four steps and starts with applying a fast-Fourier Transform (FFT) to each column (*i.e.* lateral spatial positions along the shear wave propagation direction) of the shear wave data (see Fig. 1). This step resulted in a 2D complex-valued function $G(k_m, t_i)$ where k_m and t_i are the m^{th} spatial frequency and the i^{th} time point, respectively. In other words, for each spatial frequency K_m , the complex-valued function $G(k_m, t_i)$ yielded a sequence of phasors; one for each time point (see step 1 in Fig. 2). In the second step, a set of pre-determined θ values, which typically ranged from 0.1 m/s to 10m/s with a pre-determined increment, was used to test how the phasors can be aligned. Formally, each $G(k_m, t_i)$ was multiplied by $e^{2\pi j \theta_n t_i k_m}$ where θ_n was the set of pre-determined θ between 0.1 m/s to 10 m/s. In the third step, a discrete form of Eqn. (4) was used to determine the optimal θ_n that had best aligned all phasors. The discrete form of Eqn. (4) reads,

$$A_{k_m}[\theta_n] = \left| \sum_{i=-T/2}^{T/2} e^{2\pi j \theta_n t_i^k m} G(k_m, t_i) \right| \quad (7)$$

As illustrated in step 3 in Fig. 2, we found one θ_n associated with the maximum complex modulus of A_{k_m} and that θ_n became an estimate of $\hat{c}$ for the spatial frequency k_m . In the fourth step, we adopted a "compounding" approach. In other words, if the number of available spatial frequencies was p , the estimated group SWS $\hat{c}$ corresponded to the peak in $\sum_{m=1}^p A_{k_m}$. This compounding often produced a sharper peak in $\sum_{m=1}^p A_{k_m}$ by which $\hat{c}$ could be estimated (see Fig. 2).

2) FDSM-temporal Implementation—The FDSM-temporal implementation is similar to the FDSM-spatial implementation. Only these differences are described in order to make our presentation concise. In the first step, the FFT was applied to each row (*i.e.* time domain) of the shear wave data. Hence, we got a 2D complex-valued function $G(f_m, x_i)$ where f_m and x_i were the m^{th} frequency component and the i^{th} lateral location along the shear wave propagation direction, respectively. In the second step, each $G(f_m, x_i)$ was multiplied by $e^{2\pi j \theta_n x_i f_m}$ where θ_n was the same set of pre-determined θ . In the third step, a discrete form of Eqn. (6) was adopted,

$$A_{f_m}[\theta_n] = \left| \sum_{i=-X/2}^{X/2} e^{2\pi j \theta_n x_i f_m} G(f_m, x_i) \right| \quad (8)$$

Of note, in the FDSM-temporal implementation, Eqn. (8) was summed over all lateral spatial locations. In the fourth step, we also used a "compounding" approach over the frequency domain because the excitation source considered in the current study was a broadband acoustic pushing pulse (see step for in Fig. 2). In order to utilize identical spectrums between the proposed method and the comparison methods outlined later, the frequencies used in the compounding were from 0 Hz up to the Nyquist frequency (*i.e.* half the sampling frequency; see Table I).

B. Validation and Comparison

The proposed FDSM method was validated using a combination of simulated shear wave data, ultrasonic shear wave measurements collected from a commercial phantom (Model 049, CIRS Inc., VA) and *ex-vivo* tissue. Table I summarizes the relevant data acquisition parameters used for each experiment.

1) Simulations of SWE—A transient finite element (FE) simulation of SWE in a homogeneous numerical phantom was performed to obtain a noise-free time-resolved displacement field. The numerical phantom (*i.e.* Model 1 in [21]) had a uniform shear modulus of 1.59 kPa, corresponding to an expected shear wave speed of 1.26 m/s. A single

simulated acoustic push excited the tissue at the center of the region-of-interest. The simulated shear wave motion field used for SWS tracking was composed of a rectilinear grid of particle displacement data with 0.1 mm-by -0.1 mm spacing between points. In order to keep this study concise, interested readers are referred to our prior publication for details [21]. It is worth noting that procedures used were largely consistent with a suggested guideline by Palmeri et al. [26], except that 10-node tetrahedral elements were used. The point shear wave excitation scheme was selected because it is consistent with the data acquisition settings used for tissue-mimicking phantom measurements below. Only the displacement data in line with the axial depth of the excitation source was used for SWS estimation.

