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Simultaneous Auto-calibration and Gradient Delays Estimation (SAGE) in non-Cartesian Parallel MRI using Low-rank Constraints

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Abstract

Purpose—To correct gradient timing delays in non-Cartesian MRI while simultaneously recovering corruption-free auto-calibration data for parallel imaging, without additional calibration scans.

Methods—The calibration matrix constructed from multi-channel k-space data should be inherently low-rank. This property is used to construct reconstruction kernels or sensitivity maps. Delays between the gradient hardware across different axes and RF receive chain, which are relatively benign in Cartesian MRI (excluding EPI), lead to trajectory deviations and hence data inconsistencies for non-Cartesian trajectories. These in turn lead to higher rank and corrupted calibration information which hampers the reconstruction. Here, a method named Simultaneous Auto-calibration and Gradient delays Estimation (SAGE) is proposed that estimates the actual k-space trajectory while simultaneously recovering the uncorrupted auto-calibration data. This is done by estimating the gradient delays that result in the lowest rank of the calibration matrix. The Gauss-Newton method is used to solve the non-linear problem. The method is validated in simulations using center-out radial, projection reconstruction and spiral trajectories. Feasibility is demonstrated on phantom and in vivo scans with center-out radial and projection reconstruction trajectories.

Results—SAGE is able to estimate gradient timing delays with high accuracy at a signal to noise ratio level as low as 5. The method is able to effectively remove artifacts resulting from gradient timing delays and restore image quality in center-out radial, projection reconstruction, and spiral trajectories.

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Conclusion—The low-rank based method introduced simultaneously estimates gradient timing delays and provides accurate auto-calibration data for improved image quality, without any additional calibration scans.

Keywords

Non-Cartesian MRI; parallel imaging; gradient delays; joint estimation; radial; spiral; projection reconstruction

Introduction

Non-Cartesian MRI trajectories, such as spiral, radial, and concentric rings, allow efficient coverage of k-space and support many promising applications, including cardiac MRI [1], functional MRI [2], dynamic contrast enhanced MRI [3], ultra-short echo time imaging [4] and PET/MR applications [5]. However, non-Cartesian imaging is susceptible to gradient fidelity errors, such as those due to gradient delays in the system as well as by short-term eddy currents. The gradient timing errors usually result in spurious geometric image distortions, misaligned and emphasized edges, image blurring, streaking artifacts or ghosting. Moreover, non-Cartesian trajectories are well suited for parallel imaging acceleration. The typical oversampling of k-space in center-out non-Cartesian trajectories such as radial, spiral or cone trajectories is often used to for self/auto-calibration. Unfortunately, the delays cause significant data inconsistencies which corrupt the auto-calibration.

Gradient errors can be corrected by separate calibration measurements of physical gradient timing delays [6, 7, 8] or more sophisticated measurements of actual gradient waveforms [9, 10, 11]. These require additional acquisitions that depend on the experimental Signal to Noise Ratio (SNR). In addition, these methods assume that the errors are consistent between calibration and acquisition, which may not be an accurate assumption due to phenomena such as gradient coupling or patient motion [12]. Magnetic field monitoring is another approach that uses NMR probes placed around the imaging volume in the scanner [13, 14]. The main disadvantage of this method is that it requires specialized hardware.

There have been efforts to correct for the gradient errors retrospectively without additional calibration measurements or hardware [12, 15, 16, 17, 18, 19, 20, 21]. In the early work of Rasche *et al.* [15], artifacts induced by gradient errors for projection reconstruction trajectory were shown to be reduced by averaging the magnitude of filtered back projections. Recently, a number of promising works [12, 18, 19, 22] have shown that promoting self-consistency inherent in neighboring multi-channel k-space measurements can correct for trajectory errors, without separate trajectory calibration scans. In particular, Deshmane *et al.* proposed the GROG method [12], which estimates the trajectory errors by maximizing the signal at the DC frequency and Wech *et al.* proposed the SCITA method [18], maximizing mutual accordance of data. However, both of these methods are designed for constant gradient readouts such as in radial trajectories where gradient delays are translated into k-space trajectory shifts in the readout direction. Mani *et al.* proposed MUSSELS [23] based method to correct for radial trajectory errors by lifting the problem into a multi-shot k-space

reconstruction [24]. Peterson *et al.*, Lee *et al.* and Lobos *et al.* proposed methods [22, 25, 26] that exploit the low dimensional property of the calibration data to correct for EPI (Echo Planar Imaging) ghosting. Ianni *et al.* further proposed a more general approach, TrACR [19], to jointly estimate trajectory errors and images. This method is based on SENSE and SPIRiT parallel imaging reconstructions, and requires additional delay-free calibration Cartesian images to obtain either accurate sensitivity maps or accurate SPIRiT kernels.

