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GPU-based Green's function simulations of shear waves generated by an applied acoustic radiation force in elastic and viscoelastic models

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Abstract

Shear wave calculations induced by an acoustic radiation force are very time-consuming on desktop computers, and high-performance graphics processing units (GPUs) achieve dramatic reductions in the computation time for these simulations. The acoustic radiation force is calculated using the fast near field method (FNM) and the angular spectrum approach (ASA), and then the shear waves are calculated in parallel with Green's functions on a GPU. This combination enables rapid evaluation of shear waves for push beams with different spatial samplings and for apertures with different $f/\#$. Relative to shear wave simulations that evaluate the same algorithm on an Intel i7 desktop computer, a high performance nVidia GPU reduces the time required for these calculations by a factor of 45 and 700 when applied to elastic and viscoelastic shear wave simulation models, respectively. These GPU-accelerated simulations also compared to measurements in different viscoelastic phantoms, and the results are similar. For parametric evaluations and for comparisons with measured shear wave data, shear wave simulations with the Green's function approach are ideally suited for high-performance GPUs.

1 Introduction

Shear wave elasticity imaging (SWEI) [36] is an emerging modality for noninvasively imaging the mechanical properties of soft tissue including breast [39], liver [31], kidney [2], thyroid [37], heart [10], and blood vessels [11]. In SWEI, an acoustic radiation force push beam is applied to soft tissue, and the transient shear wave data is then collected with high frame-rate ultrasound [36]. Other methods for ultrasound elastography include acoustic radiation force imaging (ARFI) [25], supersonic shear imaging (SSI) [4], spatially modulated ultrasound radiation force (SMURF) [19], crawling wave spectroscopy (CWS) [14, 15], and comb-push ultrasound shear elastography (CUSE) [38].

Numerical simulation approaches are useful for validating and improving shear wave imaging techniques. At present, most of the numerical simulations of shear waves generated

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by an applied acoustic radiation force employ the finite element (FEM) method. FEM models have been developed for ARFI imaging [30], for simulating the shear wave response to acoustic radiation force in breast [24] and liver [31], for modeling shear wave propagation in a viscoelastic model [18], for evaluating thermal effects of an applied acoustic radiation force in soft tissue [29], and other applications [12, 28, 34]. Other simulation options include the finite difference method [17,27], the pseudospectral method [32], and the Green's function approach [5, 6, 8, 13]. The Green's function approach, which is applicable to simulations of elastic [6] and viscoelastic media [5], is appropriate when shear wave calculations are needed at only a few observation points in space, as opposed to other numerical methods that calculate the entire transient response.

Despite the advantages for a small number of points, the Green's function approach is very time-consuming for shear wave simulations on a desktop computer. To accelerate these calculations, a more efficient calculation approach is needed. Since Green's functions are readily evaluated in parallel, these calculations are ideal for graphics processing units (GPUs). The GPU has previously been established as an effective approach for accelerating medical ultrasound calculations using the angular spectrum method [41], the k-space simulation method [40], and other calculations [3, 9, 26, 33, 40, 41]. Similar performance enhancements are anticipated for shear wave calculations on the GPU.

After Navier's equation and the Green's function approach are introduced, a GPU-based parallel algorithm that calculates the shear wave motion induced by an acoustic radiation force is described for simulations in elastic and viscoelastic models. The computation times of serial and parallel Green's function calculations are evaluated and compared on a desktop computer and on a high-performance GPU. The convergence of the Green's function calculations is evaluated for push beams with different spatial samplings and with different $f/\#$. In addition, simulated shear waves are compared with shear waves measured in different elastic and viscoelastic shear wave phantoms. The results suggest that the Green's function approach is ideal for simulating shear waves with high-performance GPUs.

2 Theory

2.1 Navier's Equation

Shear waves induced by an acoustic radiation force in an elastic, isotropic, homogeneous, and nearly incompressible medium are described by Navier's equation [1]

$$(\lambda + 2\mu) \nabla (\nabla \cdot \mathbf{u}) - \mu \nabla \times (\nabla \times \mathbf{u}) + \mathbf{f} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}, \quad (1)$$

where $\mathbf{u}$ is the particle displacement, $\mathbf{f}$ is the body force, and t is the time. In Eq. 1, the bulk modulus and shear modulus are indicated by λ and μ , respectively, and ρ is the density. When the Kelvin-Voigt model for viscous loss is included, Eq. 1 becomes [5, 7]

$$\left(\lambda + 2\mu + (\eta_p + 2\eta_s)\frac{\partial}{\partial t}\right)\nabla(\nabla \cdot \mathbf{u}) - \left(\mu + \eta_s\frac{\partial}{\partial t}\right)\nabla \times (\nabla \times \mathbf{u}) + \mathbf{f} = \rho\frac{\partial^2 \mathbf{u}}{\partial t^2}, \quad (2)$$

where the parameters η_p and η_s denote the bulk viscosity and shear viscosity, respectively.

2.2 Green's functions calculations

The Green's function model for Eq. 1 calculates the displacement induced by a spatially and temporally impulsive body force. When the acoustic radiation force push is applied in the z -direction, the z -component of the Green's function that describes the resulting displacement is represented by [1]

$$g_z(\mathbf{r}, t) = g_z^P(\mathbf{r}, t) + g_z^S(\mathbf{r}, t) + g_z^C(\mathbf{r}, t), \quad \text{where} \quad (3)$$

$$g_z^P(\mathbf{r}, t) = \frac{1}{4\pi\rho c_p^2} \frac{z^2}{r^3} \delta\left(t - \frac{r}{c_p}\right), \quad (4)$$

$$g_z^S(\mathbf{r}, t) = \frac{1}{4\pi\rho c_s^2} \left(1 - \frac{z^2}{r^2}\right) \frac{1}{r} \delta\left(t - \frac{r}{c_s}\right), \quad (5)$$

$$g_z^C(\mathbf{r}, t) = \frac{1}{4\pi\rho} \left(\frac{3z^2}{r^2} - 1\right) \frac{1}{r^3} t \left[H\left(t - \frac{r}{c_p}\right) - H\left(t - \frac{r}{c_s}\right) \right]. \quad (6)$$

In Eqs. 3 – 6, $g_z^P(\mathbf{r}, t)$, $g_z^S(\mathbf{r}, t)$, and $g_z^C(\mathbf{r}, t)$ represent the compressional, shear, and the coupling components, respectively, of the Green's function for the elastic model in Eq. 1, where $\mathbf{r}$ denotes a spatial point in Cartesian coordinates, the compressional wave speed is $c_p = \sqrt{(\lambda + 2\mu)/\rho}$, and the shear wave speed is $c_s = \sqrt{\mu/\rho}$. In the above expressions, $\delta(\cdot)$ denotes the Dirac delta function, $H(\cdot)$ denotes the Heaviside (unit step) function, $r = |\mathbf{r}|$ is the distance between the source point and the observation point, and z is the distance between the source point and the observation point in the z -direction. An approximate viscoelastic Green's function model is [5]

$$g_z(\mathbf{r}, t) = g_z^P(\mathbf{r}, t) + g_z^S(\mathbf{r}, t) + g_z^C(\mathbf{r}, t), \quad \text{where} \quad (7)$$

$$g_z^P(\mathbf{r}, t) = \frac{1}{4\pi\rho c_p} \frac{1}{\sqrt{2\pi\nu_p t}} \frac{z^2}{r^3} e^{-(c_p t - r)^2/(2\nu_p t)}, \quad (8)$$

$$g_z^S(\mathbf{r}, t) = \frac{1}{4\pi\rho c_s} \frac{1}{\sqrt{2\pi\nu_s t}} \left(1 - \frac{z^2}{r^2}\right) \frac{1}{r} e^{-(c_s t - r)^2/(2\nu_s t)}, \quad (9)$$

$$g_z^C(\mathbf{r}, t) = \frac{1}{4\pi\rho} \left(\frac{3z^2}{r^2} - 1\right) \frac{1}{r^3} [I_p(\mathbf{r}, t) + I_s(\mathbf{r}, t)], \quad (10)$$

$$I_p(\mathbf{r}, t) = \frac{\sqrt{\nu_p t}}{\sqrt{2\pi c_p}} \left[e^{-tc_p^2/(2\nu_p)} - e^{-(c_p t - r)^2/(2\nu_p t)} \right] + \frac{t}{2} \left[\operatorname{erf}\left(\frac{c_p t}{\sqrt{2\nu_p t}}\right) - \operatorname{erf}\left(\frac{c_p t - r}{\sqrt{2\nu_p t}}\right) \right], \quad (11)$$

$$I_s(\mathbf{r}, t) = \frac{\sqrt{\nu_s t}}{\sqrt{2\pi c_s}} \left[e^{-tc_s^2/(2\nu_s)} - e^{-(c_s t - r)^2/(2\nu_s t)} \right] + \frac{t}{2} \left[\operatorname{erf}\left(\frac{c_s t}{\sqrt{2\nu_s t}}\right) - \operatorname{erf}\left(\frac{c_s t - r}{\sqrt{2\nu_s t}}\right) \right], \quad (12)$$

where $\nu_p = (\eta_p + 2\eta_s)/\rho$, $\nu_s = \eta_s/\rho$, and $\operatorname{erf}(\cdot)$ denotes the error function.