2) Phantom and Ex-Vivo Tissue Experiments—A research ultrasound system (V1 system, Verasonics Inc., WA) equipped with an L7-4 linear array transducer (ATL, Phillips Inc., WA) was used for the creation of one single push-pulse excitation. This was followed by ultrafast imaging of the resulting waveforms (see Table I). The acoustic push-pulse had a push-duration of 203 microseconds at 4.1 MHz. Ultrafast ultrasonic imaging frames were constructed from coherent compounding of three angles of plane waves (4, 0 and -4 degrees). Ultrasound echo data were collected after the L7-4 linear array had been turned back to imaging mode and pulsed at 5 MHz.

Ultrasound measurements were collected from a commercial general-purpose ultrasound phantom (Model 049, CIRS Inc., VA). The L7-4 transducer was positioned such that the induced shear wave would propagate through an elastically homogeneous portion of the phantom and a linear stage was used to position the transducer at 11 different elevational positions along the phantom with a 2-mm spacing between each position. This was done in order to vary the speckle pattern between each acquisition location, as it is known to affect the measured shear wave [27]. Therefore, 11 independent measurements were collected at each excitation voltage listed in Table I. Axial data was averaged over a 0.5 mm interval centered at the focal point of the push-pulse.

Ex-vivo tissue experiments were collected from fresh pig liver acquired from a local butcher and on beef muscle tissue purchased from a local supermarket. Thermally coagulated liver tissues were created through a double immersion heating method [28]. The double immersion method uniformly heated the liver tissue at 60°C so that SWS estimates as well as their stiffness obtained from these thermally coagulated liver tissues were projected to increase [28]. In the muscle tissue, in order to vary the projected SWS, ultrasound measurements were taken both parallel and perpendicular to the muscle fiber directions. The combination of measurements collected from beef muscle tissue (perpendicular and parallel to the muscle fiber) and from coagulated and “normal” pig liver tissues permitted a wide range of SWS (2 - 10 m/s) to be used for the evaluation of the proposed FDSM method.

During experiments, all tissues were placed in a water bath containing degassed phosphate-buffered saline (PBS) solution. The PBS solution was at room temperature (approximately 21°C). In all tissues, visual and ultrasonic B-mode imaging inspections were performed to select tissue regions for shear wave measurements that were free of major vasculature, bile ducts and other heterogeneities. Shear wave measurements were collected at 4 different

locations within the “normal” and thermally-coagulated liver tissues. In the beef muscle tissues, measurements were collected in 4 different locations: two for the direction parallel to the fiber and the other two for the direction perpendicular to the fiber direction. In each location, 5 data acquisitions were performed. Consequently, 60 independent SWS measurements were obtained through these *ex vivo* tissue experiments.

3) Data Analysis and Benchmark Algorithms—All collected ultrasound data were processed using a phase-based 1D auto-correlation algorithm [24], which is commonly utilized within the SWE literature [29], [30]. In order to avoid outliers among ultrasonically measured displacements, a two-dimensional median filter (approximately $0.3\text{ mm} \times 0.3\text{ mm}$) was applied to filter out large errors in $u(x, t)$, similar to that reported in other shear wave tracking studies [31], [32]. Both implementations (see Section III-A) of the proposed FDSM method were used to estimate SWS values. As a part of this feasibility study, the same shear wave data were also tracked using four other SWS estimation approaches: The Radon-Sum method [12], the TTP-RANSAC method [13], a cross-correlation based TOF approach adapted from [14] with modification to suit the single-pulse excitation data (see Section III-B2) and a 2D-FT approach adapted to produce a single SWS estimation in a comparable manner to the proposed FDSM method.

