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Properties of a Joint Reconstruction Method for Edge-Illumination X-Ray Phase-Contrast Tomography

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Abstract

Edge illumination X-ray phase-contrast tomography (EIXPCT) is an emerging technique for estimating the complex-valued refractive index distribution of an imaged object. A conventional EIXPCT system requires measurement of multiple images at each tomographic view angle, leading to prolonged data acquisition times. Recently, a joint reconstruction (JR) method has been developed to enable single-shot EIXPCT imaging without restrictive assumptions related to the object, imaging geometry or hardware. In the JR method, estimates of the refractive index distribution are recast as the solution to a nonlinear optimization problem. While a preliminary study has demonstrated the potential usefulness of the JR method, its numerical properties remain largely unexplored. In this project, the convexity, cross-talk properties and noise properties of the JR method are investigated.

Keywords

Joint image reconstruction; Phase-contrast tomography; Single-shot imaging

1 Introduction

Edge illumination X-ray phase-contrast tomography (EIXPCT) is a novel phase-contrast tomography technology for estimating the complex-valued refractive index distribution ($n = 1 - \delta + i\beta$) of an object imaged with laboratory-based X-ray sources [7,9–11,14]. Here, δ denotes the deviation from unity of the real-valued refractive index distribution and β denotes the absorption distribution. Hereafter, for simplicity, δ will be referred to as the refractive index distribution. Compared with conventional computed tomography (CT), which estimates only the X-ray attenuation property, EIXPCT provides additional information about the refractive index distribution.

The conventional EIXPCT system setup [9,14] is shown in Fig. 1. The incident X-ray beam from the source is partially blocked by a sample mask. Then, the beam interacts with the object immediately following the sample mask. This process creates phase shifts in the wavefront, which can also be interpreted as refraction of the X-ray beam in geometric optics. After propagation in free space, the X-ray beam is either blocked by the detector mask or detected by the detector. A refracted beam component thus can be either detected or undetected depending on the positions of the two blocking masks. Consequently, the

information about refraction angles is encoded in the measurement data. Because the refraction angle is proportional to the first-order derivative of the phase shift [10], the phase information can be retrieved afterwards.

Conventional reconstruction methods for EIXPCT employ a two-step strategy [7, 9]. First, a phase retrieval step is implemented by processing two or more measured intensity images acquired as described above to separate refractive index distribution information from absorption distribution information and obtain the sinograms of both distributions. In the second step, image reconstruction methods are employed to estimate independently the distributions of both β and δ based on the sinograms from the first step. The most common choice is the filtered back projection (FBP) method. Due to the nature of this approach, at least two images are acquired at each tomographic view angle, which prolongs data acquisition times and potentially increases the imaging dose.

To simplify and speed up the data acquisition process, single-shot methods that take only one image per tomographic view angle are highly desirable [3, 5, 14]. Several different methods for single-shot EIXPCT have been proposed. Some exploit special hardware so that with one X-ray exposure, it is possible to obtain information equivalent to that of multiple images. Das et al. proposed a method that employs a polychromatic X-ray source and an energy-sensitive detector [3]. If there are N energy bins, then effectively N images are obtained for one X-ray exposure, compensating for the loss of information normally provided from other mask positions. Endrizzi et. al. presented a similar idea utilizing specially fabricated, “multi-way” masks [6]. The conventional mask places the absorbing septa periodically with equal gaps between any neighboring septa, while the multi-way mask has multiple groups of absorbing septa with equal gaps within each group but non-uniform gaps between groups. The absorbing septa are designed to have a one-to-one correspondence with the detector pixels, so different pixels in the measured image obtained by single-shot imaging can be divided and regrouped into multiple images based on their corresponding septa group, thus achieving the effect of acquiring multiple images per X-ray exposure. These methods can be effective, but their special hardware requirements add to the cost and complexity of the system.

Other single-shot methods employing the basic system setup are available, but they rely on specific assumptions about the object or imaging geometry. For example, the reverse projection technique [8] designed by Hagen et al. makes use of the conjugate-view symmetry in a parallel beam tomography geometry. However, it is not applicable to more general imaging geometries like the widely employed fan-beam and cone-beam geometries. For general geometries, Diemoz et. al. proposed the single-shot reconstruction method that considers the ratio of the refractive index (δ) and absorption (β) to be a constant for the entire object [4, 5]. The quasi-homogeneous assumption is useful for single-material objects or objects containing multiple materials with similar δ/β ratios but may fail for complex compositions of materials with vastly varying δ/β ratios. To address this problem, Zamir et al. recently proposed a new reconstruction method capable of reconstructing multi-material objects [13]. However, this method still relies on the prior knowledge of all the materials constituting the object. The method also assumes that one material is encased by another and that the encasing material has a slowly varying projected thickness.

