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METHODS AND CONSIDERATIONS TO DETERMINE SPHERE CENTER FROM TERRESTRIAL LASER SCANNER POINT CLOUD DATA

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Abstract

The Dimensional Metrology Group (DMG) at the National Institute of Standards and Technology (NIST) is performing research to support the development of documentary standards within ASTM E57 committee. This committee is addressing the point-to-point performance evaluation of a subclass of 3D imaging systems called Terrestrial Laser Scanners (TLSs) which are laser-based and use spherical coordinate system. This paper discusses the usage of sphere targets for this effort and methods to minimize the errors due to the determination of their centers. The key contributions of this paper include the methods to segment sphere data from TLS point cloud and the study of some of the factors that influence the determination of sphere centers.

Keywords

Point cloud segmentation; sphere fitting; Terrestrial laser scanners; 3D imaging systems; Documentary standards

1 INTRODUCTION

Terrestrial Laser Scanners (TLSs) are instruments that can measure 3D coordinates of objects at high speed resulting in high density 3D point cloud data without the necessity of a cooperative target. There are some data quality issues that are unique to TLS systems. The scans from a TLS are often affected by material and optical properties of the surfaces they are scanning and by the objects in the immediate vicinity of the object being measured. Additionally, the probing direction of a TLS (or the laser beam) is not necessarily orthogonal to the surface of the object being measured. This is unlike coordinate measuring machines

(CMMs) and laser trackers, where the probing point is mostly normal and offset to the measured surface.

In most applications, point cloud data from TLSs are segmented and substitute geometries (planes, spheres, cylinders etc.) are fit to this data to determine features of interest such as distances between objects, size, concentricity, parallelism etc. Spheres are of particular interest to users of TLS as these are preferred targets to register multiple TLS scans. This is because the sphere's geometry appears the same regardless of the direction of the scan. However, any error in determining the sphere centers will be propagated as registration errors. It is therefore necessary to ensure that the sphere center obtained from the scan data is as close to its true geometric center as possible to minimize registration related errors. This is one of the motivations for this work.

Another significant motivation for this work is to support an ongoing standards activity within the ASTM E57 committee on 3D imaging systems [1]. The goal of the committee is to develop documentary standards for the point-to-point performance evaluation of laser-based 3D imaging systems such as TLSs that acquire data in a spherical coordinate system. The Dimensional Metrology Group (DMG) at the National Institute of Standards and Technology (NIST) is actively engaged in this effort. The currently proposed tests call for the evaluation of point-to-point performance of these systems by determining the measurement error between two derived points* at a number of positions in the TLS's work volume. The targets recommended for a majority of the proposed tests (non-ranging tests) are spheres. To adequately evaluate the performance of a TLS, it is important to ensure that the errors obtained during these tests are due to the TLS and not due to data processing errors. To achieve this, it is necessary to first understand the influence factors that affect the calculated sphere center and then develop robust data processing methods to segment and obtain the sphere center.

In this context, this paper discusses novel methods that were developed at NIST and National Research Council of Canada (NRC) to minimize the errors due to data processing and calculating the sphere center from the data obtained from a TLS. Since the true geometric center is an unknown quantity, simulations and studies have been performed to evaluate the effect of various influence factors on the determination of true center. Where possible, experimental data has also been used, primarily to assess the effect of the algorithms on the variability in the sphere center. The key contributions of this paper are in the area of data segmentation methods and the factors that influence the determination of sphere centers.

2 REVIEW OF SPHERE FITTING ALGORITHMS

Algorithms to fit spheres to 3D point cloud data have been studied extensively. Forbes [2] discussed a variety of algorithms to fit spheres and other geometries. While using these algorithms, it is assumed that the 3D data set has been properly segmented, i.e., it is free of

* A derived point is a point obtained from a group of measured points on an artifact. It is dependent on the artifact geometry and the fitting algorithms used. Examples include a sphere center obtained by fitting a sphere to points measured on the sphere's surface; the apex of a pyramid obtained by the intersection of three or more planes by measuring the planar surfaces of the pyramid.

any spurious points that do not belong to the sphere. Methods to segment and extract data representative of a sphere from a larger dataset are discussed in the next section. These segmenting methods require knowledge of fitting spheres, hence some of the common fitting algorithms are reviewed here.

Most of the sphere fitting algorithms discussed in this paper rely on least-squares minimization methods to obtain their center. There are other methods such as a minimum zone sphere, maximum inscribed sphere, minimum circumscribed sphere, etc. that are not a part of this discussion.

The equation of a sphere in Cartesian coordinate system is given by:

$$(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2 = r^2, \quad 1$$

where, (x, y, z) is the center of the sphere (x_p, y_p, z_p) is a point on the sphere and r is the radius of the sphere. The next few sub-sections will describe some of the most widely used objective functions for fitting spheres and determining sphere centers and radii. Specifically, they will describe the definitions of two important fitting algorithms followed by a means to obtain a good initial estimate that can be used as the first step in solving them.

2.1 Orthogonal non-linear least-squares method

A non-linear least-squares algorithm is a method that attempts to minimize the sum of squares of the orthogonal distances (called residuals) between a given set of points and the best fitted sphere surface. Many software suites have built-in iterative algorithms to solve such nonlinear least-squares problems. Examples include Gauss-Newton algorithm and Levenberg–Marquardt algorithm (implemented in [3], for example). Discussion about these algorithms is beyond the scope of this paper, however all these methods require the formulation of an objective function which involves the parameters to be minimized. We have found that Levenberg-Marquardt algorithm is more robust than Gauss-Newton algorithm when used for fitting data to spheres and this is also documented here [4]. The objective function for a sphere to use a nonlinear least-squares algorithm is given by equation 2 and is called the orthogonal error function.

$$F = \sum_{i=1}^n f_i^2 = \sum_{i=1}^n (\sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} - r)^2. \quad 2$$

In the above equation, there are four unknown parameters (x, y, z, r) and as such this minimization method is known as a “unconstrained orthogonal non-linear least squares algorithm” or “orthogonal unconstrained fit” for short. If the radius of the sphere is known and fixed, then there will be only three unknown parameters (x, y, z) to be obtained and this minimization method is termed as “orthogonal constrained fit” for short.