Modifications to the cross-correlation and 2D-FT methods are briefly described below for the sake of completeness. In our modified version of the cross-correlation method, temporal shear wave profiles were cross-correlated for all pairs of lateral points within the lateral window with spacings of 1mm, 2mm, 3mm and 4mm from each other. The SWS was then calculated by collecting and inverting the slope of a linear regression applied to the lateral spacing of each pair against the time delays associated with the maximum correlation coefficient. Distinct from [14], we applied the RANSAC linear fitting to estimate this slope in place of weighted least-squares fitting. Hereafter, the cross-correlation-based TOF method is referred as to the X-corr method. The X-corr method was added to the comparison methods because, like the proposed FDSM method, it utilizes the full shape of the (time-domain) wave-form in its estimation and should, therefore, be considered as a counterpart of the proposed method. In both the TTP-RANSAC and X-corr methods, 5000 and 0.93 milliseconds were used by the RANSAC to determine outliers. In the adapted 2D-FT approach, similar to [19] the shear wave data was transformed into a k-space representation via a 2D fast Fourier Transform. The resulting 2D amplitude spectrum was collected and a threshold filter was applied using -10 dB of the largest k-space signal amplitude as the threshold. Peaks in the k-space representation were then collected and used to calculate a frequency dependent SWS (i.e. phase SWS) via the relationship $c = \lambda f_t = \frac{f_t}{f_s}$, where λ is the associated wavelength, and f_t and f_s are the temporal and spatial frequencies respectively. The bandwidth used for SWS estimation was from 120 Hz, corresponding to the lowest frequency above DC from the Fast Fourier Transform. ALL phase SWS values collected were then simply averaged over this bandwidth to arrive at a group SWS estimation. Averaging of this bandwidth makes this adapted 2D-FT method comparable to the estimation arrived at using the “compounding” used in the proposed FDSM method (see Fig. 2, particularly, step 4).

4) Performance Evaluation—All data processing and algorithmic implementations were conducted using Matlab (Version 2016a, Mathworks Inc., MA). In order to assess the robustness of the proposed FDSM method in the presence of measurement noise, signal quality was varied in both the point SWE simulation and the measurements collected from the commercial phantom experiment and the *ex-vivo* tissue. First, in the simulation data, Gaussian measurement noise was added to the noise-free finite element-simulated data. The additive noise was scaled to a ratio of the largest peak-to-peak amplitude of the simulated shear wave data (*i.e.* the absolute difference between the largest and smallest particle displacement for a given lateral location). The peak signal-to-noise ratio then could be calculated by

$$PSNR = \frac{(A_{peak-to-peak})^2}{P_{noise}} = \frac{(d_{max} - d_{min})^2}{\sigma_{noise}^2} \quad (9)$$

Peak signal-to-noise ratio, rather than the typical root-mean-squared definition, was selected because it is independent of the choice of temporal signal length, *i.e.* the number of virtual imaging frames used in the acquisition. 1000 realizations of each peak signal-to-noise ratio (PSNR) were generated for this study.

In order to verify the simulation findings, measurements were conducted in the commercial CIRS phantom with varying signal strengths. It was assumed that sources of noise within the system remained at a constant level. The signal quality could then be modulated by varying the signal energy. This was accomplished by varying the peak-to-peak voltage used for acoustic push-pulse excitation as suggested by others [15]. Excitation voltages were varied from 25-to-60 volts (see Table I).

To assess the variation of estimated SWS by each method as a function of signal quality, the root mean squared deviation was calculated as follows:

$$RMSD = \sqrt{\frac{\sum_{i=1}^n (\hat{c}_i - c_Q)^2}{n}} \quad (10)$$

Here i denotes the individual SWS estimates and Q denotes the average estimated SWS measured by the method at the highest signal quality for the given measurement (*i.e.* 35 dB for the simulation data and 60V for the phantom and *ex vivo* tissue experiments).

IV. Results

Using the CIRS phantom data, the effect of the increment used in θ on the estimation of SWS was investigated. As shown in Fig. 3, once the incremental θ approached 0.01 m/s and 0.005 m/s for the FDSM-temporal and FDSM-spatial implementations, respectively, the impact of this incremental θ was minimal ($< 0.5\%$). Thus, the above-mentioned two incremental θ values were used for all results reported below.

Results for the point SWE simulation are shown in Fig. 4. Overall, with the increase of PSNR, the SWS estimates obtained from the TTP-RANSAC, X-corr, 2D-FT, FDSM-temporal and FDSM-spatial methods converged to the underlying SWS value. However, the estimated SWS showed a small positive bias (approximately 0.1 m/s) when the Radon-Sum method was used, regardless of the PSNR value. When the PSNR was low (5-10 dB), the TTP-RANSAC, 2D-FT and X-corr methods tended to significantly overestimate the SWS. The overestimation obtained from the X-corr method could be as high as approximately 3.0 m/s or 250% at the PSNR of 5 dB. As shown in Fig. 4b, in the same range of PSNR, variances among SWS estimates obtained from both FDSM-temporal and FDSM-spatial implementations were relatively small. However, the estimate variances was significantly high for the TTP-RANSAC, 2D-FT and X-corr methods.

