Analysis of a self-propelling sheet with heat transfer through non-isothermal fluid in an inclined human cervical canal

Ahsan Walait¹ · A. M. Siddiqui² · M. A. Rana¹

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Abstract The present theoretical analysis deals with biomechanics of the self-propulsion of a swimming sheet with heat transfer through non-isothermal fluid filling an inclined human cervical canal. Partial differential equations arising from the mathematical modeling of the proposed model are solved analytically. Flow variables like pressure gradient, propulsive velocity, fluid velocity, time mean flow rate, fluid temperature, and heat-transfer coefficients are analyzed for the pertinent parameters. Striking features of the pumping characteristics are explored. Propulsive velocity of the swimming sheet becomes faster for lower Froude number, higher Reynolds number, and for a vertical channel. Temperature and peak value of the heat-transfer coefficients below the swimming sheet showed an increase by the increment of Brinkmann number, inclination, pressure difference over wavelength, and Reynolds number whereas these quantities decrease with increasing Froude number. Aforesaid parameters have shown opposite effects on the peak value of the heat-transfer coefficients below and above the swimming sheet. Relevance of the current results to the spermatozoa transport with heat transfer through non-isothermal cervical mucus filling an inclined human cervical canal is also explored.

Keywords Swimming sheet · Heat transfer · Non-isothermal fluid · Inclined channel · Spermatozoa transport

✉ Ahsan Walait
ahsanwalait@gmail.com

¹ Department of Mathematics and Statistics, Riphah International University, Islamabad, Pakistan

² Department of Mathematics, Pennsylvania State University, York Campus, 1031 Edgecomb Avenue, York, PA 17403, USA

1 Introduction

The word spermatozoon is derived from the Greek word “sperma”, meaning seed. It is a uniflagellar male reproductive cell that contains 23 out of 46 chromosomes of a normal cell with limited life span. Spermatozoa produced in human testicles are released from the penis. A healthy spermatozoon is composed of head, neck, mid piece, and tail. The mid piece and sperm tail represent a flagellum that provides the propulsive force, allowing locomotion of the spermatozoon cell as it moves toward the point of fertilization in the female genital tract [1, 2]. During coitus, about 182 million sperm are deposited into the vaginal tube. They quickly contact the cervical mucus and enter the cervix to avoid vaginal acid and immune responses. As the spermatozoa are large in number and are close to each other, they travel in union. It is therefore possible to approximate their motion by a sheet swimming through fluid within a two-dimensional channel [3–14]. We will consider the motion of a swimming sheet with heat transfer through non-isothermal fluid filled in an inclined two-dimensional channel. This geometry accounts for the inclined human cervical canal filled with non-isothermal cervical mucus as a temperature distributor.

Over the decades, the study of swimming of microscopic organisms has been a constant source of inspiration for many researchers and scientists. Taylor [3] proposed a mathematical model for the swimming of a microorganism in the form of an infinite flexible sheet suspended in a linearly viscous fluid. He presented a two-dimensional model of spermatozoa locomotion by considering the zero assumption of Reynolds number, propulsive waves to be of sinusoidal nature and of smaller amplitude. Taylor applied the perturbation technique up to the fourth order, assuming wave amplitude to be a small parameter to compute the propulsive velocity of the sheet. However, Taylor’s analysis was limited only to zero Reynolds number. This limitation was removed by Reynolds [4]. He included local inertia and predicted that the swimming speed increases above the limiting value given by Taylor at zero frequency. Tuck [5] modified the Reynolds formula in which he neglected the simultaneous effects of convection, which induced a non-uniform mean second-order flow with direction such as to oppose the propulsion. Shack and Lardner [6] modeled the microscopic organisms as a two-dimensional sheet swimming with low Reynolds numbers between two rigid walls. They assumed the large wavelength of propulsive waves passing through the sheet as compared to the channel spacing, but the amplitude of the propulsive waves is arbitrary. They found analytical solutions for the propulsive velocity and rate of energy dissipated. Smelser et al. [7] presented a mathematical model for Odeblad’s theory [8] in which they proposed a mechanism for the spermatozoa transport through the cervical canal. They idealize the geometry as self-propagating waves of lateral displacement in which channel walls are allowed to vibrate as right-traveling waves, left-traveling waves, and standing waves. Shukla et al. [9] studied the swimming of microorganisms by considering the dynamic interaction of the micelles and wall of the channel with the moving spermatozoa through the cervical canal. They concluded that the propulsive velocity and work done by the swimming sheet increase when the dynamic interaction increases. Sinha et al. [10] extended the work for micropolar fluid as mucus filling the cervical canal and analyzed the effects of micropolar parameters on propulsive velocity and rate of work by the swimming sheet. Shukla et al. [11] proposed a mathematical model to study the motion of a swimming sheet through the cervical canal. They considered peristaltic and longitudinal waves propagating along the channel walls. They analyzed that the peristaltic wave propagating in a direction opposite to the motion of the swimming sheet causes a reduction

in the propulsive velocity of the sheet. Philip and Chandra [12] adopted microcontinuum approach to study the motion of the swimming sheet through the cervical canal by considering the micropolar fluid as cervical mucus. They carried out their analysis under the assumption that the waves travelling along the channel walls and in the self propelling sheet of spermatozoa are independent of time in the moving frame of reference. They showed that the propulsive velocity of the swimming sheet of spermatozoa in linearly viscous cervical mucus is greater than micropolar fluid, and the peristaltic wave on the wall of the channel caused a reduction in the propulsive velocity. Radhakrishnamacharya and Sharma [13] considered the same problem [12] for linearly viscous cervical mucus and analyzed the effect of the viscosity variation of the mucus across the channel. They observed that the propulsive velocity of the swimming sheet decelerated as the viscosity of the peripheral layer increased. Pak and Lauga [14] modeled the geometry for both spermatozoa locomotion and ciliary transport in Stokes flow.

In the male reproductive system, outside the abdominal cavity there exists a pair of testes in which spermatozoa are produced. Spermatozoa require about $2\text{--}3^\circ\text{C}$ less temperature than the normal body temperature for survival [15]. Temperature of spermatozoa is regulated in testes because of their outside location. If the testes remain too close to the body, then it is very likely that the increase in temperature will harm the formation of the spermatozoa. This is the reason why testes are carried in an external pouch, called a scrotum, rather than within the abdominal cavity. They normally remain slightly cooler than the body temperature to facilitate spermatozoa production. In the female reproductive tract, cervical mucus facilitates spermatozoa by giving an ideal temperature distribution for the survival of spermatozoa in the cervical canal.

Despite the existence of large body of literature on spermatozoa transport through the cervical canal, no previous attempts have been made to study the self-propulsion of spermatozoa through the inclined cervical canal with heat transfer. Numerous researchers have investigated peristaltic flow with heat transfer [16–26]. Motivated from the literature (cited in [3–26]), we theoretically provide the biomechanics of the self-propulsion of spermatozoa with heat transfer through non-isothermal cervical mucus filled in an inclined cervical canal. We propose a biomechanical model that considers the cervical canal as a two-dimensional channel and impose two traveling waves propagating along the flexible walls of the channel. The lower and upper walls of the channel are maintained at the same temperature. Temperatures of the swimming sheet and channel walls are different. We further assume spermatozoa in the form of an infinite flexible sheet, which moves itself by passing propulsive waves down its length in a direction opposite to the direction of propagation of the traveling waves, the non-isothermal cervical mucus as non-isothermal linearly viscous fluid and sheet of spermatozoa swimming through the cervical mucus. We will derive the expressions for propulsive velocity, fluid velocity, time mean flow rate, fluid temperature, and heat-transfer coefficients in the moving frame of reference. We will estimate the quantitative effects of emerging parameters on these flow variables via graphs.