2.2 Directional non-linear least-squares method

Witzgall et al. [5] and Franaszek et al. [6] described a directional error function to fit sphere data that is obtained from TLSs with low signal-noise ratio at long ranges. This error function is based on minimizing the distance between the measured point U_j and its projection onto the sphere surface that is closest to the TLS as shown in Figure 1.

The minimization function for this method is given by

$$F = \left(\frac{1}{n}\right) \sum_{i=1}^n f'_i, \quad 3$$

where

$$f'_i = \begin{cases} \left(p_i - \sqrt{R^2 - q_i^2} - r_i\right)^2 & \text{if } q_i < R \\ (p_i - r_i)^2 + (q_i - R)^2 & \text{if } q_i \geq R. \end{cases}$$

Here U is the center of the sphere (in Figure 1), O is the origin, R is the radius of the sphere, r_i is the length of the vector OU_i , p_i is the scalar projection of vector OU in the direction of the vector OU_i and q_i is the shortest distance from the sphere center U to the vector OU_i .

In this paper, the algorithms based on the directional error function are termed as “directional unconstrained fit” if the radius of the sphere is unknown and “directional constrained fit” if the radius is known.

2.3 Linear least-squares method

The two fits described in sections 2.1 and 2.2 are typically solved by iterative methods that require an initial estimate of the parameters, which is then refined iteratively to converge on the correct fit. Such an initial estimate can be obtained by minimizing the objective function described by equation 4 [2]:

$$F = \sum_{i=1}^n f_i^2 = \sum_{i=1}^n ((x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2 - r^2)^2. \quad 4$$

The function f_i in equation 4 can be expanded to a function given by equation 5.

$$f_i = -2x_i x - 2y_i y - 2z_i z + (x_i^2 + y_i^2 + z_i^2) + (x^2 + y^2 + z^2 - r^2). \quad 5$$

Equation 5 above has four unknown parameters x , y , z , r and they can be obtained by minimizing F by solving a linear least-squares system $AX = B$, where

$$A = \begin{bmatrix} -2x_1 & -2y_1 & -2z_1 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -2x_n & -2y_n & -2z_n & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \\ \rho \end{bmatrix}, \text{ and } B = \begin{bmatrix} -(x_1^2 + y_1^2 + z_1^2) \\ \vdots \\ -(x_n^2 + y_n^2 + z_n^2) \end{bmatrix}, \quad 6$$

and $\rho = (x^2 + y^2 + z^2 - r^2)$. The parameters (x, y, z, ρ) can be solved by performing the matrix operation described in equation 7 [7] provided that four or more number of points are obtained on the sphere ($n \geq 4$). These parameters can also be solved by using linear solver functions available in many software suites.

$$X = [A^T A]^{-1} [A^T B]. \quad 7$$

The radius can then be determined as $r = \sqrt{(x^2 + y^2 + z^2 - \rho)}$.

3 METHODS TO SEGMENT SPHERE FROM 3D SCAN DATA AND OBTAIN THE SPHERE CENTER

The data obtained from TLSs may be very noisy or have spurious data points that do not belong to the object that is being scanned. A typical scan of a sphere mounted on a plate is shown in Figure 2. It is apparent that there are some points in the scan data that are not points on the sphere of interest and these have to be discarded to obtain a reliable sphere center. Depending on the parameters of the scan (distance to the sphere, TLS settings, sphere size, material, etc.), some of the scans may not have a sufficient number of points on the sphere surface to reliably calculate their sphere center as illustrated in Figure 3.

The next few sub-sections will illustrate the various segmentation methods that were explored to extract data corresponding to a sphere from 3D scan data. It should be noted that all the illustrations in this section are a 2D representation of a 3D operation. The techniques proposed in this section were developed after analyzing thousands of sphere datasets acquired from six different TLSs. This data was acquired during an instrument runoff [8] performed as part of the ongoing ASTM E57 standards work. The methods we propose here were tested on those datasets to ensure their broad applicability across different TLSs.

3.1 Closest point method

The closest point method, which we describe here, serves not only as a segmentation technique but also as a method to determine an initial estimate for the center of the sphere to be used with other methods. This method has the following prerequisites:

1. The sphere is mounted in such a way that there is no mounting apparatus between the sphere and the TLS.

2. The scan data corresponding to the sphere is separated from its surroundings in such a way that only one sphere exists per dataset. Some spurious points corresponding to the mount behind the sphere are acceptable.
3. The coordinate system of the dataset and that of the TLS are coincident; i.e., the scan data is in the reference frame of the TLS without any transformation of data.

The procedure to obtain the initial center (C_0) of the sphere using this method is as follows and illustrated in Figure 4:

1. The median distance d between the origin of the TLS (O) and N points on the sphere closest to O is calculated. The number of points N in this case is empirically obtained and could be either a fixed number or a fixed percentage of the points (e.g. N is lesser of 500 points or 5% of the number of points in the dataset).
2. Points are selected from the dataset that fall within a distance of $d + 0.5R_{CAL}$ from the origin, where R_{CAL} is calibrated radius of the sphere[†]. The resulting dataset is considered as the segmented data belonging to the sphere.
3. Estimates for the sphere center (C_0) can be determined by fitting a sphere to data selected in the previous step using an orthogonal constrained fit algorithm.

