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Traceable Calibration and Demonstration of a Portable Dynamic Force Transfer Standard

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Abstract

In general, the dynamic sensitivity of a force transducer depends upon the mechanical system in which it is used. This dependence serves as motivation to develop a dynamic force transfer standard, which can be used to calibrate an application transducer *in situ*. In this work, we SI-traceably calibrate a hand-held force transducer, namely an impact hammer, by using a mass suspended from a thin line which is cut to produce a known dynamic force in the form of a step function. We show that this instrument is a promising candidate as a transfer standard, since its dynamic response has small variance between different users. This calibrated transfer standard is then used to calibrate a secondary force transducer in an example application setting. The combined standard uncertainty ($k = 2$) in the calibration of the transfer standard was determined to be 2.1 % or less, up to a bandwidth of 5 kHz. The combined standard uncertainty ($k = 2$) in the performed transfer calibration was less than 4 %, up to 3 kHz. An advantage of the transfer calibration framework presented here, is that the transfer standard can be used to transfer SI-traceable calibrations without the use of any SI-traceable voltage metrology instrumentation.

1. Introduction

The dynamic and static responses of a force transducer typically differ [1, 2], preventing the use of established static force calibration procedures [3, 4] in applications that require the measurement of fast, dynamic forces. In order to make dynamic measurements with a statically calibrated force transducer, a common approach is to restrict the use of the force transducer to the measurement of forces that vary sufficiently slowly, such that the deviation between the static and dynamic responses is expected to be within determined acceptable limits. However, many applications use force transducers in the range in which the deviation of the transducer's response from its static sensitivity may be large, for example in Charpy testing [5, 6] and in measuring cutting forces in machining [7]. Thus, there is a need to traceably calibrate the dynamic response.

In addition to the deviation between the static and dynamic response, another significant challenge in dynamic calibration of these devices is that the dynamic response is generally dependent upon the mechanical application in which it is used. For instance, in general, the dynamic response depends upon the structure to which the transducer is coupled, the properties of the couplings between the transducer and this structure, and the location at

which the force is desired to be known. Thus, there is a need to calibrate the response of the force transducer in its application setting, also referred to as an *in situ* calibration. This need for *in situ* calibration suggests that the transducer be calibrated in its application setting by use of a *transfer device* (or transfer standard) [8].

There are three main objectives in this paper. The first is to identify and present a force transducer that may be a suitable transfer standard. A handheld device known as an *impact hammer* [9] is a promising transfer standard candidate, as we will show that its dynamic response has only small variations from user to user. The second objective is to dynamically calibrate this transfer force transducer in an SI-traceable manner [8]. The third objective is to demonstrate the transfer of the calibration to another application transducer, which can then be used to make dynamic force measurements. Previously reported efforts on the dynamic calibration of impact hammers include those by Fujii [10] and Waltham and Kotlicki [11].

An impact hammer can be used to perform *in situ* calibrations provided that (1) the impact hammer is SI-traceably calibrated, and (2) the response of the impact hammer does not change between different uses and users (i.e., the dynamic response is invariant). Three commercially available impact hammers are shown in Figure 1(a) while an illustration of the basic anatomy of an impact hammer is shown in part (b). As shown in Figure 1(b), the main components of an impact hammer consist of a force transducer supported by a handle that the user holds, a head mass that assists in generating the force, often a detachable tip that can be changed to alter the generated impact waveform, and sometimes additional accelerometer(s) to compensate for inertial forces in the hammer. The impact hammers shown in Figure 1(a) have head masses of a few grams to a few hundreds of grams, but there are larger impact hammers available that have head masses of a few kilograms.

It is important to point out that the bandwidth to which an impact hammer is calibrated is not the same as the transfer bandwidth when calibrating an application transducer using the impact hammer. Rather, the maximum transfer bandwidth is a product of the impact hammer calibrated bandwidth and the bandwidth of the applied force pulse. For example, although a particular impact hammer may have a calibrated and relatively constant spectral response up to 10 kHz, one may only be capable of generating a force pulse that is short enough to contain significant frequency content up to 5 kHz.

The remainder of this manuscript is as follows. In the next section, we present the theoretical background used in the calibration of the impact hammer and the transfer calibration process. Following, the calibration arrangement used to traceably calibrate the impact hammer is presented. Section 4 goes into the details of the uncertainty analysis of both the impact hammer calibration and the transfer calibration. In Section 5, calibration data for the impact hammer are presented. Following, in Section 6, a transfer calibration is performed wherein the calibrated impact hammer is used to calibrate a force transducer in a setting that mimics one in which a force transducer may be used to make force measurements in an industrial application. Last, concluding remarks are given in Section 7. A study on how well the calibration apparatus replicates the dynamics while holding the hammer by hand is presented in Appendix A, while supplementary information regarding the calibration bandwidth is provided in Appendix B.

2. Theoretical Background and Transfer Calibration Process

2.1. Calibration of Transfer Standard

The traceable calibration of an impact hammer is expressed by the following convolution

$$v_{ts}(t) = \int_0^t a_{ts}(\tau)f(t-\tau)d\tau, \quad (1)$$

where a_{ts} is the impulse response function of the transfer standard which we wish to determine, v_{ts} is the measured output voltage, f is a traceably known force, and τ is the integration variable. Determining a_{ts} from v_{ts} and f , where the quantities are related through equation (1), is referred to as the *deconvolution problem* [12]. Provided that a_{ts} is linear and time-invariant, the convolution given by equation (1) can be transformed into a multiplication in the Laplace domain with complex frequency s . In this work, lower case letters represent time domain quantities, while upper case letters denote their Laplace domain equivalents. For example, force is transformed as $F(s) = \mathcal{L}\{f(t)\}$, where $\mathcal{L}$ is the Laplace operator. The Laplace representation of equation (1) takes the form

$$V_{ts}(s) = A_{ts}(s)F(s). \quad (2)$$

The quantity $A_{ts}(s)$, the Laplace transform of the time domain impulse response $a_{ts}(t)$, is called the transfer function or frequency domain response function of the transfer standard. For the calibrations presented here, we apply a force by suspending a known mass in a known gravitation field by a thin line, whereupon the line is cut and a dynamic step force is applied to the transducer [13]. The applied step force takes the form

$$f(t) = \begin{cases} -f_o & t < 0 \\ 0 & t \geq 0 \end{cases} \Leftrightarrow F(s) = \frac{f_o}{s}. \quad (3)$$

