On Studying Common Factor Dominance and Approximate Unidimensionality in Multicomponent Measuring Instruments With Discrete Items

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Abstract

This article outlines a procedure for examining the degree to which a common factor may be dominating additional factors in a multicomponent measuring instrument consisting of binary items. The procedure rests on an application of the latent variable modeling methodology and accounts for the discrete nature of the manifest indicators. The method provides point and interval estimates (a) of the proportion of the variance explained by all factors, which is due to the common (global) factor and (b) of the proportion of the variance explained by all factors, which is due to some or all other (local) factors. The discussed approach can also be readily used as a means of assessing approximate unidimensionality when considering application of unidimensional versus multidimensional item response modeling. The procedure is similarly utilizable in case of highly discrete (e.g., Likert-type) ordinal items, and is illustrated with a numerical example.

Keywords

binary (scored) item, common factor, common factor dominance, interval estimation, global factor, local factor, multidimensionality, unidimensionality

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Educational and psychological researchers are frequently confronted with multi-item measuring instruments, such as tests, scales, self-reports, subscales, testlets, or inventories (referred to generically as “instruments” below), which may have been initially developed to be unidimensional but are likely tapping also into additional traits, constructs, or abilities (cf. Reise, Cook, & Moore, 2015). This is oftentimes the case when an instrument measures predominantly a single (global) factor, which is common to all its items, along with several additional factors of local nature (i.e., “minor” factors shared by smaller subsets of items). For instance, Raykov and Pohl (2013a) discussed several research settings where scholars were concerned with measuring instruments that possibly resulted from series of successful revisions yet inadvertently tapped into what may be seen as minor subabilities in addition to constructs of main concern. In such situations, researchers are typically interested in the degree to which they can treat an instrument as primarily evaluating a single factor, and specifically whether (a) that common factor can be viewed as “dominating” the other factors, and hence (b) the instrument can be seen for some purposes as approximately unidimensional (cf. Reckase, 2016).

A methodology that allows one to address this query of common factor dominance (CFD) is latent variable modeling (LVM; e.g., B. O. Muthén, 2002). Recently, Raykov and Pohl (2013a, 2013b) discussed two LVM-based approaches that permit scholars to respond to certain aspects of this theoretical and empirically important question. Their procedures, however, were developed for the case of (approximately) continuous indicators (i.e., for instruments consisting of components that could be treated as continuous). However, the majority of contemporary educational and behavioral research deals with highly discrete ordinal items, such as binary or binary scored measures or Likert-type items with only a few possible response options. In these cases, the methods in Raykov and Pohl (2013a, 2013b) cannot be recommended, as they may yield misleading estimates, tests results, and substantive interpretations.

This note is concerned with an LVM approach addressing the aforementioned issues (a) and (b) when the instrument components are not (approximately) continuous but are highly discrete and ordinal instead. In the remainder, a readily applicable procedure is discussed that furnishes the point and interval estimates of the proportion explained variance by all factors that is due to a global factor common to all items, and of that proportion due to some or all other, local factors. Part of the motivation behind the method consists in the fact that it is not dependent on sample size, unlike a statistical test of unidimensionality. With this property, the approach may be considered a useful addition to that overall test of unidimensionality for a multi-item measuring instrument, which is frequently conducted when using LVM for studying its latent structure (e.g., Raykov & Marcoulides, 2011). The method can also be seen as a conceptual extension of that in Raykov and Pohl (2013a) to the more frequently encountered case of discrete ordinal items in empirical research, and thus intends to aid behavioral and social scholars pursuing the query if a given instrument can be treated for certain aims as unidimensional. Last but not least, the procedure is based

on several CFD indexes that can be viewed as effect size measures of the degree of violation of unidimensionality, which are not only point but also interval estimated for a studied population.