The rest of the paper is organized into six sections: the problem is formulated in Section 2; Section 3 deals with the solution of the problem and includes expressions for propulsive velocity, fluid velocity, time mean flow rate, fluid temperature, and heat-transfer coefficients. Quantitative effects of the emerging parameters are discussed through graphs in Section 4. Section 5 provides the relevance of the results obtained to the spermatozoa transport with heat transfer through non-isothermal cervical mucus filled in an inclined human cervical canal. Finally, concluding remarks are given in Section 6.

Fig. 1 Propulsion of swimming sheet of spermatozoa through non-isothermal fluid-filled inclined cervical canal

2 Formulation of the problem

We consider the transport of spermatozoa through non-isothermal physiological fluid filled in an inclined human cervical canal. The cervical canal of the female reproductive tract is considered to be a two-dimensional channel with flexible walls $\hat{H}_1$ and $\hat{H}_2$. The width of the channel is $2h$. The upper and lower channel walls and the swimming sheet are maintained respectively at temperatures $\hat{T}_w$ and $\hat{T}_s$ (here $\hat{T}_w \neq \hat{T}_s$). The phenomenon representing motion of the spermatozoa through the cervical canal of human procreation system is shown in Fig. 1. The approximated schematic diagram of the self-propelling swimming sheet along with heat transfer through an inclined channel is shown in Fig. 2. The Cartesian coordinate system has been chosen where the $\hat{X}$ -axis is taken along the channel and $\hat{Y}$ -axis normal to the channel walls. In the schematic diagram, the channel is inclined at an angle θ with the horizontal. The walls of the channel are assumed to be excited by the two traveling waves of finite amplitude propagating from the uterus to the vagina. Spermatozoa propel themselves in the form of an infinite flexible sheet, which forwards itself by passing the propulsive waves down its length in the backward direction. The swimming sheet of spermatozoa propels with propulsive velocity $\hat{V}_p$ in the negative axial direction,

whereas the traveling waves along the channel walls propagate in the positive axial direction. Furthermore, it is assumed that the traveling waves of wavelength λ propagating along the channel walls with speed C and self-propulsive wave generated by the self-propulsion of swimming sheet are in synchronization in the moving frame of reference.

$$\dot{H}_1(\dot{X}, \dot{t}) = h + a_1 \sin \frac{2\pi}{\lambda} (\dot{X} - C\dot{t} + \dot{V}_p \dot{t}), \quad (1)$$

$$\dot{H}_0(\dot{X}, \dot{t}) = a_0 \sin \frac{2\pi}{\lambda} (\dot{X} - C\dot{t} + \dot{V}_p \dot{t}), \quad (2)$$

$$\dot{H}_2(\dot{X}, \dot{t}) = -h + a_2 \sin \frac{2\pi}{\lambda} (\dot{X} - C\dot{t} + \dot{V}_p \dot{t}), \quad (3)$$

If $\dot{P}$ is the dynamic pressure, $\dot{V}^\pm = (\dot{U}^\pm, \dot{V}^\pm)$ is the velocity field, $\dot{T}^\pm$ is the temperature, μ is the viscosity, g is the gravitation force per unit mass, k is thermal conductivity, and C_p is the specific heat constant, then the Navier–Stokes equations along with the continuity equation, governing the two-dimensional flow of an incompressible fluid, considering

thermal effects under the influence of gravitational force in laboratory frame of reference are given by

$$\partial_{\dot{Y}} \dot{V}^{\pm} = -\partial_{\dot{X}} \dot{U}^{\pm}, \quad (4)$$

$$\rho \dot{D}_t \dot{U}^{\pm} = -\partial_{\dot{X}} \dot{P}^{\pm} + \mu \nabla^2 \dot{U}^{\pm} - \rho g \sin \theta, \quad (5)$$

$$\rho \dot{D}_t \dot{V}^{\pm} = -\partial_{\dot{Y}} \dot{P}^{\pm} + \mu \nabla^2 \dot{V}^{\pm} + \rho g \cos \theta, \quad (6)$$

$$\rho C_p \dot{D}_t \dot{T}^{\pm} = k \nabla^2 \dot{T}^{\pm} + \dot{\Phi}^{\pm}, \quad (7)$$

where $\dot{D}_t = \partial_t + U \partial_{\dot{X}} + V \partial_{\dot{Y}}$ is the material derivative, $\nabla^2 = \partial_{\dot{X}}^2 + \partial_{\dot{Y}}^2$ is the Laplace operator, and $\dot{\Phi}^{\pm} = 2\mu (\partial_{\dot{X}} \dot{U}^{\pm})^2 + 2\mu (\partial_{\dot{Y}} \dot{V}^{\pm})^2 + \mu (\partial_{\dot{X}} \dot{V}^{\pm} + \partial_{\dot{Y}} \dot{U}^{\pm})^2$ is the viscous dissipation term in energy equation. In the present analysis, we represent the region between the swimming sheet and the upper wall ($\dot{H}_0 \leq \dot{Y} \leq \dot{H}_1$) with the superscript (+) and region between the swimming sheet and lower wall ($\dot{H}_2 \leq \dot{Y} \leq \dot{H}_0$) with the superscript (-). The swimming sheet of spermatozoa is self-propelling and massless, therefore forces exerted by the fluid on it must balance for its motion to be of constant velocity $\dot{V}_p$. Mathematically, we can write it as

$$\int_A (S^+ + S^-) dA = 0, \quad (8)$$

where S^+ and S^- are the resultant forces acting on the upper and lower surfaces of the swimming sheet, respectively, and A is the surface area of the spermatozoa. Equation (8) is referred to as the condition of equilibrium [6, 7, 9–13].

We considered the physiological fluid as incompressible and non-isothermal linearly viscous fluid. Motion is due to the traveling waves propagating along the channel walls, gravitational force, and self-propelling wave generated by the self propulsion of swimming sheet. The swimming sheet propagates with velocity $C - \dot{V}_p$ with respect to the laboratory frame of reference. Now, if we introduce a frame moving with the velocity $C - \dot{V}_p$ in the positive axial direction, then the sheet would appear to be stationary, and flow will be in the steady state. The normalized transformation is defined to transform the equations of motion, energy equation, boundary condition, and surface profiles of the channel wall and the swimming sheet from a dimensional fixed frame to normalized moving frame of reference as:

$$\begin{aligned} x &= \frac{\dot{X} - (C - \dot{V}_p)t}{\lambda}, \quad u = \frac{\dot{U} - (C - \dot{V}_p)}{C}, \quad t = \frac{Ct}{\lambda}, \quad v = \frac{\lambda \dot{V}}{Ch}, \quad \delta = \frac{h}{\lambda} \\ y &= \frac{\dot{Y}}{h}, \quad p = \frac{\dot{P} h^2}{\lambda C \mu}, \quad V_p = \frac{\dot{V}_p}{C}, \quad \epsilon_0 = \frac{a_0}{h}, \quad \epsilon_1 = \frac{a_1}{h}, \quad \epsilon_2 = \frac{a_2}{h}, \quad T = \frac{\dot{T} - \dot{T}_s}{\dot{T}_w - \dot{T}_s} \\ Re &= \frac{\rho ch}{\mu}, \quad Fr = \frac{c^2}{gh}, \quad Br = \frac{\mu c^2}{k(T_w - T_s)}, \quad Pr = \frac{\mu}{k C_p}. \end{aligned} \quad (9)$$