We note that the impact hammer uses a piezoelectric crystal as the sensing element, which does not have a DC response. Thus when the mass is suspended, the output voltage exponentially decays towards 0 V, whereupon the line is cut and the step load is applied. The negative sign in equation (3) is a matter of convention as the impact hammer is calibrated in compression. From equation (3), it follows that the transfer function may be obtained as

$$A_{ts}(s) = \frac{1}{f_o}sV_{ts}(s). \quad (4)$$

It is important to point out that the form of the step in equation (3) assumes an infinite slope, and consequently an infinite bandwidth, of the applied force profile. In practice, the step force has finite slope and an unknown shape, which will limit the calibration bandwidth of the transducer and create an additional-frequency dependent uncertainty component. For the

applied step forces in this arrangement, $f_o = mg$ where m is the mass of the deadweight and g is the local acceleration due to gravity. Under the assumption of a linear, time-invariant system, the product $sV_{ts}(s)$ can be considered one quantity, namely the Laplace transform of the time-derivative of the voltage signal $d(v_{ts})/dt$. The time domain impulse response function a_{ts} is then determined by using the inverse Laplace transformation of equation (4), which yields

$$a_{ts}(t) = \frac{1}{f_o} \frac{dv_{ts}}{dt}. \quad (5)$$

We derive the impulse response from numerical, time domain differentiation of the step response.

2.2. Transfer Calibration

The calibration of an application force transducer using a transfer transducer may be expressed as

$$v_t(t) = \int_0^t a_t(\tau) f(t - \tau) d\tau, \quad (6)$$

where v_t is the measured time-dependent voltage response of the application transducer in its application setting, f is the force applied to the application transducer and the transfer standard, and a_t is the impulse response of the application transducer that we wish to determine. As before, we transform equation (6) to the frequency domain as

$$V_t(s) = A_t(s)F(s). \quad (7)$$

The applied force f (or F) does not need to be explicitly known in order to determine A_t , and thus perform the transfer calibration. During the transfer calibration, the applied force must be common to the application transducer and the transfer standard, and equation (2) can be used in conjunction with equation (7) to determine the transfer function of the application transducer A_t as,

$$A_t(s) = \frac{V_t(s)}{F(s)} = A_{ts}(s) \left(\frac{V_t(s)}{V_{ts}(s)} \right) = A_{ts}(s) V_r(s), \quad (8)$$

where $V_r(s) \equiv V_t(s)/V_{ts}(s)$ (subscript “r” for “ratio”) for notational convenience.

2.3. Summary of the Transfer Calibration Process

The steps yielding a traceably-calibrated application transducer using the impact hammer as a transfer standard are summarized as follows:

1. **Calibration of the transfer standard:** Apply a known, traceable dynamic force $F(s)$ to the impact hammer and measure its response $V_{ts}(s)$ to determine $A_{ts}(s)$.
2. **Application of a common force:** Use the impact hammer to apply a common force to the application transducer and the impact hammer, measure the responses of the application transducer and the impact hammer, and construct the voltage ratio $V_t(s)/V_{ts}(s)$.
3. **Calibration of the application transducer:** Determine the impulse response function $A_t(s)$, given the transfer standard impulse response $A_{ts}(s)$ and the measured voltage ratio $V_t(s)/V_{ts}(s)$, using equation (8).

3. Calibration Arrangement for the Transfer Standard

3.1. Calibration Configuration of the Impact Hammer

The calibration arrangement used to apply the step load to the impact hammer is shown in Figure 2. An impact hammer will be a suitable transfer standard only if its impulse response has only small variations between different uses and users. To minimize the variation in the response from user to user, it is important that the impact hammer be much stiffer than the hand holding it [14]. From a beam modelling perspective, the hand holding the impact hammer can be considered a weak spring (or soft spring) supported boundary condition [15]. Although impact hammers are designed and constructed to have an invariant response between different users, small deviations may still exist and will be considered in the uncertainty analysis. The calibration apparatus must likewise secure the impact hammer in a way that produces a soft boundary condition, so that the condition in which the impact hammer is calibrated and the condition in which it is used are sufficiently similar such that the impulse response does not change. Soft boundary conditions also render the impact hammer response insensitive to changes in the effective clamping length. A study on how well the clamping mechanism mimics the hand-held configuration is reported in Appendix A.

3.2. Calibration Procedure for the Impact Hammer

The calibration procedure is as follows. As shown in Figure 2(a), the impact hammer is secured into a clamping mechanism that realizes the soft boundary conditions using a combination of wire-coil springs, damping material, and pneumatic isolation. A weight is suspended from the impact hammer using a thin nylon line. When the weight is suspended, the impact hammer will tilt due to the soft clamping boundary conditions. Fine-pitch angle adjustment screws are used to adjust the clamping mechanism and impact hammer so that the nylon line, and consequently the applied force vector, are aligned with the head of the impact hammer. The nylon line is cut with a pair of sharp snips that have been fitted with an electrical switch that triggers a digital voltmeter (Agilent 3458A) ¹ to record 1 second of the voltage output signal from the impact hammer at a sampling rate of 50 kHz. When the nylon line is cut, the weight is detached which applies a step load to the impact hammer. The

¹Certain commercial equipment, instruments, or materials are identified in this article in order to describe the experimental procedure adequately. Such identification is not intended to imply recommendation or endorsement by the National Institute of Standards and Technology, nor is it intended to imply that the materials or equipment identified are necessarily the best available for the purpose.

voltage output from the impact hammer is amplified before it is recorded. For these measurements we consider the impact hammer and amplifier to be a single composite instrument that is being calibrated; thus, when later performing the transfer calibration of the working force transducer, the same amplifier must be used.

The trigger switch of the snips starts the recording of voltage sampled by the digital voltmeter (DVM) before the wire is cut, and within the 1 second of data collection, the response of the impact hammer is recorded. There is no synchronization between the precise time when the applied force step occurs and the recording of the impact hammer response (i.e. the time between the trigger signal and the force step is not a precisely known or repeatable quantity) and this will result in an uncertainty in the phase of the response, which will be addressed in subsection 4.1.6. The entire response of the impact hammer occurs within a 10 ms time window, so the other 990 ms of data are neglected. When analyzing the data in the frequency domain through Fourier transformation, the 10 ms time window equates to a 100 Hz frequency resolution.

3.3. Nature of the Cut and Calibration Bandwidth

In a separate set of measurements, the amount of time to close the snips over the diameter of the nylon line was found to be a few hundred milliseconds; however, the rise time of the applied force step is much shorter (see Appendix B). From these results, we infer that during the cut, the line is increasingly cut until the effective cross sectional area is reduced to a level at which the stress in the line exceeds the tensile strength of the material and rapid fracture occurs, creating the force step.

