Published in final edited form as:

Sens Imaging. 2018 December ; 19: . doi:10.1007/s11220-018-0198-7.

Reduction of Angularly-Varying-Data Truncation in C-Arm CBCT Imaging

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Abstract

C-arm cone-beam computed tomography (CBCT) has been used increasingly as an imaging tool for yielding 3D anatomical information about the subjects in surgical and interventional procedures. In the clinical applications, the limited field-of-view (FOV) of C-arm CBCT can lead to significant data truncation, resulting in image artifacts that can obscure low contrast tumor embedded within soft-tissue background, thus limiting the utility of C-arm CBCT. The truncation issue can become serious as most of the surgical and interventional procedures would involve devices and tubes that are placed outside the FOV of C-arm CBCT and thus can engender angularly-varying-data truncation. Existing methods may not be adequately applicable to dealing with the angularly-varying truncation. In this work, we seek to reduce truncation artifacts by tailoring optimization-based reconstruction directly from truncated data, without performing pre-reconstruction data compensation, collected from physical phantoms and human subjects. The reconstruction problem is formulated as a constrained optimization program in which a data-derivative- ℓ_2 -norm fidelity is included for effectively suppressing image artifacts caused by the angularly-varying-data truncation, and the generic Chambolle–Pock algorithm is tailored to solve the optimization program. The results of the study suggest that an appropriately designed optimization-based reconstruction can be exploited for yielding images with reduced artifacts caused by angularly-varying-data truncation.

Keywords

Computed tomography; Image reconstruction; Optimization program; Primal-dual algorithm; Angularly-varying-data truncation; Data derivative

1 Introduction

C-arm cone-beam computed tomography (CBCT) is used frequently as a tomographic imaging tool for providing 3D anatomical information about the subjects in surgical and interventional procedures [1–9]. While it is of utility for revealing high contrast anatomic

structures, there exist increased needs for localizing soft-tissue tumors in images of C-arm CBCT procedure guidance and possible complication detection. In current clinical C-arm CBCT, the flat-panel detector forms only a field of view (FOV) of limited size, thus resulting in significant data truncation. Image artifacts caused by data truncation can obscure the identification of low-contrast tumors in surgical and interventional procedures. This truncation issue becomes increasingly serious as the surgical and interventional procedures often involve devices and tubes that are placed outside the FOV of C-arm CBCT, thus engendering angularly-varying-data truncation at different projection views.

Ad hoc methods such as pre-reconstruction data compensation [10–17] have been investigated for reducing truncation artifacts in C-arm CBCT imaging. While these methods have some success in a limited number of situations, they may not be adequately applicable to the angularly-varying-data truncation considered. More importantly, these methods may introduce additional artifacts as a result of their inadequate data compensation, especially for clinically relevant situations in which some high-contrast interventional devices/tubes are present outside of the FOV.

Optimization-based reconstruction methods may be exploited for reducing image artifacts as a result of data truncation. In particular, evidence exists suggesting that optimization-based reconstruction incorporating a data-derivative divergence term appears to be effective in minimizing image artifacts caused by angularly-varying-data truncation [18, 19]. The resulting reconstruction appears to behave analogously to images reconstructed with analytic methods referred to as the Λ - or local-tomography [20–23], and the data-derivative divergence term helps the recovery of detailed structures relative to standard optimization-based reconstruction and improves reconstruction robustness against angularly-varying truncated projection data. In this work, we seek to reduce image artifacts observed in the reconstructions obtained with the current clinical algorithm by applying an optimization-based reconstruction method [18, 19] directly to angularly-varying-truncation data collected from physical phantoms and human subjects in neurological interventional procedures.

As mentioned above, the image reconstruction is designed specifically as a constrained optimization program, which contains a data fidelity of ℓ_2 -norm form and a constraint on the image total-variation (TV), thus forming a non-smooth, convex program [24–26]. We show that the appropriate design of the overall data fidelity is a key to effectively reduce the angularly-varying-truncation artifacts in C-arm CBCT. Specifically, we demonstrate that a data-derivative- ℓ_2 -norm fidelity can be included in the overall data fidelity for effectively suppressing the image artifacts caused by the angularly-varying-data truncation. The optimization program is designed as a convex program, and thus algorithms exist that can be used for solving the program. In the work, the generic CP algorithm that can solve mathematically non-smooth, convex program is tailored to solve our designed optimization program [18, 19, 24, 25]. The results of the study suggest that an appropriately designed optimization-based reconstruction with a data-derivative fidelity and the corresponding CP algorithm can be exploited for yielding image reconstruction with reduced artifacts directly from truncated data collected in CBCT imaging in neurological interventional procedures.