After making use of the normalized transformation (9) into (4)–(7), we arrive at

$$\partial_y v^{\pm} = -\partial_x u^{\pm}, \quad (10)$$

$$Re \delta Du^{\pm} = -\partial_x p^{\pm} + \nabla^2 u^{\pm} - \frac{Re}{Fr} \sin \theta, \quad (11)$$

$$Re \delta^3 Dv^{\pm} = -\partial_y p^{\pm} + \delta^2 \nabla^2 v^{\pm} + \frac{\delta Re}{Fr} \cos \theta, \quad (12)$$

$$Re Pr \delta DT^{\pm} = \nabla^2 T^{\pm} + Br \phi^{\pm}, \quad (13)$$

where $D = (u^\pm \partial_x + v^\pm \partial_y)$ is the transformed normalized material derivative, $\nabla = \delta^2 \partial_x^2 + \partial_y^2$ is the transformed normalized Laplace operator, $\phi = 2\delta^2 (\partial_x u^\pm)^2 + 2\delta^2 (\partial_y v^\pm)^2 + (\delta^2 \partial_x v^\pm + \partial_y u^\pm)^2$ is the transformed normalized viscous dissipation term of energy equation, Fr is the Froude number, Re is the Reynolds number, δ is the wave number, Br is the Brinkmann number, and Pr is the Prandtl number, respectively. For the self-propulsion of the swimming sheet through a linearly viscous fluid, $Re < 1$ and $\delta \ll 1$ as reported in the literature [3, 6, 7, 9–13]. Therefore, applying the long wavelength approximation ($\delta \rightarrow 0$), equations of motion and energy (11)–(13) take the form:

$$\partial_y v^\pm = -\partial_x u^\pm, \quad (14)$$

$$\partial_y^2 u^\pm = P^\pm + \frac{Re}{Fr} \sin \theta, \quad (15)$$

$$\partial_y^2 T^\pm = -Br (\partial u^\pm)^2, \quad (16)$$

where $P^\pm = \partial_x p^\pm$ is the pressure gradient. Equations (15) and (16) represent four second-order and inhomogeneous linear partial differential equations for both (+) and (−) regions.

Physical boundary conditions associated with (14–16) for the upper region $y \in [h_0(x), h_1(x)]$ and the lower region $y \in [h_2(x), h_0(x)]$ in normalized form are defined as

$$u^+ = V_p - 1, \quad T^+ = 1, \quad v^+ = 2\pi\epsilon_1 (V_p - 1) \cos 2\pi x \text{ at } y = h_1, \quad (17)$$

$$u^\pm = -1, \quad T^\pm = 0, \quad v^\pm = 2\pi\epsilon_0 (V_p - 1) \cos 2\pi x \text{ at } y = h_0, \quad (18)$$

$$u^- = V_p - 1, \quad T^- = 1, \quad v^- = 2\pi\epsilon_2 (V_p - 1) \cos 2\pi x \text{ at } y = h_2, \quad (19)$$

where normalized transformed surfaces of the self-propelling sheet h_0 and traveling waves propagating along the channel walls h_1 and h_2 , respectively, are given by

$$h_0(x) = \epsilon_0 \sin(2\pi x), \quad (20)$$

$$h_1(x) = 1 + \epsilon_1 \sin(2\pi x), \quad (21)$$

$$h_2(x) = -1 + \epsilon_2 \sin(2\pi x), \quad (22)$$

in which ϵ_0 is the normalized amplitude of propulsive sheet, ϵ_1 and ϵ_2 , respectively, are the normalized amplitudes of traveling waves propagating along the upper and lower walls of the channel.

3 Solution to the problem

Upon solving a couple of partial differential equations (15) subject to the boundary conditions (17)–(19), we get the fluid- velocities profiles above and below the swimming sheet of the form:

$$u^+(x, y) = f_1(x, y)V_p + f_2(x, y) \left(P^+ + \frac{Re}{Fr} \sin \theta \right) - 1, \quad (23)$$

$$u^-(x, y) = g_1(x, y)V_p + g_2(x, y) \left(P^- + \frac{Re}{Fr} \sin \theta \right) - 1, \quad (24)$$

where

$$\begin{aligned} f_1(x, y) &= \frac{y - h_0}{H_1}, & f_2(x, y) &= \frac{(y - h_0)(y - h_1)}{2}, \\ g_1(x, y) &= \frac{y - h_0}{H_2}, & g_2(x, y) &= \frac{(y - h_0)(y - h_2)}{2}, \end{aligned}$$

in which $H_i = h_i - h_0$. It is inferred from (23) and (24) that the flow of fluid is pressure-driven flow under the combined influence of the self-propulsion of the swimming sheet and the traveling waves propagating along the walls of the channel. The propulsive velocity V_p of the swimming sheet is unknown in these equations, which must be determined by the condition of equilibrium (8). Motivated by the reported literature [6, 7, 9–13], we are interested in investigating the propulsive velocity of the swimming sheet, which can be achieved through u -component¹ of the velocity only. Therefore, the velocity component v is of least interest to us and we omit the expression for v . However, it can easily be derived by making use of the u component of the velocity into continuity equation (14) along with concern boundary conditions (17)–(19).

Flow rates q^+ and q^- for the upper and lower regions, respectively, are defined by

$$q^+ = \int_{h_0}^{h_1} u^+ dy, \quad q^- = \int_{h_2}^{h_0} u^- dy. \quad (25)$$

Equation (25), after making use of (23) and (24), yields the following expressions for pressure gradient in terms of flow rate and propulsive velocity:

$$P^+ = \frac{q^+}{F_2} - V_p \frac{F_1}{F_2} + \frac{H_1}{F_2} - \frac{Re}{Fr} \sin \theta, \quad (26)$$

$$P^- = \frac{q^-}{G_2} - V_p \frac{G_1}{G_2} - \frac{H_2}{G_2} - \frac{Re}{Fr} \sin \theta, \quad (27)$$

where

$$F_1 = \frac{H_1}{2}, \quad F_2 = -\frac{H_1^3}{12}, \quad G_1 = -\frac{H_2}{2}, \quad G_2 = \frac{H_2^3}{12}.$$

It is worth noting from the integration of continuity equation that the two fluxes q^+ and q^- are constants. Further, it is evident that the pressure difference over a wavelength Δp is the same for the two regions. Therefore, integration of pressure gradients (26) and (27) over a wavelength gives relations of flow rates above and below the swimming sheet in terms of propulsive velocity and pressure difference over wavelength:

$$q^+ = \frac{\Delta p}{I_{11}} + \frac{V_p I_{12}}{I_{11}} + \frac{Re \sin \theta}{I_{11} Fr} - \frac{I_{13}}{I_{11}}, \quad (28)$$

$$q^- = \frac{\Delta p}{I_{21}} + \frac{V_p I_{22}}{I_{21}} + \frac{Re \sin \theta}{I_{21} Fr} + \frac{I_{23}}{I_{21}}, \quad (29)$$

where

$$\begin{aligned} I_{11} &= \int_0^1 \frac{1}{F_2} dx, & I_{12} &= \int_0^1 \frac{F_1}{F_2} dx, & I_{13} &= \int_0^1 \frac{H_1}{F_2} dx, \\ I_{21} &= \int_0^1 \frac{1}{G_2} dx, & I_{22} &= \int_0^1 \frac{G_1}{G_2} dx, & I_{23} &= \int_0^1 \frac{H_2}{G_2} dx. \end{aligned}$$

¹When we set $\theta \rightarrow 0$ in (23) and (24), we recover velocity profiles for linearly viscous fluid through horizontal channel [12].