As part of the calibration procedure, and to determine the calibration bandwidth, we perform multiple averages with different diameter nylon lines (0.22 mm to 0.74 mm) and different weight mass values (1.73 kg and 4.46 kg). These different combinations will yield different initial tensile stresses in the line and can result in different fracture dynamics, wave speeds and dispersion. These will result in variation of the exact force step profile applied to the impact hammer. When analyzing the calibration data, we determine the bandwidth up to which this variation does not affect the response. For the impact hammer calibrations reported here, the usable bandwidth was determined to be 5 kHz.

4. Uncertainty Analysis

We perform the uncertainty analysis using the quantities represented in the frequency domain. The uncertainty in the response function amplitude of a calibrated application transducer is derived by the propagation of uncertainty of equation (8), which yields

$$u_{A_t}^2 = \left(\frac{\partial A_t}{\partial A_{ts}} \right)^2 u_{A_{ts}}^2 + \left(\frac{\partial A_t}{\partial V_r} \right)^2 u_{V_r}^2, \quad (9)$$

where u_{A_t} is the absolute uncertainty in the application transducer response function amplitude, $u_{A_{ts}}$ is the absolute amplitude uncertainty in the calibration of the transfer standard, and u_{V_r} is the absolute amplitude uncertainty in the voltage ratio during the

transfer. Within this work, u_y denotes the absolute uncertainty of the quantity y , while $\tilde{u}_y$ is the relative uncertainty of the quantity y .

4.1. Uncertainty in Transfer Standard Calibration

A summary of the uncertainties of the transfer standard calibration amplitude and phase are presented in Tables 1 and 2, respectively. The amplitude uncertainty in calibration of the transfer standard is partially derived from equation (4)

$$u_{A_{ts}}^2 = \left(\frac{1}{f_o}\right)^2 u_{V_{ts}}^2 + \left(\frac{V_{ts}}{f_o^2}\right)^2 u_{f_o}^2 + u_{A_{ts},rep}^2 + u_{A_{ts},user}^2 + u_{A_{ts},config}^2, \quad (10)$$

where $u_{V_{ts}}$ is the uncertainty in the voltage measurement and u_{f_o} is the uncertainty in the applied force. Of the last three terms, $u_{A_{ts},rep}$ accounts for non-repeatability [16] or type A variation [17] among multiple calibrations, $u_{A_{ts},user}$ is the absolute uncertainty due to user-to-user variation of the impact hammer response, and $u_{A_{ts},config}$ accounts for any differences in the determined response function of the impact hammer between its mounting in the calibration mechanism and the way it is supported in the transfer calibration.

4.1.1. Uncertainty in Calibration Voltage—The uncertainty in the voltage amplitude in equation (10) is written

$$\tilde{u}_{V_{ts}}^2 = \tilde{u}_{V_{ts},align}^2 + \tilde{u}_{V_{ts},inst}^2. \quad (11)$$

The quantity $\tilde{u}_{V_{ts},align}$ is the relative uncertainty originating from off-axis forces applied to the transducer since the wire suspending the deadweight does not coincide perfectly with the center axis of the impact hammer, and $\tilde{u}_{V_{ts},inst}$ is the relative uncertainty in the voltage measurement. An off-axis sensitivity was determined by calibrating the impact hammer with the deadweight, cut-wire method while varying the impact hammer angle at a known angle up to ± 10 deg. The mean off-axis sensitivity over 5 kHz of bandwidth was thereby determined to be $S_{oa} \approx 3.5 \times 10^{-5} \text{ V}/(\text{N} \cdot \text{deg})$. By geometrical measurements before cutting the wire, we are able to consistently center the wire with respect to the cylindrical end of the impact hammer to ± 1.5 mm, corresponding to an angle of 1.26 deg about each of the two orthogonal axes. Assuming a worst case scenario in which the wire is off both axes by the maximum 1.5 mm, the maximum angle of eccentricity of the applied force is $\theta_{max} = \sqrt{2}(1.26) \approx 1.78$ deg. The relative angular uncertainty of the output voltage is thus the frequency-dependent quantity $\tilde{u}_{V_{ts},align} = S_{oa} \theta_{max} / (V_{ts}(s)/F(s))$. For our calibrations the relative uncertainty is approximately $\tilde{u}_{V_{ts},align} \approx 3.1 \times 10^{-3}$ ($k=1$) over the entire calibration bandwidth.

The uncertainty due to the voltage measurement instrumentation $\tilde{u}_{V_{ts},inst}$ includes uncertainties of its internal reference (calibration uncertainty and drift), base DC uncertainty relative to its internal reference, and frequency-dependent gain. Uncertainties in the noise

floor of the instrument (as well as noise on the impact hammer output signal and any additional noise pick up) are accounted for in the repeatability of the multiple measurements. For the measurements presented here $\tilde{u}_{V_{ts},inst} = 1.8 \times 10^{-4}$ ($k=1$).

4.1.2. Uncertainty in Calibration Force—The uncertainty in the force in equation (10) is given to be

$$\tilde{u}_{f_o}^2 = \tilde{u}_{f_o,g}^2 + \tilde{u}_{f_o,mass}^2 + \tilde{u}_{f_o,align}^2 + \tilde{u}_{f_o,profile}^2. \quad (12)$$

In equation (12), $\tilde{u}_{f_o,g}$ is the relative uncertainty of the gravitational acceleration, $\tilde{u}_{f_o,mass}$ is the relative uncertainty of the initially suspended and subsequently detached mass, $\tilde{u}_{f_o,align}$ is the relative uncertainty due to misalignment of the applied force vector with respect to the axis of the transducer, and $\tilde{u}_{f_o,profile}$ is the relative uncertainty due to finite-bandwidth effects of the applied step force profile. The acceleration due to gravity has been well characterized in the location where the calibration was performed [16, 18] and we take $g = 9.801018 \text{ m/s}^2$ with a relative uncertainty of $\tilde{u}_{f_o,g} = 1 \times 10^{-6}$ ($k=1$).