To address these concerns, a novel joint reconstruction (JR) method for single-shot imaging has been proposed recently by our group [2]. This method involves formulation of an optimization problem that is solved by use of a batch gradient descent algorithm. The method imposes no restrictive assumptions on imaging geometry or material composition and requires no special masks or detectors. The effectiveness of the JR method has been validated with both computer-simulated and experimental datasets [2]. However, the previous study was preliminary and the numerical properties of the JR method have not been explored systematically. Therefore, the applicability and robustness of this method to a general case situation remain unclear.

The goal of this study is to investigate the numerical properties of the JR method through computer-simulation studies. Three vital aspects are considered. First, the convexity of the cost function defined in the optimization problem employed in the JR method is analyzed in Section 4. Convex optimization problems don't have local minima, so they are guaranteed to converge to a global minimum with the gradient-descent algorithm. On the contrary, a non-convex problem may become trapped in local minima. Studying the convexity property helps understand the robustness of the algorithm.

Second, the cross-talk effect between the two channels (absorption and refractive index) in JR is investigated. In general, for multi-channel reconstruction methods, because the information needed for reconstructing various channels is mixed together in the recorded measurements, mitigating cross-talk between different channels can be a challenge. In the iterative framework employed in the JR method, the estimated value of β (or δ) at one iteration can influence the estimate of δ (or β) at the next iteration. If the cross-talk effect is strong, and one channel takes suboptimal values in the previous iteration, the reconstruction of the other channel may be led in a wrong direction and the values may end up in local minima. In summary, whether the cross-talk effect is negligible or not depends on the ability of the algorithm to accurately reconstruct one channel without reliance on an accurate estimate of the other channel. This is investigated in Section 5.

Finally, an analysis of the noise statistics of the reconstructed images is also conducted. This reveals the upper bound for quality of reconstructed images in term of biases and variances. This study is presented in Section 6.

2 Theoretical background

2.1 Imaging model and data-acquisition for EIXPCT

Below, the basic physical principles that underly EIXPCT are reviewed briefly. For a more extensive description, the readers are referred to the literature [7, 9, 14].

A typical setup for an EIXPCT system is illustrated in Fig. 1. When employing the multi-shot data acquisition design required by the conventional two-step reconstruction approach, the sample mask can be translated vertically by a motor during the data acquisition process at each tomographic view angle. The motion of the sample mask allows different portions of X-rays to fall on the sensitive areas of the detectors.

Consider a 2-dimensional (2D) tomography problem. The absorption and refractive index distributions of the object are functions of the spatial co-ordinate $\mathbf{r} = (x, y)$, so they can be denoted by $\beta(\mathbf{r})$ and $\delta(\mathbf{r})$ respectively. Define the beam line $L(\theta, d)$ to be a line oriented at the tomographic view angle $\theta \in [0, 2\pi)$ and going through the detector at position d . Let the functions $A(\theta, d)$ and $\phi(\theta, d)$ denote the integral of the β and δ distributions respectively along $L(\theta, d)$:

$$A(\theta, d) = \int_{L(\theta, d)} \beta(\mathbf{r}) dl, \text{ and } \phi(\theta, d) = \int_{L(\theta, d)} \delta(\mathbf{r}) dl. \quad (1)$$

These equations describe the 2D Radon transformation of β and δ . Let ϕ_d denote the first order derivative of $\phi(\theta, d)$ with respect to the detector coordinate:

$$\phi_d = \frac{\partial \phi(\theta, d)}{\partial d}. \quad (2)$$

The measured intensity is a function of view angle θ and detector position d , and can be calculated as [9]

$$I(\theta, d) = \exp \left(-\frac{4\pi}{\lambda} A(\theta, d) \right) \left[I_{TC}(\Delta\xi) - \frac{z_{od}}{M} I'_{TC}(\Delta\xi) \phi_d(\theta, d) \right]. \quad (3)$$

Here, $I(\theta, d)$ denotes the measured intensity value at tomographic view angle θ and detector coordinate d as defined in Fig. 1, λ denotes the wavelength of the incident X-ray beam, and $M = (z_{so} + z_{od})/z_{so}$ is the magnification coefficient. The function I_{TC} represents the illumination curve that describes the relationship between the measured intensity I and the sample mask position ξ in an air scan, and I'_{TC} is the derivative of the curve with respect to ξ . The illumination curve can be obtained by a separate calibration procedure [9].