2.3 Simulating shear waves with Green's functions

To simulate shear waves generated by an acoustic radiation force, the Green's function and the acoustic radiation force are convolved in time and space by numerically evaluating the superposition integral

$$u_z(\mathbf{r}, t) = \iiint_{\mathbf{V}'} \int_{\tau} g_z(\mathbf{r} - \mathbf{r}', t - \tau) f(\mathbf{r}', \tau) d\tau d\mathbf{V}', \quad (13)$$

where $u_z(\mathbf{r}, t)$ is the displacement at $\mathbf{r}$, $g_z(\mathbf{r}, t)$ is the Green's function for either the elastic or viscoelastic model, and $f(\mathbf{r}', t)$ is the acoustic radiation force. For these calculations, the acoustic radiation force is represented by $f(\mathbf{r}', t) = f(\mathbf{r}') w(t)$, where $f(\mathbf{r}')$ is the steady-state acoustic radiation force obtained from the intensity calculated in FOCUS, and $w(t)$ describes the time dependence of the acoustic radiation force. In this model, $w(t)$ is the same at every point in space and is given by $w(t) = \operatorname{rect}[(t - T/2)/T]$. The expression $\operatorname{rect}[(t - T/2)/T]$ indicates that the 'push' beam is applied starting at time $t = 0$ and ending at time $t = T$,

where the variable T describes the duration of the push, and equal weighting is applied at each point in time.

For shear wave simulations with the elastic model, the convolution in time between the Green's function $g_z(\mathbf{r}-\mathbf{r}', t)$ in Eqs. 3 – 6 and $w(t)$ is calculated analytically. This step, which eliminates all of the delta functions from the result, is convenient for numerical calculations. The results are then given by

$$h_{z,T}(\mathbf{r}, T) = \int_{\tau} g_z(\mathbf{r}, t - \tau) w(\tau) d\tau = h_{z,T}^P(\mathbf{r}, t) + h_{z,T}^S(\mathbf{r}, t) + h_{z,T}^C(\mathbf{r}, t), \text{ where} \quad (14)$$

$$h_{z,T}^P(\mathbf{r}, t) = \frac{1}{4\pi\rho c_p^2} \frac{z^2}{r^3} \text{rect}\left(\frac{t - r/c_p - T/2}{T}\right), \quad (15)$$

$$h_{z,T}^S(\mathbf{r}, t) = \frac{1}{4\pi\rho c_s^2} \left(1 - \frac{z^2}{r^2}\right) \frac{1}{2} \text{rect}\left(\frac{t - r/c_s - T/2}{T}\right), \quad (16)$$

$$\begin{aligned} h_{z,T}^C(\mathbf{r}, t) = & \frac{1}{4\pi\rho} \left(\frac{3z^2}{r^2} - 1 \right) \frac{1}{r^3} \\ & \times \left[\frac{1}{2} \left(t^2 - \frac{r^2}{c_p^2} \right) \text{rect}\left(\frac{t - r/c_p - T/2}{T}\right) + \frac{1}{2} (2tT - T^2) H\left(t - \frac{r}{c_p} - T\right) \right. \\ & \left. + \frac{1}{2} \left(\frac{r^2}{c_s^2} - (t - T)^2 \right) \text{rect}\left(\frac{t - r/c_s - T/2}{T}\right) - \frac{1}{2} (2tT - T^2) H\left(t - \frac{r}{c_s}\right) \right], \end{aligned} \quad (17)$$

where $h_{z,T}(\mathbf{r}, t)$ represents the transient displacement of a point body force in the z -direction for the elastic model when the window function is given by $w(t) = \text{rect}[(t - T/2)/T]$, and $h_{z,T}^P(\mathbf{r}, t)$, $h_{z,T}^S(\mathbf{r}, t)$, and $h_{z,T}^C(\mathbf{r}, t)$ are the compressional, shear, and coupling components of the resulting transient displacement, respectively. The remaining 3D integral $\iiint_{\mathbf{V}'} h_{z,T}(\mathbf{r} - \mathbf{r}', t) f(\mathbf{r}') d\mathbf{V}'$ then describes the spatial convolution between $h_{z,T}(\mathbf{r}, t)$ and the steady-state acoustic radiation force $f(\mathbf{r}')$, which is calculated numerically by evaluating

$$u_z^E(\mathbf{r}, t) = \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} \sum_{k=1}^{N_z} [h_{z,T}(\mathbf{r} - \mathbf{r}', t)] f(\mathbf{r}') \Delta x \Delta y \Delta z. \quad (18)$$

In this triple sum, N_x , N_y , and N_z describe the number of spatial points in each direction for the simulated steady-state 3D acoustic radiation force, and Δx , Δy , and Δz are the spatial sampling intervals in the x , y , and z directions for the simulated acoustic radiation force. The superscript in $u_z^E(\mathbf{r}, t)$ indicates the shear wave particle displacement for the elastic model.

For shear wave simulations with the viscoelastic model, no delta functions appear in the Green's function given in Eqs. 7 – 12, so no analytical simplifications are needed. For shear wave calculations with Eqs. 7 – 12 and Eq. 13, the convolution in space is evaluated first, then the convolution in time is computed. The discretized representation of Eq. 13 for numerical calculations with the viscoelastic model is

$$u_z^V(\mathbf{r}, t) = \sum_{i=1}^{N_t} \left[\sum_{m=1}^{N_z} \sum_{n=1}^{N_y} \sum_{k=1}^{N_x} [g_z(\mathbf{r} - \mathbf{r}', t - \tau)] f(\mathbf{r}') \Delta x \Delta y \Delta z \right] w(\tau) \Delta \tau, \quad (19)$$

where N_t is the number of time samples in $w(t)$, and $\Delta \tau$ is the temporal sampling interval. The superscript in $u_z^V(\mathbf{r}, t)$ indicates the shear wave particle displacement calculated for the viscoelastic model.

3 Numerical Calculations

3.1 Acoustic radiation force

In shear wave elasticity imaging (SWEI), shear waves are generated by focused or unfocused ultrasound beams. For three-dimensional (3D) shear wave calculations in an isotropic, homogeneous simulation model, the pressure field is calculated first, and then the radiation force is obtained from the computed pressure field, where the radiation force then provides the input to Eq. 1 or Eq. 2 for calculations of the shear wave particle displacements. In each of the following simulations, the pressure, radiation force, and shear wave particle displacements are all evaluated in 3D.

The 3D pressure fields are calculated in FOCUS, the “Fast Object-oriented C++ Ultrasound Simulator” (<https://www.egr.msu.edu/~fultras-web>). In these simulations, the fast nearfield method computes the input pressure plane [20, 21], and then the angular spectrum approach determines the pressure field throughout the remaining 3D volume [42,43]. For each of the shear wave simulations evaluated here, a Philips L7-4 linear ultrasound transducer array (Philips Healthcare, Andover, MA) generates the acoustic radiation force “push”, where the array consists of 128 curved transducer elements with an elevation focus of 25 mm. The width of each ultrasound transducer element is 0.283 mm, the height is 7 mm, and the kerf between adjacent elements is 0.025 mm. The transducer array face is centered at $z = 0$ in the x - y plane, the center of each element is located on the y -axis, and the normal at the center of the array is coincident with the z -axis. After the 3D pressure is calculated, the 3D intensity is computed with $I = |p|^2 / (2\rho c_p)$, and the acoustic radiation force $f(\mathbf{r}, t)$ is determined from [25]

$$f(\mathbf{r}, t) = \frac{2\alpha I(\mathbf{r}, t)}{c_p}, \quad (20)$$

where α is the ultrasound attenuation coefficient, and $I(\mathbf{r}, t)$ is the acoustic intensity.

3.2 Parallel implementation

The convolution integral in Eq. 13 is very time-consuming when implemented as a serial calculation. For $N_x \times N_y \times N_z$ observation points and for N_t time samples, Eq. 13 is evaluated at $N_x \times N_y \times N_z \times N_t$ points in space and time. In practice, this is a problem for calculations on desktop or laptop computers for large values of $N_x \times N_y \times N_z \times N_t$. However, Green's function calculations are independent at each point in time and at each point in space, which suggests that Green's function calculations are appropriate for parallel computations and that a graphics processing unit (GPU) is ideal for this problem.