Fig. 5 displays the results from SWS measurements conducted on the CIRS phantom. These results displayed qualitatively similar results to the point-SWE simulations. Overall, in all methods, the mean SWS estimates are in good agreement and their results converged to within 4 % of each other when the acoustic pushing power had reached 60V (Fig. 5a). However, when the acoustic push power was low (< 30 V), estimation variances obtained from the Radon-Sum and FDSM-temporal methods were the lowest among all methods, while the estimation variance exhibited by the X-corr method was the highest for the TOF methods (see Fig. 5b). For the 2D-FT method, the mean SWS and RMSD dramatically diverged from its 60V value for voltages below 45V. This is due to outlier peaks in the k-space signal, which corrupt the calculated mean. In Fig. 5a, it is also easy to notice that, at high signal quality, the mean SWS values obtained from the TTP-RANSAC method were slightly higher than the other methods and the 2D-FT estimation method was lower. Regardless of the SWS method, the SWS estimates at the high SNR (inferred by the high acoustic pushing power) tended to be about 6-9 percent lower than expected based on the elasticity reported by the manufacturer (2.83 m/s for Young's modulus of 24 kPa). However, it is worth noting that this deviation is well within the acceptable variance of the phantom material reported in the literature [33].

Fig. 6 is a plot of the RMSD as a function of the excitation voltage when all *ex vivo* tissue data have been combined. These results were also in agreement with the results of the tissue-mimicking CIRS phantom, as the TTP-RANSAC and X-corr methods tended to exhibit more estimation variance at lower signal strengths, while the FDSM-temporal implementation performed comparably to the Radon-Sum method in terms of the low estimation variance. It should be noted that at 30V, one data acquisition from the "normal" liver tissue resulted in a large outlier (an SWS of 122 m/s) when the Radon-Sum method was used. Hence, this outlier was excluded. (see caption in Fig. 6 for details)

Figure 7 displays correlation plots of the combined *ex vivo* tissue data at high signal quality (*i.e.* 60V) for the FDSM-temporal and FDSM-spatial implementations, and the four comparison algorithms. Overall, estimated SWS values obtained the FDSM implementation were most consistent with those obtained by the Radon-Sum method, with the 2D-FT being a close second. Fig. 7 also indicates that the FDSM-spatial failed to obtain meaningful SWS values in the majority of experiments in which the SWS exceeded 5 m/s. Furthermore, we

found that higher estimated SWS values measured from the *ex vivo* tissue specimens had a greater variation than at lower SWS (See Fig. 7).

To evaluate the correlation between SWS estimates obtained from the proposed method and its four counterparts, the Pearson's linear correlation including P-values, scaling factors, and mean differences were computed. Table II summarizes correlative results obtained at high signal quality (60V) and intermediately low signal quality (40V) from the aggregated tissue data. Overall, good correlation ranged approximately from 0.96 to 0.99 was obtained when both implementations were compared to the four comparison methods. At the intermediately low signal quality the only the correlation between FDSM-temporal and the Radon-Sum method retained a high correlation (0.98). Again, these quantitative analyses confirmed that the FDSM-temporal implementation was most consistent with the Radon-Sum method. If results from the FDSM-temporal implementation and the Radon-Sum method as the baseline, the TTP-RANSAC and X-corr methods slightly (10%) overestimated the SWS values. At high signal quality, 2D-FT estimated essentially the same SWS as FDSM-temporal, though it showed greater variability than did the Radon Sum method. It is also important to note that the FDSM-spatial implementation significantly (30-40%) underestimated the SWS values.

Using the CIRS phantom data, we also evaluated the time needed in order to obtain one SWS estimate. As shown in Table III, the proposed method, particularly, the FDSM-temporal implementation was second only to the 2D-FT in terms of computational efficiency.

