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Heat Transfer in a Falling Laminar Liquid Film with In-Depth Radiation Absorption¹

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Abstract

A heat transfer problem involving a steady-state falling liquid film along a vertical wall with an imposed constant heat flux on the liquid film surface and an in-depth thermal radiation absorption in the film layer is analyzed. The liquid film is assumed to be normal to the incident radiative heat flux, and the heat flux is assumed to be parallel and unidirectional. Nondimensionalized governing equations are formulated and solved numerically. Sample calculations are provided. The effects of pertinent dimensionless parameters on the heat transfer process are demonstrated. The calculated results reveal that the in-depth thermal radiation absorption would play a key role in the heat transfer in the liquid film.

Keywords

falling liquid film; heat transfer; thermal radiation

1. Introduction

Heat and mass transfer in a falling liquid film along an inclined or vertical surface under the influence of gravity find many applications in chemical engineering processes, nuclear reactor safety, and fire protection engineering. Examples are wetted-wall columns, film reactors, evaporators, condensers, desalination, cooling of a heated surface, and thermal shielding of a surface from thermal radiation from fires [1-3]. Analytical and numerical studies have been reported in the literature [e.g., 4-13]. However, most, except [7], of the previous studies have not included the treatment of thermal radiation. In [7], the characteristics of heat, mass and momentum transfer of a water film falling over an inclined plate with solar radiant heating and water evaporation were investigated. The physical processes were highly coupled with conduction, convection with flow turbulence, diffusion, radiation, and phase change. In this work, heat transfer in a falling laminar liquid film with a radiative heat flux imposed on the liquid-gas interface and a simplified treatment of in-depth

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absorption of thermal radiation occurring in the liquid film without evaporation are examined. We encountered this problem while we were exploring the feasibility of using a flowing liquid film to cool the strong wall during experiments to study thermal responses of structural elements and systems to fires in a laboratory.

2. Problem formulation and analysis

Figure 1 is a schematic representation of the problem considered. We limit our discussion here to a fully developed steady-state non-wavy laminar liquid film flow over a vertical surface. All thermo-physical properties of the liquid are assumed to be constant, and that the liquid film thickness is also assumed to be constant with negligible liquid evaporation over the domain of interest. The solid surface is maintained at uniform temperature T_w . Under these assumptions, the steady-state equation of motion for the falling liquid film with negligible viscous dissipation can be simplified to [4]

$$\mu_L \frac{d^2 v_z}{dy^2} = -\rho_L g \quad (1)$$

with the following boundary conditions

$$\text{At } y = 0, \quad v_z = 0 \quad (2)$$

$$\text{At } y = \delta, \quad \frac{dv_z}{dy} = 0 \quad (3)$$

Equation (1) with the above two boundary conditions can be integrated to obtain the velocity distribution $v_z(y)$ in the falling film.

$$v_z = \frac{\rho_L g \delta^2}{2\mu_L} \left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 \right] = v_{z,\max} \left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 \right] \quad (4)$$

With

$$v_{z,\max} = \frac{\rho_L g \delta^2}{2\mu_L} \quad v_{z,m} = \frac{\int_0^W \int_0^\delta v_z dy dx}{\int_0^W \int_0^\delta dy dx} = \frac{\rho_L g \delta^2}{2\mu_L} = \frac{2}{3} v_{z,\max} \quad (5)$$

In Eq. (5), $v_{z,\max}$ is the at $y = \delta$ from Eq. (4).

The energy equation for the falling film can be written as

$$\rho_L C_p v_z \frac{\partial T}{\partial z} = k_L \left[\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] - \vec{\nabla} \cdot \vec{q}_r \quad (6)$$

If axial conduction is assumed to be unimportant and neglected when compared to axial convection, Eq. (6) can be simplified to

$$\rho_L C_p v_z \frac{\partial T}{\partial z} = k_L \frac{\partial^2 T}{\partial y^2} - \vec{\nabla} \cdot \vec{q}_r \quad (7)$$

Substituting Eq. (4) into Eq. (7),

$$\rho_L C_p v_{z, \max} \left[2 \left(\frac{y}{\delta} \right) - \left(\frac{y}{\delta} \right)^2 \right] \frac{\partial T}{\partial z} = k_L \frac{\partial^2 T}{\partial y^2} - \vec{\nabla} \cdot \vec{q}_r \quad (8)$$

with the following boundary conditions

$$T = T_w \text{ at } y = 0 \text{ and } z > 0 \quad (9)$$

$$k_L \frac{\partial T}{\partial y} + h(T_{y=\delta} - T_\infty) = q_e \text{ at } y = \delta \text{ and } z > 0 \quad (10)$$

$$T = T_o \text{ at } z = 0 \text{ and } y > 0 \quad (11)$$

In writing Eq. (10), we assume there is also a convective heat flux at the liquid-gas interface.