On the GPU, which contains a large number of independent processors, the jobs are broken into several pieces that are distributed to different processors for parallel computations. When these calculations are implemented properly, GPUs achieve a considerable speedup compared to a single serial CPU. First, the acoustic radiation force, which is pre-calculated and stored in a data file, is loaded into the CPU (host) memory. Then, the computed acoustic radiation force is copied into the GPU (device) global memory. The kernel function, which describes the superposition algorithm, is then called and the job is separated into multiple threads that are evaluated on different processors in parallel on the GPU. For each thread, the shear wave particle displacement is calculated at a single observation point and a single moment in time. Each thread obtains the acoustic radiation force from global memory, whereas the calculated shear wave particle displacement result is first stored temporarily in local memory for each thread and then copied into global memory after the calculation is completed. After all of the threads are finished, the results are copied and returned to the host memory. The CPU performs the post-processing, and then the result is stored in an output data file. These calculations are implemented using the Compute Unified Device Architecture (CUDA) programming language on an nVidia (Santa Clara, CA) Kepler K20 GPU.

4 Results

Numerical simulations are performed serially on an Intel(R) Core(TM) i7-2600 3.40GHz CPU and in parallel on an nVidia Tesla K20 GPU. In these simulations, 3D pressure fields are calculated using the fast nearfield method combined with the angular spectrum approach, and shear waves are then computed for the 3D acoustic radiation force that is obtained from the 3D pressure distribution. In each calculation of the acoustic radiation force, the compressional wave speed c_p is 1500 m/s, and the attenuation coefficient α_p is 0.5 dB/cm/MHz for the compressional wave component. The excitation frequency for the transducer array is $f_0 = 4.09$ MHz. For the shear wave simulations, the shear elasticity μ is 2.25 kPa, while the viscosity η_s is equal to 0 Pa·s for simulations with the elastic model and

1 Pa·s for simulations with the viscoelastic model. The duration of the acoustic radiation force push in each simulation is 200 μ s.

4.1 Simulated shear wave particle displacements at discrete observation points

To demonstrate the similarities and differences between shear waves simulated with the elastic and viscoelastic models, shear wave particle displacements are calculated at 8 different observation points located at the same depth. The depth selected for each observation point is the same as the depth of the focal peak for the push beam. The observation points are equally spaced in the lateral direction at this depth, where the distance from the peak of the push beam to each observation point in the x -direction ranges from 5 to 40 mm with a 5 mm increment in the $y = 0$ plane. These simulations are evaluated for an $f/2$ push beam with 40 transducer elements that is focused on-axis at a depth of 25 mm. The focal peak for this push beam is located at a depth of 22.9 mm. The push beam and the locations of the observation points are shown in Figure 1. At each observation point, the shear wave particle displacements are calculated at $t = 40$ ms with a temporal sampling frequency of 10 kHz.

Figure 2 shows the simulated shear wave particle displacements for the push beam in Figure 1, where the simulated shear wave particle displacements for the elastic and viscoelastic models with $\mu = 2.25$ kPa and $\eta_s = 0$ or 1 Pa·s are evaluated at the observation points shown in Figure 1. In Figure 2, the peaks for different observation points appear at different times. Figure 2 also shows that the peak times for the normalized shear wave particle displacements calculated with the elastic model are different from the peak times for the viscoelastic model. Furthermore, at the same observation point, the peaks calculated for the viscoelastic model arrive later than the peaks for the elastic model. In addition, the differences in the peak times between successive observation points are not the same for simulations with the elastic model and the viscoelastic model. For shear wave simulations with the elastic model, the differences in the peak times for adjacent observation points are 3.34 ms, 3.37 ms, 3.36 ms, 3.35 ms, 3.37 ms, 3.35 ms, and 3.38 ms for the observation points with x coordinates equal to (5mm and 10mm), (10mm and 15mm), (15mm and 20mm), (20mm and 25mm), (25mm and 30mm), (30mm and 35mm), and (35mm and 40mm), respectively. For simulations of shear waves with the viscoelastic model, the peak time differences for the same pairs of observation points are 3.69 ms, 3.61 ms, 3.54 ms, 3.42 ms, 3.33 ms, 3.30 ms, and 3.29 ms, respectively. The values of the full width at half maximum (FWHM) for the shear wave particle displacements calculated at these same locations are 1.06 ms, 1.02 ms, 0.99 ms, 0.97 ms, 1.00 ms, 1.02 ms, 1.06 ms, and 1.08 ms for the elastic model, and the FWHM values at these same observation points are 5.50 ms, 6.70 ms, 7.33 ms, 7.68 ms, 7.94 ms, 8.22 ms, 8.54 ms, and 8.89 ms for the viscoelastic model.

4.2 Computation times for serial and parallel calculations

To evaluate the computation times of the serial and parallel implementations, the simulations are programmed and evaluated in C and CUDA, respectively. Computation times are determined for shear wave simulations with the elastic and viscoelastic models using the push beam shown in Figure 1. In these simulations, the number of observation points ranges

from 20 to 200 with an increment of 20. The observation points are sampled uniformly between 5 mm and 40 mm in the x direction, the y -coordinate is equal to zero, and the z coordinate is equal to 22.9 mm. Thus, the end points are the same as in Figure 1, but the region in between is more densely sampled. For each observation point, the shear wave particle displacements are calculated at 400 temporal samples for 40 ms, which yields a temporal sampling rate of 10 kHz. Eq. 13 is evaluated once at each temporal sample and at each observation point, so the total number of spatial and temporal points at which the Green's functions are superposed ranges from 8,000 (20 spatial points $\times$ 400 temporal points) to 80,000 (200 spatial points $\times$ 400 temporal points). Furthermore, the discretized 3D acoustic radiation force contains $109 \times 55 \times 218 = 1,306,910$ source points, so the total number of Green's function evaluations with Eqs. 7 – 12 or Eqs. 14 – 17 ranges from 1.05×10^{10} to 1.05×10^{11} in these simulations. The serial and parallel codes are evaluated and the computation times are measured and stored. The serial C code is implemented on a desktop computer with an Intel(R) Core(TM) i7-2600 3.40GHz CPU, and the parallel CUDA code is implemented on an nVidia Tesla K20 GPU at Michigan State University's high performance computing center (<https://icer.msu.edu/>).

Figure 3 describes the computation time versus the number of observation points for elastic and viscoelastic shear wave simulation models. The results in Figure 3 show that the parallel implementation is much faster than the serial implementation. For the shear wave simulations with the elastic model, the parallel implementation is about 45 times faster than the serial implementation, while for shear wave simulations with the viscoelastic model, the parallel implementation is about 700 times faster than the serial implementation. Moreover, the simulations of shear waves with the elastic model are much faster than the simulations of shear waves with the viscoelastic model for both the serial and parallel implementations. For serial calculations, the simulation of shear waves with the elastic model is about 50 times faster than the simulation of shear waves with the viscoelastic model. In the parallel implementation, the simulation of shear waves with the elastic model is around 2 times faster than the simulation of shear waves with the viscoelastic model. Figure 3 also indicates that these calculations are consistently faster with the elastic model because of the additional time required to compute the error function in Eqs. 7 – 12.

4.3 Convergence of Green's function calculations

To assess the numerical convergence of these Green's function calculations, several simulations of shear waves induced by push beams with different spatial sampling rates and different $f/\#$ are evaluated. Different ultrasound beams are simulated by selectively activating contiguous groups of transducer elements. For unfocused beam calculations, 12 elements are activated with equal phases and amplitudes. For focused beam calculations, the number of active array elements varies with the $f/\#$. Specifically, 54, 40, and 27 elements are excited and focused on-axis at a depth of 25 mm for values of the $f/\#$ equal to 1.5, 2, and 3, respectively. Figure 4 depicts four different simulated ultrasound beams in the $y=0$ plane. Figure 4a shows the focused ultrasound beam with $f/1.5$, Figure 4b shows the focused ultrasound beam with $f/2$, Figure 4c shows the focused ultrasound beam with $f/3$, and Figure 4d shows the unfocused beam (f/∞). The $f/2$ push beam shown in Figure 4b is the same configuration depicted in Figure 1.