Background, Notation, and Assumptions

In this article, we assume that a set of binary or binary scored items are given, which are denoted $y_1, y_2, \dots, y_p$ ($p > 2$) and represent the components of a measuring instrument under consideration (cf. Raykov & Pohl, 2013a; see also the “Conclusion” section for an extension). All items are presumed to load on a common (global) factor denoted g , and possibly on one or more additional (local) factors designated $f_1, \dots, f_q$ ($0 < q < p$). The local factors $f_1, \dots, f_q$ are interpretable as sources of variance in the subsets of items loading on them, which are not captured by the common factor that represents the variance common to all p items. No limit is placed on the number of items that load on a local factor other than being at least 2, and not all items need to be loading also on a local factor within the overall model that is similarly assumed identified. For the latter to be the case, the common factor g is stipulated as being uncorrelated with any of the q additional factors, and in order to avoid potential empirical underidentification, slow convergence, inadmissible solutions, or excessive latent correlation standard errors (unless with very large samples), the covariance matrix of the local factors is assumed to be diagonal. (In case $q = 1$, the last assumption is obviously not needed.)

With the preceding discussion in mind, the model that is posited in the studied population is as follows (cf. B. O. Muthén, 1984):

$$\underline{y}^* = \Lambda \underline{\eta} + \underline{\delta}, \quad (1)$$

where $\underline{y}^* = (y_1^*, \dots, y_p^*)'$ symbolizes the $p \times 1$ vector of assumed latent normal variables underlying the respective items, $\Lambda = [\lambda_{ij}]$ is the associated $p \times (q + 1)$ factor loading matrix, $\underline{\eta} = (g, f_1, \dots, f_q)'$ is the $(q + 1) \times 1$ vector comprising the global and local factors that are all assumed with 0 mean, and $\underline{\delta}$ is the pertinent $p \times 1$ residual term vector with mean 0 that is assumed uncorrelated with $\underline{\eta}$ (underlining denotes vector and priming denotes transposition in this article). Furthermore, a vector of item thresholds (one per item) is treated as a set of nuisance parameters in what follows, which are estimated along with all other model parameters (for an example, see illustration section and Table 1 there). All $q + 1$ factors in Equation (1) are assumed to have unit variance, and in the last q columns of Λ there are some free parameters at appropriate places and their remaining elements are zeros.

As discussed in the literature (e.g., Raykov & Pohl, 2013a, 2013b), Model (1) may be developed based on prior research and/or existing theory in a given substantive domain, or derived through previous exploratory factor analysis—carried out on an independent sample(s) from the same population—or may reflect a hypothesis that is to be examined, possibly in relation to other rival hypotheses. As indicated earlier, this article is concerned with Model (1) itself rather than any of those earlier

Figure 1. Model (1) with $p = 9$ items in an overall psychometric test and $q = 1$ local factor, which is used in the illustration section. Dependent variables are presented within rectangles, to emphasize the explained variance concept used instrumentally in the article, and short one-way arrows stand for residual terms in the vector $\underline{\delta}$ in Equation (1); see main text.

activities, and thus considers it as given; that is, we assume below that Model (1) is (a) plausible for a studied population, (b) identified, and (c) of concern to a researcher interested in quantifying possible CFD (essential unidimensionality) and/or violations of unidimensionality. To give an example, a special case of Model (1) with $p = 9$ items and $q = 1$ local factor is displayed in Figure 1.

In the illustration section, we use the model in Figure 1 on empirical data.

Evaluation of Proportion of Variance Explained by the Common or the Local Factors

Based on Model (1) and the above developments, we denote next by λ_{jg} the loading of the assumed latent normal variable underlying the j th item, y_j , on the common (global) factor g , and by λ_{js} the loading of that item on the s th local factor (cf., e.g., B. O. Muthén, 1984; we note that $\lambda_{js} = 0$ may hold for appropriate of the item by local factor combinations, that is, not all items need to load on a local factor; $j = 1, \dots, p$, $s = 1, \dots, q$).