Assuming (a) the liquid layer being gray, (b) no scattering and emission in the flowing gray medium, (c) a non-emitting solid surface, (d) the axial (z -direction) convective heat transfer being dominant over the radiative contribution in that direction and the temperature change being slow enough in the axial direction that $q_{r,z}/z \ll q_{r,y}/y$, (e) the incoming radiation at the liquid surface be parallel and unidirectional, and (f) $q_{r,x}/x = 0$ for a two-dimensional problem, then the divergent of the radiative flux vector can be simplified to [14]

$$\vec{\nabla} \cdot \vec{q}_r = - \frac{\partial q_{r,y}}{\partial y} = - q_e \frac{d}{dy} \exp[-a(\delta - y)] = - a q_e \exp[-a(\delta - y)] \quad (12)$$

Introducing the following dimensionless variables

$$\begin{aligned}
\theta &\equiv \frac{T}{T_\infty} & \xi &\equiv \frac{y}{\delta} & \zeta &\equiv \frac{z}{\delta} \\
Re &\equiv \frac{\delta v_{z,m} \rho_L}{\mu_L} = \frac{2\delta v_{z,\max} \rho_L}{3\mu_L} & Pr &\equiv \frac{\nu_L}{\alpha_L} = \frac{\mu_L C_p}{k_L} & Pe &\equiv Re Pr \\
\tau &\equiv a\delta & Nu &\equiv \frac{h\delta}{k_L} & Q_e &\equiv \frac{\delta q_e}{k_L T_\infty}
\end{aligned}$$

With Eq. (12), Eq. (8) can be expressed in terms of the above dimensionless variables.

$$\frac{3}{2}Pe[2\xi - \xi^2]\frac{\partial\theta}{\partial\zeta} = \frac{\partial^2\theta}{\partial\xi^2} + Q_e\tau\exp[-\tau(1-\xi)] \quad (13)$$

The boundary conditions, Eq. (9), (10), and (11), can be written in the following dimensionless forms.

$$\theta = \theta_{\xi=0} \text{ at } \xi = 0 \text{ and } \zeta > 0 \quad (14)$$

$$\frac{\partial\theta}{\partial\xi} + Nu(\theta_{\xi=1} - \theta_{\xi\rightarrow\infty}) = Q_e \text{ at } \xi = 1 \text{ and } \zeta > 0 \quad (15)$$

$$\theta = \theta_o \text{ at } \zeta = 0 \text{ and } \xi > 0 \quad (16)$$

The dimensionless temperature, $\theta(\zeta, \xi)$, in the falling liquid film is thus governed by four dimensionless parameters, Pe , Nu , Q_e , and τ .

Equation (13) together with Eqs. (14), (15), and (16) was solved numerically by marching forward in the ζ direction using a forward-difference approximation for the first derivative of θ with respect to ζ and using the Crank-Nicolson scheme [15] for the second derivative of θ with respect to ξ . At each ζ step, $\theta(\xi)$ was obtained using a tridiagonal matrix algorithm [15]. Through successive grid refinement, convergence was achieved with $\zeta = 0.05$ and $\xi = 0.025$; these grid sizes were used in the following simulations.

3. Results and discussion

The following dimensionless parameters were used in the benchmark simulation: $Pe = 10$, $Nu = 1$, $Q_e = 0.3$, $\tau = 1$, and $\theta_{\zeta=0} = \theta_{\xi=0} = \theta_{\xi\rightarrow\infty} = 1$. These parameters were obtained using $k_L \approx 1$ W/m-K, $\delta \approx 0.01$ m, $q_e \approx 10^4$ W/m², $T_w = T_0 = T_\infty \approx 300$ K, $a \approx 10^2$ m⁻¹, $Pr \approx 10$, and $Re \approx 1$. For illustration, we assume constant $Nu (= 1)$ although $Nu = f(Re, Pr)$ used to simulate the cases with could be included in the numerical calculations if the convective heat transfer correlation is known. Table 1 summarizes the dimensionless

parameters used to simulate the cases with $\xi \in [0,1]$ and $\zeta \in [0,100]$ to obtain the dimensionless temperature contours $\theta(\zeta, \xi)$. Figure 2(a) shows the calculated dimensionless temperature distribution within the liquid film for the benchmark case.

At small ζ , the temperature profile is clearly influenced by the axial convective heat transfer. As ζ increases, the effect of axial convection diminishes.

Figure 2 shows the comparison of the dimensionless temperature profile with ($\tau = 1$) and without in-depth radiation absorption ($\tau = 0$) in the liquid film. In both cases, the temperature profiles are similar, but the temperatures in the liquid film are lower without in-depth radiation absorption.

The way in which the temperature distribution in the liquid film is affected by increasing Pe from 10 to 100 is indicated in Fig. 3(b). Comparing to the benchmark case ($Pe = 10$), the effect of axial convection is extended further downstream with a higher $Pe (= 100)^2$. Figure 3(c) shows the contrasting results of decreasing Pe from 10 to 0.1. In this case, the effect of axial convection becomes less important and is only extended to a very small ζ downstream.