An operator $\mathcal{Q} : \mathbb{L}^2(\mathbb{R}^2) \otimes \mathbb{L}^2(\mathbb{R}^2) \rightarrow \mathbb{L}^2([0, 2\pi) \times \mathbb{R})$ is employed to represent the overall imaging model that maps the object properties $\beta(\mathbf{r})$ and $\delta(\mathbf{r})$ to the measured intensity function $I(\theta, d)$. For simplicity, the arguments of functions are dropped, so the mapping relationship can be denoted by $I = \mathcal{Q}(\beta, \delta)$. Two operators $\mathcal{H} : \mathbb{L}^2(\mathbb{R}^2) \rightarrow \mathbb{L}^2([0, 2\pi) \times \mathbb{R})$ and $\mathcal{D} : \mathbb{L}^2(\mathbb{R}^2) \rightarrow \mathbb{L}^2([0, 2\pi) \times \mathbb{R})$ denote the 2D Radon transformation and the derivative of Radon transformation along the d coordinate so that $A = \mathcal{H}(\beta)$ and $\phi_d = \mathcal{D}(\delta)$. In terms of these quantities, the imaging process can be written as

$$\mathcal{I}(\beta, \delta) = \exp \left(-\frac{4\pi}{\lambda} \mathcal{H}(\beta) \right) \left[I_{TC}(\Delta\xi) - \frac{z_{od}}{M} I'_{TC}(\Delta\xi) \mathcal{D}(\delta) \right]. \quad (4)$$

2.2 JR method in its continuous form

The inverse problem is to solve for the absorption distribution β and refractive index distribution δ given measurement data. In this section, the problem is described in a continuous setting. Let I_m denote the measured intensity where the tomographic view angle ranges from 0 to 2π and the detector position $d \in (-\infty, \infty)$. In contrast to the FBP method that is typically employed in the conventional two-step method, in the JR method, the absorption and refractive index distributions are estimated by solving an optimization problem. The cost function considered is of the form

$$f(\beta, \delta) = \|I_m - \mathcal{J}(\beta, \delta)\|^2 + R(\beta, \delta). \quad (5)$$

Here, $R(\beta, \delta)$ is a regularization term that can be designed freely.

A gradient descent algorithm can be employed to solve the optimization problem. The gradients of the cost function with respect to the absorption and refractive index distributions are [2]:

$$\frac{\partial f}{\partial \beta} = 2\mathcal{J}'_{\beta}*[I_m - \mathcal{J}(\beta, \delta)] + R'_{\beta}(\beta, \delta), \quad (6)$$

$$\frac{\partial f}{\partial \delta} = 2\mathcal{J}'_{\delta}*[I_m - \mathcal{J}(\beta, \delta)] + R'_{\delta}(\beta, \delta), \quad (7)$$

where $\mathcal{J}'_{\beta}$ and $\mathcal{J}'_{\delta}$ denote the gradient of $\mathcal{Q}(\beta, \delta)$ with respect to β and δ . The quantities $\mathcal{J}'_{\beta}^*$ and $\mathcal{J}'_{\delta}^*$ denote the adjoint operators corresponding to $\mathcal{J}'_{\beta}$ and $\mathcal{J}'_{\delta}$, respectively. Similarly, R'_{β} and R'_{δ} denote the gradient of $R(\beta, \delta)$ with respect to β and δ , respectively. The operators $\mathcal{J}'_{\beta}^*$ and $\mathcal{J}'_{\delta}^*$ can be analytically calculated as

$$\mathcal{J}'_{\beta}^* = -\frac{4\pi}{\lambda}\mathcal{H}^*\left[I_{TC}(\Delta\xi) - \frac{z_{od}}{M}I'_{TC}(\Delta\xi)\mathcal{D}(\delta)\right]\exp\left(-\frac{4\pi}{\lambda}\mathcal{H}(\beta)\right), \quad (8)$$

$$\mathcal{J}'_{\delta}^* = -\mathcal{D}^*\left[\frac{z_{od}}{M}I'_{TC}(\Delta\xi)\right]\exp\left(-\frac{4\pi}{\lambda}\mathcal{H}(\beta)\right), \quad (9)$$

where $\mathcal{H}^*$ and $\mathcal{D}^*$ are the adjoints of $\mathcal{H}$ and $\mathcal{D}$.

2.3 JR method in its discrete form

In practice, digital detectors are employed so the measured intensities are discrete instead of continuous. In addition, in order to formulate the reconstruction process as a numerical optimization problem, the β and δ distributions to be estimated should also be discretized.