For each push beam, the acoustic radiation force is calculated with spatial samplings of 4λ , 2λ , λ , $\lambda/2$, $\lambda/4$, $\lambda/8$, $\lambda/16$, and $\lambda/18$, where λ is the wavelength of the compressional wave ($\lambda = c_p/f_0 = 0.3667$ mm). The push beams with spatial samplings less than or equal to $\lambda/2$ are calculated with FOCUS as described in Section 3, and the push beams that are more coarsely sampled are obtained by down-sampling the push beam that is computed with $\lambda/2$ sampling. The push beam sampled at 4λ , which is the most coarsely sampled push beam, consists of $N_x^s \times N_y^s \times N_z^s = 13 \times 7 \times 27 = 2,457$ source points, while the most finely sampled push beam sampled at $\lambda/18$ consists of $N_x^s \times N_y^s \times N_z^s = 981 \times 491 \times 1963 = 945,520,173$ source points. The simulated shear wave particle displacement induced by the $\lambda/18$ sampled push beam is used as the reference, and the simulated shear wave particle displacements induced by all of the other push beams are compared with the reference shear wave particle displacement to evaluate the convergence of these Green's function calculations. The percent difference between the two shear wave particle displacement waveforms is defined as

$$\% \text{ difference} = \frac{\|u(\mathbf{r}, t) - u_{ref}(\mathbf{r}, t)\|}{\|u_{ref}(\mathbf{r}, t)\|} \times 100\%, \quad (21)$$

where $u(\mathbf{r}, t)$ is the simulated shear wave particle displacement induced by a push beam at a single observation point, $u_{ref}(\mathbf{r}, t)$ is the reference shear wave particle displacement induced by the push beam sampled at $\lambda/18$ at the same observation point, and $\|\cdot\|$ denotes the L^2 -norm of the time-domain waveform.

For the shear wave particle displacement simulated by each push beam, the percent difference is evaluated at 8 observation points. For each push beam, the observation points are all located at the same depth, and the depth selected for each observation point is the same as the depth of the focal peak. For the push beams with $f/1.5$, $f/2$, $f/3$, and f/∞ , these depths are equal to 24.5 mm, 22.9 mm, 20.0 mm, and 16.6 mm, respectively. The observation points are equally spaced in the lateral direction, where the distance from the peak of the push beam to each observation point in the x -direction ranges from 5 to 40 mm with a 5 mm increment in the $y = 0$ plane. The largest percent difference is retained to represent the worst case value across these 8 locations. The percent difference is evaluated for simulated shear waves with the elastic model using $\mu = 2.25$ kPa and $\eta_s = 0$ Pa·s and for simulated shear waves with the viscoelastic model using $\mu = 2.25$ kPa and $\eta_s = 1$ Pa·s for the four push beams with $f/1.5$, $f/2$, $f/3$, and f/∞ described in section 3.1.

The results obtained with the elastic model are illustrated in Figure 5 for the four push beams evaluated with different spatial samplings. In each of these simulations, the percent difference decreases when the spatial sampling rate increases. For the $f/1.5$ push beams sampled at λ , the percent difference is 16.4%, the percent difference is 7.04% when the spatial sampling is $\lambda/2$, and the percent difference is 2.50% when the spatial sampling of the push beam is $\lambda/16$. The percent differences for the $f/2$ push beams sampled at λ , $\lambda/2$, and $\lambda/16$ are 13.5%, 4.53%, and 0.98%, respectively. The percent differences for the $f/3$ push beams sampled at λ , $\lambda/2$, and $\lambda/16$ are 9.96%, 3.63%, and 0.23%, respectively. The percent

differences for the unfocused push beams sampled at 4λ , $\lambda/2$, and $\lambda/16$ are 6.00%, 2.70%, and 0.29%, respectively.

The percent differences in the shear wave particle displacements simulated for the viscoelastic model with $\mu = 2.25$ kPa and $\eta_s = 1$ Pa·s are displayed in Figure 6 for the same four push beams. In Figure 6, the percent difference also tends to decrease as the spatial sampling increases. For the f/1.5 push beam sampled at λ , the percent difference is 6.15%, the percent difference is 4.74% when the spatial sampling is $\lambda/2$, and the percent difference is 1.00% when the spatial sampling of the push beam is $\lambda/16$. The percent differences for the f/2 push beam sampled at λ , $\lambda/2$, and $\lambda/16$ are 2.56%, 1.26%, and 0.40%, respectively. The percent differences for the f/3 push beam sampled at λ , $\lambda/2$, and $\lambda/16$ are 1.15%, 0.92%, and 0.19%, respectively. The percent differences for the unfocused push beam sampled at λ , $\lambda/2$, and $\lambda/16$ are 1.35%, 0.83%, and 0.34%, respectively.

4.4 Comparisons between simulated and measured shear waves

Shear waves are also measured in viscoelastic shear wave phantoms and compared with the GPU-simulated shear waves. The measurements are performed with a Verasonics ultrasound system (Verasonics Inc., Redmond, WA) and an L7-4 linear ultrasound transducer array, which is the same transducer array that is described and modeled in section 3.1. To induce the shear waves, an f/2 push beam in Figure 1 that excites 40 contiguous elements is focused at 25 mm with a center frequency of 4.09 MHz. The push beam is applied for 200 μ s, and then the Verasonics system is switched to plane wave imaging mode for shear wave measurements. In-phase/quadrature (IQ) data is measured for 10 ms with a frame rate of 11.76 kHz in a two-dimensional (2D) region ranging from -19.40 mm to 19.40 mm in the lateral direction and from 0 to 39.58 mm in the axial direction with a spatial sampling of 0.154 mm in each direction. Then, shear wave particle velocities are obtained from the IQ data by calculating one-dimensional (1D) autocorrelations between consecutive frames [16]. A 2D median filter with a 3 by 3 kernel is then applied to the measured shear wave particle velocities for each frame. Three different viscoelastic shear wave phantoms from Computerized Imaging Reference Systems (CIRS), Inc (Norfolk, Virginia, US) are used for these measurements, namely shear wave phantoms E3962-1, E3962-2, and E3962-3.

After the shear waves in the viscoelastic shear wave phantoms are measured, the results are compared to simulated shear waves evaluated within the same 2D region at three different points in time. For phantoms E3962-1 and E3962-2, the shear wave particle displacements are displayed at 3 ms, 6 ms, and 9 ms, while for phantom E3962-3, the shear wave particle displacements are displayed at 2 ms, 3 ms, and 4 ms. Different time points are required for phantom E3962-3 because the shear elasticity for this phantom is much larger, so the shear waves propagate out of the field of view much more quickly. The shear waves are simulated for several different combinations of the shear elasticity and shear viscosity. For phantom E3962-1, the shear elasticity and the shear viscosity are relatively small, so the shear elasticity in the simulations ranges from 0.1 to 0.5 kPa with an increment of 0.1 kPa, and the shear viscosity ranges from 0 to 0.5 Pa·s with an increment of 0.1 Pa·s. For shear wave phantoms E3962-2 and E3962-3, the shear elasticity in the simulations ranges from 2 to 5 kPa with an increment of 0.1 kPa, and the shear viscosity ranges from 0 to 2.5 Pa·s with an

increment of 0.1 Pa·s. For each combination of parameters, the simulation of the shear wave particle velocity in a 2D plane at one point in time is completed in 26.3 s on an nVidia Tesla K20 GPU. After the shear wave particle velocities are simulated on the GPU, the peak values of the measured and simulated shear wave particle velocities are both normalized to 1, and the percent difference between the simulated and measured shear wave particle velocities is evaluated with Eq. 21, where $u(\mathbf{r}, t)$ represents the simulated shear wave particle velocity in a 2D plane at time t , $u_{ref}(\mathbf{r}, t)$ represents the measured shear wave particle velocity in the same 2D plane at time t , and $\|\cdot\|$ denotes the L^2 -norm. The percent difference is calculated for each combination of the shear elasticity and shear viscosity parameters, and the combination of shear wave parameters that provides the minimum percent difference is selected for each 2D plane.

Figure 7 shows the measured shear wave particle velocities in phantom E3962-1 and the corresponding simulated shear wave particle velocities. For all three points in time, the material parameters that minimize the percent difference are $\mu = 0.3$ kPa and $\eta_s = 0.1$ Pa·s. In Figure 7(a), the location of the peak shear wave particle velocity in the right half of the image is $(x, z) = (2.15 \text{ mm}, 22.18 \text{ mm})$, while in Figure 7(d), the location of the peak shear wave particle velocity in the right half of the image is $(x, z) = (2.00 \text{ mm}, 23.56 \text{ mm})$. Also, in Figure 7(a), the full width at half maximum (FWHM) of the shear wave in the x direction evaluated at the peak on the right is 1.08 mm, and in Figure 7(d), the FWHM of the shear wave in the x direction evaluated at the peak on the right is 1.14 mm. In Figure 7(b), the location of the peak shear wave particle velocity in the right half of the image is $(x, z) = (3.69 \text{ mm}, 22.79 \text{ mm})$, while in Figure 7(e), the maximum location of the peak shear wave particle velocity in the right half of the image is $(x, z) = (3.85 \text{ mm}, 22.95 \text{ mm})$. In Figure 7(b), the FWHM of the shear wave in the x direction evaluated at the peak on the right is 1.55 mm, and in Figure 7(e), the FWHM of the shear wave in the x direction evaluated at the peak on the right is 1.57 mm. In Figure 7(c), the location of the peak shear wave particle velocity in the right half of the image is $(x, z) = (5.24 \text{ mm}, 21.25 \text{ mm})$, while in Figure 7(f), the location of the peak shear wave particle velocity in the right half of the image is $(x, z) = (5.54 \text{ mm}, 22.48 \text{ mm})$. In Figure 7(c), the FWHM of the shear wave in the x direction evaluated at the peak on the right is 1.98 mm, and in Figure 7(f), the FWHM of the shear wave in the x direction evaluated at the peak on the right is 1.91 mm. The measured and simulated peak locations and FWHM values for shear wave phantom E3962-1 are summarized in Table 1.