V. Discussion

The proposed FDSM-temporal implementation obtained results most consistent with the established Radon-Sum method. We think both methods utilize the concept of characteristic lines. On the one hand, the Radon-Sum method aims at implicitly finding local peaks along a characteristic line through the Radon sums in the time domain. On the other hand, the proposed FDSM method explicitly uses the concept of characteristic lines. In this sense, we expect both methods perform comparably. However, in terms of dealing with outlier displacement estimates, they are different. As argued by Rouze et al. [12], in the Radon-Sum method, since only a (Radon) sum is used to compare trajectories to determine the SWS, "isolated" outlier displacement estimates will contribute only to the sum through a limited number of spatial locations. Thus, these outliers are unlikely to corrupt the determination of an optimal trajectory. In contrast, the proposed FDSM method uses the full shape of the waveform as opposed to just the peak of the waveform in the Radon-Sum method. Also, the proposed FDSM method performs a summation over the frequency domain (see Eqns. 4 and 6), given each trial θ . Since energy associated with the outlier displacements due to noise should be reasonably spread over a large frequency range, the summation process over the frequency domain makes the SWS estimation robust.

As acknowledged by Rouze et al. [12], the Radon-Sum method may not be able to accurately estimate a SWS value that is below its detectable minimum SWS. This explains why there is an existence of a small bias in Fig. 4a. In another occasion, the use of the

Radon-Sum method resulted in a large outlier SWS (122 m/s), while other methods including the proposed FDSM method was able to avoid this significant error. Although it does not mean that the proposed FDSM is more robust than the Radon-Sum method, this definitively suggests that the proposed method is different and could be an alternate method for robust SWS estimation.

A limitation of the proposed FDSM method is that the use of FFT in its calculation makes it vulnerable to spectral leakage and windowing effects. For a single impulsive acoustic excitation as was used here, this means that distortions in the phase-spectrum may occur in the regions where the waveform is just entering the spatial or temporal window. This effect seems more prominent in the FDSM-spatial implementation and explains why the FDSM-spatial implementation failed to estimate the SWS in the majority of *ex vivo* tissue specimens when these tissues became stiff (see Fig. 7). We stipulate that the FDSM-spatial implementation failed in stiff tissues for the following reason. As the tissue became stiffer, the wavelength of the shear wave increased, thereby requiring a large distance (*i.e.* the spatial window length mentioned above) in order to capture the full wave for the Fourier Transform. Extending the lateral window might help mitigate these effects. This possibility was not investigated in order to keep this work concise. In the future, the influence and strategies of mitigating this shortcoming will be further studied.

It is interesting to note that the TTP-RANSAC and X-corr methods appeared to be less robust as compared to the FDSM-temporal implementation and the Radon-Sum method. This observation was largely derived from the fact that both had a tendency to deviate substantially away from its high signal estimate when the signal quality was reduced (see Figs. 5b and 6). The theoretical advantage of cross-correlation is the use of the full shape of the waveform to estimate arrival time instead of the waveform maximum used in TTP. However, the time displacement-time curves could change significantly and therefore do not satisfy stationarity. Practically, using the correlation to track time-delays among non-stationary signals is ineffective. Now referring to the TTP-RANSAC method, the determination of outliers will be decided among 5000 trial solutions with a pre-determined threshold. The number of outliers will increase, as the shear wave signal strength decreases. Consequently, the RANSAC methodology for outlier detection may not be as effective. If outliers are not effectively removed, the estimation of SWS by the TTP-RANSAC method will be less accurate.

As stated before, the proposed FDSM method is based on the concept of the characteristic line (see Eqn. 2). Thus, theoretically, the proposed method can only be used to estimate the SWS values of plane shear waves. Among known SWE modalities, only the supersonic shear imaging [9] can be approximated as a plane shear wave within the image plane. Transient elastography and point SWE [9] generate spherical shear waves. Although its applicability with point SWE has been investigated in this study, the proposed method will have to be tested in situations in which the plane shear wave assumption may be violated. Furthermore, we assumed that, within the entire region of interest, there is only one shear wave. In the future, the potential of the FDSM method for spatial mapping of SWS in a heterogeneous medium will be studied. Certainly, we expect that more technical developments are needed.