In this work, we propose a gradient delay estimation method [27] that is general to non-Cartesian trajectories. We name the method Simultaneous Auto-calibration and Gradient delay Estimation (SAGE). Unlike previous methods, it does not require prior GRAPPA/SPIRiT calibration or any prior direction specifications. We make use of the fact that uncorrupted multi-channel data inherently lives in low dimensional subspace [28, 29, 30, 31] and data corruption induces inconsistencies that violate this property. Instead of solving for the k-space trajectory deviations directly, which can be non-uniform for oscillating or ramping gradients, we directly solve for the actual gradient delay. Explicitly we find the delays that result in the lowest rank of the calibration matrix which is constructed from central k-space data. We formulate the joint estimation as a low-rank constrained problem, and solve it using the Gauss-Newton method. We estimate the gradient delay values as well as the uncorrupted auto-calibration data simultaneously. Our proposed method can be viewed as an extension work of the Simultaneous Auto-calibration and K-space Estimation (SAKE) method [30], a calibration-less parallel imaging method that exploits the low dimensional subspace property of multi-channel data to reconstruct missing k-space measurements. Here, the method is extended to also solve for possible delays.

Theory

Low Rank Property in Parallel Imaging

Multi-channel k-space measurements are inherently coupled because of the spatially localized coil sensitivities. Such correlations can be exploited by explicitly solving for sensitivity maps [31]. In methods such as GRAPPA, SPIRiT, ESPIRiT, LORAK, MUSSELS and SAKE [28, 29, 31, 32, 23, 30], the correlations within multichannel k-space data can be exploited by constructing a block-Hankel matrix from a calibration region [29, 30].

Typically, a fully sampled region in the center of k-space, *i.e.* the auto-calibration (AC) region is used for calibration. By sliding a window across the AC region, we can construct a calibration matrix, denoted as $A = HFx$, where H is the block-Hankel structured operator, x is the multichannel low resolution image of auto-calibration region, and F is the multichannel Fourier transform operator of the auto-calibration region. The columns of A are shifted blocks of the AC area shown in Fig 1, leading to a matrix structure known as block-Hankel. Under the reasonable assumption that coil sensitivities are smooth in space with compact k-space support, it has been shown that a calibration matrix A has low rank [30, 32].

Such low-rank property has been explicitly exploited in SAKE [30] and similarly in LORAK [32], as well as MUSSELS [23], which seek to reconstruct a full k-space from undersampled, multichannel dataset using a structured low-rank matrix completion method, formulated in Equation 1.

$$\begin{aligned}
& \text{minimize } \|Dx - y\|_2^2 \\
& \text{subject to } A = HFx \quad (1) \\
& \text{rank}(A) \leq k
\end{aligned}$$

where D is the undersampled multichannel Fourier transform operator.

Instead of filling in missing k-space, we aim to exploit this low-rank property of the auto-calibration matrix to estimate gradient delays of non-Cartesian data. The basic idea behind the proposed method is to find the gradient delays such that minimize the rank of the calibration matrix.

Solve for Gradient Timing Delays with Low Rank Constraints

For non-Cartesian imaging, gradient delays usually cause inconsistent deviations in k-space between repetitions. For some trajectories, such as EPI, radial (projection reconstruction) the delays in the gradients will cause a uniform shift in k-space along the readout. For more general trajectories such as spirals, radial center-out, cones, rosette etc, the effect in k-space is more complicated. Therefore solving for the gradient delays directly is much simpler. Our hypothesis is that inconsistent trajectory deviation results in a calibration matrix with a higher rank. Supporting Fig S1 demonstrates in simulation that gradient delays increase the rank of the calibration matrix substantially so that searching for the sub-space can actually remove them. Motivated by this hypothesis, we formulate solving for the gradient delay estimation problem as a low-rank constrained optimization.