Figure 8 shows the measured shear wave particle velocities in phantom E3962-2 and the corresponding simulated shear wave particle velocities. For all three points in time, the material parameters that minimize the percent difference are $\mu = 2.3$ kPa and $\eta_s = 0.1$ Pa·s. In Figure 8(a), the location of the peak shear wave particle velocity in the right half of the image is $(x, z) = (4.93 \text{ mm}, 22.33 \text{ mm})$, while in Figure 8(d), the location of the peak shear wave particle velocity in the right half of the image is $(x, z) = (5.08 \text{ mm}, 23.41 \text{ mm})$. In Figure 8(a), the full width at half maximum (FWHM) of the shear wave in the x direction evaluated at the peak on the right is 1.23 mm, and in Figure 8(d), the FWHM of the shear wave in the x direction evaluated at the peak on the right is 1.17 mm. In Figure 8(b), the location of the peak shear wave particle velocity in the right half of the image is $(x, z) =$

(9.70 mm, 23.56 mm), while in Figure 8(e), the location of the peak shear wave particle velocity in the right half of the image is $(x, z) = (9.55 \text{ mm}, 23.10 \text{ mm})$. Also, in Figure 8(b), the FWHM of the shear wave in the x direction evaluated at the peak on the right is 1.62 mm, and in Figure 8(e), the FWHM of the shear wave in the x direction evaluated at the peak on the right is 1.59 mm. In Figure 8(c), the location of the peak shear wave particle velocity in the right half of the image is $(x, z) = (14.17 \text{ mm}, 21.71 \text{ mm})$, while in Figure 8(f), the location of the peak shear wave particle velocity in the right half of the image is $(x, z) = (14.32 \text{ mm}, 22.48 \text{ mm})$. In Figure 8(c), the FWHM of the shear wave in the x direction evaluated at the peak on the right is 1.80 mm, and in Figure 8(f), the FWHM of the shear wave in the x direction evaluated at the peak on the right is 1.94 mm. The measured and simulated peak locations and FWHM values for shear wave phantom E3962-2 are collected in Table 2.

Figure 9 shows the measured shear wave particle velocities in phantom E3962-3 and the corresponding simulated shear wave particle velocities. For the measured and simulated shear waves at 2 ms, the material parameters that minimize the percent difference are $\mu = 4.9 \text{ kPa}$ and $\eta_s = 1.5 \text{ Pa}\cdot\text{s}$. For the measured and simulated shear waves at 3 ms and 4 ms, the material parameters that minimize the percent difference are $\mu = 4.5 \text{ kPa}$ and $\eta_s = 1.5 \text{ Pa}\cdot\text{s}$. In Figure 9(a), the location of the peak shear wave particle velocity in the right half of the image is $(x, z) = (6.00 \text{ mm}, 19.25 \text{ mm})$, while in Figure 9(d), the location of the peak shear wave particle velocity in the right half of the image is $(x, z) = (5.85 \text{ mm}, 19.40 \text{ mm})$. In Figure 9(a), the full width at half maximum (FWHM) of the shear wave in the x direction evaluated at the peak on the right is 3.04 mm, and in Figure 9(d), the FWHM of the shear wave in the x direction evaluated at the peak on the right is 3.37 mm. In Figure 9(b), the location of the peak shear wave particle velocity in the right half of the image is $(x, z) = (7.39 \text{ mm}, 17.25 \text{ mm})$, while in Figure 9(e), the location of the peak shear wave particle velocity in the right half of the image is $(x, z) = (8.01 \text{ mm}, 17.71 \text{ mm})$. In Figure 9(b), the FWHM of the shear wave in the x direction evaluated at the peak on the right is 3.55 mm, and in Figure 9(e), the FWHM of the shear wave in the x direction evaluated at the peak on the right is 4.24 mm. In Figure 9(c), the location of the peak shear wave particle velocity in the right half of the image is $(x, z) = (10.93 \text{ mm}, 15.25 \text{ mm})$, while in Figure 9(f), the location of the peak shear wave particle velocity in the right half of the image is $(x, z) = (10.47 \text{ mm}, 15.86 \text{ mm})$. In Figure 9(c), the FWHM of the shear wave in the x direction evaluated at the peak on the right is 4.62 mm, and in Figure 9(f), the FWHM of the shear wave in the x direction evaluated at the peak on the right is 4.81 mm. The measured and simulated peak locations and FWHM values for shear wave phantom E3962-3 are assembled in Table 3.

5 Discussion

5.1 Diffraction, attenuation, and dispersion

Figure 2 demonstrates the complicated interaction between diffraction, attenuation, and dispersion in the shear wave particle displacements induced by a 3D acoustic radiation force. In Figure 2, the peak amplitude of the shear wave particle displacement decreases as the observation point moves away from the push beam due to the diffraction of the shear wave.

For shear wave simulations with the elastic model, the peak time differences between successive observation points are approximately the same. The values of the FWHM are also similar at each observation point, which suggests that the elastic model experiences no significant waveform spreading due to dispersion in these locations.

For simulations of shear waves with the viscoelastic model, the amplitude of the shear wave particle displacement decreases due to the combined effects of diffraction and attenuation, so the amplitude decays more rapidly. With the viscoelastic model, the differences between the peak times for successive observation points decrease monotonically as the observation points move away from the push beam. This suggests that the propagation speed of the shear wave particle displacement decreases as the shear wave propagates due to the frequency dependence of the phase velocity. The value of the FWHM grows larger as the distance from the peak of the push beam increases, which demonstrates that the viscoelastic model is also dispersive, in contrast to the elastic model.

5.2 Computation time

To obtain the combination of μ and η_s values that produced the best agreement with the measured viscoelastic shear wave phantom data in Figures 7–9, an exhaustive search that evaluated the objective function in Eq. 21 across multiple frames of shear wave data evaluated at different time points was performed. This would not have been possible without a high-performance GPU or some other high-performance computing solution. For Figure 7, the GPU-accelerated simulations were evaluated for 5 values of μ and 6 values of η_s at 11 time points, and for Figure 8, the GPU-accelerated simulations were evaluated for 31 values of μ and 26 values of η_s at 11 time points. For Figure 9, the simulation results that were calculated for Figure 8 were used again, so there was no need to compute those results a second time. Thus, 9196 simulations of the shear wave particle displacement were evaluated for these three figures. On an nVidia Tesla K20 GPU, the time for a single shear wave simulation is 26.3 seconds, and the total computation time was less than 3 days. However, the same single shear wave calculation takes 5.27 hours on a desktop computer with an Intel(R) Core(TM) i7-2600 3.40GHz CPU, which means that these same computations would require more than 5 years on our Intel i7 desktop computer. Thus, the GPU achieves a significant reduction in the computation time, which suggests that these Green's function calculations are ideal for this high-performance GPU. During the construction of these three figures, we considered the possibility of showing more than three panels, and only integer time increments (in milliseconds) were considered. We ultimately decided to evaluate Figures 7 and 8 at the same three points in time such that the motion of the shear wave was effectively represented while the peak of the shear wave particle velocity remained within the field of view. Figure 9 required different time points to maintain the peak and the surrounding region within the field of view. Having access to all of the simulation data in advance was essential as we evaluated several different options for these three figures.

5.3 Convergence

Figures 5 and 6 show that the percent difference is larger for shear wave particle displacements calculated using push beams with smaller $f/\#$. Smaller percent differences are achieved in simulations of arrays with larger $f/\#$ because for push beams with lower $f/\#$, the

acoustic radiation force is more concentrated in the focal region, so finer spatial samplings are required to capture the high frequency components in the spatial domain. Figures 5 and 6 also indicate that, when the spatial sampling of the push beam is greater than or equal to $\lambda/2$, the percent difference for the elastic model is usually somewhat higher than the percent difference for the viscoelastic model. However, for push beams sampled more finely than $\lambda/2$, the percent differences are similar for shear wave simulations evaluated with the elastic and viscoelastic models.