Because of the earlier assumption of unit latent variances and uncorrelated factors, from Equation (1), it follows that the proportion π_g of the total overall variance in the latent normal variables in $\underline{y}^*$ underlying the p items $y_1, y_2, \dots, y_p$, which is explained by the common factor g , is

$$\pi_g = \frac{\sum_{1 \leq j \leq p} \lambda_{jg}^2}{\sum_{1 \leq j \leq p} \lambda_{jg}^2 + \sum_{1 \leq j \leq p; 1 \leq s \leq q} \lambda_{js}^2}, \quad (2)$$

where the range of the running index (j or s) is correspondingly stated beneath the summation sign. The index π_g in Equation (2) is proposed in this article as a measure

of CFD, or approximate (essential) unidimensionality, for instruments consisting of binary (binary scored) or ordinal items. (Its point and interval estimation is discussed below; see also the conclusion section for indicated extension to more general discrete items.)

Similarly, the proportion π_l of the total overall variance in the latent normal variables underlying the p items, which is explained by the local factors, is

$$\pi_l = \frac{\sum_{1 \leq j \leq p; 1 \leq s \leq q} \lambda_{js}^2}{\sum_{1 \leq j \leq p} \lambda_{jg}^2 + \sum_{1 \leq j \leq p; 1 \leq s \leq q} \lambda_{js}^2} = 1 - \pi_g. \quad (3)$$

With the previous proposal in Equation (2) in mind, the index π_l in Equation (3) could be considered a measure of lack of unidimensionality, or lack of CFD, in a given multi-item instrument complying with the generic Model (1).

If a researcher is interested in the degree to which some but not all local factors contribute to possible lack of unidimensionality, or lack of CFD, in an instrument under consideration, then as an index of this degree for a prespecified set S of local factors one can use

$$\pi_S = \frac{\sum_{1 \leq j \leq p; s \in S} \lambda_{js}^2}{\sum_{1 \leq j \leq p} \lambda_{jg}^2 + \sum_{1 \leq j \leq p; s \in S} \lambda_{js}^2}, \quad (4)$$

where the sum in the numerator and the second term in the denominator are taken over all local factors included in S .

We stress that the proposed indexes π_g , π_l , and π_S are defined at the population level (and thus do not depend on sample size; for convenience and simplicity of reference, in the rest of this article, we call these three variance proportions “CFD indexes”). In addition, we emphasize that the information about instrument (approximate or essential) unidimensionality, CFD, or lack thereof, which is contained in any of the quantities π_g , π_l , and π_S , does not depend on whether a particular null hypothesis is rejected or retainable when tested, or true or false in a studied population (assuming as mentioned that Model 1 is plausible for the latter).

Point and Interval Estimation of the Common Factor Dominance Indexes

In a given empirical study within the setting underlying this article (see earlier assumptions), after Model (1) is fitted to the analyzed data and found plausible using a method appropriate for the item distributions such as weighted least squares (WLS; L. K. Muthén & Muthén, 2016), the above CFD indexes are estimated correspondingly as (with a hat denoting the estimate of the quantity beneath, which is obtained thereby):

$$\hat{\pi}_g = \frac{\sum_{1 \leq j \leq p} \hat{\lambda}_{jg}^2}{\sum_{1 \leq j \leq p} \hat{\lambda}_{jg}^2 + \sum_{1 \leq j \leq p; 1 \leq s \leq q} \hat{\lambda}_{js}^2}, \quad (5)$$

$$\hat{\pi}_l = \frac{\sum_{1 \leq j \leq p; 1 \leq s \leq q} \hat{\lambda}_{js}^2}{\sum_{1 \leq j \leq p} \hat{\lambda}_{jg}^2 + \sum_{1 \leq j \leq p; 1 \leq s \leq q} \hat{\lambda}_{js}^2} = 1 - \hat{\pi}_g, \text{ and} \quad (6)$$

$$\hat{\pi}_S = \frac{\sum_{1 \leq j \leq p; s \in S} \hat{\lambda}_{js}^2}{\sum_{1 \leq j \leq p} \hat{\lambda}_{jg}^2 + \sum_{1 \leq j \leq p; s \in S} \hat{\lambda}_{js}^2}, \quad (7)$$

