RESEARCH

A dynamical model for the number of infertile couples

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Abstract

In this paper a dynamic model is suggested in order to predict treatment results for infertility of couples. For this purpose, five basic groups of couples are examined. These groups are: susceptible couples, patient couples, couples under treatment taking medicine only, those under treatment using surgery and cured ones. The main aim is to find an asymptotically stable free equilibrium point for this model. The method benefits from scrutinizing the dynamical model deeply. We show that there is another equilibrium point that is not signed stable. In addition, we solve this model numerically via Rung–Kutta method and sketch appropriate graphs for the solutions thus obtained.

Keywords: Infertility, Asymptotically stable, Free equilibrium point, Sign stable

Introduction

Infertility is defined as the disability of a couple in reproduction despite having a sexual relationship during 1 year without using any contraceptive. The population of infertile couples in Iran has been 10–15% of couples married more than 1 year [1], during the last 10 years. This problem can be related to the female or male partner or both. About 40% of infertility cases are related to men, 40% to women, 10% to both of them, and in 10% the reason is not clear. In the last 10% of couples although none of them have problem or disability, for some unclear causes they are unable in reproduction [2].

Infertility has been a mystery for a long time such that it has been the subject of research among the researchers. At first, they related it to religion and art. The primary men were unable in understanding the complex process of reproduction so they related it with invisible and magic interpretations. Nevertheless, problems regarding the connection between the quality of sexual intercourse and birth have existed in primary civilizations such as Egypt, Greece, and Babylon. The primary considered having the ability of reproduction as the salient capacity

and in most times infertility was regarded as a problem related to the female. With gradual scientific advancements, infertility changed from its primary form. The main changes occurred in Renaissance period. One of the most important actions that were done at this time was the exact anatomy of genital organ. Also, the surgery of women disease, removing pelvis adhesiveness, considering the womb tubes and being assured that they are open by using Endoscopy, Laparoscopy achieved a salient progress in this period. Using stimulus ovulation drugs, laboratory reproduction, and other Assistant Reproductive Technology (ART) were also applied. In 1978, the first production of laboratory reproduction (Brown Louise) was born by S.R. Edward and P. Stepto in the North-West of England. The dream became true! Nowadays, using the findings of reproduction biology industry and genetics, studying and treatment of infertility has found a remarkable progress.

Six written notes of Hippocrates in 360–370 B.C. reflected the common medical knowledge of about 400 years B.C. These writings included discussion about birth, the science of diagnosis, and recognizing the causes of diseases, different disorders related to infertility, and the ways of diagnosis and treatment of disease.

In the middle centuries, the achievements obtained in infertility are not noticeable. Churches related pregnancy and birth with myth and religious beliefs. Based on the beliefs of that time, the distinct disconformity of the

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vagina and external genital organ of man was recognized as the main reason for infertility.

In the sixteenth century, scientific observations led to the extension of science and medicine. In 1538, Anderes and Salious accomplished the distinct anatomy of woman's genital organ. Their student, Gabriel Fallopius, analyzed one section of woman's genital organ called fallopian tubes. His action was considered as a significant step in the infertility science. In seventeenth and eighteenth centuries, sperm cell was recognized and its role in reproduction process was explained. Spallanzani (1729–1799) was the first scientist who analyzed the infertility mechanism in a book entitled "Artificial Reproduction". In 1780, he showed that touching ovocyte and spermatozoa leads to zygosis. He was the first who analyzed the artificial zygosis of dogs. Also, he was a pioneer in accomplishing the inoculation in man by placing man's sperm sufferings from Hypospadias into woman's vagina. In 1805, the first Endoscopy action was performed by Bozzini. For the first time, Hysteroscopy was done in 1869 by Leoni Panta and in 1879 Nitze's contact lens was added to Hysteroscopy to improve lightness and view point. The Nitze's Hysteroscopy was applied 29 years later in 1908.