The selection of points within $d+0.5R_{CAL}$ distance in step#2 was empirically determined and is approximately equal to selecting points within a cone angle of 120° (described in section 3.2). The initial sphere center estimate may also be obtained using other methods, either manually or automatically, using the software provided by the TLS manufacturer or any other third party software.

3.2 Cone angle method

The cone angle method of segmentation (Figure 5) requires an initial center, which may be obtained using the method described in section 3.1 and it involves the selection of a conical region of data on the sphere. The use of cone angle based segmentation eliminates points measured at high angles of incidence (at the outer periphery of the sphere). It also ensures that the scan data is symmetric in the non-ranging directions which minimizes the error in the calculated sphere center in those directions. This method may be implemented using the following sequence of steps:

1. An initial sphere center C_0 is determined using the closest point method or other methods as described in section 3.1
2. A vector C_0O is constructed between the origin O and the center C_0
3. Points P_i are included in the segmented data (where P_i is the i^{th} point in the dataset) if the acute angle β_i between the vector C_0P_i and C_0O is less than or equal to $\theta/2$. Here, θ is the cone opening angle (or “cone angle”) and is empirically chosen to be 120° [†] and will be discussed in section 6.4.

[†]The sensitivity of the results due to an error in the calibrated radius is discussed in section 6.7

4. An orthogonal constrained fit is performed on the resulting dataset and the standard deviation of the residuals (σ_R) is calculated. Points are excluded from this dataset when the absolute values of the corresponding residuals exceed $3\sigma_R$.
5. The remaining points in step#4 are used to calculate the sphere center using an orthogonal constrained fit.

3.3 Double cone angle method

A double cone angle method (Figure 6) extends the cone angle method described in section 3.2 to further exclude spurious points that may sometimes be included in the cone angle method. The following procedure is used to perform the double cone angle method:

1. An initial sphere center C_0 is determined using the closest point method or other methods as described in section 3.1
2. A vector C_0O is constructed between the origin O and the center C_0 .
3. The angle α is calculated where $\alpha = 2\tan^{-1}\left(\frac{R_{CAL} \sin(\theta/2)}{d_s - R_{CAL} \cos(\theta/2)}\right)$ and $d_s = |C_0O|$, which is the 3D distance between the origin and the initial center C_0 .
4. Points P_i are included in the segmented data (where i is the i^{th} point in the dataset) if both the following conditions are met:
 - a. the acute angle β_i between the vector C_0P_i and C_0O is less than or equal to $\theta/2$ (θ is the cone angle and is empirically chosen to be 120°) and
 - b. the acute angle between vector P_iO and C_0O is less than or equal to $\alpha/2$.
5. An orthogonal constrained fit is performed on the resultant dataset and the standard deviation of the residuals (σ_R) is calculated. Points are excluded from this dataset when the absolute values of the corresponding residuals exceed $3\sigma_R$.
6. The remaining points in step#5 are used to calculate the sphere center using an orthogonal constrained fit.

3.4 Cone-cylinder method

A cone-cylinder method (Figure 7) is similar to the double cone angle method, except that the second region of selection is a cylinder instead of a cone. The following procedure is used to implement this method.

1. An initial sphere center C_0 is determined using the closest point method or other methods as described in section 3.1
2. A vector OC_0 is constructed between the origin O and the center C_0
3. Points P_i are included in the segmented data (where P_i is the i^{th} point in the dataset) if both the following conditions are met:

[‡]The cone angle of 120° is used in a 2D context and is equal to π steradians in 3D. The usage of the 2D angles useful while calculating the angles between various vectors in these segmentation methods.

- a. the acute angle β_i between the vector C_0P_i and C_0O is less than or equal to $\theta/2$ (θ is the cone angle and is empirically chosen to be 120°) and
 - b. The distance between the points and line C_0O is less than or equal to $R_{CAL} \sin(\theta/2)$ (i.e., within a cylinder of radius $R_{CAL} \sin(\theta/2)$)
4. An orthogonal constrained fit is performed on the resultant dataset and the standard deviation of the residuals (σ_R) is calculated. Points are excluded from this dataset when the absolute values of the corresponding residuals exceed $3\sigma_R$.
 5. The remaining points in step#4 are used to calculate the sphere center using an orthogonal constrained fit. The use of the cylindrical region in this method or a second cone region in the double cone angle method ensures that the points on the outer periphery of the sphere are excluded, resulting in a lower σ_R and thereby ensuring greater confidence in the sphere center determination process.

3.5 Radius correction method

The radius correction method is useful when the TLS data obtained from a sphere is symmetrical in the non-ranging directions and it attempts to correct the error in the sphere center calculated by the sphere fitting algorithms (orthogonal unconstrained fit and/or the orthogonal constrained fit).

3.5.1 Squishing and flaring of spheres—The quality of the scan data of a sphere from some TLSs varies over the sphere surface [9] and is typically worse at the outer periphery. When datasets from such targets are fit to a sphere using an orthogonal unconstrained fit, the resulting radius (R_{UNC}) may be different from its calibrated radius (R_{CAL}). As a result, the spheres may appear “squished” when $R_{UNC} < R_{CAL}$ or “flared” when $R_{UNC} > R_{CAL}$.

When the sphere appears “squished”, the calculated center is displaced towards the TLS relative to its true geometric center, as shown in Figure 8. Also, when the sphere appears “flared”, the calculated center is displaced away from the TLS relative to its true geometric center. In most cases it was observed that the sphere appears to be “squished”, rather than “flared”.