We organize the paper as follow. Data acquisition from a physical head phantom and a patient by use of a clinical C-arm CBCT system is described in Sect. 2. An image-TV-constrained, weighted ℓ_2 -norm data-fidelity-minimization reconstruction is proposed for the C-arm CBCT imaging and the tailored CP reconstruction algorithm along with the convergence conditions are presented in Sect. 3. Image-reconstruction results from physical phantom data and patient data are presented in Sect. 4, followed by discussion and remarks about our study in Sect. 5.

2 Data Acquisition

In this work, we collected projection data by using a C-arm system (Canon Medical System Infinix™-i, Japan) in which the X-ray source has distances of 700 mm and 1100 mm to the rotation center and to the detector. The flat-panel detector consists of a 1024×1024 array of detector bins with a size of $0.29 \times 0.29 \text{ mm}^2$, which is rebinned often to a 512×512 array with a bin size of $0.58 \times 0.58 \text{ mm}^2$, and thus forms a circular FOV of diameter 18.7 cm, as shown in Fig. 1. With this system, C-arm CBCT data were collected from a physical head phantom and a patient subject at 247 views over a short-scan angular range of $\sim 194^\circ$. Therefore, the dataarray sizes for both the phantom and patient studies are $512 \times 512 \times 247$. The standard calibration scheme of the clinical C-arm system was performed on data prior to the image reconstruction.

While the FOV may be sufficient to cover the scanned subject, some interventional devices, which may or may not touch directly the subject, are presented often outside the FOV during the intra-operative imaging process. For example, as shown in Fig. 1, a metal tube placed outside the FOV is projected onto the detector at 0° view without any truncation, whereas its projection is outside the detector at 90° -view, thus resulting in data truncation. Clearly, the degree of the tube truncation depends upon the projection views, i.e., it varies as a function of projection views and is thus referred to as the angularly-varying truncation. It is this type of truncations that causes the prominent streak artifacts observed in clinical reconstructions, as demonstrated in Sect. 4 below.

3 Optimization-Based Reconstruction

We summarize below the optimization program, reconstruction algorithm, and practical convergence conditions that form the necessary components of the optimization-based reconstruction [18, 19].

3.1 Optimization Program

The problem of image reconstruction considered is formulated as a constrained optimization program [19, 27]:

$$\mathbf{f}^\star = \underset{\mathbf{f}}{\operatorname{argmin}} D_W(\mathbf{g}_m, \mathcal{H}\mathbf{f}) \text{ s.t. } \|\mathbf{f}\|_{TV} \leq t_1 \text{ and } f_j \geq 0, \quad (1)$$

where $\mathbf{f}$ and $\mathbf{g}_m$ denote image vector of size N and measured-data vector of size M , respectively; $\mathcal{H}$ the system matrix of size $M \times N$; $\|\mathbf{f}\|_{TV}$ the image total-variation (TV); $t_1 >$

0 a constraint parameter on the image TV, and f_j entry j of image vector $\mathbf{f}$; and $D_W(\mathbf{g}_m, \mathcal{H}\mathbf{f})$ the data divergence. In the work, in an attempt to reduce the angularly-varying truncation, we consider [18]

$$D_W(\mathbf{g}_m, \mathcal{H}\mathbf{f}) = \|W_c(\mathcal{H}\mathbf{f} - \mathbf{g}_m)\|_2^2, \quad (2)$$

where weighting matrix W_c is given by

$$W_c = c\mathcal{I} + (1 - c)\mathcal{D}_u, \quad (3)$$

and $\mathcal{I}$ denotes the identity matrix of size $M \times M$, and matrix $\mathcal{D}_u$ of size $M \times M$ represents a two-point-difference operation (i.e., an approximation of the derivative) carried out along the horizontal axis of the detector, and c a parameter that weights the contributions of $(\mathcal{H}\mathbf{f} - \mathbf{g}_m)$ and its derivative to the data divergence. It can be observed that $D_W(\mathbf{g}_m, \mathcal{H}\mathbf{f})$ becomes the standard data ℓ_2 -norm when $c = 1$, and the data-derivative ℓ_2 -norm when $c = 0$. The derivative operation is analogous to the derivative operation involved in local tomography that enhances the structural boundaries in an image [20–23]. It has been shown [18, 19] that the appropriate incorporation of data derivative $\mathcal{D}_u$ can suppress effectively truncation artifacts in reconstructed images.

While the form of the optimization program is given by Eq. (1), the complete specification of the program depends also upon $\mathcal{H}$, c , and t_1 , which are referred to as program parameters. Clearly, program parameters can significantly impact the program and thus its solution (i.e., the reconstruction). In each of phantom- and patient-data studies below, we discuss the selection of the program parameters specific to the study.