In 1885, Schenck suggested the reproducing of mammals new ovule as rabbit and guinea pig. During the years 1878–1880, the results obtained in laboratory zygosis in sea animals laboratories were presented. In 1891, Tleape-Walter studied the zygotic of rabbit from fallopian tubes by washing and transferring it into surrogate mother. In 1915, the Swedish scientist "Jacobaeus" accomplished the first laparoscopy on man by using Terocaro Canolapnomo and the existed Peritonea. He also observed lenity by Nitze's cystoscope. In 1911, for the first time, Berheim considered peritonea in the U.S.A. In 1913, Honer Max (1873–1947) introduced Post Coital Test (PCT) as a reliable method for studying the causes of women's infertility. In 1920, Rubin used Co_2 as the blower substance to tubes for checking their openness. Later, this experiment was introduced as the most important result in assessment of reproduction in the first half of the twentieth century.

Methodology

In this paper, a mathematical model is presented for describing the infertility of couples. For this purpose, all of the infertile couples at time t are shown with $N(t)$ and are divided into five groups. The first group contains the infertility susceptible couples; those who are disable in reproduction during the first year of their marriage, despite having intercourse without using any contraceptive. Some of these couples may have no problem. Patients are denoted by $P(t)$; those under treatment without surgery were shown by $T_1(t)$; patients under treatment that at least one of them is under surgery were

shown by $T_2(t)$; and cured couples were shown by $C(t)$. So, we have $N(t) = S(t) + P(t) + T_1(t) + T_2(t) + C(t)$. This model is named SPT_1T_2C .

In the next section, a model is obtained for infertility of couples by dynamical system methods and R_0 (the number of uncured couples) is obtained.

In the third section, the free equilibrium point of the constructed model is obtained and its asymptotical stability is considered.

In the fourth section, the equilibrium point Q_* is proved the sign unstable.

Finally, the model is solved numerically using the Rung–Kutta method and, the obtained data are summarized in several graphs and analyzed.

This method is summarized in Fig. 1.

A dynamical model for infertility of couples

We begin this section by introducing some important constants. We denote the amount of the returning illness in every infertile couple by β . Infertile couples who quit because of no pregnancy are denoted by α and couples entering the treatment are shown by λ .

The rate of cured couples in $P(t)$ group is shown by γ ; treatment rate of the couples under treatment (without surgery) at $T_1(t)$ or the couples under treatment using medicine and without surgery at time t is shown by δ_1 ; and treatment rate in couples who were under treatment through surgery at $T_2(t)$ is shown by δ_2 . The fraction of $\delta_1 T_1$ of couples who are cured at time t is h . So, $(1 - h)$ is the fraction of $\delta_1 T_1$ that is used in the surgery treatment. Thus, the average of infertile couples whose illness is returned at time t is equal to βN . The possibility of illness returning in susceptible couple is equal to $\frac{S(t)}{N(t)}$. Therefore, the number of new patients at time t is equal to $\beta N \frac{S(t)}{N(t)}$ [3]. The number of infertile couples that whose illness may return at time t is equal to $\beta(\delta_1 T_1 + \delta_2 T_2)S$. The amount of new susceptible couples at t is λN ; the number of death in susceptible couples is αS and the number of couples who left the treatment unsuccessfully is D (Fig. 2).

$$\text{So } S' = \lambda N - \alpha S - \beta(\delta_1 T_1 + \delta_2 T_2) \cdot S$$

The treatment rate in patient groups is shown by γ and the of recovery rate among couples who are under treatment without surgery in γP is shown with ν . Patient couples who are under medicine treatment without surgery are shown with $\nu \gamma P$ and finally the number of couples who used surgery is shown by $(1 - \nu) \gamma P$. The number of patients who do not receive treatment is shown as ηP . Also, ηP is divided into two groups. The first group contains those who become pregnant without taking any medicine which are shown as $r \eta P$, where r is the rate of recovery in ηP . The second group $(1 - r) \eta P$, include those patients who cannot get

pregnancy due to old age and menopause; and thus are assigned as group *D*. Finally, the leaving patients without any treatment are denoted by αP (Fig. 3).

$$\text{Hence } P' = \beta(\delta_1 T_1 + \delta_2 T_2)S - (\alpha + \gamma + \eta)P.$$

If h denotes the recovery rate in group $\delta_1 T_1$, then the number of couples who received medicine methods and were cured is equal to $h\delta_1 T_1$. Also, $(1-h)\delta_1 T_1$ is the group of couples who used surgery for treatment. The rate of recovery in group $(1-h)\delta_1 T_1$ is equal to k . Therefore, $(1-k)(1-h)\delta_1 T_1$ patient couples who have gone under surgery but did not recover enter group *D*. The rate of untreated couples αT_1 entering group is *D* (Fig. 4).