3.5.2 Effect of squishing and flaring on sphere center—Consider a sphere whose true center is S , and is measured by a TLS whose origin is at O (Figure 8a). When a sphere is fit to the TLS data using an orthogonal unconstrained fit, let the resulting sphere center be C_U . It is possible that C_U may not coincide with S and may be farther or closer to the TLS than the true geometric center S . When an orthogonal constrained fit is used, let C_C represent the sphere center obtained. Though the use of a constrained fit reduces the magnitude of this issue, it may again not result in the true center S . If the data obtained from scanning a sphere is symmetrical in the non-ranging direction, then the points C_U and C_C lie on (or very close to) the vector OS . The error in the location of the sphere center is approximately equal to $k R = k(R_{CAL} - R_{UNC})$. Here the value k was obtained from a simulation described next.

This simulation models the shape of the sphere data in the form of an ellipsoid with its center at a distance (d_c) of 6.5 m from the origin. The semi-principal axes of the ellipsoid were of length r_a in the ranging direction and r_b and r_c in the non-ranging directions. The value of r_a was the nominal radius of the sphere (r_{NOM}). The values of r_b and r_c were chosen to reflect the squishing of the spheres ($r_a = r_{NOM} = 0.050$ m, $r_b = 0.049$ m, $r_c = 0.049$ m). Gaussian noise was added to the dataset in the ranging direction (whose one standard deviation was $0.2 \text{ mm} + d_c \times 40 \times 10^{-6}$) and data within a 120° cone angle was segmented. These ellipsoids were generated 100 times and were then fit using an orthogonal unconstrained fit and the unconstrained radius (r_{UNC}^i) and the center distance error (d_{UNC}^i) were calculated for each of the 100 datasets. The value k is the ratio of the RMS value of the center distance error ($d_{UNC}^i - d_c$) to the RMS value of the radial error ($r_{UNC}^i - r_{NOM}$) and was found to be 0.985 for an orthogonal fit and 0.965 for a directional fit. It should be noted that the value of k may vary based on the underlying assumptions used in this simulation.

3.5.3 Implementation of the radius correction method—The relationship between the radial error and the center location error discussed above can be used to perform a correction in the unconstrained center location and obtain an improved estimate of the sphere center. The procedure to implement this method is as follows:

1. The closest point method is used to segment the data and obtain the initial estimate of the sphere center C_0 . An orthogonal unconstrained fit is performed on the segmented data to obtain the unconstrained radius R_{UNC} .
2. A vector V joining the origin O and the initial sphere center C_0 is constructed.
3. The correction in center location $k \ R = k \times (R_{CAL} - R_{UNC})$ is calculated.
4. The coordinates of the 3D point along the direction of V located at a distance of $|OC_0| + k \ R$ from the TLS origin O is calculated. This point is considered as the corrected center of the sphere (C_R).
5. The data is segmented again by discarding points that exceed $|OC_R| - 0.5R_{CAL}$.
6. An orthogonal constrained fit is performed on the resultant dataset in step#5 and the standard deviation of the residuals (σ_R) is calculated. Points are excluded from this dataset when the absolute values of the corresponding residuals exceed $3\sigma_R$. This dataset is used for calculation of RMS values of the residuals.

3.6 Iterative usage of segmentation methods

In the four segmentation methods (described in sections 3.2 through 3.5), the initial center is an approximation and may not be accurate. Any error in this initial center could result in the segmented data containing spurious points that do not belong to the sphere. To overcome this, the following iterative method is proposed.

1. An initial estimate of the center is obtained using the closest point method or other methods as described in section 3.1. This is called the “initial center”.
2. One of the procedures to segment data as described in sections 3.2 through 3.5 is performed and the sphere center determined.

3. The sphere center from step#2 is used as the initial center and the segmentation method in step#2 is repeated.
4. The iterations may be terminated when the distance (e_C^i) between the center obtained in the i^{th} iteration and the center obtained in the preceding iteration is less than a predetermined error value. The iterations may also be terminated when a predetermined number of iterations have been performed. The center obtained in the final iteration is considered as the “final center” of the sphere.

4 FACTORS INFLUENCING TLS DATA QUALITY

In the previous sections, we briefly reviewed sphere fitting algorithms and then discussed a variety of segmentation methods to eliminate spurious points and outliers. In order to determine the suitability of these methods, it is important to understand the various sources of errors that can influence the quality of scans and therefore the determination of the sphere center.

There are a variety of factors that influence the quality of the scans. These can be broadly categorized as 1) TLS parameters, 2) Target parameters, 3) Environmental parameters and 4) Data acquisition parameters and 5) Post-processing methods. These factors are described below.

1. TLS parameters: The quality of data depends on a variety of parameters that relate to the TLS. These include a) construction of the TLS and its error sources, b) measurement principle, c) angular resolution, d) field of view, e) coverage, f) laser beam width, g) beam divergence, h) laser wavelength, and i) scan speed
2. Target parameters: The quality of the data also depends on the target, which primarily relates to the quality of the return signal to the TLS from the target. Some of the target parameters are a) reflectance factor (material, color, surface finish etc.) b) form error and c) size
3. Environmental parameters: The environment in which the target is scanned also affects the quality of the data. Some of these parameters include a) background reflectivity, b) mounting apparatus, and c) temperature, pressure and humidity.
4. Data acquisition parameters: The TLS settings and the setup used for the scan affects the data quality. These include a) range to the target, b) scan speed, c) coverage of the data on the sphere, d) number of points on the sphere and e) filtering.
5. Post-processing methods: As discussed in the earlier sections, the sphere center varies based on the method used to process the data. Some of the parameters that affect the determination of sphere center are a) fitting algorithms (which affects the sphere center though not the data quality itself), and b) segmenting methods

Discussion on each of these parameters is beyond the scope of this paper. During a scan, many of the factors described above can be controlled to be a constant value and some factors may vary depending on a variety of scan conditions. In this paper, we explore the

effect of some of the controllable factors through simulation and experimental studies. In section 5, we discuss the setup and the measurements performed on spheres using three different TLSs. In section 6, we discuss the results of the simulations and experiments that show the impact of these influence factors on the sphere center determination.