Suppose the β distribution is discretized by sampling at N vertices on a Cartesian grid. The absorption can be represented by ordering and grouping all the samples into a single vector $\beta = [\beta_1, \beta_2, \dots, \beta_N]^T \in \mathbb{R}^N$. Similarly, the δ distribution can be discretized and denoted by $\delta = [\delta_1, \delta_2, \dots, \delta_N]^T \in \mathbb{R}^N$. When imaging the object, P tomographic view angles and Q detector positions are chosen so that a total of PQ measured intensities are obtained. The vector $I(\beta, \delta) \in \mathbb{R}^{PQ}$ is an ordered collection of all of the intensity values simulated by the imaging model for a given δ and β distribution. The 2D discrete Radon transform operator $H \in \mathbb{R}^{PQ \times N}$ and its derivative $D \in \mathbb{R}^{PQ \times N}$ are constructed accordingly. Now, the discretized imaging model can be described as

$$[I(\beta, \delta)]_i = \exp \left(-\frac{4\pi}{\lambda} [H\beta]_i \right) \left[I_{TC}(\Delta\xi) - \frac{z_{od}}{M} I'_{TC}(\Delta\xi) [D\delta]_i \right], \quad (10)$$

where $i = 1, 2, \dots, PQ$ and $[\cdot]_i$ denotes the i -th component of the vector in the brackets.

Denote the measured intensity data by $I_m \in \mathbb{R}^{PQ}$. The distributions δ and β can be jointly estimated as [2]

$$(\hat{\beta}, \hat{\delta}) = \arg \min_{\beta, \delta} f(\beta, \delta) = \arg \min_{\beta, \delta} \|I_m - I(\beta, \delta)\|^2 + R(\beta, \delta), \quad (11)$$

where $f(\beta, \delta)$ describes the cost function. The gradients of the cost function can be computed as

$$\nabla_{\beta} f(\beta, \delta) = 2I'_{\beta}{}^*(I(\beta, \delta) - I_m) + \nabla_{\beta} R(\beta, \delta), \quad (12)$$

$$\nabla_{\delta} f(\beta, \delta) = 2I'_{\delta}{}^*(I(\beta, \delta) - I_m) + \nabla_{\delta} R(\beta, \delta), \quad (13)$$

where $I'_{\beta}{}^* \in \mathbb{R}^{N \times PQ}$ and $I'_{\delta}{}^* \in \mathbb{R}^{N \times PQ}$ are the discretized versions of $I'_{\beta}{}^*$ and $I'_{\delta}{}^*$, respectively. The results of the adjoint operators applied to a small vector $\epsilon \in \mathbb{R}^{PQ}$ are

$$I'_{\beta}{}^* \epsilon = -\frac{4\pi}{\lambda} H^* a \text{ and } I'_{\delta}{}^* \epsilon = -D^* b, \quad (14)$$

where H^* and D^* are adjoint operators of H and D , and

$$[a]_i = \left(I_{TC}(\Delta\xi) - \frac{z_{od}}{M} I'_{TC}(\Delta\xi) [D\delta]_i \right) \exp \left(-\frac{4\pi}{\lambda} [H\beta]_i \right) \mathbf{e}_i, \quad (15)$$

$$[b]_i = \frac{z_{od}}{M} I'_{TC}(\Delta\xi) \exp \left(-\frac{4\pi}{\lambda} [H\beta]_i \right) \mathbf{e}_i, \quad i = 1, 2, \dots, PQ. \quad (16)$$

3 Computer-simulation studies

3.1 Numerical phantom design

The numerical properties of the JR method were investigated via computer-simulation studies. A numerical phantom consisting of five ellipses was employed in the studies, as shown in Fig. 2. The absorption and refractive index describe physical properties of tissues. The β and δ values change when the tissue type changes. While their values differ, the β and δ distributions will share a common set of discontinuities. The β and δ values were designed to be non-homogeneous, which means the β/δ ratio varies for different portions of the object. The physical size of the phantom was 2.56 cm $\times$ 2.56 cm. When digitized, the phantom was modeled as a 256 pixel $\times$ 256 pixel digital object where each pixel had a dimension of 0.01 cm.

Another group of five numerical phantoms was generated separately for the convexity analysis. These phantoms were generated by placing together several randomly generated Gaussian ellipses, where the number of the ellipses, the center, axis lengths, peak absorption and refractive index values for each ellipse were chosen randomly from different Gaussian distributions.

3.2 Simulation of measurement data

The measured intensities were simulated based on Eqn.(3). The incident monochromatic X-ray beams had a wavelength of 0.2 nm. Tomographic measurement data were acquired every 0.5 degree. In total, 720 views were imaged, and at each view angle, 400 measured intensities were obtained. Neighboring measurements have a gap of 0.01 cm in between. The illumination curve was modeled based on a previous experimental calibration [8] and was fitted to a Gaussian distribution $I_{TC} = 0.13 + 0.87 \exp(-\xi^2/(1.74 \times 10^{-10} \text{m}^2))$ (arbitrary unit).