The force equilibrium condition (8) upon using the stress–strain relationship and long-wavelength approximation yields

$$\int_0^1 [p] dx = 0, \quad (30)$$

$$\int_0^1 \left[\frac{\partial u}{\partial y} \right]_{y=h_0} dx = - \int_0^1 \frac{dh_0}{dx} [p] dx, \quad (31)$$

where $[*]$ denotes the difference in quantity $'*'$ evaluated above and below the swimming sheet. Evaluating the numerical integral (31), we acquire

$$V_p I_{31} - q^+ I_{32} + q^- I_{33} - I_{34} = 0, \quad (32)$$

in which

$$\begin{aligned} I_{31} &= \int_0^1 \left(\frac{F_1}{F_2} \left(\frac{H_1}{2} + h_0 \right) - \frac{G_1}{G_2} \left(\frac{H_2}{2} + h_0 \right) + \frac{1}{H_1} - \frac{1}{H_2} \right) dx, \\ I_{32} &= \int_0^1 \frac{1}{F_2} \left(h_0 + \frac{H_1}{2} \right) dx, \quad I_{33} = \int_0^1 \frac{1}{G_2} \left(h_0 + \frac{H_2}{2} \right) dx, \\ I_{34} &= \int_0^1 \left(\frac{H_1}{F_2} \left(\frac{H_1}{2} + h_0 \right) + \frac{H_2}{G_2} \left(\frac{H_2}{2} + h_0 \right) \right) dx. \end{aligned}$$

Eliminating q^+ and q^- from (32) by the substitution of (28) and (29), we get the relation for propulsive velocity of the swimming sheet V_p in terms of pressure difference over wavelength:

$$V_p = - \left(\Delta p + \frac{Re}{Fr} \sin \theta \right) \frac{I_{42}}{I_{41}} - \frac{I_{43}}{I_{41}}, \quad (33)$$

where

$$I_{41} = I_{31} - \frac{I_{12}I_{32}}{I_{11}} + \frac{I_{22}I_{33}}{I_{21}}, \quad I_{42} = \frac{I_{33}}{I_{21}} - \frac{I_{32}}{I_{11}}, \quad I_{43} = \frac{I_{13}I_{32}}{I_{11}} + \frac{I_{23}I_{33}}{I_{21}} - I_{34}.$$

Total flow rate is computed by adding flow rates of the fluid above the swimming sheet (28) and below the swimming sheet (29) and then in turn upon using the expression of propulsive velocity from (33), one obtains

$$q = \left(\Delta p + \frac{Re}{Fr} \sin \theta \right) \left(I_{51} - \frac{I_{42}I_{52}}{I_{41}} \right) - \frac{I_{43}I_{52}}{I_{41}} + I_{53}, \quad (34)$$

where

$$I_{51} = \frac{1}{I_{11}} + \frac{1}{I_{21}}, \quad I_{52} = \frac{I_{12}}{I_{11}} + \frac{I_{22}}{I_{21}}, \quad I_{53} = \frac{I_{23}}{I_{21}} - \frac{I_{13}}{I_{11}}.$$

Flow rate Q in the laboratory frame of reference is related to flow rate q in the moving frame of reference through the following relation [12]:

$$Q = q + \frac{(1 - V_p)(\dot{H}_1 - \dot{H}_2)}{h}. \quad (35)$$

The time mean flow rate in the laboratory frame of reference is calculated as

$$\bar{Q} = q + 2(1 - V_p) = \left(\Delta p + \frac{Re}{Fr} \sin \theta \right) I_{61} + I_{62},$$

in which

$$I_{61} = I_{51} - \frac{I_{42}I_{52}}{I_{41}} + \frac{2I_{42}}{I_{41}}, \quad I_{62} = I_{53} + 2 - \frac{I_{43}I_{52}}{I_{41}} + \frac{2I_{43}}{I_{41}},$$

, which represents the equation of straight line.

Solving partial differential equations (16) along with concern boundary conditions (17)–(19), we acquire temperature distribution of fluid above and below the swimming sheet of the form:

$$T^+(x, y) = m(x, y) - m_0(x) + \frac{(1 - m_1(x) + m_0(x))(y - h_0)}{H_1}, \quad (36)$$

$$T^-(x, y) = n(x, y) - n_0(x) + \frac{(1 - n_2(x) + n_0(x))(y - h_0)}{H_2}, \quad (37)$$

where

$$m(x, y) = -\frac{Br \left(\frac{V_p}{H_1} + \left(y - \frac{h_0+h_1}{2} \right) R_1 \right)^4}{12R_1^2}, \quad R_1(x) = P^+ + \frac{Re \sin \theta}{Fr},$$

$$n(x, y) = -\frac{Br \left(\frac{V_p}{H_2} + \left(y - \frac{h_0+h_2}{2} \right) R_2 \right)^4}{12R_2^2}, \quad R_2(x) = P^- + \frac{Re \sin \theta}{Fr},$$

$$m_0(x) = -\frac{Br \left(\frac{V_p}{H_1} - \frac{H_1 R_1}{2} \right)^4}{12R_1^2}, \quad n_0(x) = -\frac{Br \left(\frac{V_p}{H_2} - \frac{H_2 R_2}{2} \right)^4}{12R_2^2},$$

$$m_1(x) = -\frac{Br \left(\frac{V_p}{H_1} + \frac{H_1 R_1}{2} \right)^4}{12R_1^2}, \quad n_2(x) = -\frac{Br \left(\frac{V_p}{H_2} + \frac{H_2 R_2}{2} \right)^4}{12R_2^2}.$$

Heat-transfer coefficients $Z^\pm$ for upper and lower region at the sheet are defined by

$$Z^\pm = (\partial_y T^\pm)(\partial_x h_0) \quad (38)$$

which upon using temperature profiles of the fluid above and below the swimming sheet (36) and (37) provides

$$Z^+ = \frac{2\pi\epsilon (1 - m_1(x) + m_0(x) + H_1 M(x)) \cos 2\pi x}{H_1}, \quad (39)$$

$$Z^- = \frac{2\pi\epsilon (1 - n_2(x) + n_0(x) + H_2 N(x)) \cos 2\pi x}{H_2}, \quad (40)$$

where

$$M(x) = -\frac{Br \left(\frac{V_p}{H_1} - \frac{H_1 R_1}{2} \right)^3}{3R_1}, \quad N(x) = -\frac{Br \left(\frac{V_p}{H_2} - \frac{H_2 R_2}{2} \right)^3}{3R_2}.$$

4 Results and discussion

The main goal of the current section is to provide numerical evaluation of the analytical results obtained in the previous section through graphs (see Figs. 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17 and 18). We developed numerical codes in MAPLE to estimate the quantitative effects of the amplitudes of the traveling waves generated in upper ϵ_1 and lower

Fig. 3 Pressure gradient versus horizontal distance for different values of **a** ϵ_1 with $\epsilon_2 = 0.1$, $\epsilon = 0.01$, $\theta = \frac{\pi}{6}$, $\Delta p = 0.2$, $Fr = 0.5$ and $Re = 0.1$ and **b** ϵ_2 with $\epsilon_1 = 0.1$, $\epsilon = 0.01$, $\theta = \frac{\pi}{6}$, $\Delta p = 0.2$, $Fr = 0.5$, and $Re = 0.1$

ϵ_1 channel walls, amplitude of the swimming sheet ϵ , pressure difference over wavelength Δp , Froude number Fr , Reynolds number Re , and inclination θ on pressure gradients $P^\pm$, velocities of the fluid $u^\pm$, propulsive velocity of the swimming sheet V_p and time mean flow rate $\bar{Q}$. Furthermore, effects of Brinkmann number Br along with the above-mentioned parameters on temperature distribution $T^\pm$ and heat-transfer coefficients of swimming sheet $Z^\pm$ for upper and lower regions are also discussed.

4.1 Pressure gradient

Effects of the amplitudes of traveling waves generated in upper ϵ_1 and lower ϵ_2 channel walls, amplitude of the swimming sheet ϵ , pressure difference over wavelength Δp , inclination θ , Froude number Fr , and Reynolds number Re on pressure gradient are observed in Figs. 3–5 by graphical evaluation of (26–27). Figure 3 displays the effects of ϵ_1 and ϵ_2 . Pressure gradient above the sheet is independent of ϵ_2 whereas below the sheet it is independent of ϵ_1 . The magnitude of the pressure gradient above the sheet is an increasing function of ϵ_1 , while below the sheet it is an increasing function of ϵ_2 . Figure 4a, which exhibits the effects of ϵ , indicates that the magnitude of the pressure gradient is a decreasing function of ϵ . From Figs. 4b, 5a and b, it is noted that pressure gradient is independent of θ , Fr , and Re . None of these (θ , Fr and Re) made any impact on the pressure gradient.