$$\text{Therefore, } T_1' = v\gamma P - (\alpha + \delta_1)T_1.$$

The number of couples who used surgery for treatment is equal to $\delta_2 T_2$. Couples under surgery without recovery group *D* are αT_2 (Fig. 5).

$$\text{Hence } T_2' = k(1-h)\delta_1 T_1 + (1-v)\gamma P - (\alpha + \delta_2)T_2.$$

The number of people who quit due to nonpregnancy is $(1-r)\eta P$ and $(1-k)(1-h)\delta_1 T_1$. Here, λN denotes the number of infertile couples who used treatment methods and entered the susceptible couple at time t (Fig. 6).

Hence $C' = h\delta_1 T_1 + r\eta P + \delta_2 T_2 - \alpha C$.

Based on what is stated we have the following system which we call it $SPT_1 T_2 C$ model.

$$\begin{cases} S' = \lambda N - \alpha S - \beta(\delta_1 T_1 + \delta_2 T_2)S \\ P' = \beta(\delta_1 T_1 + \delta_2 T_2)S - (\alpha + \gamma + \eta)P \\ T_1' = v\gamma P - (\alpha + \delta_1)T_1 \\ T_2' = k(1-h)\delta_1 T_1 + (1-v)\gamma P - (\alpha + \delta_2)T_2 \\ C' = h\delta_1 T_1 + r\eta P + \delta_2 T_2 - \alpha C \\ N' = (\lambda - \alpha)N - (1-k)(1-h)\delta_1 T_1 - (1-r)\eta P \end{cases}$$

For finding R_0 we replace $N(t)$, and $S(t)$ with S_0 and we deduce the following system.

$$R_0 = \text{Trace}(FT^{-1}) = \frac{\beta\delta_1 S_0 v\gamma(\alpha + \delta_2) + \beta\delta_2 S_0 (v\gamma k\delta_1(1-h) + (\alpha + \delta_1)\gamma(1-v))}{(\alpha + \gamma + \eta)(\alpha + \delta_1)(\alpha + \delta_2)}.$$

This linear system is divided into two parts. The first matrix is shown by F , referred to as returning matrix, and the second matrix is T , referred to as the affected matrix. Hence, if

$$F = \begin{pmatrix} 0 & \beta\delta_1 S_0 & \beta\delta_2 S_0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and

$$T = \begin{pmatrix} \alpha + \gamma + \eta & 0 & 0 \\ -v\gamma & \alpha + \delta_1 & 0 \\ -(1-v)\gamma & -k(1-h)\delta_1 & \alpha + \delta_2 \end{pmatrix}$$

then

$$T^{-1} = \begin{pmatrix} \frac{1}{\alpha + \gamma + \eta} & 0 & 0 \\ \frac{v\gamma}{(\alpha + \gamma + \eta)(\alpha + \delta_1)} & \frac{1}{(\alpha + \delta_1)} & 0 \\ \frac{v\gamma k\delta_1(1-h) + (\alpha + \delta_1)\gamma(1-v)}{(\alpha + \gamma + \eta)(\alpha + \delta_1)(\alpha + \delta_2)} & \frac{(1-h)k\delta_1}{(\alpha + \delta_1)(\alpha + \delta_2)} & \frac{1}{(\alpha + \delta_2)} \end{pmatrix}$$

Therefore

$$FT^{-1} = \begin{pmatrix} A & B & \frac{\beta\delta_2 S_0}{(\alpha + \delta_2)} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where

$$A = \frac{\beta\delta_1 S_0 v\gamma}{(\alpha + \gamma + \eta)(\alpha + \delta_1)} + \beta\delta_2 S_0 \left(\frac{v\gamma k\delta_1(1-h) + (\alpha + \delta_1)\gamma(1-v)}{(\alpha + \gamma + \eta)(\alpha + \delta_1)} \right) \quad (1)$$

and

$$B = \frac{\beta\delta_1 S_0}{(\alpha + \delta_1)} + \frac{\beta\delta_2 S_0(1-h)k\delta_1}{(\alpha + \delta_1)(\alpha + \delta_2)}.$$

R_0 is the trace of FT^{-1} . Thus [3]

Analyzing the dynamical model of couples' infertility based on asymptotic stability

Based on the biological hypothesis, we deduced $SPT_1 T_2 C$ model.