For simplicity, we assume the central k-space data is fully sampled, so we can ignore undersampling effects. This assumption holds for most non-Cartesian trajectories, such as radial, spiral, even when these are overall undersampled for parallel imaging. Further discussion of undersampling effect on the proposed method is described in the Discussion section. In addition, we assume timing delays for the gradient axes are independent. We further assume that the same delays occur in every time-repetition (TR) throughout the scan.

Problem description

Throughout the paper we use the following notation: $\vec{\Delta t}$: estimated delays on different axes

x_i : low resolution Cartesian image of the auto-calibration region from coil i

y_{ij} : central k-space data that is used for calibration from coil i , repetition j

$D_j(\vec{\Delta t})$: multichannel non-uniform fast Fourier transform (NUFFT) operator at repetition j , parameterized by k-space shifts cause by the delays $\vec{\Delta t}$

F : multidimensional (Cartesian) fast Fourier transform operator (FFT)

H : a linear operator that constructs a structured block-Hankel calibration matrix from corresponding Cartesian k-space data

N_C : the total number of coils

N_P : the total number of repetitions (*e.g.* the number of projections for radial, or the number of interleaves for spirals)

k : the rank of the data matrix, known as a priori or estimated [30], which is bounded by block size and coil bandwidth

Reformulating Equation 1 to include the gradient delays and non-Cartesian trajectories results in the following inverse problem formulation:

$$\begin{aligned} & \underset{\vec{\Delta t}, x}{\text{minimize}} && \sum_{j=1}^{N_P} \sum_{i=1}^{N_C} \|D_j(\vec{\Delta t})x_i - y_{ij}\|_2^2 \\ & \text{subject to} && A = HFx \\ & && \text{rank}(A) \leq k \end{aligned} \quad (2)$$

where both x and $\vec{\Delta t}$ are unknowns.

Algorithms

The objective function contains a non-linear function of the variables, in addition to low-rank constraints. We therefore choose an alternating minimization approach, which alternates between minimizing the objective function and imposing low-rankness. In addition we use the Gauss-Newton method to optimize the non-linear objective. The process is illustrated in Fig 1.

First, we fix the gradient delay $\vec{\Delta t}$ and solve for the coil images x :

$$\begin{aligned} & \underset{x}{\text{minimize}} && \sum_{j=1}^{N_P} \sum_{i=1}^{N_C} \|D_j(\vec{\Delta t})x_i - y_{ij}\|_2^2 \\ & \text{subject to} && A = HFx \\ & && \text{rank}(A) \leq k \end{aligned} \quad (3)$$

A low-rank projection method is used to approximate the solution of x , which is performed by hard-thresholding the singular values of the calibration matrix, and projecting back to image space:

$$\begin{aligned} A &= HFD^\dagger y \\ U\Sigma V^T &= A \\ \bar{A} &= U\text{HardThresh}(\Sigma)V^T \\ x &= F^\dagger H^\dagger \bar{A} \end{aligned} \quad (4)$$

where † denotes the pseudo inverse operator. Note the inverse non-uniform Fourier transform is computationally intensive when solving directly. Instead, $D^\dagger y$ can be approximately solved iteratively using conjugate-gradient.

After solving for x , $\vec{\Delta t}$ is approximately solved using the Gauss-Newton method with first-order approximation:

$$\underset{\Delta(\vec{t})}{\text{minimize}} \quad \sum_{j=1}^N \sum_{i=1}^N \left\| D_j(\vec{\Delta t}) x_i - y_{ij} \right\|_2^2 \quad (5)$$

Concretely, the following first order approximation expansion of the NUFFT operator D is used in the Gauss-Newton method:

$$\begin{aligned} D_j(\vec{\Delta t}) &\approx D_j(0) + \vec{\Delta t} \cdot \nabla \vec{D}_j \quad (6) \\ &= D_j(0) + \begin{bmatrix} \Delta t_x & \Delta t_y \end{bmatrix} \begin{bmatrix} \frac{\partial D}{\partial t_x} \\ \frac{\partial D}{\partial t_y} \end{bmatrix} \end{aligned}$$