These point estimates are readily obtained using the increasingly popular LVM software Mplus (L. K. Muthén & Muthén, 2016), and the source code needed for that activity is provided in the appendix (where it is used on the example data set of the illustration section). Application of the popular delta method (e.g., Raykov & Marcoulides, 2004), which is also incorporated in Mplus, yields thereby a large-sample standard error associated with each of the CFD indexes π_g , π_l , and π_S . To obtain an approximate (i.e., large-sample) confidence interval (CI) for any of them, we first note that each CFD index is a proportion and thus a parameter bounded between 0 and 1. For this reason, one can use the initial monotone transformation method in Raykov and Marcoulides (2011) to obtain a CI for any of these three CFD indexes at a prespecified confidence level. This method is implemented in the R-function “ci.cfd” provided also in the appendix. Alternatively, one can use the popular bootstrap approach for this purpose, which is similarly incorporated in Mplus (e.g., Efron & Tibshiriani, 1993).

We exemplify next the outlined procedure for studying CFD and related violations of instrument unidimensionality.

Illustration on Data

In this section, we demonstrate the utility and applicability of the above estimation approach for the CFD indexes (Equations 2-4) using a numerical example. Suppose that for an ability test consisting of $p = 9$ binary items administered to $n = 1,512$ persons, Model (1) with $q = 1$ local factor loading on the third and last two of the items (see Figure 1) is of interest to a researcher concerned with quantifying the extent to which there may be CFD and possible violation of unidimensionality. (The raw data can be obtained from the authors on request.)

We begin by fitting Model (1) using the WLS estimation method (see the Mplus source code for Step 1 in the appendix). This model is found to be associated with the following fit indexes: $\chi^2 = 31.469$, degrees of freedom (df) = 24, $p = .141$, and

root mean square error of approximation (RMSEA) = .014 with a 90% CI (.0, .027). These tenable indexes suggest that Model (1) (Figure 1), which underlies this article, is a tenable means of description and explanation of the analyzed data. We can thus proceed to Step 2 of the outlined procedure, in which we point and interval estimate the CFD indexes π_g , π_l , and π_s (in this example, $\pi_l = \pi_s$ as $q = 1$). To this end, we introduce as “external” model parameters each of the two CFD indexes of concern here, which is accomplished in the second Mplus file in the appendix. The model fitted in this second step has identical goodness-of-fit indexes as the one reported above, since no restrictions are imposed on its parameters that have consequences for the relevant polychoric correlation matrix of the nine analyzed items and lead to loss of degrees of freedom. The parameter estimates, standard errors, t values, and p values associated with the fitted model in this second step, including the two CFD indexes as “external” parameters, are presented in Table 1.

As seen from Table 1, the CFD indexes (Equations 2 and 3) are estimated as follows (standard errors in parentheses following estimate): $\hat{\pi}_g = 0.775$ (0.074), and $\hat{\pi}_l = 0.225$ (0.074). Using the R-function “ci.cfd” in the appendix, we obtain the following 95% CIs for these respective proportions explained variance (associated index within parenthesis): $CI(\pi_g) = (.600, .888)$, and $CI(\pi_l) = (.122, .400)$.