4.2 Fluid velocity

For different values of amplitudes of the traveling waves generated in upper ϵ_1 and lower ϵ_2 channel walls, the amplitude of the swimming sheet ϵ , inclination θ , Froude number Fr , and Reynolds number Re , we plotted the expression (23–24) of fluid velocity versus normal distance in Figs. 6–8. Propulsion of the swimming sheet is taken along the $-x$ -axis. Moreover, traveling waves propagate along the inclined cervical canal in the direction of x -axis. Thus, a negative sign of the fluid velocity in these figures (Figs. 6–8) represents the

Fig. 4 Pressure gradient versus horizontal distance for different values of **a** ϵ with $\epsilon_2 = 0.1$, $\epsilon_1 = 0.1$, $\theta = \frac{\pi}{6}$, $\Delta p = 0.2$, $Fr = 0.5$, and $Re = 0.1$ and **b** θ with $\epsilon_1 = 0.1$, $\epsilon = 0.01$, $\epsilon_2 = 0.1$, $\Delta p = 0.2$, $Fr = 0.5$ and $Re = 0.1$

direction of motion of the fluid. From these figures, it is worth noting that fluid velocity profiles intersect at the same point due to the continuity of fluid velocity in both regions. The point of intersection of the fluid velocity above the swimming sheet lies at the trajectory of swimming sheet $y = h_0$, where both velocities meet at $u^\pm = -1$. This point of concurrency graphically provides the verification of the boundary condition (18). Furthermore, through these figures it is also reveals that fluid velocities along the upper and lower channel walls

Fig. 5 Pressure gradient versus horizontal distance for different values of **a** Fr with $\epsilon_1 = 0.1$, $\epsilon = 0.01$, $\epsilon_2 = 0.1$, $\theta = \frac{\pi}{6}$, $\Delta p = 0.2$, and $Re = 0.1$ and **b** Re with $\epsilon_1 = 0.1$, $\epsilon = 0.01$, $\epsilon_2 = 0.1$, $\Delta p = 0.2$, $Fr = 0.5$ and $\theta = \frac{\pi}{6}$

Fig. 6 Fluid velocity versus normal distance for different values of **a** ϵ_1 with $\epsilon_2 = 0.1$, $\epsilon = 0.01$, $x = 0.25$, $\theta = \frac{\pi}{6}$, $\Delta p = 0.2$, $Fr = 0.5$ and $Re = 0.1$ and **b** ϵ_2 with $\epsilon_1 = 0.1$, $\epsilon = 0.01$, $\theta = \frac{\pi}{6}$, $\Delta p = 0.2$, $x = 0.25$, $Fr = 0.5$ and $Re = 0.1$

are equal $u^-(h_2) = u^+(h_1)$, which graphically verifies the boundary conditions (17) and (19).

Fluid velocities above and below the swimming sheet are strongly dependent upon ϵ_1 and ϵ_2 , respectively, as can be seen in Fig. 6. It is demonstrated in Fig. 6a that fluid velocity above the swimming sheet is maximal in the absence of an upper traveling wave. Motion of the fluid towards the $-x$ -direction reduces as ϵ_1 is incremented. Minimal fluid velocity below the swimming sheet is observed in Fig. 6b in the absence of a lower traveling wave.

Fig. 7 Fluid velocity versus normal distance for different values of **a** ϵ with $\epsilon_2 = 0.1$, $\epsilon_1 = 0.1$, $x = 0.25$, $\theta = \frac{\pi}{6}$, $\Delta p = 0.2$, $Fr = 0.5$ and $Re = 0.1$, and **b** θ with $\epsilon_1 = 0.1$, $\epsilon = 0.01$, $\epsilon_2 = 0.1$, $x = 0.25$, $\Delta p = 0.2$, $Fr = 0.5$ and $Re = 0.1$

Fig. 8 Fluid velocity versus normal distance for different values of **a** Fr with $\epsilon_1 = 0.1$, $\epsilon = 0.01$, $\epsilon_2 = 0.1$, $x = 0.25$, $\theta = \frac{\pi}{6}$, $\Delta p = 0.2$, and $Re = 0.1$ and **(b)** Re with $\epsilon_1 = 0.1$, $\epsilon = 0.01$, $\epsilon_2 = 0.1$, $\Delta p = 0.2$, $x = 0.25$, $Fr = 0.5$ and $\theta = \frac{\pi}{6}$

Furthermore, increment in ϵ_2 assists the motion of fluid towards the $-x$ -direction. It seems from Fig. 6a and b that fluid velocities above and below the swimming sheet are minutely dependent upon ϵ_2 and ϵ_1 , respectively. This minute impact is due to the changes that appear in the propulsive velocity of the swimming sheet. Minute enhancement in fluid velocities below and above the swimming sheet with the increment in ϵ_1 and ϵ_2 , respectively, in the direction of motion of swimming sheet is evident in Fig. 6. We plotted Fig. 7 to observe effects of the ϵ and θ on fluid velocities. Figure 7a renders that fluid velocity above the swimming sheet is enhanced, whereas fluid velocity below the swimming sheet is reduced when ϵ increases. Figure 7b shows that fluid velocities above and below the swimming sheet diminish as θ is incremented. Figure 8 shows the effects of the Re and Fr on fluid velocities. It is inferred from Fig. 8a that fluid velocities increment as Fr increases. A decrease in fluid velocities with an increase of Re is speculated in Fig. 8b. The impact of Re and Fr on fluid velocities near the channel walls is observed to be strengthened.

4.3 Propulsive velocity

Figures 9–11 represent the graphical interpretation of the propulsive velocity (33). Profiles are displayed for different values of amplitudes of the traveling waves generated in upper ϵ_1 and lower ϵ_2 channel walls, pressure difference over wavelength Δp , inclination θ , Froude number Fr , and Reynolds number Re . It is disclosed from these profiles (Figs. 9–11) that propulsive velocity of the swimming sheet enhances when $|\epsilon|$ increases. One can also speculate that enhancement in the amplitude of a propulsive wave either in phase or out of phase to the traveling waves of channel walls causes increment in velocity of the sheet in the negative axial direction. The effects of ϵ_1 and ϵ_2 on propulsive velocity are shown in Fig. 9. Propulsive velocity of the swimming sheet is reduced with an increase in the ϵ_1 and ϵ_2 . On more close observation, we explore that the increasing amplitude of traveling waves in the walls of channel impede the motion of the sheet. Propulsive velocity of the sheet in the absence of traveling waves in the channel walls is maximal. Amplitudes of the waves

Fig. 9 Propulsive velocity versus amplitude of the swimming sheet for different values of **a** ϵ_1 with $\epsilon_2 = 0.1$, $\epsilon = 0.01$, $\theta = \frac{\pi}{6}$, $\Delta p = 0.2$, $Fr = 0.5$, and $Re = 0.1$ and **b** ϵ_2 with $\epsilon_2 = 0.1$, $\epsilon = 0.01$, $\theta = \frac{\pi}{6}$, $\Delta p = 0.2$, $Fr = 0.5$, and $Re = 0.1$

traveling along the walls of the channel show symmetric behavior of propulsive velocity, i.e., $V_p(\epsilon_1, \epsilon_2) = V_p(\epsilon_2, \epsilon_1)$. It is also revealed that a reduction in the propulsive velocity becomes more significant as the ϵ_1 and ϵ_2 is incremented to large values. Figure 10a shows the effects of pressure difference over wavelength on propulsive velocity of the sheet. Enhancement in propulsive velocity of the sheet when Δp increases is observed in Fig. 10a. The self-propelling sheet for pressure rise over wavelength $\Delta p \geq 0$ advances in the negative axial direction, whereas it moves backward to positive axial direction for pressure drop over wavelength $\Delta p < 0$. Thus, by controlling the pressure difference over wavelength,