$D_j(0)$ is the nominal NUFFT operator without timing delays. The gradient of the NUFFT operator $\nabla \vec{D}$ in Equation 6 can be calculated using the chain rule and the frequency derivative property of Fourier transform [33], that is, applying a linear ramp filter on low resolution image x and then performing the forward NUFFT operation:

$$\begin{bmatrix} \frac{\partial D}{\partial t_x} \\ \frac{\partial D}{\partial t_y} \end{bmatrix} x = D(-jLx) \begin{bmatrix} \frac{\partial k_x}{\partial t_x} \\ \frac{\partial k_y}{\partial t_y} \end{bmatrix}, \text{ where } L \text{ is the ramp filter and } k_x/k_y \text{ are k-space trajectory in x/y axis}$$

respectively. Equation 6 describes the 2D problem for solving the gradient timing delays on the x and y axes, and it can be easily extended into the 3D case.

This approximation results in the following closed form to solve for $\vec{\Delta t}$:

$$\underset{\vec{\Delta t}}{\text{minimize}} \quad \sum_{j=1}^N \sum_{i=1}^N \left\| \left(D_j(0) + \vec{\Delta t} \cdot \nabla \vec{D}_j \right) x_i - y_{ij} \right\|_2^2 \quad (7)$$

$$\vec{\Delta t} = \begin{bmatrix} \Delta t_x \\ \Delta t_y \end{bmatrix} = \sum_{j=1}^N \sum_{i=1}^N \text{Re} \left[\frac{(\nabla \vec{D}_j x_i) * (y_{ij} - D_j(0) x_i)}{(\nabla \vec{D}_j x_i) * (\nabla \vec{D}_j x_i)} \right] \quad (8)$$

The gradient timing delays $\vec{\Delta t}$ can then be used to update k-space coordinates k_x and k_y for the next iteration. If the k-space trajectory's analytic expression is not known, then interpolation can be used to update the k-space coordinates in practice. Pseudo code is provided in the Supporting Material (Supporting Table S1).

Methods

Reconstruction and Data Processing

Image reconstruction and post-processing were carried out by Berkeley Advanced Reconstruction Toolbox (BART), including non-uniform fast Fourier Transform (NUFFT), and ESPIRiT [31] calibration. All reconstructions and simulations were performed on a laptop equipped with an Intel Core i7 CPU (2 GHz, Santa Clara, CA) with 8GB RAM. Studies were conducted on a 7.0 T GE clinical scanner (GE Healthcare, Waukesha, WI, USA.) with 32-channel head coil and 3.0 T clinical scanner (Philips Healthcare, Best, Netherlands) with 16-channel torso coil. All the volunteer studies conducted were approved by our institutional IRB.

In the spirit of reproducible research, we provide MATLAB (The MathWorks, Natick, MA) demonstration code to reproduce some of the results described in this article. The software can be downloaded from: <https://github.com/mikgroup/DelayRecon>.

Simulation Validation

To validate the algorithm, simulations on a 256×256 brain image with 8-channel simulated sensitivity profiles were performed, both of which can be downloaded from ISMRM 2015 Sunrise Course on The Image Reconstruction Pipeline: <http://gadgetron.sf.net/sunrise>. The simulation was performed using 2D center-out radial (trapezoid waveforms with ramp up sampling), projection reconstruction (PR) trajectory (full-spoke) and multi-shot time-optimal spiral [34] trajectory. The center-out radial trajectory was comprised of 804 radial spokes, each containing 256 readout points. The PR trajectory included 402 projections and each projection had 380 readout points. The spiral trajectory comprised of 60 interleaves, each with 1182 readout points. Gaussian white noise was added. To validate SAGE, we artificially created delays in x and y gradient axes individually, each ranging from ranging from -2 to 2 samples, to generate corrupted k-space data. Here we denote the positive gradient delay as the time delay from the start of ADC window to the start of readout part of played-out gradient waveform.

Images were reconstructed with the nominal trajectory as well as the corrected coordinates estimated by SAGE. In addition, the estimated delays were compared to the ground truth. The sensitivity maps were computed from the reconstructed and corrected calibration data by ESPIRiT [31].

For different trajectories, the study was repeated for white matter SNR values of 50, 25, and 5 in order to validate the robustness of SAGE against noise. Each experiment was repeated 10 times to compute the mean and standard deviation of the estimated delay values.