Based on the presented results, one can conclude that a range of plausible population values for the proportion total variance explained by the global factor could stretch from .6 through almost .9, whereas for the proportion total variance explained by the local factor that range could stretch from approximately .1 through .4. These relatively wide CIs (owing to the binary nature of the analyzed items, at a sample size that is not sufficiently large to render considerably shorter CIs, given the level of complexity of the fitted model), do not allow a clearer conclusion whether one may consider the nine-item test under consideration as evaluating one dominant common factor, or alternatively also an additional second factor that is relevant to be taken into account in subsequent work with the overall measuring instrument (nine-item test). One may therefore suggest that it would be better to defer any further interpretation to the substantive researcher in the subject-domain area of relevance (or replication study with a larger sample size). Specifically, they would then have to decide whether (a) to place more emphasis on the fact that based on the above CI results it is plausible to suggest that the global (common) factor could be explaining up to about 90% of the total variance in the items (assumed underlying latent variable variances), and thus conclude in favor of approximate unidimensionality or a single dominant factor behind the used nine-item instrument or (b) to place more emphasis on the fact that it is plausible to suggest that the global (common) factor could only be explaining less than two thirds of the total item variance (assumed underlying latent variable variances) and thus may be seen less clearly as dominating if at all the local factor loads on the third and last two items. Whether a content researcher decides for (a) or (b) may well depend also on the “loss” (risk for subsequent research or given purposes) associated with an incorrect decision in either case. Last but not least, we stress that in a particular empirical setting a researcher may not necessarily

Table 1. Parameter Estimates, Standard Errors, *t* Values, and *p* Values Associated With Model (1) (Software Output Format).

Parameter	Estimate	SE	Estimate/SE	Two-tailed <i>p</i> value
GLOBAL BY				
ITEM1	0.607	0.041	14.897	.000
ITEM2	0.425	0.041	10.386	.000
ITEM3	0.414	0.046	9.019	.000
ITEM4	0.417	0.041	10.069	.000
ITEM5	0.548	0.049	11.159	.000
ITEM6	0.518	0.043	12.084	.000
ITEM7	0.328	0.045	7.212	.000
ITEM8	0.448	0.049	9.109	.000
ITEM9	0.263	0.047	5.662	.000
LOCAL BY				
ITEM3	0.162	0.083	1.943	.052
ITEM8	0.617	0.269	2.295	.022
ITEM9	0.356	0.162	2.191	.028
GLOBAL WITH				
LOCAL	0.000	0.000	999.000	999.000
Thresholds				
ITEM1\$I	−0.294	0.033	−8.988	.000
ITEM2\$I	0.005	0.032	0.154	.877
ITEM3\$I	−0.783	0.036	−21.686	.000
ITEM4\$I	0.205	0.032	6.322	.000
ITEM5\$I	0.847	0.037	23.018	.000
ITEM6\$I	0.407	0.033	12.257	.000
ITEM7\$I	0.580	0.034	16.909	.000
ITEM8\$I	−0.975	0.038	−25.353	.000
ITEM9\$I	−0.541	0.034	−15.905	.000
Variances				
GLOBAL	1.000	0.000	999.000	999.000
LOCAL	1.000	0.000	999.000	999.000
New/additional parameters				
EV_GLOBA	1.840	0.111	16.513	.000
EV_LOCAL	0.534	0.224	2.380	.017
PI_G	0.775	0.074	10.418	.000
PI_L	0.225	0.074	3.021	.003

Note. EV_GLOBA = total explained variance (in assumed underlying latent normal variables, γ^* , by global factor); EV_LOCAL = total explained variance (in underlying latent normal variables) by local factor; PI_G = π_g ; PI_L = π_l (see Equations 2 and 3).

need to decide eventually for (a) or (b), but could instead elect to use the test in question as a two-dimensional measuring instrument in following analyses, for instance by including both its factors (or possibly estimated scores on them) in subsequent LVM say relating this nine-item test with its explicit structure to other dependent or explanatory variables (cf. Raykov & Pohl, 2013a, 2013b).