Fig. 10 Propulsive velocity versus amplitude of the swimming sheet for different values **a** Δp with $\epsilon_1 = 0.1 = \epsilon_2$, $\epsilon = 0.01$, $\theta = \frac{\pi}{6}$, $Fr = 0.5$ and $Re = 0.1$, and **b** θ with $\epsilon_1 = 0.1 = \epsilon_2$, $\epsilon = 0.01$, $\Delta p = 0.2$, $Fr = 0.5$, and $Re = 0.1$

Fig. 11 Propulsive velocity versus amplitude of the swimming sheet for different values **a** Fr with $\epsilon_1 = 0.1 = \epsilon_2$, $\epsilon = 0.01$, $\Delta p = 0.2$, $\theta = \frac{\pi}{6}$, and $Re = 0.1$, and **b** Re with $\epsilon_1 = 0.1 = \epsilon_2$, $\epsilon = 0.01$, $\Delta p = 0.2$, $Fr = 0.5$, and $\theta = \frac{\pi}{6}$

one can control the speed as well as the direction of motion of the sheet. Propulsive velocity of the swimming sheet for $\Delta p = 0$ is sandwiched between the propulsive velocity of the swimming sheet for $\Delta p < 0$ and $\Delta p > 0$. Figure 10b shows the effects of θ on propulsive velocity of the sheet. An increase in θ results in an increase in propulsive velocity of the sheet. In fact, propulsive velocity of the swimming sheet will be at a maximum for the vertical channel and at a minimum for the horizontal channel. Figure 11 discloses effects of the Fr and Re on propulsive velocity of the sheet. Figure 11a shows that velocity of the self-propulsive sheet decreases as Fr is increased. Effects of Re can be delineated from

Fig. 12 Pressure rise versus time mean flow rate for different values of **a** ϵ_1 with $\epsilon_2 = 0.1$, $\epsilon = 0.01$, $\theta = \frac{\pi}{6}$, $Fr = 0.5$ and $Re = 0.1$ and **b** ϵ_2 with $\epsilon_2 = 0.1$, $\epsilon = 0.01$, $\theta = \frac{\pi}{6}$, $Fr = 0.5$ and $Re = 0.1$

Fig. 13 Pressure rise versus time mean flow rate for different values of **a** ϵ with $\epsilon_2 = 0.1, \epsilon_1 = 0.1, \theta = \frac{\pi}{6}, Fr = 0.5$, and $Re = 0.1$ and **b** θ with $\epsilon_1 = 0.1, \epsilon_2 = 0.1, \epsilon = 0.01, Fr = 0.5$, and $Re = 0.1$

Fig. 11b. This figure depicts an increase in propulsive velocity of the sheet with increasing Re . In other words, one may say that the sheet moves with greater propulsive velocity when the value of Re is large enough. Minimal velocity of the sheet is observed in Reynolds free flow.

Comparison of the current analysis with the existing literature [3, 6, 11, 12] is made in Table 1 in the form of numeric data. In the particular environment (i.e., when $\theta = 0$), propulsive velocity of the swimming sheet agrees with the propulsive velocity calculated by Shack

Fig. 14 Pressure rise versus time mean flow rate for different values of **a** Fr with $\epsilon_2 = 0.1, \epsilon_1 = 0.1, \epsilon = 0.01, \theta = \frac{\pi}{6}, Fr = 0.5$, and $Re = 0.1$ and **b** Re with $\epsilon_1 = 0.1, \epsilon_2 = 0.1, \epsilon = 0.01, Fr = 0.5$, and $\theta = \frac{\pi}{6}$

Fig. 15 Temperature versus normal distance for different values of **a** ϵ_1 with $\epsilon_2 = 0.1$, $\epsilon = 0.01$, $\theta = \frac{\pi}{6}$, $Re = 0.1$, $Fr = 0.05$, $\Delta p = 0.2$, and $Br = 1.0$ **b** ϵ_2 when $\epsilon_1 = 0.1$, $\epsilon = 0.01$, $\theta = \frac{\pi}{6}$, $Re = 0.1$, $Fr = 0.05$, $\Delta p = 0.2$, and $Br = 1.0$

and Lardner [6], Philip and Chandra [12], and Shukla et al. [11]. It is delineated that in the free channel case $\Delta p = 0$, as $\epsilon \rightarrow 1$ then $V_p \rightarrow 1$, that is dimensional propulsive velocity of the swimming sheet, $\hat{V}_p$ approaches the wave speed C of the sinusoidal waves traveling along the channel walls. Shack and Lardner [6] observed the same result. It is revealed from this table that the magnitude of propulsive velocity calculated by Taylor [3] is lesser

Fig. 16 Temperature versus normal distance for different values of **a** ϵ with $\epsilon_1 = 0.1$, $\epsilon_2 = 0.1$, $Re = 0.1$, $Fr = 0.05$, $\Delta p = 0.2$, $Br = 1.0$ and $\theta = \frac{\pi}{6}$ **b** inclination θ with $\epsilon_1 = 0.1$, $\epsilon_2 = 0.1$, $\epsilon = 0.01$, $Re = 0.1$, $Fr = 0.05$, $\Delta p = 0.2$, and $Br = 1.0$

Fig. 17 Temperature versus normal distance for different values of **a** Fr with $\epsilon_1 = 0.1$, $\epsilon_2 = 0.1$, $\epsilon = 0.01$, $\Delta p = 0.2$, $Re = 0.1$, $Br = 1.0$, and $\theta = \frac{\pi}{6}$ **b** Re with $\epsilon_1 = 0.1$, $\epsilon_2 = 0.1$, $\epsilon = 0.01$, $Fr = 0.05$, $\Delta p = 0.2$, and $Br = 1.0$

as compared to the magnitude of propulsive velocity calculated by Philip and Chandra [12], Shukla et al. [11], Shack and Lardner [6] and in the present analysis. Taylor [3] analyzed the swimming sheet in an unbounded domain of fluid. It is worth mentioning here that the propulsive velocity through the channel (in bounded domain) was found to be greater than in the unbounded domain of the fluid, as is shown in Table 1.

Fig. 18 Temperature versus normal distance for different values of **a** Br with $\epsilon_1 = 0.1$, $\epsilon = 0.01$, $\epsilon_2 = 0.1$, $\Delta p = 0.2$, $Re = 0.1$, $Fr = 0.05$, and $\theta = \frac{\pi}{6}$ **b** Δp with $\epsilon_1 = 0.1$, $\epsilon_2 = 0.1$, $\epsilon = 0.01$, $Re = 0.1$, $\theta = \frac{\pi}{6}$, $Fr = 0.05$, and $Br = 1.0$

Table 1 Comparison of the present analysis with [6, 11, 12] and [3] reported propulsive velocity of the spermatozoa

Propulsive velocity of spermatozoa when $\epsilon_1 = \epsilon_2 = 0.0$, $\Delta p = 0.0$			
ϵ	V_p	V_p	V_p
	Present analysis	Philip and Chandra [12]	Shukla et al. [11]
0.0	0.0000000000	0.0000000000	0.0000000000
0.1	0.02941176684	0.02941175591	0.0294117646
0.3	0.2288135593	0.2288135630	0.2288135596
0.5	0.5000000001	0.4999999963	0.4999999996
0.7	0.7424242344	0.7424242344	0.7424242396
0.9	0.9274809048	0.9274809048	0.9274809134
1.0	0.9990968157	0.9990968157	0.9992163442
	Shack and Lardner [6]	Taylor [3]	
0.0	0.0000000000	0.0000000000	
0.1	0.0294117647	0.0049406250	
0.3	0.2288135593	0.0401906250	
0.5	0.5000000001	0.0878906250	
0.7	0.7424242425	0.1024406250	
0.9	0.9274809159	0.0154406250	
1.0	0.9993327774	-0.0923780600	