3.2 Reconstruction Algorithm

The problem of image reconstruction is thus tantamount to solving Eq. (1), which is a convex but non-smooth problem. Therefore, we will use a primal-dual algorithm by tailoring the generic CP algorithm to solve the non-smooth convex problem in our reconstruction, and summarize below the tailored CP algorithm for which the detailed derivation can be found in Refs. [18, 19].

Using two positive parameters λ and ν , we change the form of Eq. (1) to

$$\mathbf{f}^\star = \arg \min_{\mathbf{f}} \frac{1}{2} \lambda \|W_c(\mathcal{H}\mathbf{f} - \mathbf{g}_m)\|_2^2 \quad \text{s.t.} \quad \|\nu\mathbf{f}\|_{TV} \leq \nu t_1 \quad \text{and} \quad f_i \geq 0, \quad (4)$$

which specifies a feasible solution set identical to that defined by optimization problem in Eq. (1). It can readily be shown that the constrained optimization program in Eq. (4) can be recast into an unconstrained problem as

$$\mathbf{f}^\star = \arg_{\mathbf{f}} \min \left\{ \frac{1}{2} \lambda \|W_c(\mathcal{H}\mathbf{f} - \mathbf{g}_m)\|_2^2 + \delta_{\text{Diamond}(\nu t_1)}(\|\nabla \mathbf{f}\|_{\text{MAG}}) + \delta_P(\mathbf{f}) \right\}, \quad (5)$$

where ∇ depicts a gradient matrix of size $3N \times N$ with its transpose given by $\nabla^T = (\nabla_x^T, \nabla_y^T, \nabla_z^T)$; superscript “ T ” indicates a transpose operation; ∇_x , ∇_y , and ∇_z are matrices of size $N \times N$ representing two-point differences along x -, y -, and z -axis, respectively, yielding vectors $\nabla_x \mathbf{f}$, $\nabla_y \mathbf{f}$, and $\nabla_z \mathbf{f}$ of size N , which in turn form vector $\nabla \mathbf{f}$ of size $3N$ in a concatenated form in the order of x , y , and z ; $\|\nabla \mathbf{f}\|_{\text{MAG}}$ denotes a vector of size N in which entry i is given by $(\|\nabla \mathbf{f}\|_{\text{MAG}})_i = \sqrt{(\nabla \mathbf{f})_i^2 + (\nabla \mathbf{f})_{i+N}^2 + (\nabla \mathbf{f})_{i+2N}^2}$; and indicator functions $\delta_P(\mathbf{x})$ and $\delta_{\text{Diamond}(\nu t_1)}(\mathbf{x})$ are defined as

$$\begin{aligned} \delta_P(\mathbf{x}) &\equiv \begin{cases} 0 & \mathbf{x} \geq 0 \\ \infty & \text{Otherwise,} \end{cases} \quad (6) \\ \delta_{\text{Diamond}(\nu t_1)}(\mathbf{x}) &\equiv \begin{cases} 0 & \|\mathbf{x}\|_1 \leq \nu t_1 \\ \infty & \|\mathbf{x}\|_1 > \nu t_1 \end{cases}. \end{aligned}$$

Now, our problem of image reconstruction becomes to solve the unconstrained problem in Eq. (5). As discussed above, since the objective function in Eq. (5) includes non-smooth functions such as the image-TV term, we use a specific primal-dual algorithm tailored from the generic CP algorithm [24, 25] to solve the optimization program in Eq. (5).

The generic CP algorithm solves a primal minimization problem of the form

$$\mathbf{x}^\star = \argmin_{\mathbf{x}} \{F(K\mathbf{x}) + G(\mathbf{x})\}, \quad (7)$$

where K denotes a linear transform, and functions F and G are convex. The pseudo codes of the preconditioned generic CP algorithm [24, 25, 28] that solves the primal minimization problem in Eq. (7) are given in Table 1:

where $|\cdot|$ takes the absolute value of each entry of a matrix, vectors $\mathbf{1}_X$ and $\mathbf{1}_Y$ of the sizes identical to that of $\mathbf{x}$ and $\mathbf{y}$ have all components set to 1.

In attempt to tailor the generic CP algorithm to solve our optimization problem in Eq. (5) [or, equivalently, Eq. (1)], we need to carry out two key steps: (1) identification of the concrete forms of the linear transform K and the convex functions F and G ; and (2) calculations of the proximal mappings of functions F^* and G^* in Lines 6 and 7 of Table 1, where symbol “ $*$ ” refers to convex conjugate [29].