The quantity $\tilde{u}_{f_o,mass}$ represents the relative uncertainty associated with the mass which is suspended from the transducer and has three components. Specifically, they are the uncertainty of the determination of the mass of the suspended deadweight, an uncertainty due to the buoyancy correction, and an uncertainty in the mass of the portion of the wire that suspends the deadweight and that remains with the weight after the cut occurs. The uncertainty of the deadweight mass and the buoyancy correction are negligible compared with the nylon line mass uncertainty. In order to determine the uncertainty of the wire mass, a total of 91 cut portions of wire were weighed together and found to have a mass of 37.98 g, giving an average mass value of 0.4174 g. This average line mass is corrected for and the uncertainty is taken to be half the value of the correction. In the worst case, the uncertainty will be largest for the 1.73 kg mass, and the relative uncertainty for the mass is $\tilde{u}_{f_o,mass} = 1.2 \times 10^{-4}$ ($k=1$).

The relative uncertainty due to misalignment takes the form

$$\tilde{u}_{f_o,align} = (1 - \cos \theta_{max}), \quad (13)$$

where θ_{max} is the maximum angle of the applied mass given in the previous section. Using the same geometry considerations as with the voltage uncertainty due to misalignment, the relative uncertainty is $\tilde{u}_{f_o,align} = 4.8 \times 10^{-4}$ ($k=1$).

The uncertainty in the force profile $\tilde{u}_{f_o,profile}$, which originates from a finite slope of the step function, is frequency-dependent and is as large as 4.7×10^{-3} ($k=1$) at 5 kHz. This uncertainty corresponds to the attenuation of the Fourier transform of a half-sine pulse of a duration of 10 μs at 5 kHz. Measurements used to determine the 10 μs pulse width and the calculation of the uncertainty are presented in Appendix B.

4.1.3. Non-Repeatability—The non-repeatability $u_{A_{ts,rep}}$ in equation (10) is determined as the standard deviation of the mean of a set of repeated calibration measurements, using different diameter nylon lines and deadweight masses. This term accounts for different sources of variability in the calibration including: any motions of the mass and string (which can be considered non-zero initial conditions in equations (1) and (2)) as well as any variability in cutting the wire. This also accounts for any electrical noise that may be present in the measurement. The relative uncertainty due to non-repeatability $\tilde{u}_{A_{ts,rep}}$ is frequency-dependent and ranged from 2.4×10^{-3} to 7.2×10^{-3} ($k = 1$) over a 5 kHz bandwidth.

4.1.4. Uncertainty due to User-to-User Variability—The impact hammer is held and swung by a user in performing a transfer calibration. The support (i.e. boundary condition) provided to the impact hammer can vary from user to user (and from use-to-use by a given individual), causing a variation in the impact hammer transfer function. This potential change in transfer function gives rise to an additional uncertainty component that we denote $\tilde{u}_{A_{ts,user}}$. A measurement was performed wherein three different users struck a reference transducer between 600 and 650 times each, and the response from the impact hammer and reference transducer were simultaneously recorded. When striking the reference transducer² with the impact hammer, each strike is not identical in magnitude due to inconsistencies in the swing speed that the user generates each time. On impacts with too low swing speed, the force amplitudes generated were not large and the signal-to-noise ratio was low. On impacts with too high swing speed, the force amplitude was large and the impact hammer would over-range and be clipped. Therefore, the 550 files (each file representing one impact) were parsed by selecting only the files for which: (1) the voltage from the impact hammer did not suffer from any clipping (5 V saturation limit), (2) the impact was large enough to generate a sufficient signal-to-noise ratio (a peak voltage greater than 1.75 V), and (3) no “double-taps” occurred. These criteria yielded 200 to 300 acceptable impacts, depending on the user.

The experimental setup used for the user-to-user variability test is similar to the one shown in Figure 6, but with a piezo-electric transducer (PCB Model 208C02) with an impact cap used in place of the transducer and additional mass shown in the figure. Although each user impacts the reference transducer with a different force amplitude, the voltage ratio between the impact hammer and the reference transducer should ideally be the same. However, since each user holds the impact hammer and strikes the reference transducer differently, the mean voltage ratio from each user may have frequency-dependent variations, as well as a potential amplitude scaling difference that originates from cosine errors. We wish to evaluate only the frequency-dependent variations and not the amplitude scaling differences, which are accounted for elsewhere (see Section 4.2.3). Therefore, in order to isolate the frequency-dependent user-to-user variability, we scale the data in the following manner. The averaged voltage ratio response in the frequency domain (up to 5 kHz) of all the impacts for the i^{th} user is denoted $(V_{ts}/V_{ref})_i$. We scale the mean of each user by a single constant, denoted a_i which is intended to correct for the cosine error, so that we can evaluate only the frequency-

²Within this work *reference transducer* refers to a transducer that is used to characterize the (a) user-to-user variability and (b) force step width, while *application transducer* is the working transducer which is finally calibrated using the impact hammer (transfer calibration).

dependent differences between each user. The value of the a_i for each user is determined through the following optimization problem

$$\underset{a_i}{\text{minimize}} \sum_{i=1}^{n_{\text{users}}-1} \left\{ \left(\frac{V_{\text{ts}}}{V_{\text{ref}}} \right)_0 - a_i \left(\frac{V_{\text{ts}}}{V_{\text{ref}}} \right)_i \right\}. \quad (14)$$

In equation (14), the quantity $(V_{\text{ts}}/V_{\text{ref}})_0$ is a vector of averaged voltage ratio at a given frequency from an arbitrarily selected user and the a_i are the coefficients being determined, which account for the cosine error from different average impact angles from different users. For reference, the values of a_i ranged over $\{0.995, 0.988\}$ which correspond to impact angles (see equation (18) in Section 4.2.3) ranging over $\{5.8^\circ, 8.7^\circ\}$.

The data, scaled to correct for angular differences, are presented in Figure 3 (a) while the standard deviation is shown in (b). The standard deviation of the scaled data is taken as the uncertainty associated with the user-to-user variability, which is denoted u_{user} for amplitude and $u_{\text{user,phase}}$ for phase. Note that in Section 4.1.3, we used the standard deviation of the mean since the different wire and mass combinations should yield the same impact hammer transfer function (i.e., the transfer function is linear and invariant), but here the standard deviation is used since the transfer function is likely different from user to user. These uncertainties are frequency dependent (see Figure 3 (b)), and dependent upon the response of the reference transducer used in making this comparison amongst users. The average relative standard deviation of the amplitude up to 5 kHz ranged from 4.7×10^{-4} to 3.5×10^{-3} . There was no measurable difference in the standard deviation of the phase up to the same frequency. It may be expected that the support provided by a human hand does not affect the response at high frequencies. We expect that this uncertainty term would be reduced by taking data with additional users.