Similar to other TOF methods [11], [12], [13], the proposed FDSM method is applicable to behaviors of shear wave propagation that are similar to shear wave propagation in linearly elastic media. However, biological tissues are viscoelastic. Thus, the frequency-dependent shear wave dispersion may be a potential confounder. In a strongly dispersive medium, especially given a broadband excitation pulse, the SWS can no longer be assumed to be frequency independent and the signal compounding approach (*i.e.* the fourth step described in Section III-A) used to incorporate the full spectrum of the waveform is no longer valid. Furthermore, given the possibility of shear wave scattering due to the presence of heterogeneities [34], the SWS estimates may also depend on the spatial frequency content. These influences on the accuracy of our method are yet to be assessed. Nevertheless, based on the tested data, the proposed FDSM-temporal implementation performed as well as the Radon-Sum method in the *ex vivo* tissues, which are heterogeneous and dispersive biological media. This coupled with the strong agreement with the adapted 2D-FT at high signal quality suggests the compounding approach functions as a robust averaging process in dispersive media, with a resulting group SWS that is compatible with TOF based estimations.

Based on data shown in Table III, we also found that two RANSAC-based methods (*i.e.* TTP-RANSAC and X-corr methods) were the least efficient. Because the mathematical procedures (see Section III-A) involved in the proposed FDSM method, such as the FFT and complex additions and multiplications, are not computationally expensive. Indeed, the similarity in computational process is likely why the proposed FDSM method and the 2D-FT approach were computationally less expensive as compared to the other methods. The computational efficiency of the proposed FDSM method is proportional to the incremental θ . In this preliminary study, the incremental θ was very fine (*e.g.* 0.005-0.01 m/s). However, if the sub-sample estimation similar to that used in the speckle tracking or time-delay estimation [35] was used, we can still significantly improve the computational efficiency. Consequently, given more optimization and technical developments, the proposed FDSM method could be a real-time algorithm. Noise occurring during *in vivo* SWE scanning is complex. Physiological motion, speckle tracking errors, thermal effect and ultrasound system dependent factors [15] all introduce noise that impacts the SWS measurements. In this study, the proposed FDSM method has been evaluated using computer-simulated data with additive noise. However, the additive noise was Gaussian, which is probably only appropriate for the noise due to the speckle tracking error. Our work is in progress to further evaluate the proposed FDSM method and this ongoing work includes the utility of more suitable noise models with more realistic simulated SWE data [21].

Lastly, while aspects of FDSM and the 2D-FT share some conceptual similarities, it is worth contrasting some of the distinguishing characteristics between the two methods. The fundamental difference is theoretical. The proposed FDSM method intends to use 1D spectral amplitudes to align the phasors (see Fig. 2). Given the elastic medium assumption, this process naturally yields the group SWS. The strong agreement between the FDSM-temporal implementation and the Radon-Sum method, a TOF method, suggests that the above-mentioned phasor alignment process is comparable to the general underlying concept involving TOF-based methods. In contrast, the 2D-FT method and its variations (*e.g.* [19]) has been used to obtain phase SWS values and therefore, the frequency-dependent shear

wave dispersion in biological media can be estimated given the acoustic radiation forces as the stimuli. However, the 2D-FT method requires the adoption of some ad hoc approaches to infer the group SWS from phase SWS data. In this study, we used simple averaging as such an ad hoc approach and this approach is consistent with our assumption of elastic media made for the proposed FDSM method. Other approaches that may require a constitutive material model in order to convert phase SWS data to a group SWS have also been proposed [19]. Nevertheless, preliminary results shown in Section IV (see Table II and Fig. 7) demonstrated that the proposed FDSM method has merits and therefore, warrants further investigations and comparisons to the 2D-FT method and its variations [17], [18], [19], [20], particularly, in biological media.

VI. Conclusion

This study presented a novel approach to estimate the SWS. The proposed FDSM algorithm was tested using computer-simulated data and phantom and *ex vivo* tissue experiments. Overall, the performance of the proposed FDSM-temporal implementation was most consistent (*i.e.* correlation of 0.99 and the scale factor of 1.03) with the established Radon-Sum method for a range of signal quality tested. In comparison, both TTP-RANSAC and X-corr methods had a tendency to deviate substantially away from its high signal estimate when the signal quality was reduced. We also observed that the FDSM-spatial implementation did not perform well during *ex vivo* tissue experiments, particularly, when the tissue was stiff. Future developments and extensions of the proposed approach in conjunction with biological tissues are proposed as our future work.

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Appendix A

The one-dimensional wave equation for a transverse wave propagating in an elastic medium can be expressed as the partial differential equation [22]:

$$\frac{\partial^2 u(x, t)}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u(x, t)}{\partial t^2} \quad (11)$$

where $u(x, t)$ is the particle displacement in the medium and is a function of the spatial and temporal dimensions x and t respectively. The term c corresponds to the velocity of the wave as it travels through the medium, *i.e.* the SWS in this study.