For discussion convenience, let $\mathbf{a}$ and $\mathbf{b}$ denote two vectors of sizes kN and N , where $k \geq 1$ is a positive integer. Let a_j denote the j th entry of $\mathbf{a}$, $j = 1, 2, \dots, kN$, and b_j the j th entry of $\mathbf{b}$, $j = 1, 2, \dots, N$, respectively. Their multiplication and division are defined below as

$$\begin{aligned} (\mathbf{a}\mathbf{b})_j &= a_j b_{j_m} \\ (\mathbf{a}/\mathbf{b})_j &= a_j / b_{j_m}, \end{aligned}$$

where $j = 1, 2, \dots, kN$, and $j_m = \text{mod}(j, N)$ indicates the remainder of j dividing by N , and $b_{j_m} \neq 0$. In particular, when $k = 1$, $\mathbf{a}$ and $\mathbf{b}$ are of the same size N , and vectors $\mathbf{a}\mathbf{b}$ and $\mathbf{a}/\mathbf{b}$ thus are of size N depicting element-wise multiplication and division. We also use $\mathbf{a}^\top \mathbf{b}$ to denote their inner product for vectors $\mathbf{a}$ and $\mathbf{b}$ of an identical dimension.

It can be shown [18, 19] that Eq. (5) can be interpreted as a primal problem with the form in Eq. (7) by using the following assignments:

$$\mathbf{x} = \mathbf{f}, \quad K = \begin{pmatrix} W_c \mathcal{H} \\ \nu \nabla \end{pmatrix}, \quad (8)$$

and

$$\begin{aligned} F(K\mathbf{x}) &= F_1(\mathbf{r}) + F_2(\mathbf{z}) \\ &= \frac{1}{2} \lambda \|\mathbf{r} - W_c \mathbf{g}_m\|_2^2 + \delta_{\text{Diamond}(\nu t_1)}(\|\mathbf{z}\|_{\text{MAG}}), \\ G(\mathbf{x}) &= \delta_p(\mathbf{f}), \end{aligned} \quad (9)$$

where $\mathbf{r} = W_c \mathcal{H} \mathbf{f}$, $\mathbf{z} = \nu \nabla \mathbf{f}$, $\|\mathbf{z}\|_{\text{MAG}}$ depicts a vector of size N in which entry i is given by $(\|\mathbf{z}\|_{\text{MAG}})_i = \sqrt{z_i^2 + z_{i+N}^2 + z_{i+2N}^2}$, and z_i indicates entry i of vector $\mathbf{z}$.

Using F and G are determined Eq. (9), one can analytically derive the proximal mappings of functions F^* and G (see Appendix A of Ref. [19]), which are given by

$$\text{prox}_{\Sigma} [F^*](\mathbf{p}, \mathbf{q}) = \text{prox}_{\Sigma_1} [F_1^*](\mathbf{p}) + \text{prox}_{\Sigma_2} [F_2^*](\mathbf{q}) \quad (10)$$

and

$$\text{prox}_{\mathcal{G}} [G](\mathbf{f}) = \text{pos}(\mathbf{f}), \quad (11)$$

where

$$\begin{aligned}\text{prox}_{\Sigma_1}[F_1^*](\mathbf{p}) &= (\mathbf{p} - \Sigma_1 W_c \mathbf{g}_m) / (\mathbf{1}_D + \Sigma_1 / \lambda) \\ \text{prox}_{\Sigma_2}[F_2^*](\mathbf{q}) &= \mathbf{q}(\mathbf{1}_V - \Sigma_2 \mathbf{s} / \|\mathbf{q}\|_{\text{MAG}}),\end{aligned}$$

$\mathbf{p}$ and $\mathbf{q}$ are vectors of sizes M and $3N$, vector $\Sigma_1 = \mathbf{1}_D / \|\mathbf{W}_c \mathcal{H} \mathbf{1}_I\|$ of size M , vectors $\mathbf{1}_D$ and $\mathbf{1}_I$ of sizes M and N with all components set to 1, vector $\mathbf{1}_V$ of size $3N$ with entries set to 1; vector $\Sigma_2 = \mathbf{1}_V / 2\nu$ of size $3N$, vector $\mathbf{s} = \text{ProjectOnto}_{\ell_1 \text{Ball}_{\nu t_1}}(2\nu \mathbf{q}|_{\text{MAG}})$, and operator $\text{ProjectOnto}_{\ell_1 \text{Ball}_{\nu t_1}}$ projects vector $2\nu \mathbf{q}|_{\text{MAG}}$ of size N onto the ℓ_1 -ball of scale νt_1 [18, 30], operator $\text{pos}(\mathbf{f}) = 0$ and f_j for $f_j \leq 0$ and $f_j > 0$, respectively, enforcing the non-negativity constraint, vector $\mathcal{T} = \mathbf{1}_I / (\|\mathcal{H}^T W_c^T \mathbf{1}_D + \nu \|\nabla^T \mathbf{1}_V\|)$ of size N .

Using Eqs. (10) and (11) to replace $\text{prox}_{\Sigma}[F^*](\mathbf{y})$ and $\text{prox}_{\mathcal{T}}[G](\mathbf{x})$ in Lines 6 and 7 of Table 1, the pseudo codes are obtained for the tailored CP algorithm specifically for solving the program in Eq. (5), or equivalently, the program in Eq. (1).