4.1.5. Uncertainty due to Difference between Calibration and Use

Configurations—Similar to the impact hammer transfer function being susceptible to change due to the variability between users (Section 4.1.4), the impact hammer transfer function may change from the calibration mechanism described in Section 3 to the hand-held configuration. This potential change in transfer function gives rise to an additional uncertainty component that we denote $\tilde{u}_{A_{\text{ts,config}}}$. A study was performed to measure how well the calibration apparatus, shown in Figure 2, replicates the conditions in which the impact hammer will be used to make a transfer calibration, namely being held by hand. These results are summarized in Appendix A in Figure 10. Within the calibration bandwidth of 5 kHz, we found overall agreement between the handheld and calibration configurations, although there is some question at the lowest frequencies (up to 300 Hz, see Appendix A). Relatively large angular uncertainty limited the comparison, and further studies are warranted in order to better compare the impact hammer behavior in the two configurations. At this time, we do not include an uncertainty contribution from possible differences in transfer function between the two configurations.

4.1.6. Phase Uncertainty in Transfer Standard Calibration—The phase of an idealized impulse occurring at $t = 0$ is zero at all frequencies. The phase of the impact hammer response is therefore determined by taking the phase of the Fourier Transform of the derived impulse response function, and subtracting it from zero over all frequencies. The precise time that the force is applied to the cap of the impact hammer during calibration is not known, and will result in an uncertainty in the phase response. The time that the force is applied to the cap is approximately determined by rescaling the individual step responses by determining the point where there is a 20 % change in amplitude and setting the cut time ($t = 0$) at 4 samples earlier (corresponding to 80 μ s). This selected time-shift ensured that the system remains *causal*, which requires that the impact hammer output cannot lead the applied force (i.e. that the response phase be less than or equal to 0 at all frequencies). The scaled data are shown in Figure 4, in which the step response is shown in part (a) and the derived impulse response is provided in part (b). The step responses presented in Figure 4 were generated with a 4.45 kg mass and a 0.74 mm nylon line. The time scaling is confirmed to be repeatable since the location of maximum amplitude in the impulse response occurs at the same instance of time for the 12 data collections shown in Figure 4. The phase uncertainty of the calibration is taken to be

$$u_{A_{ts}, \text{phase}}^2 = u_{\text{phase,rep}}^2 + u_{\text{phase,res}}^2 + u_{\text{phase,user}}^2 + u_{\text{phase,config}}^2. \quad (15)$$

In equation (15), $u_{\text{phase,rep}}$ is the uncertainty due to the repeatability of the phase response, given by one standard deviation of the mean, and $u_{\text{phase,res}}$ is the uncertainty representing the timing of the cut due to the sampling resolution. For $u_{\text{phase,res}}$, we take an uncertainty of half the sample time $\pm t/2$, where the sample time t is one over the sample frequency ν_s , written as $t = 1/\nu_s$. This corresponds to a frequency-dependent phase uncertainty of

$$u_{\text{phase,res}} = (2\pi\nu) \frac{\Delta t}{2} = \pi \frac{\nu}{\nu_s}, \quad (16)$$

where ν and ν_s are in units of Hz. Although $u_{\text{phase,res}}$ becomes rather large at high frequencies, this part of the phase uncertainty (linear slope) only causes an uncertainty in the timing of the force pulse (e.g. in a transfer calibration), and not a distortion of the force pulse. The absolute uncertainty $u_{\text{phase,user}}$ is the phase difference from user to user, while $u_{\text{phase,config}}$ accounts for any phase differences between the calibration and the transfer configuration. Both of these uncertainties were found to be negligible.

In this section, we have presented the uncertainty of the absolute phase; however, often only the deviations of the phase response from a linear trend are of interest, since the deviations will affect the shape of the time domain response while the linear trend corresponds only to an overall time delay. Although not shown, after subtracting out the linear trend from each phase, the standard deviation of the mean of the residuals encompasses zero except at low frequencies (below 1000 Hz). Thus we do not observe a significant phase distortion between 1 kHz and 5 kHz.

4.2. Uncertainty in the Transfer Calibration Voltage Ratio Amplitude

During the transfer calibration, a common force is applied to the application transducer by the transfer standard and their output responses are recorded. The uncertainty on the ratio of the voltage amplitudes is written:

$$\tilde{u}_{V_r}^2 = \tilde{u}_{V_r, \text{ch2ch}}^2 + \tilde{u}_{V_r, \text{rep}}^2 + \tilde{u}_{V_r, \text{ang}}^2. \quad (17)$$

The quantity $\tilde{u}_{\text{ch2ch}}$ is the relative uncertainty in the channel-to-channel gain ratio of the instrument, $\tilde{u}_{V_r, \text{rep}}$ accounts for the type A uncertainty in multiple transfer calibrations, and $\tilde{u}_{V_r, \text{ang}}$ accounts for any uncertainty in the voltage measurement due to angular impacts of the impact hammer. As shown in equation (9), the total uncertainty of the transfer calibration combines the uncertainty contributions described in this section and the uncertainty of the transfer standard calibration, described in Section 4.1. A summary of the amplitude uncertainties in the transfer calibration are presented in Table 3, while the phase uncertainty is presented in Table 4.

4.2.1. Uncertainty in Channel-to-Channel Gain—For the example transfer calibration presented in Section 6, one instrument was used, namely an oscilloscope (LeCroy HRO 66Zi). The channel-to-channel gain was measured, and is applied as a correction to the voltage ratio. This relative correction (difference from 1) is measured as ranging up to 4.0×10^{-3} with a relative uncertainty $\tilde{u}_{V_r, \text{ch2ch}}$ of 1.6×10^{-3} , which equates to a relative uncertainty in the voltage ratio that ranges up to 6.4×10^{-6} ($k = 1$).

4.2.2. Uncertainty in Voltage Ratio due to Non-Repeatability—In equation (17), $\tilde{u}_{V_r, \text{rep}}$ is the standard deviation of the mean of the repeated measurements. This term accounts for all sources of noise, as well other sources of random variation such as impact-to-impact differences due to transducer cross-axis and moment sensitivity. For the measurements in this work (see Figure 8), $\tilde{u}_{V_r, \text{rep}}$ was determined from 370 measurements and ranged from 7.2×10^{-4} to 1.0×10^{-1} ($k = 1$) up to a bandwidth of 5 kHz, and from 7.2×10^{-4} to 5.1×10^{-3} ($k = 1$) up to a bandwidth of 3 kHz.