4.4 Pumping characteristics

The swimming sheet propels itself with propulsive velocity in the negative axial direction, whereas traveling waves propagate along the inclined channel walls in a direction opposite to the direction of motion of the sheet in positive axial direction. In our study, the main direction of flow is along the swimming sheet, so we take the negative pumping rate as increasing pumping rate. Figures 12–14 explain (3), which characterize the pumping mechanism in negative axial direction. Through these figures, we examine effects of the amplitudes of traveling waves generated in upper ϵ_1 and lower ϵ_2 channel walls, inclination θ , Froude number Fr , and Reynolds number Re on the pumping rate. Pressure difference over wavelength versus time mean flow rate plane is divided into four conventional quadrants. Quadrant *I* represents the peristaltic pumping region (when $\bar{Q} > 0$ and $\Delta p > 0$), quadrant *II* denotes the retrograde pumping region (when $\bar{Q} < 0$ and $\Delta p > 0$), quadrant *III* is the self-propulsive pumping region (when $\bar{Q} < 0$ and $\Delta p < 0$), and quadrant *IV* is designed to represent the augmented pumping region (when $\bar{Q} > 0$ and $\Delta p < 0$). In the peristaltic pumping region, a positive value of $\bar{Q}$ is entirely due to the traveling waves in the channel walls after overcoming the pressure difference, gravitation, and self-propulsion of the swimming sheet. The pressure difference assists the flow due to the traveling waves in the channel wall in the augmented pumping region. Self-propulsive pumping region is the region in which pressure difference favors the flow but due to the dominance of self-propulsive waves and gravitation over traveling waves, the flow takes place in the negative axial direction. In retrograde pumping region, flow takes place in the direction of the swimming sheet. Figure 12 depicts the variation of pumping rate with the ϵ_1 and ϵ_2 . A reduction

in the pumping rate with increasing ϵ_1 and ϵ_2 in propulsive, augmented, and retrograde pumping regions is noted. Further, it is inferred that the flow in the peristaltic pumping region disappears due to the dominance of propulsive motion and gravitation over traveling wave propagating along the channel. Figure 13a displays effects of the ϵ on pumping rate. From this figure, it is speculated that the pumping rate increases in the augmented, retrograde, and propulsive pumping regions as ϵ is incremented. Figure 13b, which illustrates the effects of θ , infers that pumping rate enhances in all four regions when θ increases. Figure 14 presents effects of the Fr and Re on pumping rate. The pumping rate diminishes in the augmented, retrograde, and propulsive pumping regions with increasing Fr , as is delineated in Fig. 14a. Through Fig. 14b, we observe that Re has the same impact on pumping rate as we observed in Fig. 13b for θ .

4.5 Temperature distribution and heat-transfer coefficients

Physical behavior of the amplitudes of traveling waves generated in upper ϵ_1 and lower ϵ_2 channel walls, amplitude of the swimming sheet ϵ , inclination θ , Froude number Fr , Reynolds number Re , Brinkmann number Br , and pressure difference over wavelength Δp on temperature of the fluid and heat-transfer coefficients for lower and upper regions at the swimming sheet are depicted in Figs. 15, 16, 17, 18, 19, 20, 21, 22, 22, 23, 24, 25, and 26. These figures are graphical forms of (36, 37, 39), and (40).

Figure 15 was prepared to show the effects of the ϵ_1 and ϵ_2 on temperature of the fluid. Temperature of the fluid above the swimming sheet is found to be decreased when ϵ_1 increases in Fig. 15a. On the other hand, temperature of the fluid below the swimming sheet increases with increasing ϵ_2 , as is seen in Fig. 15b. Moreover, it is also seen in Fig. 15 that temperatures of the fluid above and below the swimming sheet are independent of ϵ_2 and ϵ_1 respectively. Figure 16 explores influences of the ϵ and θ on temperature of the fluid. Figure 16a shows that the temperature of the fluid below the swimming sheet intensifies, whereas temperature of the fluid above the swimming sheet reduces as ϵ is incremented. An increase in temperature of the fluid above and below the swimming sheet with an increase in the θ is speculated in Fig. 16b. Figure 17 is plotted to see effects of the Fr and Re on temperature of the fluid. By the increase of Fr , temperatures of the fluid above and below the swimming sheet diminish, as is observed in Fig. 17a while opposite effects of Re are seen on these temperatures in Fig. 17b. Figure 18 displays the effects of Br and Δp on temperature of the fluid. This figure infers that temperatures of the fluid above and below the swimming sheet are observed to be maximal for higher values of Br and Δp . Figures 19–26 illustrate the behavior of the heat-transfer coefficients at the swimming sheet. Heat-transfer coefficients Z^- and Z^+ have symmetrical bell-shaped and inverted bell-shaped curve behavior due to the self-propulsive motion of the swimming sheet. It is noticed from Figs. 19–26 that $|Z^+|$ maps on Z^- . The peak value of the bell-shaped heat-transfer coefficient curve Z^- becomes enhanced as the ϵ , Br , Re , θ , and Δp are increased, while it is reduced with the increase in Fr as indicated from (a) parts of Figs. 19–24. Similarly, one can speculate likewise impact of the ϵ , Br , Re , θ , Δp and Fr on peak value of inverted bell-shaped heat-transfer coefficient $|Z^+|$ from (b) parts of Figs. 19–24. Figures 25 and 26 delineate the impact of ϵ_1 and ϵ_2 on heat-transfer coefficients for upper (+) and lower (–) regions of the fluid. It is disclosed from these figures that heat-transfer coefficients of the fluid for the upper and lower regions, respectively, are independent of ϵ_2 and ϵ_1 . From Figs. 25b and 26a, it is worth mentioning that $Z^-(\epsilon_1, \epsilon_2) = |Z^+(\epsilon_2, \epsilon_1)|$. In other words, one may say that amplitudes of the traveling waves generated in the channel walls have symmetric impact on heat-transfer coefficients Z^- and $|Z^+|$. Moreover, heat-transfer coefficients Z^- and $|Z^+|$

Fig. 19 Heat-transfer coefficient at the swimming sheet for different values of ϵ with $\epsilon_1 = 0.1$, $\epsilon_2 = 0.1$, $Re = 0.1$, $Fr = 0.05$, $\Delta p = 0.2$, $Br = 1.0$, and $\theta = \frac{\pi}{6}$ for **a** Z^- and **b** Z^+

along the horizontal distance decreases on $[0, \frac{1}{4}] \cup [\frac{1}{2}, \frac{3}{4}]$ while increases on $[\frac{1}{4}, \frac{1}{2}] \cup [\frac{3}{4}, 1]$ with the increase in ϵ_2 and ϵ_1 , respectively.

5 Applications to spermatozoa transport

Spermatozoa transport through the human cervical canal is an important step towards the process of human procreation. A dynamic interaction between the swimming spermatozoa and the mucus are necessary and sufficient for the passage of spermatozoa through the

Fig. 20 Heat-transfer coefficient at the swimming sheet for different values of Br with $\epsilon_1 = 0.1$, $\epsilon = 0.01$, $\epsilon_2 = 0.1$, $\Delta p = 0.2$, $Re = 0.1$, $Fr = 0.5$, and $\theta = \frac{\pi}{6}$ for **a** Z^- and **b** Z^+

Fig. 21 Heat-transfer coefficient at the swimming sheet for different values of Fr with $\epsilon_1 = 0.1$, $\epsilon_2 = 0.1$, $\epsilon = 0.01$, $\Delta p = 0.2$, $Re = 0.1$, $Br = 1.0$, and $\theta = \frac{\pi}{6}$ for **a** Z^- and **b** Z^+

cervical canal. The results obtained in the previous section may be applied to the spermatozoa transport with heat transfer through non-isothermal cervical mucus filled in an inclined human cervical canal.