3.3 Practical Convergence Conditions

The tailored CP algorithm summarized above can mathematically converge a convex optimization program. Without loss of generality, we use three quantitative metrics below to define the necessary convergence conditions:

$$\begin{aligned}\bar{D}_W(\mathbf{f}_n) &= \|\mathbf{W}_c(\mathcal{H}\mathbf{f}_n - \mathbf{g}_m)\|_2^2 / \|\mathbf{W}_c(\mathcal{H}\mathbf{f}_1 - \mathbf{g}_m)\|_2^2 \quad (12) \\ \overline{\text{TV}}(\mathbf{f}_n) &= \left| \|\mathbf{f}_n\|_{\text{TV}} - t_1 \right| / t_1 \\ \overline{\text{cPD}}(\mathbf{f}_n) &= \|\text{cPD}(\mathbf{f}_n) / \text{cPD}(\mathbf{f}_1)\|, \end{aligned}$$

where $\bar{D}_W(\mathbf{f}_n)$ and $\overline{\text{TV}}(\mathbf{f}_n)$ denote the normalized data divergence and image-TV at iteration n , and $\overline{\text{cPD}}(\mathbf{f}_n)$ the normalized conditional primal-dual gap is defined as [24]

$$\text{cPD}(\mathbf{f}_n) = \frac{1}{2}\lambda \|\mathbf{W}_c(\mathcal{H}\mathbf{f}_n - \mathbf{g}_m)\|_2^2 - \left[-\frac{1}{2\lambda} \|\mathbf{p}_n\|_2^2 - \mathbf{g}_m^T \mathbf{W}_c^T \mathbf{p}_n - \nu t_1 \|\mathbf{q}_n|_{\text{MAG}}\|_{\infty} \right],$$

and $\|\cdot\|_{\infty}$ denotes the largest entry of the vector. Using the metrics defined, one obtains necessary mathematical convergence conditions, given by

$$\left| \frac{\partial \bar{D}(\mathbf{f}_n)}{\partial n} \right| \rightarrow 0 \quad (13)$$

$$\overline{\text{TV}}(\mathbf{f}_n) \rightarrow 0$$

$$\overline{\text{cPD}}(\mathbf{f}_n) \rightarrow 0.$$

The mathematical convergence conditions can be satisfied by the tailored CP algorithm summarized above. However, it is unlikely that they can be met in numerical studies due to the limited computer precision. Therefore, practical convergence conditions are needed for numerical studies. Based upon the mathematical convergence conditions, we define below practical convergence conditions for specific real-data studies considered:

$$\left| \frac{\partial \bar{D}(\mathbf{f}_n)}{\partial n} \right| < 10^{-3} \quad (14)$$

$$\overline{\text{TV}}(\mathbf{f}_n) < 10^{-3}$$

$$\overline{\text{cPD}}(\mathbf{f}_n) < 10^{-3}.$$

4 Results

Although the mathematical form of the optimization program is given by Eq. (1), the complete specification of the program depends upon the program parameters involved. For a given set of parameters, without loss of generality, we first performed reconstruction studies to verify that the practical convergence conditions in Sect. 3.3 can be achieved by the tailored CP algorithm. Then, we will illustrate the strategy and scheme to determine the program parameters for each of the studies involving physical phantom and patient subject, followed by presenting the results obtained for the physical phantom and patient subjects considered by applying the tailored CP algorithm to solving the optimization program completely specified by selected parameters.

4.1 Practical Convergence Conditions

Without loss of generality, we first perform a study using the physical head phantom with parameter $t_1 = 18,000$ to demonstrate that the practical convergence conditions in Eq. (14) above can be reached. In Fig. 2, we display metrics $\left| \frac{\partial \bar{D}(\mathbf{f}_n)}{\partial n} \right|$, $\overline{\text{TV}}(\mathbf{f}_n)$, and $\overline{\text{cPD}}(\mathbf{f}_n)$ calculated from reconstructions of the physical phantom as functions of iteration number n , along with the convergence reconstruction of the physical head phantom shown in Fig. 3b. It can be observed that the tailored CP algorithm takes about 900 iterations to satisfy the practical convergence conditions in Eq. (14). This observation can also be made for other subjects and

4.2 Determination of Program Parameters

As shown in Eq. (1), there exist several important program parameters for completely specifying the optimization program, including system matrix, image array (i.e., voxel shape and size), modulation parameter c , and TV-constraint parameter t_1 .