In all except for two of the curves in Figures 5 and 6, the percent difference decreases monotonically as the spatial sampling decreases. The exceptions are in Figure 6, where the percent difference for push beams sampled at $\lambda/2$ is smaller than the percent difference for push beams sampled at $\lambda/4$ and $\lambda/8$. Similar behavior is also observed in the convergence of the fast nearfield method for rectangular and circular pistons [20, 21], where the overall trend is that the percent difference is decreasing, but local minima are also observed in several locations. To determine whether the local minima in Figure 6 are unique to the parameter combinations evaluated here, additional simulations of shear waves in viscoelastic models were performed using $f/1.5$ push beams evaluated with different values of the shear elasticity and shear viscosity. Four different parameter combinations (not shown) were evaluated: $\mu = 2.25$ kPa and $\eta_s = 2$ Pa·s, $\mu = 2.25$ kPa and $\eta_s = 0.1$ Pa·s, $\mu = 6$ kPa and $\eta_s = 1$ Pa·s, and $\mu = 1$ kPa and $\eta_s = 1$ Pa·s. For each parameter combination, the percent difference for push beams with spatial samplings of $m\lambda$ with $m = 1$ to 4 with an increment of 1 and λ/n with $n = 2$ to 18 with an increment of 1 were evaluated. In all of these simulations, a local minimum for the percent difference was also observed for the push beam sampled at $\lambda/2$. In particular, the percent difference for the shear wave particle displacement simulated using the push beam sampled at $\lambda/2$ was consistently smaller than the push beam sampled from 4λ to λ and from $\lambda/3$ to $\lambda/8$ in each of these results. This suggests that $\lambda/2$ is a reasonable default value for the spatial sampling, especially for push beams with smaller $f/\#$ evaluated with the viscoelastic model.

For shear wave simulations with the elastic model, the percent difference is below 5% for the $f/2$, $f/3$, and f/∞ push beams when the spatial sampling is $\lambda/2$, and the percent difference is below 5% for the $f/1.5$ push beam when the spatial sampling is $\lambda/4$. For shear wave simulations evaluated with the viscoelastic model, the percent difference is below 5% for the $f/2$, $f/3$, and f/∞ push beams when the spatial sampling is 2λ , and the percent difference is below 5% for the push beam with $f/1.5$ when the spatial sampling is $\lambda/2$. If a smaller percent difference is desired, e.g., 1%, then a much finer spatial sampling is required. For shear wave simulations evaluated with the elastic model, the percent difference is greater than 1% for the $f/1.5$ push beam when the spatial sampling is greater than or equal to $\lambda/16$, the percent difference is below 1% for the $f/2$ push beam when the spatial sampling is $\lambda/16$, the percent difference is below 1% for the $f/3$ push beam when the spatial sampling is $\lambda/8$, and the percent difference is below 1% for the f/∞ push beam when the spatial sampling is equal to $\lambda/4$. For shear wave simulations evaluated with the viscoelastic model, the percent difference is 1% for the $f/1.5$ push beam when the spatial sampling is $\lambda/16$, the percent difference is below 1% for the $f/2$ push beam when the spatial sampling is $\lambda/16$, and the percent difference is below 1% for the $f/3$ and f/∞ push beams when the spatial sampling is

$\lambda/2$. This information is potentially important for simulations of shear waves because the accuracy of the computed shear wave depends on the sampling of the push beam.

5.4 Comparisons between measured and simulated shear waves

Figure 7 shows images of measured shear wave particle velocities in a phantom where the shear viscosity is relatively small, which is confirmed through comparisons with simulated shear wave data. Table 1 shows that the z coordinates of the measured peak values on the right side of the images in Figure 7 vary slightly as the time increases, whereas the z coordinates of the simulated peaks decrease by a small amount with increasing time. The x coordinates of the peaks increase with increasing time as expected, and the percent differences between the simulated and measured values (using the measured values as the references) are 7%, 4.3%, and 5.7% at times $t = 3$ ms, 6 ms, and 9 ms, respectively. These differences are relatively small, suggesting that there is good agreement between the measured and simulated coordinates of the peak shear wave particle velocities. The percent differences between the measured and simulated FWHM values at the peaks (using the measured values as the references) are 5.6%, 1.3%, and 3.5% at times $t = 3$ ms, 6 ms, and 9 ms, respectively, which also indicates good agreement between the measured and simulated FWHM values of the shear wave particle velocities evaluated at the peaks.

Figure 8 contains another example of shear wave particle velocities measured in a phantom where the shear viscosity is relatively small. Table 2 shows that the z coordinates of the measured peak values on the right side of the images in Figure 8 again vary slightly as the time increases, whereas the z coordinates of the simulated peaks also decrease by a small amount with increasing time. The percent differences between the simulated and measured values of the x coordinates (again using the measured values as the references) of the peaks are 3%, 1.5%, and 1.1% at times $t = 3$ ms, 6 ms, and 9 ms, respectively. These differences are also small, which indicates close agreement between the measured and simulated coordinates of the peak shear wave particle velocities. The normalized differences between the measured and simulated FWHM values at the peaks (again using the measured values as the references) are 4.9%, 1.9%, and 7.8% at times $t = 3$ ms, 6 ms, and 9 ms, respectively, which also indicates good agreement between the measured and simulated FWHM values of the shear wave particle velocities evaluated at the peaks. The measured and simulated images at each time point are also similar.

Figure 9 shows an example of shear wave particle velocities measured in a phantom where the shear viscosity is larger. Table 3 indicates that the z coordinates are decreasing as a function of time in the measured and simulated shear wave data. The percent differences between the simulated and measured values of the x coordinates (once again using the measured values as the references) of the peaks are 2.5%, 8.4%, and 4.2% at times $t = 2$ ms, 3 ms, and 4 ms, respectively. The normalized differences between the measured and simulated FWHM values at the peaks (once again using the measured values as the references) are 10.9%, 19.4%, and 4.1% at times $t = 2$ ms, 3 ms, and 4 ms, respectively. The percent differences in the x coordinates are comparable to those observed for the other two shear wave phantoms, but the percent differences in the first two FWHM values for phantom E3962-3 reach double digits and are therefore noticeably larger than any of the values

observed in the other two shear wave phantoms. The shear elasticity and the shear viscosity are also much larger in shear wave phantom E3962-3.

Overall, there is good qualitative agreement between the measured and simulated images shown in Figures 7–9. In particular, the locations of the measured and simulated peak particle velocities are similar in most of these images, as indicated by the values provided in Tables 1–3. The widths of the measured and simulated shear wave particle velocities at the peak are also approximately the same in Figures 7 and 8, which is reinforced by the measured and simulated FWHM values listed in Tables 1 and 2. However, the measured and simulated widths of the shear wave particle velocity at the peak are somewhat different in Figure 9, which is substantiated by the values in Table 3. Possible explanations for these differences are that a different model is required (e.g., a fractional Voigt model) to achieve a better match with the experimental data, particularly for viscoelastic shear wave phantoms with greater attenuation and dispersion such as E3962-3. A different objective function that emphasizes agreement in the FWHM at the expense of diminished agreement at the peak location might also yield different results. The resolutions of these and other related issues are topics of ongoing research.

5.5 Intended applications

These GPU-accelerated shear wave simulations are intended for several different types of calculations. For example, these accelerated shear wave simulations are capable of quickly evaluating different push beams at different depths for different material parameter combinations. These simulations are also directly applicable to parametric characterizations that identify the sources of errors in various methods for estimating shear wave parameters, including the time-of-flight approach [22, 31, 35, 39, 44] and the k-space approach [23]. In addition, these accelerated simulations are also expected to enhance new model-based approaches that minimize the difference between measured and simulated shear waves and thereby obtain more accurate estimates of shear wave parameters. In each of these applications, hardware acceleration is essential for rapid and/or repeated shear wave calculations because these simulations are otherwise excessively time-consuming.

6 Conclusion

Shear waves induced by an acoustic radiation force are evaluated for elastic and viscoelastic models using Green's functions on a high-performance GPU. The 3D acoustic radiation force is first computed using the fast nearfield method and the angular spectrum approach in FOCUS, and then shear waves are simulated by superposing the contributions from all of the source points in the 3D push beam, which is an ideal calculation for a GPU. Diffraction effects are observed in the simulated shear wave particle displacements in both elastic and viscoelastic models, and the shear viscosity introduces additional dispersion and attenuation. Comparisons between the computation times for serial and parallel implementations show that an nVidia Tesla K20 GPU accelerates these Green's function calculations by factors of 45 and 700 for shear wave simulations in elastic and viscoelastic models, respectively, relative to serial calculations on a desktop computer with an Intel(R) Core(TM) i7-2600 3.40GHz CPU. When these Green's functions are evaluated for push beams with different

spatial samplings, the results show that higher spatial sampling rates are required for push beams with lower f/λ . Measured and simulated shear waves in elastic and viscoelastic shear wave phantoms are also compared, and the results suggest that the Green's function simulation approach provides an effective simulation model for shear waves in elastic and viscoelastic shear wave phantoms.