5 TEST SETUP

To study the impact of some of the influence factors on the sphere center determination, we established a test setup for obtaining sphere scan data. This involved scanning a grid of nine spheres (as illustrated in Figure 9) ten times using TLSs from three different manufacturers (270 sphere datasets). The TLSs were configured to ensure that the number of points measured over the sphere surface remains approximately equal (≈ 1500 points). Some relevant specifications of the three TLSs are given in Table 1.

The TLSs were placed at ≈ 6.5 m away from the grid of spheres. Each of the spheres in the grid is spaced ≈ 1.5 m from each other in both the horizontal and vertical directions. The spheres used in this setup were commercially obtained and have a near-lambertian surface that is friendly to TLSs. To minimize any points from the mounting apparatus, the supports and mounting plates holding the spheres were covered using flock paper (depicted in Figure 10).

The spheres have a nominal radius of 50 mm and have a form error of ≈ 0.01 mm. The choice of the size/radius of the sphere was based on a) their ability to be scanned by the TLS in its work volume resulting in sufficient data quantity and quality needed for these tests, and b) their commercial availability. All the spheres were calibrated on a coordinate measuring machine and the mean value of the radii was 50.001 mm and the standard deviation was 6.6 μm . For simplifying the comparison of various factors influencing the determination of the sphere center in this paper, a radius of 50 mm was used as the calibrated radius (R_{CAL}) for all the spheres. The effect of errors in the calibrated radius is insignificant [13] compared to the other TLS related errors and will be discussed later in section 6.7.

6 EFFECT OF VARIOUS INFLUENCE FACTORS ON SPHERE CENTER

There are a variety of factors that influence the quality of TLS scan data and therefore the sphere center, as discussed in section 4. Theoretically there are several combinations of these factors that could be studied, however such an exhaustive study is beyond the scope of this paper. We therefore draw from our experience in studying thousands of TLS datasets to consider select combinations of parameters. In this section, data processing factors that affect the determination of sphere center will be discussed in detail. This will be done by varying one parameter at a time and studying its effect on the quality of data or the movement of the sphere center.

6.1 Segmentation methods

The four segmentation methods described in sections 3.2 through 3.5 were used to segment data obtained by scanning the nine spheres using three TLSs. To compare the segmentation methods objectively, a metric was adopted. The resultant segmented data was fit to a sphere

using the orthogonal constrained fit and the root-mean-square (RMS) values of the residuals from the fit were recorded for each of the 27 datasets (3 TLSs $\times$ 9 spheres). The summation of these RMS values provides us a metric that can be used to compare these segmentation methods. It should be noted that RMS values alone may not be enough to gauge the quality of segmentation. RMS values may be high in a few cases described below:

- a. Data with high noise may result in higher RMS value, but still result in the determination of a center very close to the true center of the sphere.
- b. Consider a dataset with high noise in the peripheral regions of the scanned sphere compared to the region closer to the TLS. Discarding the high noise peripheral points from this dataset may result in lower RMS value but may not result in a center that is closer to the true center of the sphere.
- c. Data with low noise that is not in the shape of a sphere (e.g. an ellipsoid) when fit to a sphere may result in high RMS value.

Since the true geometric center is an unknown quantity, the data obtained after performing the segmentation method was observed visually. This was to ensure that points that do not belong to the sphere are discarded.

All the 27 datasets were processed using a radius of 50 mm, with a 120° (for the cone angle based methods) and the segmentation was performed iteratively 5 times (as described in section 3.6) and Table 2 lists the values for the $\Sigma(\text{RMS})$ values for various segmentation methods. It may be noted that the double cone angle method and the cone-cylinder method have comparatively lower values of ΣRMS indicating lower number of spurious points in the segmented datasets. It is likely that a small number of measured points may also be discarded in this process, however the effect of such a low number of points is insignificant. It should also be noted that $\Sigma(\text{RMS})$ values may depend on the TLS measurement uncertainty and may be lower for some TLSs compared to others. However, this metric shows the overall effectiveness of the algorithm for the data from all the TLSs used in this study.

6.1.1 Number of iterations—As mentioned in section 3.6, the iteration process (used for segmenting sphere datasets) can be terminated after a predetermined error value e_C^i or after a certain number of iterations. It was observed that most spheres do not require more than 5 iterations. To illustrate this, the 270 sphere datasets were processed using the Cone-Cylinder method (with a radius of 50 mm and cone angle of 120°) and e_C^i values were recorded for each iteration. Table 3 shows the number of spheres which showed improvement in their center value after each iteration. It is possible that the e_C^i in the final iteration may never be exactly 0 μm , but an infinitesimally small value ($< 1 \text{ nm}$) that can be assumed to be zero for all practical purposes.

6.1.2 Initial center—A calculation was performed to understand the sensitivity of the sphere's final center on the location of the initial center. In this calculation, all the 27 datasets (9 spheres from 3 TLSs) were segmented using the cone-cylinder method (with

radius of 50 mm, a 120° cone angle and iterated 5 times). During the segmentation process, the initial center was perturbed by a known distance in various directions and the final center was recorded. The calculation indicated that even a 10 mm perturbation in the initial center is insufficient to have a significant effect on the final center of the sphere.