Before the disease the free equilibrium point is $Q_0 = (S_0, 0, 0, 0, S_0)$, and we have the following result.

$$\begin{pmatrix} P' \\ T_1' \\ T_2' \end{pmatrix} = \begin{pmatrix} 0 & \beta\delta_1 S_0 & \beta\delta_2 S_0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} P \\ T_1 \\ T_2 \end{pmatrix} - \begin{pmatrix} \alpha + \gamma + \eta & 0 & 0 \\ -v\gamma & \alpha + \delta_1 & 0 \\ -(1-v)\gamma & -k(1-h)\delta_1 & \alpha + \delta_2 \end{pmatrix} \begin{pmatrix} P \\ T_1 \\ T_2 \end{pmatrix}$$

Theorem 3.1 If $R_0 < 1$ then:

- (a) If $\lambda - \alpha > 0$ then, Q_0 is the free equilibrium point which is locally stable but it is not asymptotically stable.
 (b) If $\lambda - \alpha = 0$ then, the free equilibrium point is unstable.
 (c) If $\lambda - \alpha < 0$ then, Q_0 is locally asymptotically stable.

Proof

- (a) The linearization matrix of the model SPT_1T_2C at the point Q_0 is

$$\begin{pmatrix} -\alpha & 0 & -\beta\delta_1S_0 & -\beta\delta_2S_0 & \lambda \\ 0 & -(\alpha + \gamma + \eta) & \beta\delta_1S_0 & \beta\delta_2S_0 & 0 \\ 0 & v\gamma & -(\alpha + \delta_1) & 0 & 0 \\ 0 & (1-v)\gamma & (1-h)k\delta_1 & -(\alpha + \delta_2) & 0 \\ 0 & -(1-r)\eta & -(1-k)(1-h)\delta_1 & 0 & \lambda - \alpha \end{pmatrix}.$$

The characteristic equation of this matrix is :
 $-(\alpha + \lambda')(\lambda - \alpha - \lambda')[-v\gamma(-\beta\delta_1S_0(\alpha + \delta_2 + \lambda') - \beta\delta_2S_0(1-h)k\delta_1) - (\alpha + \delta_1 + \lambda')[(\alpha + \gamma + \eta + \lambda') - (\alpha + \delta_2 + \lambda') - \beta\delta_2S_0(1-v)\gamma]] = 0.$ So $-(\alpha + \lambda')$

$$(\alpha + \delta_2 + \lambda') - \beta\delta_2S_0(1-v)\gamma] = 0. \text{ So } -(\alpha + \lambda')$$

$$(\lambda - \alpha - \lambda')[\lambda'^3 + [(3\alpha + \delta_2) + \gamma + \eta + \delta_1]\lambda'^2 +$$

$$[-v\gamma\beta\delta_1S_0 + (\alpha + \gamma + \eta)(\alpha + \delta_2) + (\alpha + \delta_1)(\alpha + \delta_2) +$$

$$(\alpha + \delta_1)(\alpha + \gamma + \eta) - \beta\delta_2S_0(1-v)\gamma]\lambda' + [-v\gamma\beta\delta_1S_0$$

$$(\alpha + \delta_2) - v\gamma\beta\delta_0\delta_2\delta_1k(1-h) + (\alpha + \delta_1)(\alpha + \gamma + \eta$$