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References

1. Aki K, Richards PG. Quantitative Seismology. Geology (University Science Books).: Seismology. University Science Books; 2002.
2. Amador C, Urban MW, Chen S, Greenleaf JF. Shearwave dispersion ultrasound vibrometry (SDUV) on swine kidney. *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*. 2011; 58(12):2608–2619.
3. Asen JP, Buskenes JI, Nilsen CIC, Austeng A, Holm S. Implementing capon beamforming on a GPU for real-time cardiac ultrasound imaging. *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*. 2014; 61(1):76–85.
4. Bercoff J, Tanter M, Fink M. Supersonic shear imaging: a new technique for soft tissue elasticity mapping. *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*. 2004; 51(4): 396–409.
5. Bercoff J, Tanter M, Muller M, Fink M. The role of viscosity in the impulse diffraction field of elastic waves induced by the acoustic radiation force. *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control*. 2004; 51(11):1523–1536.
6. Callé S, Remenieras J, Matar OB, Hachemi ME, Patat F. Temporal analysis of tissue displacement induced by a transient ultrasound radiation force. *The Journal of the Acoustical Society of America*. 2005; 118(5):2829–2840. [PubMed: 16334661]
7. Catheline S, Gennisson JL, Delon G, Fink M, Sinkus R, Abouelkaram S, Culioli J. Measurement of viscoelastic properties of homogeneous soft solid using transient elastography: an inverse problem approach. *The Journal of the Acoustical Society of America*. 2004; 116(6):3734–3741. [PubMed: 15658723]
8. Chatelin S, Charpentier I, Corbin N, Meylheuc L, Vappou J. An automatic differentiation-based gradient method for inversion of the shear wave equation in magnetic resonance elastography: specific application in fibrous soft tissues. *Phys Med Biol*. 2016; 61(13):5000–5019. [PubMed: 27300107]
9. Chen J, Yiu B, So HKH, Yu ACH. Real-time GPU-based adaptive beam-former for high quality ultrasound imaging. 2011 IEEE International Ultrasonics Symposium (IUS); 2011. 474–477.
10. Couade M, Pernot M, Messas E, Bel A, Ba M, Hagege A, Fink M, Tanter M. In vivo quantitative mapping of myocardial stiffening and transmural anisotropy during the cardiac cycle. *IEEE Transactions on Medical Imaging*. 2011; 30(2):295–305. [PubMed: 20851788]
11. Couade M, Pernot M, Prada C, Messas E, Emmerich J, Bruneval P, Criton A, Fink M, Tanter M. Quantitative assessment of arterial wall biomechanical properties using shear wave imaging. *Ultrasound in Medicine & Biology*. 2010; 36(10):1662–1676. [PubMed: 20800942]
12. Fahey BJ, Nightingale KR, Nelson RC, Palmeri ML, Trahey GE. Acoustic radiation force impulse imaging of the abdomen: demonstration of feasibility and utility. *Ultrasound in Medicine & Biology*. 2005; 31(9):1185–1198. [PubMed: 16176786]

13. Gennisson JL, Catheline S, Chaffai S, Fink M. Transient elastography in anisotropic medium: application to the measurement of slow and fast shear wave speeds in muscles. *J Acoust Soc Am*. 2003; 114(1):536–541. [PubMed: 12880065]
14. Hah Z, Hazard C, Mills B, Barry C, Rubens D, Parker K. Integration of crawling waves in an ultrasound imaging system. part 2: signal processing and applications. *Ultrasound in Medicine & Biology*. 2012; 38(2):312–323. [PubMed: 22178168]
15. Hazard C, Hah Z, Rubens D, Parker K. Integration of crawling waves in an ultrasound imaging system. part 1: system and design considerations. *Ultrasound in Medicine & Biology*. 2012; 38(2): 296–311. [PubMed: 22178166]
16. Kasai C, Namekawa K, Koyano A, Omoto R. Real-time two-dimensional blood flow imaging using an autocorrelation technique. *IEEE Transactions on sonics and ultrasonics*. 1985; 32(3):458–464.
17. Langdon J, Mercado K, Dalecki D, McAleavey S. Compensating for Scholte waves in single track location shearwave elasticity imaging. *The Journal of the Acoustical Society of America*. 2015; 137(4):2364–2364.
18. Lee KH, Szajewski BA, Hah Z, Parker KJ, Maniatty AM. Modeling shear waves through a viscoelastic medium induced by acoustic radiation force. *International Journal for Numerical Methods in Biomedical Engineering*. 2012; 28(6–7):678–696. [PubMed: 25364845]
19. McAleavey SA, Menon M, Orszulak J. Shear-modulus estimation by application of spatially-modulated impulsive acoustic radiation force. *Ultrasonic Imaging*. 2007; 29(2):87–104. [PubMed: 17679324]
20. McGough RJ. Rapid calculations of time-harmonic nearfield pressures produced by rectangular pistons. *Journal of the Acoustical Society of America*. 2004; 115(5):1934–1941. [PubMed: 15139602]
21. McGough RJ, Samulski TV, Kelly JF. An efficient grid sectoring method for calculations of the near-field pressure generated by a circular piston. *The Journal of the Acoustical Society of America*. 2004; 115(5):1942–1954. [PubMed: 15139603]
22. McLaughlin J, Renzi D. Shear wave speed recovery in transient elastography and supersonic imaging using propagating fronts. *Inverse Problems*. 2006; 22(2):681.
23. Nenadic IZ, Urban MW, Zhao H, Sanchez W, Morgan PE, Greenleaf JF, Chen S. Application of attenuation measuring ultrasound shearwave elastography in 8 post-transplant liver patients. 2014 IEEE International Ultrasonics Symposium (IUS); 2014. 987–990.
24. Nightingale KR, Nightingale RW, Palmeri ML, Trahey GE. A finite element model of remote palpation of breast lesions using radiation force: Factors affecting tissue displacement. *Ultrasonic Imaging*. 2000; 22(1):35–54. [PubMed: 10823496]
25. Nightingale KR, Palmeri ML, Nightingale RW, Trahey GE. On the feasibility of remote palpation using acoustic radiation force. *The Journal of the Acoustical Society of America*. 2001; 110(1): 625–634. [PubMed: 11508987]
26. Novotny PM, Stoll JA, Vasilyev NV, Pedro J, Dupont PE, Zickler TE, Howe RD. GPU based real-time instrument tracking with three-dimensional ultrasound. *Medical image analysis*. 2007; 11(5): 458–464. [PubMed: 17681483]
27. Orescanin M, Wang Y, Insana MF. 3-D FDTD simulation of shear waves for evaluation of complex modulus imaging. *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*. 2011; 58(2):389–398.
28. Palmeri ML, McAleavey SA, Trahey GE, Nightingale KR. Ultrasonic tracking of acoustic radiation force-induced displacements in homogeneous media. *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control*. 2006; 53(7):1300–1313.
29. Palmeri ML, Nightingale KR. On the thermal effects associated with radiation force imaging of soft tissue. *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control*. 2004; 51(5): 551–565.
30. Palmeri ML, Sharma AC, Bouchard RR, Nightingale RW, Nightingale KR. A finite-element method model of soft tissue response to impulsive acoustic radiation force. *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control*. 2005; 52(10):1699–1712.