6.2 Sphere fitting algorithms

As discussed in section 2, we have explored two fitting algorithms, one based on orthogonal and the other based on directional error function. To compare these, the sphere datasets were processed using the segmentation methods that use the directional error function (instead of orthogonal error function) to perform the sphere fit.

These datasets were processed using a radius of 50 mm, a cone angle of 120° and segmentation was iterated 5 times. For the purpose of comparing the two sphere fitting algorithms we used the same metric used in section 6.1. i.e., the $\Sigma(\text{RMS})$ values were calculated using orthogonal constrained fit on the segmented data obtained using directional fitting.

The results of the metric for directional error function are listed in Table 4 and indicate that it does not significantly improve $\Sigma(\text{RMS})$ values for all the four segmentation methods. However, both the algorithms (orthogonal and directional) in conjunction with the double cone angle and the cone-cylinder methods perform better when compared to the other two segmentation methods.

6.3 Number of points

To understand the relationship between the number of points and the error in determining the sphere center, a simulation was performed on 27 sphere datasets obtained from three different TLSs.

The scans from the TLSs were segmented using the cone-cylinder method (with radius of 50 mm, a 120° cone angle and iterated 5 times) to obtain datasets free of any spurious points. The simulation involved constructing two sub-sampled spheres with fixed number of points (N). The points were extracted from the segmented datasets, but randomly picked for both the spheres while ensuring the selected points are spread out on the surface of the sphere. Initially 10 points were selected on these spheres (N=10) and progressively increased to 1300 points in increments of 10. This process was repeated 30 times and each time the simulation generates a different sphere with N number of points. These datasets were then fit using an orthogonal constrained fitting algorithm and the distance between these two virtual spheres was calculated. Each of the 30 repetitions results in 30 distance values. These distances were averaged to obtain a mean distance error between the spheres (d_1). The nominal distance error between each of these pairs of spheres should be 0 mm. Figure 11 shows the variation in the distance(d_1) between these two virtual spheres as a function of the number of points (N) on their surfaces. Multiple curves of the same color/style correspond to the nine spheres on the grid.

Point spacing from scans on a target is not random and is typically in a grid like fashion at least in one direction (horizontal or vertical). To understand the effect of increasing the

number of points on a sphere (while maintaining the pattern of the scan), a simulation was performed. Two virtual spheres were generated with a radius of 50 mm placed at a distance (d_c) of 6.5 m from the origin and a random Gaussian noise (whose one standard deviation was $0.2 \text{ mm} + d_c \times 40 \times 10^{-6}$) in the ranging direction. Sphere#1 was constructed with ≈ 1300 points and the number of points on Sphere#2 is varied. Sphere#2 was constructed 100 times and the distance between the two spheres is calculated and averaged at each point density. Figure 12 shows that the distance between both the spheres decrease with increasing number of points, very similar to Figure 11.

Based on these plots, it can be concluded that higher number of points minimize the error in the determination of the center, but this benefit diminishes after a certain number of points.

6.4 Choice of cone angle and/or coverage

Coverage, in this paper, is defined as the fraction of the surface area of the sphere that can be scanned using a TLS. Some of the factors that affect the coverage are the distance of the target to the TLS, target size, the return intensity of the signal and the angle of incidence of the laser beam with the target surface. When a TLS scans a sphere, the return signal from the outer periphery of the target is relatively weak, owing to the fact that both the incidence angle and the beam size are large. This low intensity may cause the return signal from these regions to be filtered out, resulting in data loss. Additionally, the return signal from the center region of the sphere surface visible to the TLS results in a signal that has high intensity and may saturate the TLS's sensors. This also may result in data of poor quality and higher RMS noise.

In the context of understanding TLS related error sources, Rachakonda et al. [10] observed that, as a TLS scans a flat plate target, the RMS value of the noise on the target increases with increasing incidence angle, as shown in Figure 13 (a similar effect was observed by Cheok et al. [11] for the ranging error). This may be attributed to the fact that, at larger incidence angles, the return signal intensity from the target is lower than at a 0° incidence angle. Also, the projected shape of the beam on the target is near-elliptical (covering a larger area on the target), compared to a near-circular shape (covering a lower area on the target) when the angle of incidence is 0° . For a sphere the angle of incidence varies considerably over its surface during a scan. This will result in the return signal averaged over a smaller near-circular region closest to the TLS and averaged over a larger near-elliptical region at its outer periphery.

In the case of spheres, when the coverage of the scan data on a sphere is symmetric, the sphere can be represented in 2D as a circle and the segmenting conical region as a cone with the apex at the center of the circle (as illustrated in Figure 5). In all the algorithms based on the cone angle selection of data on the sphere, the cone angle was chosen to be 120° , which was based on empirical observations of results from various TLSs. The selection of 120° cone angle was essentially a compromise between a) the high incidence angle at the outer periphery of the sphere resulting in increased noise of the measurements in these regions and b) sufficient coverage of the sphere surface for a sphere fit.

6.4.1 Cone angle vs angle of incidence—For sphere segmentation, the cone angle (θ) chosen for data segmentation and the angle of incidence (ϕ) shown in Figure 14 are related. Consider a point P on the sphere that is being scanned by the TLS (at position O). The angle of incidence of the laser beam on the sphere is given by ϕ . The angle subtended at the center of the sphere (C) with respect to the vector OC by the point P is half of the cone angle (θ). In all cases, $\phi > \theta/2$. When the distance (d) increases, the angle of incidence asymptotically approaches half the cone angle ($\phi \approx \theta/2$). This relationship is shown in Figure 15 for a cone angle $\theta = 120^\circ$.