4.2.3. Uncertainty in Voltage Ratio due to Angular Variations—Because force is a vector quantity, the force applied during transfer calibration will be subject to cosine errors. This results in an error in the measured voltage ratio, for which it is assumed that the force applied along the impact hammer axis is the same as the force applied along the application transducer axis. Provided that the responses involved are linear, these cosine errors will affect the amplitude of the frequency response function by a constant factor. This relative error is

$$\tilde{u}_{V_r, \text{ang}} = 1 - \cos(\alpha), \quad (18)$$

where α is the impact angle. An impact angle of 5° will result in a relative error of less than 0.4 % ($k = 1$). For the transfer calibration presented within this work, a conservative estimate

of an impact angle of 10° is assumed which leads to an error of 1.5 % ($k = 1$). We note that for application transducers with large cross- and moment-sensitivities, there can be non-negligible contributions from these sensitivities to the uncertainty due to the mean misalignment of applied impulses; this did not apply to the example transfer calibration reported here (Section 6).

4.2.4. Uncertainty in Transfer Calibration Phase—The transfer calibration compares the signal (voltage) at the output of the conditioning amplifier connected the application transducer, to the signal at the output of the conditioning amplifier connected to the impact hammer. As noted previously, we treat both amplifiers as integral to the respective transducers. The uncertainty in the phase of the transfer calibration is given to be

$$u_{A_t, \text{phase}}^2 = u_{V_t, \text{phase}}^2 + u_{A_{ts}, \text{phase}}^2 \quad (19)$$

The voltage ratio uncertainty $u_{V_t, \text{phase}}$ is taken as the standard deviation of the mean of the averaged voltage ratio phase responses, which leads to an absolute uncertainty of 7.4×10^{-4} rad to 6.6×10^{-3} rad ($k = 1$). The phase uncertainty in the transfer standard calibration $u_{A_{ts}, \text{phase}}$ was determined in Section 4.1.6, and ranges from 6.4×10^{-3} rad to 1.9×10^{-1} rad ($k = 1$) up to 3 kHz. The phase responses of the two anti-aliasing filters used respectively for the impact hammer and application transducer output signals were corrected for, with only a negligible contribution to the overall phase uncertainty from the uncertainty of these corrections. As noted previously, the phase uncertainty is predominantly linear in frequency, corresponding to a time-delay uncertainty for a measured force waveform.

5. Example Calibration of the Transfer Standard

In this section we present data for the calibration of an impact hammer with the apparatus described above. The time histories for twelve different cut-wire calibrations are shown in Figure 4(a), and the derived impulse response, obtained by numerical differentiation, is shown in Figure 4(b). From the data observed in Figure 4(a), the voltage response closely approximates a step, although there are some low frequency oscillations. We note that the impulse response has been multiplied by the sample time to give the same units as the step response.

The averaged Fourier amplitudes of the data presented in Figure 4(b) are provided in Figure 5(a) along with averages of three other sets of data obtained with different combinations of wire diameter and suspended mass. The phase responses of these data are also shown in Figure 5(b). In Figure 5, the dashed, yellow lines represent the uncertainty of the mean of the averaged sensitivity amplitudes and phases. The uncertainty of the mean is taken at each frequency to be the mean root-sum-of-squares (RSS) of the uncertainties of each of the four data sets, RSS combined with the non-repeatability (standard deviation of the mean) among the means of each of the four data sets. The uncertainty of each of the four data sets includes all components discussed in Section 4.1. As shown in Figure 5 (a), the amplitude sensitivities agree up to 5 kHz, then start to deviate from one another. Assuming that the

response of the impact hammer is linear and time invariant, we infer that the deviations are likely a result of variation of the input force profile resulting from the different diameters of the wires and the different initial tensile forces in the line. Therefore, we limit the calibration frequency range to 5 kHz.

6. Demonstration of the Transfer Calibration

The SI-traceably calibrated impact hammer, which was presented in Section 5, was then used to calibrate a force transducer in a simulated application setting. A photograph of the transfer calibration arrangement is shown in Figure 6 (a), and a block diagram of the arrangement is given in part (b). Here, a 1300 N capacity shear-web-design force transducer is secured to a granite block using a 1/4"-28 thread screw. A 12 mm thick stainless steel plate is attached to the sensing end of the force transducer. This configuration is meant to mimic an impact testing application, in which a force transducer is mounted to a large reaction wall or base and may have additional structure(s) attached to the impacted end. These additional structures and the reaction wall itself will affect the dynamic response of the transducer.

The force transducer output is amplified using a strain gauge bridge output amplifier. For the purposes of this demonstration, we consider the coupled transducer-amplifier system to be a single unit that is being calibrated (i.e., they need to be calibrated and used together). Similarly, the amplifier used on the impact hammer in its calibration is the same amplifier used here in the transfer calibration.

The impact hammer was used to strike the force transducer a total of 550 times, and the output of the impact hammer and force transducer were simultaneously recorded using an oscilloscope at a sample frequency of 200 kHz. Both the impact hammer and the application transducer outputs were passed through anti-aliasing filters, causing some attenuation within the calibration bandwidth which was then corrected for in post-processing by using data that characterized the frequency response of the filters. Of the 550 data sets, 370 were determined to be valid using the same criteria that were outlined in Section 4.1.4.

The average force spectrum applied during the impact is plotted in Figure 7, where the applied force was calculated from equation (2) using the output voltage from the impact hammer and its calibrated sensitivity $A_{ts}(s)$. We note that this applied, common force does not need to be determined in order to perform the transfer calibration; however, it is useful to generate a figure such as Figure 7, as it shows the bandwidth of the applied force, and the transfer calibration bandwidth limit imposed by the applied force pulse. For this demonstration, the input force amplitude is attenuated to about 21 % of its peak value at 5 kHz.

The voltage ratio of the two instruments V_t/V_{ts} is provided in Figure 8. The solid lines show the mean of the 370 impacts, while the thin lines represent 2σ (twice the standard deviation at each frequency). Using the voltage ratio, the transducer calibration is determined through equation (8). An interesting aspect of this transfer calibration is that the transfer is done through a ratio of voltages, so that if one instrument is making both voltage measurements, it

does not need to be SI-traceably calibrated; however, it must be linear and any channel-to-channel gain difference needs to be taken into account. The sensitivity of the application transducer determined by the transfer calibration is given in Figure 9, where the sensitivity amplitude is given in part (a), the relative amplitude uncertainty is shown in part (b), and the sensitivity phase is given in part (c). Although data are shown out to 5 kHz, the uncertainties increase at frequencies above 3.2 kHz due to the small mechanical response and output of the application transducer, resulting in a low signal-to-noise ratio which increases the uncertainty and limits the useable bandwidth.