$$(\alpha + \delta_2) - (\alpha + \delta_1)\beta\delta_2S_0(1-v)\gamma]] = 0$$

$$\lambda' = -\alpha \quad (2)$$

and

$$\lambda' = \lambda - \alpha \quad (3)$$

are two roots of this equation. If

$$\begin{aligned} & [\lambda'^3 + [(3\alpha + \delta_2) + \gamma + \eta + \delta_1]\lambda'^2 \\ & + [-v\gamma\beta\delta_1S_0 + (\alpha + \gamma + \eta)(\alpha + \delta_2) \\ & + (\alpha + \delta_1)(\alpha + \delta_2) + (\alpha + \delta_1)(\alpha + \gamma + \eta) \\ & - \beta\delta_2S_0(1-v)\gamma]\lambda' + [-v\gamma\beta\delta_1S_0(\alpha + \delta_2) \\ & - v\gamma\beta\delta_0\delta_2\delta_1k(1-h) + (\alpha + \delta_1)(\alpha + \gamma + \eta(\alpha + \delta_2) \\ & - (\alpha + \delta_1)\beta\delta_2S_0(1-v)\gamma]] = 0 \end{aligned} \quad (4)$$

then we solve the equation via Routh Hurwitz methods. As all of the parameters are positive then $a_1 = (3\alpha + \delta_2) + \gamma + \eta + \delta_1 > 0$. Moreover $a_0 = 1 > 0$.

We take

$$\begin{aligned} a_2 &= [-v\gamma\beta\delta_1S_0 + (\alpha + \gamma + \eta)(\alpha + \delta_2) + (\alpha + \delta_1) \\ & (\alpha + \delta_2) + (\alpha + \delta_1)(\alpha + \gamma + \eta) - \beta\delta_2S_0(1-v)\gamma] \\ \text{and } a_3 &= [-v\gamma\beta\delta_1S_0(\alpha + \delta_2) - v\gamma\beta\delta_0\delta_2\delta_1k(1-h) + \\ & (\alpha + \delta_1)(\alpha + \gamma + \eta(\alpha + \delta_2) - (\alpha + \delta_1)\beta\delta_2S_0(1-v)\gamma]. \\ \text{Since } R_0 < 1 \text{ then} \\ & \frac{\beta\delta_1S_0v\gamma}{(\alpha + \gamma + \eta)(\alpha + \delta_1)} < 1 \implies \\ & (\alpha + \gamma + \eta)(\alpha + \delta_1) - \beta\delta_1S_0v\gamma > 0. \end{aligned} \quad (5)$$

$$\text{And } \frac{\beta\delta_2S_0(1-v)\gamma}{(\alpha + \delta_2)(\alpha + \gamma + \eta)} < 1.$$

Therefore

$$(\alpha + \delta_2)(\alpha + \gamma + \eta) - \beta\delta_2S_0(1-v)\gamma > 0. \quad (6)$$

(5) and (6) implies $a_2 > 0$.

Since $R_0 < 1$ then

$$\begin{aligned} & (\alpha + \gamma + \eta)(\alpha + \delta_1)(\alpha + \delta_2) + \beta\delta_2S_0(1-h)k\delta_1v\gamma > \\ & \beta\delta_1S_0v\gamma(\alpha + \delta_2) + \beta\delta_2S_0(1-v)\gamma(\alpha + \delta_2). \\ \text{So } & ((\alpha + \gamma + \eta)(\alpha + \delta_1)(\alpha + \delta_2) - \beta\delta_2S_0 \\ & (1-h)k\delta_1v\gamma - \beta\delta_1S_0v\gamma(\alpha + \delta_2) \\ & - \beta\delta_2S_0(1-v)\gamma(\alpha + \delta_2)) > 0. \end{aligned} \quad (7)$$

Hence $a_3 > 0$.

$$\text{Moreover } \Delta_2 = \det \begin{pmatrix} a_1 & a_0 \\ a_3 & a_2 \end{pmatrix} = a_1a_2 - a_0a_3.$$

$$\begin{aligned} \text{Therefore } a_1a_2 - a_0a_3 &= (\alpha + \delta_2)^2 + (\alpha + \gamma + \eta)^2 + \\ & (\alpha + \delta_1)^2 + 2(\alpha + \delta_2)(\alpha + \gamma + \eta) + 2(\alpha + \delta_1)(\alpha + \delta_2) + 2 \\ & ((\alpha + \gamma + \eta)(\alpha + \delta_1)) + v\gamma\beta\delta_1s_0(\alpha + \delta_2) + v\gamma\beta\delta_2S_0 \end{aligned}$$

$$(1-h)k\delta_1 - (\alpha + \delta_1)(\alpha + \gamma + \eta)(\alpha + \delta_2) + (\alpha + \delta_1)\beta\delta_2S_0\gamma(1-v) > 0$$

If we take $b_1 = 2(\alpha + \delta_2)(\alpha + \gamma + \eta)$, and $b_2 = -(\alpha + \delta_1)(\alpha + \gamma + \eta)(\alpha + \delta_2)$ then $b_1 + b_2 = (\alpha + \delta_2)(\alpha + \gamma + \eta)(2 - (\alpha + \delta_1)) > 0$.