Theoretically, the angle of incidence is a good measure of the noise observed on the sphere and could be used to perform data segmentation. However, in practice, it is simpler to use the cone angle to perform the segmentation of data. Figure 16 shows the plots of RMS values of the residuals from a sphere fit (orthogonal unconstrained fit) at various cone angles for sphere data obtained from one particular TLS placed at a distance ≈ 3.2 m from the spheres (66 sphere datasets). This plot clearly shows that the RMS value increases markedly at cone angles beyond $\approx 125^\circ$.

6.5 Form error of the sphere

The form error of a sphere affects the determination of the sphere center. The calculated center of a sphere with large form error varies based on the region of the sphere that was scanned by the TLS as shown by Muralikrishnan et al. [12].

To understand the sensitivity of the sphere center with respect to the form error of a sphere, a simulation was performed by constructing a virtual sphere of nominal radius 50 mm located at a distance of 6.5 m away from a TLS. The virtual sphere was distorted in such a way that the exaggerated shape of the distortion represents a Meissner[§] body. Data points were selected from this virtual sphere that fall within a cone angle of 120° originating at the center of the sphere. This process was repeated 50 times while randomly selecting ≈ 700 points on the surface from various regions on the sphere. This selected data is then fit to a sphere using orthogonal constrained fit and orthogonal unconstrained fit to calculate the sphere centers. The RMS values of 50 sphere center movements were recorded and are shown in Figure 17 (blue and red dots). These values were then fit to a line to obtain the sensitivity of the form error on sphere center (solid lines). This plot shows that the center movement increases as a function (nearly linear) of the form error of the sphere. It also shows that the center moves approximately 0.08 times the value of the sphere's form error for constrained fit and 0.64 times for the unconstrained fit. It may be noted that the constrained fit has lower sensitivity for spheres with various form error values.

To understand the effect of the size of the sphere on the center movement, the above simulation was performed with virtual spheres of multiple sizes and it was found that the size of the sphere has no bearing on the movement of the sphere with a fixed value of form error.

[§]Meissner bodies are artifacts of constant width that are a resultant of the fabrication process of a sphere.

6.6 Constrained vs unconstrained fitting algorithm (orthogonal error function)

As discussed earlier in this paper, the determination of the sphere center can be performed using either an unconstrained or a constrained fitting algorithm. An unconstrained fitting algorithm calculates the sphere center and the radius simultaneously, while a constrained fitting algorithm calculates the sphere center by constraining the radius to a known value. The centers calculated using these two algorithms can differ for datasets that exhibit “squishing” or “flaring” (described in section 3.5). Neither of the algorithms result in the true geometric center, however Shilling et al. [13] observed that a constrained fit results in a center that has lower variability than that of an unconstrained fit.

A simulation was performed to understand the variability between the constrained and unconstrained sphere fitting algorithms. All the 270 datasets that were obtained by scanning the grid of spheres were segmented using the cone-cylinder method (with a radius of 50 mm, cone angle of 120° and iterated 5 times). Virtual spheres were created by selecting 300 random points on each of the segmented spheres while maintaining the distribution and coverage of the points on the sphere. A sphere fit was then performed using the orthogonal constrained fit and orthogonal unconstrained fit. The process was repeated 50 times and the standard deviation of the sphere center in the spherical coordinate system (the ranging, horizontal and vertical directions) were recorded.

The simulation shows that the standard deviation of the sphere center in the ranging direction using a constrained fit is always lower than that using the unconstrained fit. However, the simulations indicated that the same was not true for the horizontal and vertical directions. It was also observed in this simulation that the magnitude of the standard deviation in the ranging direction (σ_R) is about 3 times higher than either the horizontal (σ_H) or vertical direction (σ_V) for the constrained fit (i.e., $\sigma_R/\sigma_H \approx 3$ and $\sigma_R/\sigma_V \approx 3$). Even though the constrained fit is not necessarily better in all the 3 directions, since σ_H and σ_V are lower than σ_R , a constrained fit results in lower overall variation in the locating the center of the sphere.

6.7 Calibrated radius of the sphere used for the sphere fit

To estimate the effects of the error in the radius of a sphere on the determination of its center, three spheres of different radii were scanned using a TLS. The datasets were segmented using the cone-cylinder method, 120° cone angle and their corresponding nominal radii. An orthogonal constrained fit was performed on each of these spheres with a range of values for their nominal radii(r). The resulting sphere center locations were recorded in spherical coordinates. It was observed that the sphere centers moved in a linear fashion with changing radius values that were used for the sphere fit. This linear relationship with the variation in the radius(r) is given in Table 5. It may be observed that the effect of an error in the calibrated radius is more dominant in the ranging direction. For example, an error of 1 μm in the radius of the sphere (expected of typical contact probe CMM calibration of sphere radius) will result in a $\approx 1.24 \mu\text{m}$ error along the ranging direction for all the three spheres, which is relatively insignificant for TLS measurements. It should be noted that sensitivity values in Table 5 may vary with TLS, point density and sphere radii.

7 DISCUSSION OF RESULTS AND CONCLUSION

The calculation of sphere center is affected by a variety of TLS error sources, target related error sources and post-processing methods. These factors introduce uncertainty in determining the sphere center. To evaluate TLSs for their performance or to minimize the propagation errors during scan registration, it is important that errors due to the sphere target and the post-processing methods be minimized. A high quality sphere that has near-lambertian surface and low form error minimizes the error sources due to the target/sphere, but reducing the point cloud data to a derived point, i.e., sphere center, is still a challenge.