Conclusion

This note aimed to contribute to the discussion on such numerical indexes of CFD and related approximate (essential) unidimensionality in settings with single common global factors and one or more additional (local) factors, which do not depend on statistical hypothesis test results (assuming a plausible Model 1 for their strictly multidimensional latent structure). A main part of the motivation behind the proposed three CFD indexes was the well-known fact that with large samples the test of unidimensionality (instrument homogeneity) may yield spurious significance results, while alternatively with suboptimal asymptotically sample sizes that test may miss potentially important substantively violations of unidimensionality.¹

To respond to this concern, the article proposed three CFD indexes that evaluate the extent to which a common factor “dominates” the remaining local factor(s) in a measuring instrument with a complex latent structure (complying with Model 1). These indexes are also interpretable as effect size measures for CFD, approximate (essential) unidimensionality, or lack thereof. A point and interval estimation procedure was outlined within the popular LVM framework, which is readily applicable with widely circulated software and furnishes numerical estimates and plausible population value ranges for each CFD index. In particular, the method permits a researcher to obtain estimates and CIs for the proportion of total item variance (underlying assumed latent variable variance) that is explained by the global/common factor, or by some or all of the remaining (local) factors.

Although we assumed at the outset that we were dealing with binary (or binary scored) items, it is worth noting here that the description of the estimation procedure and CFD indexes contains nothing unique to the binary item case. In fact, the entire preceding discussion, with all its developments and proposed quantities, are just as applicable in the highly discrete ordinal item case, e.g., Likert-type items (cf. Raykov & Pohl, 2013a, 2013b, for the case of continuous, or approximately continuous, instrument components).

The discussed method and CFD indexes rest as mentioned on the assumption that their underlying Model (1) is plausible as well as identified; when this model is not both, the outlined approach and resulting CFD index estimates are not recommendable. While there is no “single” formal description of Model (1) that would be always applicable when violations of unidimensionality are of interest to study, the main characteristic of the setting underlying this article is that of a single common factor loading on all items, and one or more “local” factors loading each on at least two but not all items. A related limitation of the approach and proposed indexes is also the current lack of guidelines as to how specifically to interpret the numerical values of the latter, including the endpoints of their CIs. Until sufficient experience is accumulated with the present procedure and CFD measures, one may wish not to pursue a strict decision of the type “unidimensional and not multidimensional,” or vice versa, for a given multicomponent instrument, unless the CI for the CFD index π_g in Equation (2) lies entirely above .80 say when the former conclusion may arguably be possible to consider sensible.

Furthermore, the outlined procedure and unidimensionality violation measures are best used with large samples, when also the resulting CIs will be shorter and more likely to support a potentially clearer conclusion in favor of CFD or lack thereof. While currently there are no guidelines as to how to determine if a sample is large enough for the present aims, it may be argued that samples may be desirable here that would be even larger than those that may be satisfactory for the continuous instrument component methods in Raykov and Pohl (2013a, 2013b), due to the binary or highly discrete nature of the items dealt with in this article.

In conclusion, this note discusses a procedure for point and interval estimation of up to three proposed indexes of CFD that are not based on statistical hypothesis testing (other than, of course, that involved in ascertaining plausibility of the underlying model with common and local factors). The method can be readily used by researchers working with discrete ordinal item-based measuring instruments, who are concerned with evaluating the extent of their deviation from unidimensionality. Such a consideration may be particularly beneficial in empirical situations where (a) instrument construction, revision and development is pursued in the binary, binary scored, or highly discrete ordinal items case, or (b) a choice may be of interest to consider between using subsequently relatively easier to interpret unidimensional item response theory models or alternatively more complicated multidimensional item response theory models.