The auto-calibration region was 36×36 and block size was 6×6 . The rank threshold was chosen to be 64 for the 8-channel simulation. In the step from non-Cartesian data to the gridded data, we used iterative gridding [35] based operations to avoid errors from density compensation inaccuracy. The entire algorithm was stopped after reaching an increment shift size of 0.001 sample.

Phantom and In Vivo Experiment for Center-out Radial Trajectory

In order to evaluate the performance of SAGE, we performed a 2D RF-spoiled radial FLASH sequence scan on a sphere phantom first and then *in vivo* using the same acquisition. A slice-selective pulse was used for excitation and trapezoid waveforms were played out on the readout gradients, which were x and y axes in the experiments. Then, we introduced different delay combinations on the x and y gradient axes, ranging from $-8 \mu\text{s}$ to $8 \mu\text{s}$. The following prescribed parameters were used: flip angle = 4° , FOV = $20 \times 20 \text{ cm}^2$, in-plane resolution = $1 \times 1 \text{ mm}^2$, TE/TR = 3.4 ms/2 s, readout bandwidth = $\pm 125 \text{ kHz}$ ($4 \mu\text{s}/\text{sample}$), each spoke has 256 readout points and 628 spokes were acquired. The total scan time was 2 min 56 s. There are 15 samples on the ramp for trapezoid waveforms for this resolution. Since there are samples along the ramp part of trapezoid waveforms, the gradient timing delay induced trajectory deviations are not simply trajectory shifts.

The auto-calibration region was 36×36 and block size was 6×6 . The rank threshold was chosen to be 64. Only 200 spokes were used for delays estimation since the center of k-space is oversampled and it was computationally more efficient to use fewer spokes. The algorithm was stopped after reaching a increment shift size of 0.001 sample.

In Vivo Experiments for Projection Reconstruction Trajectory

We also performed a free-breathing abdominal scan using 3D “stack of stars” trajectory (PR in two dimensions with 1 dimension of phase encoding) on a healthy volunteer. The following prescribed parameters were used: flip angle = 10° , FOV = $32 \times 32 \text{ cm}^2$, in-plane resolution = $1 \times 1 \text{ mm}^2$, slice thickness = 3 mm, TE/TR = 1.2 ms/5.12 ms, readout bandwidth = $\pm 250 \text{ kHz}$ ($2 \mu\text{s}/\text{sample}$), each projection has 400 readout points and 600 projections were acquired per phase encoding step. The proposed method was applied on each axial slice after Fourier Transform in the phase encoding using similar parameter setting with the above examples. 200 projections were used to estimate gradient delays as they fully covered the auto-calibration k-space.

In addition, we also compared the proposed method with a widely-cited method [16] proposed by Block *et al.*, which is called “adaptive method” in the manuscript. The comparison result has been illustrated in Supporting Fig S3.

Results

Simulation Validation

Fig 2 shows numerical simulation results for a center-out radial trajectory with the true delay of 1 sample in the x-axis and 2 samples in the y-axis: top row of shows the sum of squares of the individual coil images and the estimated sensitivity maps from the calibration region;

The middle row depicts the reconstructed sum of squares of coil images using the uncorrected coordinates and the corresponding sensitivity maps, where the image was distorted and the sensitivity maps were corrupted; the bottom row demonstrates the reconstructed image and resulting sensitivity maps corrected by the proposed method (SAGE). The gradient delay artifacts visible near the scalp and artificial enhancement in the middle of the brain images (yellow arrows) were effectively eliminated by the proposed method (bottom row). The sensitivity maps in the presence of gradients delays also show incorrect contours (red arrows) corresponding to the skull that will corrupt parallel imaging reconstructions. These were corrected by SAGE. Quantitatively, the estimated delay values of $[0.9989 \ 2.0014]$ in $[x,y]$ are almost perfectly aligned with the ground truth $(t) = [1 \ 2]$.

Fig 3 shows the different artifact appearance for various trajectories and delays. As the yellow arrows indicate, artifacts can appear as erroneously enhanced contrast or void contrast, edge signal leakage for a center-out radial trajectory, image rotation or edge signal leakage for a spiral trajectory, and streaking artifacts for a PR trajectory. All of these artifacts and distortions indicated in Fig 3 were effectively eliminated and contrast was restored by the proposed method SAGE, considerably improving the image quality. Supporting Table S2 presents the estimated delay values in different delay scenarios for different trajectories. The estimated delay values for different trajectories have good agreements with the ground truth.