In generation of realistic measurement data, the line integrals in the Radon transform were calculated analytically, so the imaging model best matches the physical imaging process. Additionally, white Gaussian noise with a standard deviation equaling 1% of the noise-less measured intensity, which is similar to the noise level in experimental data [8], was added to the measurement data. The model mismatch between the forward imaging and reconstruction process is a source of reconstruction inaccuracy and is mitigated by the regularization in the reconstruction process.

For the convexity study where the topology of the optimization problem is of interest, whether or not the measurement data matches the real-world imaging process is irrelevant. Therefore, for ease of discussion and analysis, we generated idealized measurement data based on Eqn. (10), employing the same discretization in the Radon transformation as in the reconstruction algorithm. Noise was not added to idealized measurement data. Consequently, there was no model mismatch between the forward imaging and reconstruction processes.

3.3 Implementation of the reconstruction algorithm

In this work, a smoothed version of the total variation (TV) penalty [12] was utilized as the regularization term $R(\boldsymbol{\beta}, \boldsymbol{\delta})$. The penalty was defined to be a linear combination of the smoothed TV penalty of $\boldsymbol{\beta}$ and $\boldsymbol{\delta}$:

$$R(\boldsymbol{\beta}, \boldsymbol{\delta}) = l_1 R_{TV}(\boldsymbol{\beta}) + l_2 R_{TV}(\boldsymbol{\delta}), \quad (17)$$

where l_1 and l_2 are real-valued regularization parameters. The regularization parameters were chosen empirically by parameter sweeping.

The gradient descent algorithm was implemented for the JR method as described in [2]. The absorption and refractive index distributions were discretized into a 256×256 grid with a pixel size of 0.01 cm. A line search strategy [1] was employed for selecting step sizes in each iteration. The algorithm was stopped when the step size value dropped below a threshold of 1×10^{-20} .

4 Convexity

Because a gradient-descent algorithm was employed in the reconstruction method, the convexity of the cost function is important for analyzing convergence [1]. Because $R(\boldsymbol{\beta}, \boldsymbol{\delta})$ is typically defined as a convex function, we focus our study on the convexity of the data fidelity term $f_1(\boldsymbol{\beta}, \boldsymbol{\delta}) \equiv \|\mathbf{I}_m - \mathbf{I}(\boldsymbol{\beta}, \boldsymbol{\delta})\|^2$.

4.1 Theoretical analysis of convexity

Because f_1 is a function of $\boldsymbol{\beta}$ and $\boldsymbol{\delta}$, the problem can be analyzed by first investigating the convexity of the two functions: $f_{1\boldsymbol{\delta}}(\boldsymbol{\delta}) \equiv f_1(\boldsymbol{\beta}, \boldsymbol{\delta})$, given a fixed $\boldsymbol{\beta}$, and $f_{1\boldsymbol{\beta}}(\boldsymbol{\beta}) \equiv f_1(\boldsymbol{\beta}, \boldsymbol{\delta})$, given a fixed $\boldsymbol{\delta}$. A necessary but not sufficient condition for the original problem to be convex is that the two functions are convex.

The first function $f_{1\boldsymbol{\delta}}(\boldsymbol{\delta})$ can be expressed as

$$f_{1\boldsymbol{\delta}}(\boldsymbol{\delta}) = \|\mathbf{I}_m - [c_1 - c_2 \mathbf{D}\boldsymbol{\delta}]\|^2, \quad (18)$$

where c_1 and c_2 are real-valued constants independent of $\boldsymbol{\delta}$. Since $\mathbf{D}$ is a linear operator, this subproblem is quadratic and convex.

The second function $f_{1\beta}(\beta)$ is

$$f_{1\beta}(\beta) = \left\| \frac{\mathbf{I}_m}{c_3} - \exp\left(-\frac{4\pi}{\lambda} \mathbf{H}\beta\right) \right\|^2, \quad (19)$$

where c_3 is a positive constant independent of β . Although $\mathbf{H}$ is a linear operator, the exponential operation makes this function non-convex.

Because the second subproblem is non-convex, the original optimization problem is also non-convex and therefore the optimization may become trapped in a local minimum.