Appendix

Mplus Source Codes for Estimation of Proposed Common Factor Dominance Indexes and R-Function for Obtaining Their Confidence Intervals

```

TITLE:      FITTING MODEL (1) WITH  $p = 9$  AND  $q = 1$  (SEE FIGURE 1) . STEP 1.
DATA:      FILE = <NAME OF RAW DATA FILE > ;
VARIABLE:  NAMES = ITEM1-ITEM9 Z;
           USEVARIABLES = ITEM1-ITEM9;
           CATEGORICAL = ITEM1-ITEM9;
MODEL:     GLOBAL BY ITEM1* ITEM2-ITEM9;
           GLOBAL@1;
           LOCAL BY ITEM3* ITEM8-ITEM9;
           LOCAL@1;
           GLOBAL WITH LOCAL@0;

TITLE:      POINT AND INTERVAL ESTIMATION OF CFD INDEXES . STEP 2.
DATA:      <NAME OF RAW DATA FILE >;
VARIABLE:  NAMES = ITEM1-ITEM9 Z;
           USEVARIABLES = ITEM1-ITEM9;
           CATEGORICAL = ITEM1-ITEM9;
MODEL:     GLOBAL BY ITEM1* (L11) ! ASSIGN SYMBOLS TO MODEL PARAMETERS
           ITEM2-ITEM9 (L12-L19);
           GLOBAL@1;
           LOCAL BY ITEM3* (L21)
           ITEM8-ITEM9 (L22-L23);

```

```

LOCAL@1;
GLOBAL WITH LOCAL@0;
MODEL CONSTRAINT:
  NEW(EV_GLOBAL, EV_LOCAL, PI_G, PI_L); ! CFD INDEXES' PLACE HOLDERS.
  EV_GLOBAL = L11^2+L12^2+L13^2+L14^2+L15^2+L16^2+L17^2+L18^2+
  L19^2;!EV=explained variance
  EV_LOCAL = L21^2+L22^2+L23^2;
  PI_G = EV_GLOBAL/(EV_GLOBAL+EV_LOCAL); ! Equation (2).
  PI_L = 1 - PI_G; ! Equation (3)
OUTPUT: CINTERVAL;

```

Note. Comment follows exclamation mark. The source code for this Step 2 results from adding to the code for Step 1 the above MODEL CONSTRAINT section, on assigning parameter symbols (“L” for “Lambda”) to all factor loadings. (The indexes after the L need not comply to the popular LISREL notation; Jöreskog & Sörbom, 1996.) In the MODEL CONSTRAINT section, first place holders are created for the two sums appearing in the denominators of Equations (2) and (3), as well as for the two CFD indexes of relevance in this example, which are then computed/point estimated according to Equations (2) and (3), respectively. (For further details on the Mplus syntax, see e.g., Raykov & Marcoulides, 2006.) With their resulting point estimates and associated standard errors (see Table 1), using the R-function provided next furnishes the 95% CIs reported in the main text for these CFD indexes.

```

ci.cfd = function(r, se){ # R-function for a 95%-CI for any of the 3 CFD indexes
  l = log(r/(1-r))
  sel = se/(r*(1-r))
  ci_l_lo = 1-1.96*sel # use appropriate quantile for another confidence
  level
  ci_l_up = 1+1.96*sel
  ci_lo = 1/(1+exp(-ci_l_lo))
  ci_up = 1/(1+exp(-ci_l_up))
  ci = c(ci_lo, ci_up)
  ci
}

```

Note. This R-function is a minor modification of the R-function “ci.rel” in Raykov and Marcoulides (2011, chap. 7) for the aims of this article, and is presented here merely for the sake of completeness. (Comment follows after the symbol “#”).

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Note

1. In fact, if one fits the single-factor model to the example data in the “Illustration on Data” section using WLS (accounting for the binary nature of the items), one obtains the following fit indexes: $\chi^2 = 56.440$, $df = 27$, $p = .001$, RMSEA = .027 (.017, .037). These fit indexes—in particular the RMSEA and its CI—may well be viewed as suggesting plausibility of the unidimensionality hypothesis, particularly considering a possible interpretation of the sample size as being “large” (while not conducting the analyses reported in the illustration section). This data set therefore exemplifies that when studying unidimensionality, one needs to also examine alternatives to it even if instrument homogeneity may be found to be a plausible hypothesis overall. These alternatives could be generated, as rival latent structure hypotheses, using exploratory factor analysis (on an independent sample from the studied population), or based on prior research or available theory in a substantive area.

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