SNR in Supporting Table S2 was computed from signal to noise ratio of a white matter region in the red box in Fig 3. From the table, we can see that SAGE can estimate the gradient delay values with high accuracy compared with ground truth for white matter SNRs of 50, 25, and 5. As SNR decreases, the estimation errors (mean and standard deviation) increase. Even for very low SNR value of 5, the proposed method still effectively estimated delay values. The estimation errors are all less than 0.01 samples for radial and spiral trajectories and less than 0.05 samples for projection reconstruction trajectory.

In terms of convergence rate, the algorithms stopped around 50 iterations at the tolerance of 0.001 sample among all the simulation cases.

Phantom and In Vivo Experiments for Center-out Radial Trajectory

In center-out 2D radial trajectory experiments, we introduced a range of gradient delays ranging from $-8 \mu s$ to $8 \mu s$ on each axis compared to the scanner's calibrated gradient timings. Fig 4 shows a representative result with an introduced delay of $-4 \mu s$ delay in the x-gradient and $8 \mu s$ delay in the y-gradient for the phantom study, and $4 \mu s$ delay in x-gradient and $8 \mu s$ delay in y-gradient for the *in vivo* study. The sum of squares of the individual coil images reconstructed with nominal trajectory (on the left), manually corrected trajectory with system calibrated delays (in the middle) and estimated delays (in the right) in Fig 4. The results are displayed in the same window level for comparison. The contrast was restored and artifacts were removed by both the proposed method and system calibration. The visible difference between the two corrected images is marginal. The delays estimated by the proposed method deviated by $0.7\text{--}0.8 \mu s$ from the the system calibrations, which was consistent across the range of introduced gradient delays both in phantoms and *in vivo*.

In Vivo Experiments for Projection Reconstruction Trajectory

The proposed method was applied to different slices of a 3D stack of stars PR dataset. Images were reconstructed with nominal k-space trajectory (left column) and SAGE corrected trajectory (right column). The streaking artifacts of each slice, which are clearly visible (on the left column) in Fig 5 and also appear in the simulation results in Fig 3, are greatly reduced after the correction with the proposed method (on the right), resulting in a clearly improved image quality. The estimated delays were quite consistent from slice to slice, with minor differences ($0.4 \mu\text{s}$) observed between abdominal and chest slices. This difference may be related to varying amplitudes of respiratory motion.

SAGE was compared with the “adaptive method”, shown in Supporting Fig S3. Both methods can largely reduce the streaking artifacts for different slices and the contrast is more homogeneous across the images, clearly improving image quality. In the second slice, as pointed out by the red arrows, the adaptive method shows slightly more artifact than the proposed method. More details can be found in Supporting Materials.

Discussion

This work demonstrates a method named SAGE to correct for gradient timing errors in center-out radial, projection reconstruction and spiral k-space trajectories, with results showing clearly improved image quality. SAGE also provides accurate auto-calibration data for parallel imaging. It iteratively enforces a low-rank property in the calibration region and estimates timing delays on individual gradient axes.

One advantage of SAGE is that when sub-sample delays cannot be readily corrected for in the pulse sequence, SAGE can provide more precise estimate of the gradient delays.

SAGE attempts to correct for gradient delays. The delays can originate from physical delays between the gradient system and data acquisition components. Short-term linear eddy currents can also be approximated as a gradient delay. The proposed method can correct for delays from either source. A more detailed model could formulate the correction problem as eddy current estimation and solve for the corresponding eddy current parameters (*e.g.* time-constants and amplitude). But for some k-space trajectories (*e.g.* center-out radial), long term linear eddy currents might result in higher data correlation, in which case the low-rank model is not suitable for long-term eddy current correction. The proposed method is effective when the trajectory errors create more inconsistency (equivalently, higher rank) of the auto-calibration region. Further efforts are needed to investigate the extension to eddy currents correction for different trajectories.

In this paper, we applied SAGE when the center of k-space was fully-sampled or oversampled. However, if there is enough redundancy from multi-channel coils, then undersampling in the calibration region to some extent is also tolerable. For example, in the center-out radial trajectory simulation, with 8 channel coils and calibration size of 36×36 , we could still estimate delay values with less than 0.01 sample errors even with 40 spokes, which is equivalent to undersampling ratio of 2.8 with respect to Nyquist criterion.