The effect of Nu on the temperature distribution in the liquid film can be ascertained from Fig. 4(b). Without convective cooling at the gas-liquid interface (i.e., $Nu = 0$), the temperatures within the liquid film are higher than those in the benchmark case with cooling ($Nu = 1$). The competing effect of absence of both convective cooling ($Nu = 0$) and in-depth radiation absorption ($\tau = 0$) on the temperature profile in the liquid film is shown in Fig. 4(c). Since the temperatures are higher than those in the benchmark case, the convective cooling at the gas-liquid interface exerts more influence on the temperatures in the film over in-depth radiation absorption under the conditions examined in this study.

As expected, the effect of increasing the dimensionless incident flux Q_e at the interface also increases the temperatures in the liquid film, as illustrated in Fig. 5(b).

If $T_\infty \approx 300$ K and assuming the flowing liquid being water with a normal boiling point of $T_{nb} = 373$ K, the dimensionless liquid film temperature $\theta(\zeta, \xi)$ could reach the normal boiling point of water when $\theta(\zeta, \xi) = T_{nb}/T_\infty = 1.24$. This condition, when the boiling temperature of water is reached, is important from the perspective of fire safety and the ability of the film to cool the surface. The condition could be attained in the liquid film when in-depth thermal radiation absorption is significant, convective cooling at the liquid-gas interface is absence, and/or the imposed heat flux at the liquid-gas interface is high (see Figs. 3, 4, and 5). The analysis presented here will no longer be valid if boiling occurs in the liquid film.

4. Conclusions

Heat transfer involving a steady-state falling liquid film along a vertical wall with an imposed constant heat flux at the liquid film surface and in-depth thermal radiation

²As pointed out in Bird et al. [4], the above flow analysis is valid only when the flowing film is laminar with straight streamlines. At higher Re therefore Pe , transition to laminar flow with rippling will occur.

absorption in the film layer was formulated and analyzed. The resulting nondimensionalized governing equations were solved numerically. Under certain conditions studied, the heating of the film to boiling could occur and lead to error in the use of laminar liquid film analysis to cool a surface for fire safety applications.

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Nomenclature

a	Liquid absorption coefficient
C_p	Liquid heat capacity
g	Gravitational acceleration
h	Convective heat transfer coefficient
k_L	Liquid thermal conductivity
q_e	External heat flux applied on the liquid film surface
$\vec{q}_r$	Radiative flux vector
T	Liquid film temperature
T_0	Liquid film temperature at $z = 0$
T_w	Wall temperature
T_∞	Ambient temperature
v_z	z -component velocity
$v_{z,\max}$	Maximum z -component velocity
$v_{z,m}$	Mean z -component velocity
y	y -direction
z	z -direction
α_L	Liquid thermal diffusivity
δ	Liquid film thickness
μ_L	Liquid viscosity
ν_L	Liquid kinematic viscosity
ρ_L	Liquid density

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Fig. 1.
Schematic representation of the heat transfer problem considered.

Fig. 2. Comparison of dimensionless temperature contours, $\theta(\zeta, \xi)$, in the falling liquid film of the benchmark case ($\tau = 1$; left figure) with the case of no in-depth thermal radiation absorption ($\tau = 0$; right figure).

Fig. 3. Comparison of dimensionless temperature contours, $\theta(\zeta, \xi)$, in the falling liquid film of the benchmark case ($Pe = 10$; left figure) with the cases of increasing Peclet number ($Pe = 100$; middle figure) and decreasing Peclet number ($Pe = 0.1$; right figure).

Fig. 4.

Comparison of dimensionless temperature contours, $\theta(\zeta, \xi)$, in the falling liquid film of the benchmark case ($Nu = 1$; left figure) with the cases of decreasing Nusselt number ($Nu = 0$; middle figure) and decreasing Nusselt number with no in-depth thermal radiation absorption ($Nu = 0$ and $\tau = 0$; right figure).

Fig. 5. Comparison of dimensionless temperature contours, $\theta(\zeta, \xi)$, in the falling liquid film of the benchmark case ($Q_e = 0.3$; left figure) with the case of increasing Q_e ($Q_e = 0.6$; right figure).

Table 1

Summary of the dimensionless parameters used in the simulations

Figure	$\theta_{\zeta=0}$	$\theta_{\xi=0}$	$\theta_{\xi \rightarrow \infty}$	Q_e	Nu	Pe	τ
2(a), 3(a); 4(a); 5(a)	1	1	1	0.3	1	10	1
2(b)	1	1	1	0.3	1	10	0
3(b)	1	1	1	0.3	1	100	1
3(c)	1	1	1	0.3	1	0.1	1
4(b)	1	1	1	0.3	0	10	1
4(c)	1	1	1	0.3	0	10	0
5(b)	1	1	1	0.6	1	10	1