4.2.1 Determination of Program Parameters Other than t_1 —In our studies, scanning geometry information was incorporated into system matrix $\mathcal{H}$ in which each element was defined as the intersection length of X-ray measured and a voxel in the image array. We used detecting-bin size and data size identical to clinical parameters discussed in Sect. 2. For the purpose of comparison with the clinical FDK reconstructions, the image arrays of sizes $N = 700 \times 560 \times 50$ and $N = 750 \times 700 \times 50$ were used in head-phantom and patient reconstructions, and the voxel sizes are $0.47 \times 0.47 \times 0.47 \text{ mm}^3$ and $0.37 \times 0.37 \times 0.37 \text{ mm}^3$, respectively. With the determination of system matrix $\mathcal{H}$ and voxel size, c and t_1 are the only two remaining program parameters to be determined. In our previous [19] and current studies, it has been shown empirically that $c = 0.05$ appears to be effective in reducing image artifacts. With the selection of $c = 0.05$, parameter t_1 remains the only program parameter to be determined.

4.2.2 Determination of Program Parameter t_1 —Using the program parameters determined above, we performed multiple reconstructions with different values of t_1 for each data set of the physical phantom and patient subject, and, based upon visual inspection of the reconstructions, then selected the values of parameter t_1 yielding reconstructions with visually least artifacts. For example, we show in Figs. 3 and 4 reconstructions from physical phantom and patient data sets, respectively, for different values of parameter t_1 . It can be observed that $t_1 = 18,000$ and $t_1 = 10,000$ appear to yield reconstructions of the phantom and patient with visually least truncation artifacts. Additionally, this is a qualitative, subjective scheme for determination of t_1 , it seems to work effectively in terms of selecting t_1 for showing the degree of artifacts reduction. Furthermore, as one leading interest in the clinical procedures considered is in the detection of low-contrast anatomy in the image, we defined a contrast-resolution metric below for guiding the selection of parameter t_1 for a given data set:

$$\text{CNR} = \frac{|\mathbf{f}_s - \mathbf{f}_b|}{\sqrt{\sigma_s^2 + \sigma_b^2}}, \quad (15)$$

where $\mathbf{f}_s$ and σ_s denote the mean and standard deviation within a region of interest (ROI), and $\mathbf{f}_b$ and σ_b denote the mean and standard deviation within a selected background region. CNRs were computed in ROIs containing low-contrast insert in the phantom image and low-contrast structure in the patient image, indicated by the circle in the second panel in Figs. 3 and 4, respectively. We show in Fig. 5 the CNRs calculated from images of the phantom and patient reconstructed with different t_1 values. It can be observed that the CNR results corroborate visual inspection results by reaching roughly its maximum around the optimal t_1 value determined by use of visual inspection. Therefore, this approach is used for selecting the value of parameter t_1 for each of the studies in Sect. 4.

4.3 Results of the Phantom-Data Study

For the phantom data collected, image array of size $N = 700 \times 560 \times 512$ and the voxel size $0.47 \times 0.47 \times 0.47 \text{ mm}^3$ were used in the reconstructions. The modulate parameter $c = 0.05$ was chosen. By using the approach described in Sect. 4.2.2 with metric CNR, we selected $t_1 = 70,000$ based upon the phantom data. Using these program parameters determined, we then applied the tailored CP algorithm to reconstructing images from the phantom-data set of size $512 \times 512 \times 247$.

In Figs. 6, 7 and 8, we display the images within three transverse, coronal, and sagittal slices of the phantom reconstructed, with a narrow display window for revealing details of both low-contrast structures and artifacts. The images are convergent solutions to the optimization program in Eq. (1) obtained with about 900 iterations in the sense the practical convergence conditions in Eq. (14) are satisfied. As a result of the metal object placed outside the FOV of the system, significant streak artifacts can be observed in the reconstructions obtained with the FDK algorithm. Conversely, the artifacts appear to be considerably suppressed in the reconstructions of the tailored CP algorithm, thus some of the low-contrast structures can be identified more readily than those obscured in the FDK reconstructions. Moreover, it is interesting to observe that some of the cone-beam artifacts near the jaw region, as pointed by the arrows, in the FDK reconstructions appear to be reduced considerably in the reconstructions of the CP algorithm in Figs. 7 and 8.

4.3.1 Impact of Parameter c —As discussed above, program parameter c also plays a role in the complete specification of the optimization program in Eq. (1). The results above was obtained with $c = 0.05$ based upon our previous experience [19]. In this work, we have also carried out a study on the impact of c on artifact reduction. For example, we show in Fig. 9 images reconstructed with $c = 0.05$ and $c = 1.0$ from the same data set. It can be observed that, while $c = 1.0$ can suppress considerably the streak artifacts, prominent residual artifacts remain, and that $c = 0.05$ appears effectively to further reduce the residual artifacts. Similar results were obtained for c values that significantly differ than 0.05.