Due to the linearity of the derivative operator, Eqn. (11) can be adjusted to the following form,

$$\left(\frac{\partial}{\partial x} + \frac{1}{c} \frac{\partial}{\partial t}\right) \left(\frac{\partial}{\partial x} - \frac{1}{c} \frac{\partial}{\partial t}\right) u(x, t) = 0 \quad (12)$$

By introducing the characteristic lines $x + ct$ and $x - ct$, it can be shown [36] that the following solution satisfies Eqn. (12):

$$u(x, t) = h(x + ct) + g(x - ct) \quad (13)$$

To adapt this solution such that c is multiplied with x rather than t , we rewrite Eqn. (11) as follows,

$$\frac{\partial^2 u(x, t)}{\partial t^2} = c^2 \frac{\partial^2 u(x, t)}{\partial x^2} \quad (14)$$

Then, following the same linearity of the derivative operator, Eqn. (14) can be adjusted to

$$\left(\frac{\partial}{\partial t} + c \frac{\partial}{\partial x}\right) \left(\frac{\partial}{\partial t} - c \frac{\partial}{\partial x}\right) u(x, t) = 0 \quad (15)$$

Similarly, the general form of the solution has characteristic lines $t + \frac{1}{c}x$ and $t - \frac{1}{c}x$ resulting in the following general solution to Eqn. (15):

$$h\left(t + \frac{1}{c}x\right) + g\left(t - \frac{1}{c}x\right) \quad (16)$$

Biographies

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Fig. 1.

Example behavior of the plane shear wave propagation described by the function $g(x - ct)$. Shear wave propagation in the x - t plane shows a wave profile shifting across the x domain with increasing time t . Data were simulated using an open-source wave simulation program K-Wave [23]. The normalized amplitude of wave velocity is displayed.

Fig. 2.

A graphic illustration of steps involving the proposed FDSM method: (1) the 1-D FFT applied to a series waveforms, in this case, spatial waveforms, and produce a set of complex phasors for each spatial frequency of the waveform. The individual phasors then are associated with a given time point; (2) the set of phasors are realigned to each other by complex multiplication. The dummy variable θ_i determines the potential alignment by equation 7; (3) the phasors at each realignment are then summed. The value of θ_i associated with the maximum/peak amplitude of the realigned phasor sum is expected to be at the SWS that best matches the waveforms; (4) for a waveform with multiple frequency components and a single (group) SWS, the peak at the maximum alignment can be increased by summing, or “compounding”, it with other frequencies.

Fig. 3.

Percent difference from $1/\theta = 1000$ of estimated shear wave speed as a function of resolution of θ used in equations 7 and 8. Data shown is from a 60V acquisition from the commercial phantom experiment. The star markers denote the resolutions used for estimation for the respective implementation. The percent differences at these resolutions were respectively 0.3 % (FDSM-temporal) and -0.08 % (FDSM-spatial).

Fig. 4.

Results for point-SWE simulations with added Gaussian noise. (a) Mean SWS of each method as a function of signal quality. The mean was calculated by averaging the 1000 realizations of Gaussian noise computed at each PSNR level. (b) RMSD of simulation data for each method as defined in Eqn. 10. c_Q used for estimating the RMSD was calculated by averaging the SWS estimated at the PSNR of 35 dB.

Fig. 5.

Results for the commercial CIRS phantom. (a) Mean SWS for each method as a function of push-pulse excitation voltage. (b) RMSD of SWS estimates by each method as defined in Eqn. 10. c_Q used for calculating the RSMD (see Eq. 10) was calculated by averaging the SWS estimate from the 60V push-pulse, which yielded the highest signal quality. Based on the Young's modulus quoted from the phantom manufacture (24 kPa) the expected SWS for this phantom is 2.83 m/s ($SW S = \sqrt{\frac{E}{3\rho}}$)

Fig. 6.

A plot of RMSD values for all SWS estimation methods obtained from *ex-vivo* tissue experiments. c_Q was taken as the average of the 5 acquisitions collected at each measurement location for pig liver (“normal” and thermally-coagulated) and beef muscle tissue (parallel and perpendicular to the fiber direction) with the push-pulse excitation voltage at 60V. (* It was found that one of the 20 acquisitions collected from the “normal” pig liver produced an outlier SWS estimate from the Radon-Sum method (122 m/s) which accounted for 99.4% of the variation within this set of measurements. The particular acquisition was removed from all RMSD calculations in the above plot.)