In this paper, we explored a variety of post processing methods which were then implemented on data obtained from three different TLSs. These proposed methods were also tested on thousands of datasets from various TLSs to ensure their broad applicability. These tests (not presented in this paper) also involved the measurement of the center distances between two spheres using a reference instrument such as a laser tracker to obtain the ground truth information.

The determination of the sphere center is sensitive to some parameters more than others and this paper describes those parameters. In summary, below are the influence factors and a brief description on their effect on the sphere center determination.

1. **Segmentation Routine:** The four methods (sections 3.2 through 3.5) described in this paper work reasonably well to discard the spurious points, but the cone-cylinder and the double cone angle methods appear to be slightly more effective than the rest.
2. **Sphere fitting algorithm (error function):** The choice of the algorithms (orthogonal and directional) do not affect the variability of the sphere center. However, orthogonal sphere fitting functions are widely studied and are built into many commercial software and therefore generally preferred.
3. **Constrained vs unconstrained sphere fitting:** Constrained sphere fitting algorithm results in lower variability in determining sphere center compared to an unconstrained algorithm.
4. **Calibrated radius of the sphere:** An error in the calibrated radius of the sphere (used for sphere fitting) affects the sphere center in the ranging direction more than the non-ranging directions. In the ranging direction, the error in the center movement is about 1.24 times the error in the calibrated radius for the cases studied.
5. **Number of iterations for segmentation:** In general, five or more iterations work well to obtain the sphere center when using the cone-cylinder or the double cone angle method. However, the improvement in the sphere center should be noted to make decisions on increasing the number of iterations.
6. **Initial center used for segmentation:** For the datasets studied, the initial center could be as far away as 20 % of the nominal radius of the sphere from the final center without affecting the final value of the sphere center when using the cone-

cylinder or the double cone angle method (with five iterations) proposed in this paper.

7. **Cone angle:** Usage of a cone angle based method for segmentation will discard the data with higher noise at the outer periphery. Inclusion of sphere data over 120° may result in higher variability in the sphere center. The choice of the cone angle should be made after observing the noise on the sphere data and the actual coverage of the data over the sphere (in cases where the coverage is less than 120°).
8. **Number of points on the sphere:** In general, spheres sampled with larger number of points have lower variability in their center determination. However, as shown in Figure 11, the benefit of lowering the center location error due to larger sampling diminishes as the number of points increase.
9. **Form error of the sphere:** The form error of the sphere results in a center that has a linear relationship with the location of the center (Figure 17). This relationship could be used to specify the form error on the target to be used with TLS systems.

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Figure 2.
Typical scan data of a sphere from a TLS

(a) Higher point density and coverage of data on a sphere at 3 m from TLS

(b) Lower point density and coverage of data on a sphere at 22 m from TLS

Figure 3.

2D view of scan data of a sphere from a TLS at multiple distances

Figure 4.
Illustration of the closest point method (in 2D) to obtain an initial guess of the sphere center

Figure 5.
Illustration of the cone angle method in 2D

Figure 6.
Illustration of the double cone angle method in 2D

Figure 7.
Illustration of the Cone-cylinder method in 2D

Figure 8.

"Squishing" and "Flaring" effect and the locations of the centers with respect to the true geometric center.

Figure 9.
Illustration of a grid of spheres that were scanned using three TLSSs

Figure 10.
Commercial spheres used on the grid

Figure 11.
3D distance between spheres as a function of number of points on the spheres

Figure 12.

Distance between virtual spheres that have points spaced in a grid like fashion

Figure 13.
Orthogonal noise in the scan data of a flat plate at various incidence angles [10]

Figure 14.
Angle of incidence(ϕ) vs cone angle (θ)

Figure 15.
Relationship between incidence angle on a sphere and TLS-sphere distance

Figure 16.
Noise on multiple spheres at various cone angles

Figure 17.
Effect of the type of fit and the form error on the center location of spheres.

Table 1

Relevant specifications of the instruments used in this test

TLS	Maximum range	Ranging uncertainty (1 sigma level)
TLS#1	120 m	0.95 mm at 25 m
TLS#2	79 m	2.00 mm at 25 m
TLS#3	30 m	0.50 mm at 5 m

Table 2

Comparison of various segmentation methods (orthogonal error function) – values in mm

Segmentation method	$\Sigma(\text{RMS})$
Cone Angle Method	17.527
Double Cone Angle Method	16.856
Cone Cylinder Method	16.855
Radius Correction Method	17.677

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Table 3

Number of spheres whose centers moved with each iteration.

Iteration#	$e_C^i > 0 \mu\text{m}$	$e_C^i > 10 \mu\text{m}$
1	270	186
2	137	39
3	25	0
4	3	0
5	2	0

Table 4

Comparison of various segmentation methods (directional error function) – values in mm

Segmentation method	$\Sigma(\text{RMS})$
Cone Angle Method	17.547
Double Cone Angle Method	16.862
Cone Cylinder Method	16.855
Radius Correction Method	17.633

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Table 5

Sensitivity of the center location with respect to the error in calibrated radius δr of the sphere

Center movement direction	$r = 50 \text{ mm}$	$r = 75 \text{ mm}$	$r = 100 \text{ mm}$
Radial	$1.2413 \times \delta r$	$1.2416 \times \delta r$	$1.2430 \times \delta r$
Azimuth	$0.0001 \times \delta r$	$0.0004 \times \delta r$	$0.0003 \times \delta r$
Elevation	$0.0002 \times \delta r$	$0.0002 \times \delta r$	$0.0001 \times \delta r$