Since

$$0 < \alpha, \delta_1 < 1 \implies 0 < 2 - (\alpha + \delta_1) < 2 \implies b_1 + b_2 > 0 \quad (8)$$

then $\Delta_2 > 0$.

By using (7) and (8)

$$\Delta_3 = \det \begin{pmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ 0 & 0 & a_3 \end{pmatrix} = a_3(a_1a_2 - a_0a_3) > 0. \quad (9)$$

Hence by Hurwitz criterion the real parts of the roots of (4) are negative. If (3) is positive, then Q_0 is not locally asymptotically stable [4, 5].

(b) If (3) is zero then Q_0 is unstable.

(c) If $\lambda' = \lambda - \alpha < 0$ then the free equilibrium point Q_0 is locally asymptotically stable. $\square$

From the solution of a system of differential equations in SPT_1T_2C we can deduce that [6]

$$\begin{aligned} S_* &= \frac{(\alpha + \gamma + \eta)P_*}{\beta(\delta_1T_{1*} + \delta_2T_{2*})} \\ P_* &= \frac{(\alpha + \delta_1)}{v\gamma}T_{1*} \\ T_{2*} &= \frac{k(1-h)\delta_1T_{1*} + (1-v)\gamma\left(\frac{\alpha+\delta_1}{v\gamma}\right)T_{2*}}{(\alpha + \delta_2)} \\ N_* &= \frac{(1-k)(1-h)\delta_1 + (1-r)\eta\left(\frac{\alpha+\delta_1}{v\gamma}\right)}{(\lambda - \alpha)}T_{1*} \\ T_{1*} &= \frac{\alpha(\alpha + \eta)(\alpha + \delta_1)(\alpha + \delta_2)^2\gamma v^2(\lambda - \alpha)}{D} \end{aligned}$$

where $D = (\beta\delta_1v(\alpha + \delta_2) + \beta\delta_2k(1-h)\delta_1 + (1-v)(\alpha + \delta_1)\beta\delta_2)(\lambda v^2(1-k)(1-h)\delta_1\gamma(\alpha + \delta_2) + (1-r)\eta\lambda(\alpha + \delta_1)v\gamma(\alpha + \delta_2) - (\alpha + \gamma + \eta)(\alpha + \delta_1)(\lambda - \alpha))$

Therefore $Q_* = (S_*, P_*, T_{1*}, T_{2*}, N_*)$ is shown the other equilibrium point. The point Q_1 is in the interior of the positive space if $\lambda > \alpha$ and

$$\lambda v^2(1-k)(1-h)\delta_1\gamma(\alpha + \delta_2) + (1-r)\eta\lambda(\alpha + \delta_1)v\gamma(\alpha + \delta_2) > (\alpha + \gamma + \eta)(\alpha + \delta_1)(\lambda - \alpha).$$

Sign stable

The following results are recalled from [4].

Definition 4.1 An n by n square matrix $A = [a_{ij}]$ is said to be sign stable if every n by n square matrix $B = [b_{ij}]$ of the same sign pattern (i.e. $\text{sign}b_{ij} = \text{sign}a_{ij}$ for all $i, j = 1, 2, \dots, n$), is a stable matrix.