7. Summary

In this paper we present an SI-traceable dynamic calibration of a force transducer (an impact hammer) by means of an applied step force generated by a cut wire. This device is used as a transfer standard to calibrate an application (working) force transducer in its application setting. Such a standard is applicable to dynamic calibration of force measurement systems where it is possible to use the impact hammer to apply a force at the location of interest.

The uncertainties in the calibration of the transfer standard are frequency-dependent, and frequency response function amplitude uncertainties vary from 1.1 % to 2.1 % ($k = 2$) over the range of 0 Hz to 5 kHz. The uncertainty of the transfer of the calibration to an application transducer depends on the properties of the transducer in its application setting. In the example transfer calibration, the uncertainties increase to between 3.1 % and 3.4 % ($k = 2$) for bandwidths up to 3 kHz. Due to the limited bandwidth of the output signal of the application transducer used, the uncertainty increases up to 22 % ($k = 2$) at 5 kHz. The observed invariance of the impact hammer response from user to user makes it a suitable transfer standard for calibrating force transducers in their application settings. The largest source of uncertainty in the transfer calibration is due to uncertainty in the angle of the strike between the impact hammer and the secondary transducer. The human factors involved in use of a hand-held impact hammer warrant further studies with a larger sample of users.

In the example presented in this paper, the transfer of the calibration was performed without the need for any SI-traceable voltage measurement instrumentation.

Appendix A

Comparison of Clamping Mechanism and Hand-Held Configurations

We investigated how closely the impact hammer transfer function, when calibrated in the clamping mechanism (see Figure 2), represents the same transfer function as when it is used in performing a transfer calibration, namely when it is held by hand. Data were collected with three different users holding the impact hammer and suspending a 1.73 kg weight with the 0.74 mm diameter line and applying the force step by cutting the wire. A total of six cuts were performed for User 1 and User 2, while nine cuts were performed for User 3. The obtained transfer function amplitudes are plotted in Figure 10, along with the transfer function amplitude obtained in the calibration mechanism. The uncertainty evaluation of these measurements follows the same procedure as presented in Section 4.1, with an increased uncertainty contribution due to misalignment. For the hand-held measurements

$\theta_{\max} = 10^\circ$, which results in the uncertainty ranging from 8 % to 10 % ($k = 1$) in the 0 Hz to 5 kHz bandwidth.

The data suggests a discrepancy at the lowest frequencies (below 300 Hz) between the hand-held response and the response in the calibration mechanism, likely due to the bending mode of the impact hammer handle. The angular uncertainty can only result in overall (frequency-independent) shifts up or down of the mean of the hand held spectra with respect to the spectrum obtained in the calibration apparatus. It is not possible for such a shift to allow the uncertainty of the in-apparatus spectrum to overlap with the (unplotted) residual frequency-dependent uncertainty of the mean handheld spectrum at 0 Hz, 100 Hz, and 300 Hz. Due to the limited data set and the limitation on the comparison over most of the frequency range due to the large angular variation, we do not introduce a correction or uncertainty contribution from the difference between the two configurations. If further studies confirm an observable disagreement between the two methods of securing the impact hammer, then such a correction and uncertainty contribution will need to be included.

Figure 10.

A comparison between calibration data gathered holding the impact hammer by hand for three different users and the calibration data clamped in the calibration mechanism. The dashed lines represent the ($k = 2$) expanded uncertainties of the measurand.

Appendix B

Estimate of the Pulse Duration and Corresponding Uncertainty

Measurements were performed to characterize the time duration of the width of the applied force step incident on the impact hammer through the line, in order to determine a usable calibration bandwidth of the impact hammer, and to determine an uncertainty in the amplitude profile denoted $u_{f_o, \text{profile}}$ presented in Section 4.1.2. In these measurements, a mass was suspended from a line and secured to another force transducer (PCB Model 208C02) with a higher natural frequency (greater than 75 kHz in the absence of mass-loading effects) than the impact hammer. The line was cut and the response of the force transducer was recorded at a sampling rate of 1 MHz. This measurement was performed for the same combination of wire diameters and masses used to calibrate the impact hammer. The voltage output from the transducer is plotted in Figure 11 (a), while the derivative (impulse response) is presented in Figure 11(b). In all four cases, the period of the response of the transducer was approximately 20 μs , corresponding to a half-sine wave of 10 μs . We note that the observed pulse-width is approximately equal to half the period of the lowest transducer resonance (50 kHz in this case with an attached adapter of mass of approximately 1 g) and may be limited by the transducer resonance, so that 10 μs represents an upper bound on the pulse-width. The uncertainty due to this finite width effect is determined by constructing a half-sine pulse with a 10 μs duration, taking the Fourier Transform, and determining its deviation from unity as a function of frequency. For a 10 μs half-sine wave, the resulting ($k = 1$) uncertainty is 4.7×10^{-3} ($k = 1$) at 5 kHz.

Figure 11.

(a) Step response and (b) impulse response of a force transducer used to characterize the pulse-width of the applied force through the line. The traces have been aligned in time by setting the $t=0$ value of all step responses to the first time 50 mV is reached.

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Figure 1.

(a) Three different commercially available impact hammers and (b) anatomy of general impact hammer used to apply a known force to a structure.

Figure 2.

(a) Photograph of apparatus used for calibrating the impact hammer using a suspended deadweight, and (b) a block diagram of the measurement system.

Figure 3. Results of different users striking a reference transducer. (a) Scaled amplitude ratio for three different users and (b) standard deviation of the scaled data for all users. The dashed, yellow lines represent twice the standard deviation.

Figure 4. Data collected during the impact hammer calibration measurements. Time histories of (a) the step response and the (b) estimate of the impulse response obtained by numerical differentiation.

Figure 5.

Impact hammer calibration measurements: (a) Amplitude response and (b) phase response of the impact hammer for a combination of three different wire diameters and two different mass values. The dashed yellow lines represent the $(k=2)$ expanded uncertainty of the measurand.

(a)

(b)

Figure 6.

(a) Photograph of the transducer arrangement and (b) block diagram of the measurement system during the transfer calibration process.

Figure 7.
Averaged amplitude of the applied force during the transfer.

Figure 8.

Averaged (a) voltage ratio amplitude and (b) voltage ratio phase during the transfer. The dashed black lines represent the $(k=2)$ expanded uncertainty of the measurement.

Figure 9. Averaged (a) amplitude, (b) relative ($k = 2$) expanded uncertainty in the amplitude, and (c) phase calibration data of the transducer in the application setting. The dashed black lines represent the ($k = 2$) expanded uncertainty of the measurement.