4.4 Results of the Patient-Data Study

For the patient-data study, we also used the parameters selected in Sect. 4.2.1 and use the scheme described in Sect. 4.2.2 to determine parameter t_1 . Using these program parameters determined, we then apply the tailored CP algorithm to reconstructing images from the patient data of size $512 \times 512 \times 247$.

In the patient study, image array of size $N = 750 \times 700 \times 612$ and the voxel size $0.37 \times 0.37 \times 0.37 \text{ mm}^3$ were used in the reconstructions. By using the approach described in Sect. 4.2.2 with metric CNR, we selected $t_1 = 120,000$ based upon the patient data. Again, the modulate parameter $c = 0.05$ was chosen.

For this study, we display in Figs. 10, 11 and 12 images within three transverse, coronal, and sagittal slices of the patient reconstructed, with a narrow display window for revealing details of both low-contrast structures and artifacts. Again, the images are convergent solutions to the optimization program in Eq. (1) obtained with about 900 iterations in the

sense the practical convergence conditions in Eq. (14) are satisfied. The streak artifacts observed in the reference images, caused by the high-contrast interventional devices placed outside of the FOV, are significantly reduced in CP reconstructions. Moreover, the low-contrast intracranial structures in the CP reconstructions, such as hemorrhage and ventricle indicated by the arrow and the arrowhead in bottom panel of Fig. 10a, can be more discernible than those in the clinical FDK reconstruction. Conspicuous streak artifacts in the images obtained with the FDK algorithm are visually distracting and possibly obscure information of clinical significance; whereas the CP reconstructions suppress the artifacts.

5 Conclusion

In this work, we have investigated an optimization-based image reconstruction in C-arm CBCT used in neurological interventional procedures. In an attempt effectively to suppress streak artifacts as a result of the angularly-varying-data truncation caused by the high contrast devices/tubes placed outside the FOV of the C-arm CBCT system in the clinical procedures, we used a data-derivative fidelity term in the optimization program. The study results of phantom and patient data suggest that the data-derivative term helps the recovery of detailed structures, similar to that observed in Λ - or local-tomography [20–23], more effectively than does the standard optimization-based reconstruction without the data-derivative term, while the regular ℓ_2 -data term in the optimization program ensures the recovery of the image grey levels of various contrast anatomies.

In the study, we have collected data from physical phantom and patient subject by using clinical C-arm CBCT systems. Applying the tailored CP algorithm to the data, we then reconstructed images by solving the optimization program. Results reveal that the optimization program containing a data derivative term, along with the tailored CP algorithm, can yield image reconstructions with substantially reduced artifacts resulted from the angularly-varying-data truncation, thus improving the visualization of low-contrast structures that would otherwise be obscured in the clinical reconstructions.

As we discussed in Sect. 3, the performance of the optimization-based algorithm depends upon a number of parameters and performance metrics used. Therefore, it is necessary and important to discuss the determination of the key parameters that impact significantly image quality. In the study, TV-constraint parameter t_1 plays a key role in impacting the visualization of a low-contrast anatomy within a reconstructed image. We have designed contrast metric CNR for determination of parameter t_1 because one leading task in the clinical procedure considered is to identify the low-contrast anatomies embedded in the soft-tissue background. When different practical tasks are considered, it is likely that metrics and procedures suited to the tasks should be designed for the determination of the reconstruction parameters.

Finally, we remind that the study is not meant to produce a practical procedure for direct use in clinical applications. In fact, additional issues must be taken into consideration, such as specific clinical tasks to be performed. Instead, our reconstruction investigation entails, in essence, a survey of the multi-dimensional parameter spaces to yield insights into artifact reduction for the scenarios of practical, clinical significance. Knowledge gained in the study,

along with other important clinical task-specific considerations, can be exploited for aiding in the design of practical procedures of truly clinical-task utility and be generalized for investigating image reconstruction in other CBCT applications.

Acknowledgments

This work was supported in part by NIH R01 Grants Nos. CA182264, and EB018102. The contents of this article are solely the responsibility of the authors and do not necessarily represent the official NIH views.

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Fig. 1.

a The scanning configuration in which the dashed circle indicates the field of view (FOV) formed by the planar detector of the C-arm imaging system and a metal tube is placed outside of the FOV. The dashed lines pointed by arrows indicate the projection loci of the tube at **b** the 0°- and **c** the 90°-view. Clearly, projection data is truncated at the 90°-view as the tube projection is beyond the detector

Fig. 2.

Metrics $|\frac{\partial \bar{D}(\mathbf{f}_n)}{\partial n}|$, $\overline{TV}(\mathbf{f}_n)$, and $\overline{cPD}(\mathbf{f}_n)$ as functions of iteration number n . The values at $n = 900$ satisfy the practical convergence conditions in Eq. (14) conditions studied in the work. Therefore, the reconstructions at $n = 900$ is used as the surrogate solution to the convergent solution to the optimization program in Eq. (1).