An n by n square matrix $A = [a_{ij}]$ we can obtain an undirected graph G_A whose vertex set is $V = 1, 2, \dots, n$ and edges are $\{(i, j) : i \neq j; a_{ij} \neq 0 \neq a_{ji}; i, j = 1, 2, \dots, n\}$. Also a directed graph DA can also attach to A with the same vertex set and edges $\{(i, j) : i \neq j; a_{ij} \neq 0 \neq a_{ji}; i, j = 1, 2, \dots, n\}$. A k -cycle of DA is a set of distinct edges of DA of the form $\{(i_1, i_2), (i_2, i_3), \dots, (i_{k-1}, i_k), (i_k, i_1)\}$. Let $R_A = \{i : a_{ii} \neq 0\} \subset V$, which are the numbers for them the corresponding element in the main diagonal of the matrix is not zero. An R_A -coloring of G_A is a partition of its vertices into two sets, black and white (one of which may be empty). Such that each vertex in R_A is black, no black vertex has precisely one white neighbor, and each white vertex has at least one white neighbor. A $V-R_A$ complete matching is a set M of pairwise disjoint edges of G_A such that the set of vertices of the edges in M contains every vertex in $V-R_A$.

By applying this concepts we are now able to state the following theorem.

Theorem 4.2 [4] An n by n real matrix $A = [a_{ij}]$ is sign stable if it satisfies the following conditions:

- $a_{ij} \leq 0$ for all i, j ;
- $a_{ij}a_{ji} \leq 0$ for all $i \neq j$;
- The directed graph DA has no k -cycle for $k \geq 3$.
- In every R_A -coloring of the undirected graph G_A all vertices are black;
- The undirected graph G_A admits a $V-R_A$ complete matching.

Theorem 4.3 If $\lambda > \alpha$ then the matrix model SPT_1T_2C at the equilibrium Q_* is not sign stable.

Proof

The matrix of the linearize system model of SPT_1T_2C at the equilibrium Q_* is given by

$S_0, P_0, (T_1)_0, (T_2)_0, N_0$ and C_0 are the initial conditions.

We take $h_1 = \frac{b-a}{M}$, where $M > 0$ and $t_j = a + jh$ for $j = 0, 1, 2, \dots, M$.

For $i = 1, 2, 3, 4, 5$ we take

$$\begin{pmatrix} -\alpha - \beta(\delta_1 T_{1*} + \delta_2 T_{2*}) & 0 & -\beta\delta_1 S_* & -\beta\delta_2 S_* & \lambda \\ \beta(\delta_1 T_{1*} + \delta_2 T_{2*}) & -(\alpha + \gamma + \eta) & \beta\delta_1 S_* & \beta\delta_2 S_* & 0 \\ 0 & v\gamma & -(\alpha + \delta_1) & 0 & 0 \\ 0 & (1-v)\gamma & (1-h)k\delta_1 & -(\alpha + \delta_2) & 0 \\ 0 & -(1-r)\eta & -(1-k)(1-h)\delta_1 & 0 & \lambda - \alpha \end{pmatrix}.$$

$$K_{1,i} = h_1 f_i(t_j, S_j, P_j, (T_1)_j, (T_2)_j, N_j, C_j);$$

$$K_{2,i} = h_1 f_i\left(t_j + \frac{h_1}{2}, S_j + \frac{K_{1,1}}{2}, P_j + \frac{K_{1,2}}{2}, (T_1)_j + \frac{K_{1,3}}{2}, (T_2)_j + \frac{K_{1,4}}{2}, N_j + \frac{K_{1,5}}{2} + C_j \frac{K_{1,6}}{2}\right);$$

$$K_{3,i} = h_1 f_i\left(t_j + \frac{h_1}{2}, S_j + \frac{K_{2,1}}{2}, P_j + \frac{K_{2,2}}{2}, (T_1)_j + \frac{K_{2,3}}{2}, (T_2)_j + \frac{K_{2,4}}{2}, N_j + \frac{K_{2,5}}{2}, C_j + \frac{K_{2,6}}{2}\right);$$

$$K_{4,i} = h_1 f_i(t_j + h_1, S_j + K_{3,1}, I_j + K_{3,2}, (T_1)_j + K_{3,3}, (T_2)_j + K_{3,4}, N_j + K_{3,5}, C_j + K_{3,6}).$$

Theorem (4.3) requires

$$B = \begin{pmatrix} -1 & 0 & -1 & -1 & 1 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & -1 & -1 & 0 & 1 \end{pmatrix}.$$

Not to be sign stable. Since $a_{55} > 0$ then Theorem (4.3) implies that the matrix A is not sign stable. The previous Theorem implies that in SPT_1T_2C model the equilibrium point is not sign stable. This is a good result because the return of disease is not reversed for a long time. Accordingly, the problem of infertility remains still highly challenging in the community. $\square$