Table 1

Summary of relative amplitude uncertainties associated with the impact hammer calibration.

Origin of Uncertainty	Rel. Unc.	Unc. Type	Distr. Type	D.O.F.
Uncertainties in Voltage				
misalignments, $\tilde{u}_{V_{ts},align}$	3.1×10^{-3}	B	Uniform	∞
instrumentation, $\tilde{u}_{V_{ts},inst}$	1.8×10^{-4}	A	Gaussian	6–8
Combined Uncertainty	3.1×10^{-3}			
Expanded Uncertainty ($k = 2$)	6.2×10^{-3}			
Uncertainties in Force				
force profile [*] , $\tilde{u}_{f_{\phi},profile}$	$0 - 4.7 \times 10^{-3}$	B	Uniform	∞
misalignments, $\tilde{u}_{f_{\phi},align}$	4.8×10^{-4}	B	Uniform	∞
mass, $\tilde{u}_{f_{\phi},mass}$	1.2×10^{-4}	A	Gaussian	90
gravity, $\tilde{u}_{f_{\phi},g}$ (See Ref. [18])	1.0×10^{-6}	A	Gaussian	35
Combined Uncertainty [*]	$5.0 \times 10^{-4} - 4.7 \times 10^{-3}$			
Expanded Uncertainty [*] ($k = 2$)	$1.0 \times 10^{-3} - 9.5 \times 10^{-3}$			
Uncertainties due to Nonrepeatability				
amplitude nonrepeatability [*] , $\tilde{u}_{A_{ts},rep}$ (5 kHz bandwidth)	$2.4 \times 10^{-3} - 7.2 \times 10^{-3}$	A	Gaussian	47 (at 100 freqs.)
Expanded Uncertainty [*] ($k = 2$)	$4.9 \times 10^{-3} - 1.4 \times 10^{-2}$			
Uncertainties due to User-to-User Variability				
amplitude variability [*] , $\tilde{u}_{user}$	$4.7 \times 10^{-4} - 3.5 \times 10^{-3}$	A	Gaussian	2 (3 means at 4 freqs.)
Expanded Uncertainty [*] ($k = 2$)	$9.4 \times 10^{-4} - 7.0 \times 10^{-3}$			
Total Amplitude Uncertainty [†]				
Combined Uncertainty [*] ($k = 1$)	$5.5 \times 10^{-3} - 1.1 \times 10^{-2}$			
Expanded Uncertainty [*] ($k = 2$)	$1.1 \times 10^{-2} - 2.1 \times 10^{-2}$			

The abbreviation D.O.F. is the assumed or known number of degrees of freedom.

Quantities denoted with a star (*) are frequency dependent, and values are listed for frequencies up to 5 kHz.

[†]This is the total uncertainty in the calibration of the impact hammer demonstrated in Section 5, wherein the total uncertainty is calculated from equation (10) at each frequency.

Table 2

Summary of phase uncertainties associated with the impact hammer calibration.

Origin of Uncertainty	Abs. Unc. [rad]	Unc. Type	Distr. Type	D.O.F.
resolution, $u_{\text{phase, res}}$	$6.3 \times 10^{-3} - 3.1 \times 10^{-1}$	B	Uniform	∞
repeatability, $u_{\text{phase, rep}}$	$2.1 \times 10^{-3} - 6.5 \times 10^{-2}$	A	Gaussian	47 (at 100 freqs.)
Combined Uncertainty ($k = 1$)	$6.4 \times 10^{-3} - 3.2 \times 10^{-1}$			
Expanded Uncertainty ($k = 2$)	$1.3 \times 10^{-2} - 6.4 \times 10^{-1}$			

The abbreviation D.O.F. is the assumed or known number of degrees of freedom. All quantities are frequency dependent and are listed for frequencies up to 5 kHz.

Table 3

Uncertainties in amplitude of the transfer calibration of the application transducer, over a 3 kHz bandwidth.

Origin of Uncertainty	Rel. Unc.	Unc. Type	Distr. Type	D.O.F.
Uncertainties in Voltage				
angle of common force, $\tilde{u}_{V_r, \text{ang}} (10^\circ)$	1.5×10^{-2}	B	Uniform	∞
repeatability*, $\tilde{u}_{V_r, \text{rep}}$	$7.2 \times 10^{-4} - 5.1 \times 10^{-3}$	A	Gaussian	368 (at 1250 freqs.)
channel-to-channel, $\tilde{u}_{V_r, \text{ch2ch}}$	6.4×10^{-6}	B	Uniform	∞
Combined Uncertainty	$1.5^{-2} - 1.6^{-2}$			
Expanded Uncertainty ($k = 2$)	$3.0^{-2} - 3.2^{-2}$			
Uncertainty in Impact Hammer Calibration				
impact hammer*, $\tilde{u}_{A_{ts}}$	$5.5 \times 10^{-3} - 8.1 \times 10^{-3}$	A,B	Gaussian, Uniform	47 (at 100 freqs.), ∞
Expanded Uncertainty* ($k = 2$)	$1.1 \times 10^{-2} - 1.6 \times 10^{-2}$			
Total Amplitude Uncertainty[†]				
Combined Uncertainty* ($k = 1$)	$1.6 \times 10^{-2} - 1.7 \times 10^{-2}$			
Expanded Uncertainty* ($k = 2$)	$3.1 \times 10^{-2} - 3.4 \times 10^{-2}$			

The abbreviation D.O.F. is the assumed or known number of degrees of freedom.

Quantities denoted with a star (*) are frequency dependent.

[†]The total uncertainty is calculated from inserting the sub components into equation (9) and evaluating at each frequency.

Table 4
Uncertainties in phase of the transfer calibration over a 3 kHz bandwidth.

Origin of Uncertainty	Abs. Unc. [rad]	Unc. Type	Distr. Type	D.O.F.
transfer standard, $u_{A_{ts,phase}}$	$6.4 \times 10^{-3} - 1.9 \times 10^{-1}$	A,B	Gaussian, Uniform	47 (at 100 freqs.), ∞
voltage ratio, $u_{V_r,phase}$	$7.4 \times 10^{-4} - 6.6 \times 10^{-3}$	A	Gaussian	368 (at 1250 freqs.)
Combined Uncertainty ($k = 1$)	$4.1 \times 10^{-3} - 1.9 \times 10^{-1}$			
Expanded Uncertainty ($k = 2$)	$8.1 \times 10^{-3} - 3.8 \times 10^{-1}$			

The abbreviation D.O.F. is the assumed or known number of degrees of freedom. All quantities are frequency dependent.

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