4.2 Empirical verification of non-convexity

To demonstrate that an inappropriate initial guess can lead the optimization to a local minimum in JR, a simulation test was conducted. The idealized measurement data were simulated for one of the random phantoms and the JR method was applied. No regularization was employed because the idealized measurement data was free of model mismatch and noise. Random initial guesses for β and δ were employed in this study. The initial guess for β was a vector whose elements follow the zero-mean Gaussian distribution with standard deviation $\sigma_\beta = 1 \times 10^{-10}$, and the initial guess for δ was also a vector containing elements following zero-mean Gaussian distribution but with a different standard deviation $\sigma_\delta = 1 \times 10^{-6}$. Because the idealized measurement data were employed, if the problem is convex, a global minimum should be reached by a gradient descent algorithm where the cost is decreased towards to 0. However, for the reconstruction test case, the cost function value at convergence was 0.0049, while a successful reconstruction of the same sample (as described in the next paragraph) can reach a final cost function value of 0.0001. In addition, as shown in Fig. 3, neither reconstruction profile exactly matches the phantom profile, indicating poor quality of the reconstructed images. The high cost function value at convergence and erroneous image profiles indicate the solution was trapped in a local minimum.

Fortunately, although the optimization problem is non-convex, an accurate reconstruction can still be obtained by use of a suitably defined initial guess. For all phantoms we have ever tested, we have not observed sub-optimal solutions when zero vectors were used as the initial guesses for both β and δ . The effectiveness of using zero initial guesses to avoid local minima can be verified by reconstructing random objects. For all of the five random phantoms described in the previous section, the JR method with no regularization was applied to the idealized measurement data. Both β and δ employed zero vectors as initial guesses. The reconstructed images and their comparison to their corresponding phantoms are given in Fig. 4. After convergence, all of the reconstructed images look close to the phantom and the image profiles match the phantom profiles well, suggesting that the optimization converged without falling into a local minimum. These reconstructed images demonstrate that the JR method can reliably estimate the absorption and refractive index distributions with a zero initial guess, despite the cost function being non-convex.

5 Cross-talk effect

To fully understand how the two reconstruction channels affect each other in the JR method, two studies were designed to investigate the ability of each channel to evolve relatively independently and robustly without too much reliance on the other channel.

5.1 The effect of error in one channel upon the other channel

This study was designed to investigate how errors in one channel (in the form of noise or blurring) may affect the other. One channel was fixed at a predefined suboptimal object property map, and the other channel was reconstructed by minimizing the functions shown in either Eq. (18) or Eq. (19).

The set of realistic measurement data were employed in this test. When reconstructing the refractive index distribution, the absorption distribution was fixed and selected to be the designed phantom with Gaussian noise (denoted by the noisy property map) or Gaussian blurring (denoted by the blurred property map) added. The Gaussian noise had zero mean and a standard deviation of $\sigma_n = 10^{-9}$ rad. The Gaussian blurring kernel was a Gaussian function with standard deviation of $\sigma_b = 2 \times 10^{-4}$ m. When reconstructing the absorption distribution, the refractive index distribution was again fixed and selected to be the designed phantom with Gaussian noise or Gaussian blurring added. This time, the Gaussian noise was zero mean and has a standard deviation of $\sigma_n = 10^{-7}$ rad. The Gaussian blurring kernel was the same. The object property maps for the fixed channels were employed because they resemble the intermediate reconstructed images that can be encountered during the iterative reconstruction process with different levels of regularization.

When the refractive index was fixed to be the noisy/blurry object property maps as shown in the first column of Fig. 5, the reconstructed absorption distributions are shown in the second and third columns of Fig. 5. Similarly, when the absorption was fixed, the employed object property maps for the absorption and the corresponding reconstructed refractive index distributions are shown in Fig. 6. Based on the observed reconstructed images, when reconstructing one channel, although the fixed object property maps for the other channel may not be ideal – they either look noisy or blurry – the to-be-reconstructed channel can still obtain a good estimate that is close to the phantom. In conclusion, the error in one channel does not impose a significant influence upon the other channel in the JR method. This observation suggests that the two channels in JR are evolving on relatively independent tracks, indicating the robustness of the method.

5.2 The effect of the value of one channel upon the the gradient of the other channel

To further test the hypothesis that cross-talk is weak in JR, the gradients calculated in the gradient descent iterations were considered. For the gradient descent algorithm, the gradients of both channels were calculated based on the values of both channels from the previous iteration. Consequently, if the gradient for one channel is almost independent of the values of the other channel, the two channels can be reasonably treated as independent. In this study, we compared gradients of one channel with a zero initial guess when different object property maps of the other channel were selected: (a) the accurate property map, which takes

the values of the designed phantom; (b) the zero property map, which is all 0s for the other channel; (c) the noisy property map and (d) the blurred property map as defined in Sec. 5.1. The gradients with respect to β are shown in Fig. 7, and the gradients with respect to δ are shown in Fig. 8. A comparison of the profiles of the gradients is given in Fig. 9. It can be seen that the gradients are almost the same when the other channel employs different object property maps, indicating little cross-talk effect.