Numerical solutions of SPT_1T_2C

In this section we solve SPT_1T_2C model numerically via Rung–Kutta method [3, 5]. First we define the following functions:

$$f_1(t, S, P, T_1, T_2, N, C) = -\beta(\delta_1 T_1 + \delta_2 T_2)S - \alpha S + \lambda N;$$

$$f_2(t, S, P, T_1, T_2, N, C) = \beta(\delta_1 T_1 + \delta_2 T_2)S - (\alpha + \gamma + \eta)I;$$

$$f_3(t, S, P, T_1, T_2, N, C) = -(\alpha + \delta_1)T_1 + v\gamma P;$$

$$f_4(t, S, P, T_1, T_2, N, C) = k(1-h)\delta_1 T_1 + (1-v)\gamma P - (\alpha + \delta_2)T_2;$$

$$f_5(t, S, P, T_1, T_2, N, C) = (\lambda - \alpha)N - (1-k)(1-h)\delta_1 T_1 - (1-r)\eta P;$$

$$f_6(t, S, P, T_1, T_2, N, C) = h\delta_1 T_1 + r\eta P + \delta_2 T_2 - \alpha C;$$

We put $w_{1,j} = S_j$, $w_{2,j} = I_j$, $w_{3,j} = (T_1)_j$, $w_{4,j} = (T_2)_j$, $w_{5,j} = N_j$, and $w_{6,j} = C_j$. So

$$w_{i,j+1} = w_{i,j} + \frac{1}{6}(K_{1,i} + 2K_{2,i} + 2K_{3,i} + K_{4,i}).$$

If $M = 50$, $N_0 = 104$, $S_0 = 15$, $P_0 = 30$, $(T_1)_0 = 27$, $(T_2)_0 = 12$ and $C_0 = 20$, then in Figs. 7, 8, 9 and 10 we sketched the graphs of the solutions.

In Fig. 9, the rate of returning illness is 2.5 times as much as those in Figs. 7, 8, and 10. In Fig. 10, the rate of treatment through surgery is 3.5 times as much as those in Figs. 7, 8, and 9. At first, the curves of patients in the Figs. 9 and 10 are descending and this reduction in Fig. 10 is more than in Fig. 9. These curves descended until they reached their minimum amount. The minimum of Fig. 10 is less than the minimum of Fig. 9. At the end of the period, as compared with the beginning of the period, the number of patients in Fig. 9 are equal. In Fig. 10, at the end of the period the number of patients is more than $\frac{1}{3}$ of patients at the beginning of the period.

The cure rate in patients who used medicine in Fig. 8 is 0.3 times as much as in Fig. 9. So, at the beginning of the period in Fig. 9, the number of patients who used

medicine increased and reached its maximum. Then, it declined with a slight slope and reached its minimum. In comparison, in Fig. 9 the curve of the numbers of patients who used medicine is descending and at the end of the period, as compared with Fig. 8, the number of patients is less. It showed that with increasing the rate of treatment in patients who used medicine, the number of patients decreased considerably. So, at the beginning of the period, the number of patients in Figs. 8 and 9 is reduced and reached its minimum. Then, the curve of patients in Figs. 8 and 9 increased and in 8 it is more salient. Certainly, the changes in the curve of patients in Fig. 9 as compared with Fig. 8 occurred in a smaller span.

Similarly, the curve of patients in Figs. 7 and 8 are descending at the beginning.

In Fig. 10, the rate of treatment through surgery is 3.5 times as much as Fig. 7. So the number of cured patients in Fig. 10 is increasing. These results show that the treatment through surgery could be applied as a sufficient method.

Conclusion

In this research, the SPT_1T_2C model was considered for curing the infertility of couples. It is proved that the disease free equilibrium point Q_* for SPT_1T_2C model is locally stable and it is not asymptotically stable, when $R_0 < 1$. In the fourth section, we proved that the other

equilibrium point is sign unstable. Also, in the fifth section we considered the graphs of the model.

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Acknowledgements

The authors would like to express their thanks to Prof. Abbas Aflatoonian for his medical advices.

Received: 19 March 2018 Accepted: 24 July 2018

Published online: 18 August 2018

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