The rank threshold of the calibration matrix for SAGE was chosen according to the bound suggested by SAKE [30]. Briefly, the paper found that the rank of the calibration matrix A is bound by $\text{rank}(A) \leq (w + s - 1)^2$, where w is the block size of auto-calibration region and s is the coil bandwidth measured in k-space pixels. The paper also found that this threshold performs well experimentally. For more detail, we refer to Appendix A of [30]. In practice, estimates from other scans could also be used to predict the rank value for delay corrupted data sets.

The maximum delays that can be corrected by SAGE depend on k-space trajectories, and how much redundancy the data can provide by the multi-channel coils. As long as the delays do not result in a substantial chunk of missing data in the calibration region (which is beyond the limits of what parallel imaging can reconstruct), the proposed method can still correct for the delays.

From the simulation studies, even for very low SNR of 5, the proposed method was still able to correct for gradient timing delays with decent accuracy for different trajectories. Thus, high noise levels do not prohibit the application of the proposed method.

We note that for SAGE, it is essential to use iterative methods to approximate the inverse NUFFT operation, rather than the conventional gridding with density compensation. This is because if we use density compensation from the nominal trajectory, then the resulting gradient timing delay estimation is not accurate. Alternatively, we would need to update density compensation for each iteration, which is computationally intensive. Instead, we performed iterative gridding [35] based operation for each iteration. Since we only perform the operations in the auto-calibration region, the computation burden is not heavy. All the examples presented in the manuscript were computed within one minute for a MATLAB implementation. This could be further sped up by using a high performance programming language.

SAGE is essentially a low-rank matrix completion with alternating minimization, and does not assume convexity of the minimization problem. It is important that we start the reconstruction with an initial estimate that lies close to the global optimum to ensure a fast convergence. The gradient delays were initialized to zero in our experiments.

Experimentally, we found that the final solution is fairly consistent even if we specify a different initialization for gradient delays. One reason might be that the rank constraint set with respect to the gradient delays is convex-like as shown in Supporting Fig S1. However, it is possible that the algorithm will converge to a local minimum instead of a global minimum in certain scenarios. Therefore, the initialization of the images is still important. In our experiments, we start with an initial image reconstructed using the nominal trajectory, and have found that to produce accurate gradient delay estimates.

Conclusion

We propose a simple and robust method for estimating gradient timing delays based on a low-rank model (SAGE) that does not require any additional calibration scans. It determines

the gradient timing delays and simultaneously yields an accurate k-space auto calibration region data for parallel imaging.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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Figure 1.

Illustration of the proposed method: the gradient delay is estimated iteratively. In each iteration, the non-Cartesian data is gridded onto a Cartesian grid and then transformed into a block-Hankel structured calibration matrix (A). Low-rank constraints are applied on the calibration matrix. It is then reformatted to Cartesian data followed by the operation that transforms Cartesian data into non-Cartesian data. In the last step, Gauss-Newton method is applied to calculate the delay values for each axis.

Figure 2.

Simulation results with the proposed method: with gradient delays, the image is distorted as the yellow arrows show; the sensitivity maps are corrupted as the brain edges are voided as the red arrows show (in the middle row). The proposed method is able to remove the artifacts, restore image quality and sensitivity maps; the delay estimation is accurate compared with ground truth.

Figure 3.

Simulation results with gradient delays for radial, spiral and PR trajectories: All artifacts indicated by arrows were effectively eliminated and contrast was restored by the proposed method, considerably improving the image quality. Quantitative analysis of the gradient delay estimation accuracy is listed in the Supporting Table S2.

Figure 4. Phantom and in vivo results using center-out radial trajectory with the proposed method: Gradient delay induced artifacts are highlighted with yellow arrows in the left row. The contrast was restored and artifacts were removed by both the the proposed method (right column) and system calibration (center column).

Figure 5.

In vivo results using stack-of-stars 3D projection reconstruction trajectory: three different slices are displayed from the same scan, where the left column shows uncorrected image and the right column shows corrected images using SAGE with the estimated X and Y gradient delays. The gradient delay induced streaking artifacts (yellow arrows, also seen in Fig 3 simulations) are largely reduced for all the slices with the proposed method and the contrast is more homogeneous across the images, resulting clearly improved image quality.