6 Statistical properties of the images

In this component of study, the biases and variances of images reconstructed by use of the JR method with five different sets of regularization parameters were compared with the biases and variances of images reconstructed by use of the conventional two-step method. For the conventional method, we simulated images at two different sample mask positions for a total of 360 view angles in order to guarantee that both the JR method and the conventional method employed the same amount of measured intensity values as input. For the reconstruction step in the two-step approach where the two distributions were obtained with their corresponding sinograms, two methods were employed. The first method was the FBP method, which has been widely used for reconstruction of EIXPCT images. This method is denoted by CVN-FBP. The Ram-Lak filter was employed with high-frequency cutoffs to demonstrate the effect of regularization. The second method employed the gradient descent algorithm to solve two optimization problems with TV regularization for both channels:

$$\hat{\beta} = \arg \min_{\beta} \|S_1 - H\beta\|^2 + l'_1 R_{TV}(\beta), \quad (20)$$

$$\hat{\delta} = \arg \min_{\delta} \|S_2 - H\delta\|^2 + l'_2 R_{TV}(\delta). \quad (21)$$

where S_1 and S_2 are the sinograms corresponding to β and δ , respectively, and were obtained in the phase-retrieval step. Different regularization parameters l'_1 and l'_2 were tested. This method is denoted by CVN-TV. The regularization parameters and cutoff parameters employed are summarized in Table 1.

The bias and variances were estimated by analyzing the statistics of an ensemble of reconstructed images. An ensemble of 300 realistic measurement datasets for the same phantom with different realizations of Gaussian noise added were generated for each test case and reconstructed by the three reconstruction methods. Given the reconstructed β and δ images, the ensemble bias image and variance image are defined as

$$\text{bias}(\{u^k\}) = \left| \frac{1}{K} \sum_{k=1}^K u^k - u_0 \right|, \quad (22)$$

and

$$\text{var}(\{\mathbf{u}^k\}) = \frac{1}{K-1} \sum_{k=1}^K \left(\mathbf{u}^k - \frac{1}{K} \sum_{k=1}^K \mathbf{u}^k \right)^2. \quad (23)$$

Here, $\mathbf{u}$ can be either the reconstructed $\boldsymbol{\beta}$ or $\boldsymbol{\delta}$. The value K is the total number of samples in the ensemble which takes the value of 300 in this study, and $\mathbf{u}^k$ denotes the k -th sample ($k = 1, 2, \dots, K$). The notation $\{\mathbf{u}^k\}$ represents the whole ensemble of reconstructed $\boldsymbol{\beta}$ or $\boldsymbol{\delta}$, and $\mathbf{u}_0$ denotes the corresponding phantom image vector. The $(\cdot)^2$ and $|\cdot|$ operators describes element-wise operations.

To produce scalar-valued summary measures, the bias image and variance image were averaged over all pixels to produce two scalars describing the over-all bias and variance level of the reconstructed images. The acquired bias-variances relationships are given in Fig. 10. For all of the three methods, regularization parameters (or frequency cutoff values) should be carefully selected. Too small of a penalty value leads to high variance while too large penalty leads to high bias. Because both the bias and variance are simultaneously lower for the JR method than for the CVN-FBP method, it can be concluded that JR with proper regularization outperforms the CVN-FBP method in producing reconstructed images with higher quality.

The improvements may either come from the iterative, implicit phase retrieval process in the JR method, or the TV regularization employed. The bias-variance information from the CVN-TV method confirms that both factors are playing a role. First, the fact that CVN-TV method also outperforms CVN-FBP method suggests that the iterative reconstruction with TV regularization is effective in reducing noise compared with FBP implementing a high frequency cutoff. Second, the JR method has lower noise level of $\boldsymbol{\delta}$ compared with the conventional method even after the conventional method has employed the TV regularization. Note that the noise levels of $\boldsymbol{\beta}$ for CVN-TV and JR-TV are comparable. Therefore, switching from CVN-TV to JR-TV yields a net gain in overall image quality. This shows that the way the JR method separates the absorption and phase information is effective in improving image quality of the reconstructed refractive index distribution. The absorption-phase information separation is iteratively refined in the JR method, which is better than a one-step separation employed in the conventional method. In conclusion, both of the factors help improve image quality in the JR reconstruction.