31. Palmeri ML, Wang MH, Dahl JJ, Frinkley KD, Nightingale KR. Quantifying hepatic shear modulus in vivo using acoustic radiation force. *Ultrasound in Medicine & Biology*. 2008; 34(4): 546–558. [PubMed: 18222031]
32. Qiang B, Brigham JC, Aristizabal S, Greenleaf JF, Zhang X, Urban MW. Modeling transversely isotropic, viscoelastic, incompressible tissue-like materials with application in ultrasound shear wave elastography. *Physics in Medicine and Biology*. 2015; 60(3):1289. [PubMed: 25591921]
33. Rosenzweig S, Palmeri M, Nightingale K. GPU-based real-time small displacement estimation with ultrasound. *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*. 2011; 58(2):399–405.
34. Rouze NC, Wang MH, Palmeri ML, Nightingale KR. Finite element modeling of impulsive excitation and shear wave propagation in an incompressible, transversely isotropic medium. *Journal of Biomechanics*. 2013; 46(16):2761–2768. [PubMed: 24094454]
35. Sandrin L, Fourquet B, Hasquenoph J, Yon S, Fournier C, Mal F, Christidis C, Ziol M, Poulet B, Kazemi F, Beaugrand M, Palau R. Transient elastography: a new noninvasive method for assessment of hepatic fibrosis. *Ultrasound in medicine & biology*. 2003; 29(12):1705–1713. [PubMed: 14698338]
36. Sarvazyan AP, Rudenko OV, Swanson SD, Fowlkes JB, Emelianov SY. Shear wave elasticity imaging: a new ultrasonic technology of medical diagnostics. *Ultrasound in Medicine & Biology*. 1998; 24(9):1419–1435. [PubMed: 10385964]
37. Sebag F, Vaillant-Lombard J, Berbis J, Griscti V, Henry JF, Petit P, Oliver C. Shear wave elastography: a new ultrasound imaging mode for the differential diagnosis of benign and malignant thyroid nodules. *The Journal of Clinical Endocrinology & Metabolism*. 2010; 95(12): 5281–5288. [PubMed: 20881263]
38. Song P, Zhao H, Manduca A, Urban MW, Greenleaf JF, Chen S. Compush ultrasound shear elastography (cuse): a novel method for two-dimensional shear elasticity imaging of soft tissues. *IEEE Transactions on Medical Imaging*. 2012; 31(9):1821–1832. [PubMed: 22736690]
39. Tanter M, Bercoff J, Athanasiou A, Deffieux T, Gennisson J, Montaldo G, Muller M, Tardivon A, Fink M. Quantitative assessment of breast lesion viscoelasticity: initial clinical results using supersonic shear imaging. *Ultrasound in Medicine & Biology*. 2008; 34(9):1373–1386. [PubMed: 18395961]
40. Treeby BE, Tumen M, Cox BT. Time domain simulation of harmonic ultrasound images and beam patterns in 3D using the k-space pseudospectral method. *Medical Image Computing and Computer-Assisted Intervention–MICCAI 2011*; 2011. 363–370.
41. Varray F, Cachard C, Ramalli A, Tortoli P, Basset O. Simulation of ultrasound nonlinear propagation on GPU using a generalized angular spectrum method. *EURASIP journal on Image and Video Processing*. 2011; 2011(1):1–6.
42. Zeng X, McGough RJ. Evaluation of the angular spectrum approach for simulations of near-field pressures. *The Journal of the Acoustical Society of America*. 2008; 123(1):68–76. [PubMed: 18177139]
43. Zeng X, McGough RJ. Optimal simulations of ultrasonic fields produced by large thermal therapy arrays using the angular spectrum approach. *The Journal of the Acoustical Society of America*. 2009; 125(5):2967–2977. [PubMed: 19425640]
44. Zhao H, Song P, Urban MW, Greenleaf JF, Chen S. Shear wave speed measurement using an unfocused ultrasound beam. *Ultrasound in Medicine & Biology*. 2012

Figure 1.
The locations of the observation points relative to a push beam focused at 25 mm in the x - z plane with $y = 0$.

Figure 2.

Simulated shear wave particle displacements computed with elastic and viscoelastic models for an applied 3D acoustic radiation force push evaluated at the 8 observation points indicated in Figure 1. The solid lines describe the normalized shear wave particle displacements calculated for the elastic model with $\mu = 2.25$ kPa and $\eta_s = 0$ Pa·s, and the dashed lines describe the normalized shear wave particle displacements calculated for the viscoelastic model with $\mu = 2.25$ kPa and $\eta_s = 1$ Pa·s.

Figure 3.

Computation times for serial and parallel implementations of time domain shear wave calculations with Green's functions. The solid line with circle markers indicates the computation times for parallel simulations of shear waves with the elastic model, the dashed-dotted line with triangle markers indicates the computation times for parallel simulations of shear waves with the viscoelastic model, the dashed line with diamond markers indicates the computation times for serial simulations of shear waves with the elastic model, and the dotted line with square markers indicates the computation times for serial simulations of shear waves with the viscoelastic model.

Figure 4.

2D cross-sections of 3D ultrasound beams simulated in FOCUS. Figures (4a), (4b), (4c), and (4d) show the push beams with $f/1.5$, $f/2$, $f/3$, and f/∞ , respectively. In (4a)–(4d), each beam is normalized by the maximum intensity value.

Figure 5.

Percent differences calculated for shear wave particle displacements simulated with the elastic model. The shear elasticity and shear viscosity values in these simulations are $\mu = 2.25$ kPa and $\eta_s = 0$ Pa·s, where the push beams are calculated with different spatial samplings. The solid line with circle markers indicates the percent difference for the shear wave particle displacements induced by an unfocused push beam with f/∞ , the dotted line with square markers indicates the percent difference for the shear wave particle displacements induced by a focused push beam with $f/3$, the dash-dotted line with triangle markers indicates the percent difference for the shear wave particle displacements induced by a focused push beam with $f/2$, and the dashed line with diamond markers indicates the percent difference for the shear wave particle displacements induced by a focused push beam with $f/1.5$.

Figure 6.

Percent differences calculated for shear wave particle displacements simulated with the viscoelastic model. The shear elasticity and shear viscosity values in these simulations are $\mu = 2.25$ kPa and $\eta_s = 1$ Pa·s, where the push beams are calculated with different spatial samplings. The descriptions of the markers and line types are the same as those given in the caption for Figure 5.

Figure 7.

Measured shear wave particle velocities in phantom E3962-1 and the corresponding simulated shear wave particle velocities evaluated at three different times. Figs. 7(a), 7(b), and 7(c) show the shear wave particle velocities measured at 3 ms, 6 ms, and 9 ms, respectively. Figs. 7(d), 7(e), and 7(f) show the corresponding simulated shear wave particle velocities computed with $\mu = 0.3$ kPa and $\eta_s = 0.1$ Pa·s at the same three times.

Figure 8.

Measured shear wave particle velocities in phantom E3962-2 and the corresponding simulated shear wave particle velocities evaluated at three different times. Figs. 8(a), 8(b), and 8(c) show the shear wave particle velocities measured at 3 ms, 6 ms, and 9 ms, respectively. Figs. 8(d), 8(e), and 8(f) show the corresponding simulated shear wave particle velocities with $\mu = 2.3$ kPa and $\eta_s = 0.1$ Pa·s at the same times.

Figure 9.

Measured shear wave particle velocities in phantom and the corresponding simulated shear wave particle velocities evaluated at three different times. Figs. 9(a), 9(b), and 9(c) show the shear wave particle velocities measured at 2 ms, 4 ms, and 6 ms, respectively. Figs. 9(d), 9(e), and 9(f) show the corresponding simulated shear wave particle velocities computed with $\mu = 4.9$ kPa and $\eta_s = 1.5$ Pa·s at 2 ms, and computed with $\mu = 4.5$ kPa and $\eta_s = 1.5$ Pa·s at 3 ms and 4 ms, respectively.

Table 1

The peak locations and FWHM values of the measured and simulated shear wave particle velocities in the x direction evaluated at the peak for times $t = 3$ ms, 6 ms, and 9 ms in shear wave phantom E3962-1.

	measured peak (x, z)	simulated peak (x, z)	measured FWHM	Simulated FWHM
$t = 3$ ms	(2.15 mm, 22.18 mm)	(2.00 mm, 23.56 mm)	1.08 mm	1.14 mm
$t = 6$ ms	(3.69 mm, 22.79 mm)	(3.85 mm, 22.95 mm)	1.55 mm	1.57 mm
$t = 9$ ms	(5.24 mm, 21.25 mm)	(5.54 mm, 22.48 mm)	1.98 mm	1.91 mm

Table 2

The peak locations and FWHM values of the measured and simulated shear wave particle velocities in the x direction evaluated at the peak for times $t = 3$ ms, 6 ms, and 9 ms in shear wave phantom E3962-2.

	measured peak (x, z)	simulated peak (x, z)	measured FWHM	simulated FWHM
$t = 3$ ms	(4.93 mm, 22.33 mm)	(5.08 mm, 23.41 mm)	1.23 mm	1.17 mm
$t = 6$ ms	(9.70 mm, 23.56 mm)	(9.55 mm, 23.10 mm)	1.62 mm	1.59 mm
$t = 9$ ms	(14.17 mm, 21.71 mm)	(14.32 mm, 22.48 mm)	1.80 mm	1.94 mm

Table 3

The peak locations and FWHM values of the measured and simulated shear wave particle velocities in the x direction evaluated at the peak for times $t = 2$ ms, 3 ms, and 4 ms in shear wave phantom E3962-3.

	measured peak (x, z)	simulated peak (x, z)	measured FWHM	simulated FWHM
$t = 2$ ms	(6.00 mm, 19.25 mm)	(5.85 mm, 19.40 mm)	3.04 mm	3.37 mm
$t = 3$ ms	(7.39 mm, 17.25 mm)	(8.01 mm, 17.71 mm)	3.55 mm	4.24 mm
$t = 4$ ms	(10.93 mm, 15.25 mm)	(10.47 mm, 15.86 mm)	4.62 mm	4.81 mm