The aim of this section is to provide relevance of the results obtained in the previous section to the spermatozoa transport. Propulsive velocity of the sheet of spermatozoa increases with the increase in ϵ_1 and ϵ_2 , while it reduces when $|\epsilon|$ increases. Pressure rise facilitates the motion of the spermatozoa to fertilize an ovum, whereas for pressure drop, the spermatozoa make their exit from the cervical canal via the vagina. Spermatozoa attain maximal velocity for zero Froude number, larger value of Reynolds number, and vertical cervical

Fig. 22 Heat-transfer coefficient at the swimming sheet for different values of Re with $\epsilon_1 = 0.1$, $\epsilon_2 = 0.1$, $\epsilon = 0.01$, $Fr = 0.05$, $\Delta p = 0.2$, and $Br = 1.0$ for **a** Z^- and **b** Z^+

Fig. 23 Heat-transfer coefficient at the swimming sheet for different values of inclination θ when $\epsilon_1 = 0.1$, $\epsilon_2 = 0.1$, $\epsilon = 0.01$, $Re = 0.1$, $Fr = 0.05$, $\Delta p = 0.2$, and $Br = 1.0$ for **a** Z^- and **b** Z^+

canal. It is evident that inclination of the cervical canal during coitus maximizes the probability of pregnancy as compared to the horizontal sex position. After a few minutes of the deposition of spermatozoa in the orifice of cervix, the female should maintain the cervical canal inclined at an angle $0 < \theta < \frac{\pi}{2}$ for maximal propulsion of spermatozoa. Therefore, normally, physicians recommend to put a pillow under the hips after intercourse; actually, physicians are suggesting to increase the inclination with the horizontal to make the effective component of gravity strengthen [27].

Fig. 24 Heat-transfer coefficient at the swimming sheet for different values of Δp with $\epsilon_1 = 0.1$, $\epsilon_2 = 0.1$, $\epsilon = 0.01$, $Re = 0.1$, $\theta = \frac{\pi}{6}$, $Fr = 0.05$ and $Br = 1.0$ for **a** Z^- and **b** Z^+

Fig. 25 Heat-transfer coefficient at the swimming sheet for different values of ϵ_1 with $\epsilon_2 = 0.1$, $\epsilon = 0.01$, $\theta = \frac{\pi}{6}$, $Re = 0.1$, $Fr = 0.05$, $\Delta p = 0.2$, and $Br = 1.0$ for **a** Z^- and **b** Z^+

We tabulate Table 2 to estimate the propulsive velocity of the swimming sheet through the channel by utilizing the experimental data of Ishijima et al. [28], Martyn et al. [29], and Radhakrishnamacharya and Sharma [13]. It is speculated through the experimental data of Ishijima et al. [28] that the average swimming speed of the freely swimming spermatozoon is $40.5 \mu\text{m/s}$. The spermatozoon has an average beat frequency of 11.8 Hz, an amplitude of $11 \mu\text{m}$, and a wavelength of $26 \mu\text{m}$. The experimental values of propulsive velocity are calculated for the individual spermatozoon, which is a free swimmer. In our case, we are considering the spermatozoa in the cervical canal (a bounded domain). Propulsive velocity

Fig. 26 Heat-transfer coefficient at the swimming sheet for different values of ϵ_2 when $\epsilon_1 = 0.1$, $\epsilon = 0.01$, $\theta = \frac{\pi}{6}$, $Re = 0.1$, $Fr = 0.05$, $\Delta p = 0.2$, and $Br = 1.0$ for **a** Z^- and **b** Z^+

Table 2 Movement characteristics of human spermatozoa held with a micropipette and propulsive velocity of swimming spermatozoa

Effect of different solutions on propulsive velocity of the spermatozoa

 $a_1 = 0.0$, $a_2 = 0.0$, $\Delta p = 0.2$, $h = 1200 \mu\text{m}$, $\lambda = 4000 \mu\text{m}$ and $C = 800 \mu\text{m/s}$

Article reference	Medium	μ cP	a_0 μm	$\dot{V}_p$ $\mu\text{m/s}$
Ishijima et al. [28]	Hanks' solution	1.0×10^0	11.0	80.198
Ishijima et al. [28]	Hanks' solution with polyvinyl-pyrrolidone	3.5×10^1	4.3	80.030
Ishijima et al. [28]	Hanks' solution with methyl-cellulose	4.0×10^3	4.0	80.026
Ishijima et al. [28]	Physiological fluid	4.4×10^3	4.5	80.027
Radhakrishnamacharya and Sharma [13]		4.0×10^{-5}	5.0	80.041

of the spermatozoa in the cervical canal is approximately $80 \mu\text{m/s}$, which is greater than the reported experimental propulsive velocity of [28]. We have analyzed through Table 1 that the propulsive velocity of spermatozoa in the bounded domain is greater than the unbounded domain, as is evident from the comparison of Taylors [3] with Lardner [6], Philip and Chandra [12], Shukla et al. [11], and current analysis.

6 Conclusions

Self-propulsion of the swimming sheet with heat transfer through non-isothermal fluid filled in an inclined human cervical canal is addressed. The main findings of the presented analysis are:

- The propulsive velocity of the swimming sheet increases with the increase in ϵ_1 and ϵ_2 , while it decreases when $|\epsilon|$ increases. An increment in pressure rise increases the propulsive velocity while an increment in pressure drop reduces the propulsive velocity of the swimming sheet. Swimming sheets attain maximal velocity for zero Froude number, larger value of Reynolds number, and vertical channel.
- The magnitude of the pressure gradient $P^\pm$ remains unchanged with the variations of θ , Fr , and Re , while it decreases as ϵ is incremented. The magnitude of the pressure gradients P^+ and P^- enhances with increasing ϵ_1 and ϵ_2 , respectively.
- The fluid velocity $u^\pm$ decreases with an increase in Re and θ , while it increases with an increment in the Fr . Fluid velocity u^- increases when ϵ_2 increases, while it decreases with increasing ϵ . Fluid velocity u^+ enhances with the increase in ϵ , whereas it shows diminishment with the increase in ϵ .
- The pumping rate enhances with the increment in θ , Re , and ϵ , while it lessens with the increment in ϵ_1 , ϵ_2 and Fr in propulsive, augmented, and retrograde pumping regions. Moreover, in the peristaltic pumping region, the pumping rate increases, as Re and θ are increased.
- Increment in θ , Re , Br , and Δp results in an increase in the fluid temperature, while increment in Fr causes reduction in the fluid temperature. It is found that the fluid temperature above the swimming sheet is minimal for larger values of ϵ_1 , ϵ . The temperature of the fluid below the swimming sheet is maximal for larger values of ϵ and ϵ_2 .

- It is revealed that $|Z^+|$ maps on Z^- . The peak of the bell-shaped heat-transfer coefficients intensify for larger values of ϵ , Br , Re , θ , and Δp , and smaller values of Fr . Amplitudes of the channel walls have a symmetric impact on Z^- and $|Z^+|$.

The aforementioned findings find applications to spermatozoa transport with heat transfer through non-isothermal cervical mucus filled in an inclined human cervical canal.

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Compliance with Ethical Standards

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AHSAN WALAIT, A. M SIDDIQUI, M. A RANA

Informed Consent Informed consent was obtained from all individual participants included in the study.

Research involving human participants and/or animals For this type of study formal consent is not required. This article does not contain any studies with human participants or animals performed by any of the authors.

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