7 Conclusion

In this project, the properties of a previously proposed JR reconstruction method for EIXPCT were explored. The optimization problem is non-convex, but the accurate reconstructed images of random phantoms suggests that the non-convexity will not be a significant obstacle if zero initial guesses are employed. This conclusion guides the selection of initial guesses in the JR algorithm. During the reconstruction process of JR, the two channels exhibit little cross-talk effect, which suggests the robustness and reliability of the algorithm. Finally, the noise statistics of reconstructed images were studied and compared.

JR with proper regularization yields smaller reconstruction error compared with conventional methods due to its iterative nature of implicit phase retrieval as well as its regularization. This observation demonstrates the advantage of the JR method over the conventional two-step approach.

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Fig. 1.

A typical setup for an EIXPCT system. S is the X-ray source. Two masks are employed: one sample mask denoted by A_1 , and one detector mask denoted by A_2 that has exactly the same period as the detector's pixel size P . The distance between the sample mask and the source is denoted by z_{so} , and that between the detector and the sample mask is denoted by z_{od} . The object can be rotated and the tomographic view angle is denoted by θ . The detector pixel's location is denoted by d . The sample mask A_1 can be moved vertically and the relative position of the sample mask to the detector mask is described by ξ .

Fig. 2.
The phantoms describing the (a) absorption distribution and (b) refractive index distribution.

Fig. 3.

The reconstructed images and the corresponding profiles corresponding to a random phantom when a noisy initial guess is employed. (a) The reconstructed absorption distribution. (b) The profiles (at 100-th row as indicated by the dotted line) of the reconstructed and the actual absorption distributions. (c) The reconstructed refractive index distribution. (d) The profiles (at 100-th row) of the reconstructed and the actual refractive index distributions. The mean square errors (MSE) are indicated below the figures.

Fig. 4.

The reconstructed images and their corresponding profiles for 5 random phantoms. Each column represents: (a) the reconstructed absorption distribution, (b) the profiles (at 100-th row) of the reconstructed and the actual absorption distributions, (c) the reconstructed refractive index distribution, (d) the profiles (at 100-th row) of the reconstructed and the actual refractive index distributions. The mean square errors (MSE) are indicated below the figures.

Fixed property map of δ Reconstructed β A profile of reconstructed β **Fig. 5.**

When the refractive index distribution δ was fixed at a erroneous object property map as shown in the first column (noisy property map in the first row, blurry property map in the second row), the absorption distribution β was reconstructed. The reconstructed absorption distribution images are shown in the second column and their corresponding profiles (at 100-th row) are shown in the third column.

Fig. 6.

When the absorption distribution β was fixed at a erroneous object property map as shown in the first column (noisy property map in the first row, blurry property map in the second row), the refractive index distribution δ was reconstructed. The reconstructed refractive index distribution images are shown in the second column and their corresponding profiles (at 100-th row) are shown in the third column.

Fig. 7.

When β takes a zero vector as the initial guess, the gradient of the JR cost function with respect to β are displayed when different fixed property maps are employed for δ

Fig. 8.

When δ takes a zero vector as the initial guess, the gradient of the JR cost function with respect to δ are displayed when different fixed property maps are employed for β

Fig. 9. Profiles through the gradient maps in (a) Fig. 7 and (b) Fig. 8.

Bias-variance plot for reconstructed β Bias-variance plot for reconstructed δ

Fig. 10.

The bias-variance plot for reconstructed β and δ . For the CVN-FBP methods, different high frequency cutoff values were selected for the Ram-Lak filter in the FBP algorithm. For the CVN-TV method, different regularization parameters were employed for the reconstruction of β and δ separately. For the JR method, different sets of regularization parameters were employed. The ratio between the β penalty (l_1) and δ penalty (l_2) was kept constant ($l_2/l_1 = 0.002$). The parameters tested are summarized in Table 1. Note that for JR-TV, the ratio of l_1 and l_2 is a constant so only the l_2 parameters are listed.

Table 1

The regularization parameters employed in the bias-variance analysis (from no cutoff to large cutoff for CVN-FBP, from small regularization to large regularization for CVN-TV and JR TV, from right to left for all three methods as shown in the figure)

cutoff frequency (CVN-FBP)	TV parameter for β (CVN-TV)	TV parameter for δ (CVN-TV)	TV parameter for δ (JR-TV)
1, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3	1×10^{-16} , 3×10^{-16} , 1×10^{-15} , 3×10^{-15} , 1×10^{-14}	1×10^{-13} , 3×10^{-13} , 1×10^{-12} , 3×10^{-12} , 1×10^{-11}	4×10^2 , 2×10^3 , 4×10^3 , 1×10^4 , 5×10^4