Fig. 7.

Correlation plots of SWS estimates from all *ex-vivo* tissue measurements at high signal quality (*i.e.* 60V push-pulse excitation). Correlation plots show SWS estimates from FDSM-Temporal (top row) and FDSM-spatial (bottom row) relative to the three comparison algorithms. The dashed line denotes equality between the correlated methods. The markers denote liver tissue before thermal coagulation (blue squares), liver tissue after thermal coagulation (orange diamonds), beef muscle with transducer perpendicular to the muscle fiber (yellow triangles) and parallel to muscle fibers (purple-circles)

TABLE I

A Summary of Relevant Parameters for SWE Simulations and Tissue-mimicking Phantom Experiments

Parameter	pSWE simulation	Commercial Phantom	Tissue Experiments
Push-Pulse Duration (μs)	1000	203	203
Excitation Voltages (V)	NA	25, 30, 35, 40, 45, 55, 60	30, 40, 50, 60
Frame Rate (Hz)	10000	6757	6757
Lateral Pixel Width (mm)	0.1	0.075	0.075
Lateral Window Length (mm)	10	10	10
Focal Depth (mm)	12.2	14.8	14.8
F/#	1	0.8	0.8
Distance from source to ROI (mm)	2	1.4	1.4

TABLE II

Correlative Statistics for combined tissue data at 60V and 40V. (see figure 7) All reported correlations were statistically significant ($p < 0.01$) except for those denoted by an asterisk(*). the mean scaling ratio was calculated as the ratio of the respective fdsm implementation to the comparison method. Note: The correlation statistics for FDSM-spatial were calculated excluding the series of misfitted SWS estimations illustrated in Fig. 7.

Comparison	linear correlation coefficient	mean scaling ratio	mean difference (mean $\pm$ SD; m/s)
60V			
FDSM-temporal vs. Radon-Sum	0.994	1.033 ± 0.0774	0.073 ± 0.256
FDSM-temporal vs. TTP-RANSAC	0.986	1.096 ± 0.250	0.249 ± 0.399
FDSM-temporal vs. X-Corr	0.986	1.111 ± 0.116	0.394 ± 0.488
FDSM-temporal vs. 2D-FT	0.980	1.003 ± 0.119	0.095 ± 0.527
FDSM-spatial vs. Radon-Sum	0.977	0.630 ± 0.064	-1.01 ± 0.558
FDSM-spatial vs. TTP-RANSAC	0.959	0.671 ± 0.069	-0.861 ± 0.580
FDSM-spatial vs. X-Corr	0.966	0.668 ± 0.074	-0.822 ± 0.375
FDSM-spatial vs. 2D-FT	0.964	0.593 ± 0.079	-1.120 ± 0.431
40V			
FDSM-temporal vs. Radon-Sum	0.976	1.05 ± 0.154	$0.116 \pm .489$
FDSM-temporal vs. TTP-RANSAC	0.886	1.09 ± 0.188	0.020 ± 1.383
FDSM-temporal vs. X-Corr	0.847	1.24 ± 0.398	0.265 ± 1.373
FDSM-temporal vs. 2D-FT	0.201*	0.629 ± 0.375	-4.89 ± 4.522
FDSM-spatial vs. Radon-Sum	0.501	0.599 ± 0.193	-1.267 ± 1.457
FDSM-spatial vs. TTP-RANSAC	0.582	0.623 ± 0.204	-1.184 ± 1.367
FDSM-spatial vs. X-Corr	0.132*	0.686 ± 0.352	-1.407 ± 2.507
FDSM-spatial vs. 2D-FT	-0.300^*	0.296 ± 0.266	-6.489 ± 4.797

TABLE III

Computational time (mean $\pm$ SD) of each method. Computational times were assessed during the processing of the CIRS phantom data. 5000 iterations were used for two RANSAC-based methods: TTP-RANSAC and X-corr methods.

Method	computational time (s)
FDSM-temporal	0.51 ± 0.02
FDSM-spatial	1.04 ± 0.03
TTP-RANSAC	4.20 ± 0.11
Radon-Sum	2.74 ± 0.07
X-Corr	4.71 ± 0.16
2D-FT	0.21 ± 0.04