Fig. 3.

Convergent reconstructions from the head-phantom data within a transverse slice reconstructed by use of the tailored CP algorithm with parameter $t_1 = 12,000$ (**a**), $18,000$ (**b**), and $25,000$ (**c**), along with the corresponding reference (**d**). The region enclosed by the circle is the selected ROI for computing the contrast-resolution metric, CNR. Display window: $[0, 200]$ HU

Fig. 4.

Convergent reconstructions from the patient data within a transverse slice reconstructed by use of the tailored CP algorithm with parameter $t_1 = 6000$ (a), 10,000 (b), and 15,000 (c), along with the corresponding reference image (d). The region enclosed by the circle is the selected ROI for computing the contrast-resolution metric, CNR. Display window: [-150, 200] HU

Fig. 5. CNRs computed within the ROIs enclosed by the circles shown in Figs. 3 and 4 for the head phantom (**a**) and patient (**b**), along with the CNRs (dashed lines) of the corresponding reference images, as functions of t_1 . The values of t_1 indicated by the square symbols are selected as the optimal value of t_1 for image reconstruction

Fig. 6. Reconstructed (row 1) and reference (row 2) images of the head phantom within transverse slices specified by $z = 37.6$ mm (a), 12.2 mm (b), and -41.8 mm (c), respectively. Display window: [0, 200] HU

Fig. 7. Reconstructed (row 1) and reference (row 2) images of the head phantom within coronal slices specified by $x = -12.7$ mm (**a**), 18.8 mm (**b**), and 44.2 mm (**c**), respectively. Display window: $[0, 200]$ HU. The arrow in the reference image indicates the cone-beam artifacts.

Fig. 8. Reconstructed (row 1) and reference (row 2) images of the head phantom within sagittal slices specified by $y = -24.9$ mm (**a**), 12.2 mm (**b**), and -44.2 mm (**c**), respectively. Display window: $[0, 200]$ HU. The arrow in the reference image indicates the cone-beam artifacts

Fig. 9.

Images of the head phantom within slices specified by $z = 37.6$ mm (**a**), $z = 12.2$ mm (**b**), $y = -24.9$ mm (**c**), and $x = -12.7$ mm (**d**) reconstructed with $c = 0.05$ (row 1) and $c = 1.0$ (row 2). Display window: $[0, 200]$ HU

Fig. 10. Reconstructed (row 1) and reference (row 2) images of the patient subject within transverse slices specified by $z = 18.5$ mm (**a**), -22.2 mm (**b**), and -51.1 mm (**c**), respectively. Display window: $[-150, 200]$ HU. The arrow and arrowhead in the reference image indicate the hemorrhage and ventricle, respectively

Fig. 11. Reconstructed (row 1) and reference (row 2) images of the patient subject within sagittal slices specified by $y = -62.9$ mm (**a**), -2.2 mm (**b**), and 20.4 mm (**c**), respectively. Display window: $[-150, 200]$ HU

Fig. 12. Reconstructed (row 1) and reference (row 2) images of the patient subject within coronal slices specified by $x = 5.6$ mm (**a**), -2.2 mm (**b**), and -46.2 mm (**c**), respectively. Display window: $[-150, 200]$ HU

Table 1

Pseudo codes of the preconditioned CP algorithm [24, 28]

```

1:  $\Sigma \leftarrow \mathbf{1}_{Y'}/(|K|\mathbf{1}_X)$ ;  $\mathcal{F} \leftarrow \mathbf{1}_{X'}/(|K|\mathbf{1}_Y)$ 
2:  $\theta \leftarrow 1$ ;  $n \leftarrow 0$ 
3: INITIALIZE  $\mathbf{x}_0$  and  $\mathbf{y}_0$  to zero
4:  $\mathbf{x}_0 \leftarrow \mathbf{x}_0$ 
5: REPEAT
6:  $\mathbf{y}_{n+1} \leftarrow \text{prox}_{\Sigma}[F^*](\mathbf{y}_n + \Sigma K \mathbf{x}_n)$ 
7:  $\mathbf{x}_{n+1} \leftarrow \text{prox}_{\mathcal{F}}[G](\mathbf{x}_n - \mathcal{F} K^T \mathbf{y}_{n+1})$ 
8:  $\mathbf{x}_{n+1} \leftarrow \mathbf{x}_{n+1} + \theta(\mathbf{x}_{n+1} - \mathbf{x}_n)$ 
9:  $n \leftarrow n + 1$ 
10: UNTIL the convergence conditions are satisfied
11: OUTPUT: image  $\mathbf{f}_{\text{conv}} \leftarrow \mathbf{f}_{n+1}$ 

```