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Traceable Dynamic Calibration of Force Transducers by Primary Means

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Abstract

We describe an apparatus for traceable, dynamic calibration of force transducers using harmonic excitation, and report calibration measurements of force transducers using this apparatus. In this system, the force applied to the transducer is produced by the acceleration of an attached mass, and is determined according to Newton's second law, $F = ma$. The acceleration is measured by primary means, using laser interferometry. The capabilities of this system are demonstrated by performing dynamic calibrations of two shear-web-type force transducers up to a frequency of 2 kHz, with an expanded uncertainty below 1.2 %. We give an accounting of all significant sources of uncertainty, including a detailed consideration of the effects of dynamic tilting (rocking), which is a leading source of uncertainty in such harmonic force calibration systems.

Keywords

Force calibration; Force measurement; Dynamic Calibration

1. Introduction

A need for dynamic calibration of force transducers has existed for many years [1–8]. This need has gained increased prominence as engineering demands have grown more stringent [9], and a number of national metrology institutes have undertaken the development of dynamic force calibration systems [7, 10–14]. Applications that may benefit from dynamic force calibration include aerodynamic force measurements, fatigue testing, measurement of forces during machining processes, automotive crashworthiness and robotic manufacturing, among others. We describe here a system for primary, SI-traceable sinusoidal force calibration that has been implemented at the National Institute of Standards and Technology (NIST), U.S.A. The system is largely similar to two systems recently reported in Refs. [11,15], although some details are different. The system is capable of performing primary calibrations in the frequency range of 10 Hz – 2 kHz [‡] with force amplitudes up to 2 kN.

[‡]So far we have demonstrated its operation in the frequency range 100 Hz to 2 kHz, as reported in Section 4.

The calibration system applies a sinusoidal acceleration to a mass, which is attached to the force transducer. The applied force is determined from Newton's 2nd law

$$F(t) = \frac{d}{dt}(mv(t)) = ma(t) \text{ for constant mass } m, \text{ where } F \text{ is the applied force, } v \text{ and } a \text{ are}$$

respectively the mass velocity and acceleration, and t is time. The acceleration is determined by an interferometric measurement of displacement amplitude at the excitation frequency. A small correction is made to account for the non-uniform distribution of acceleration throughout the mass. The sinusoidal excitation is provided by an electrodynamic shaker to which the transducer-mass assembly is attached. While under excitation, the voltage output V of the transducer is measured, and the frequency-dependent complex sensitivity is determined as the ratio between the voltage and force, $S(\omega) = |S(\omega)|e^{i\phi(\omega)} = V(\omega)/F(\omega)$, where ω is the angular frequency in units of radians per second. For strain-gauge-based transducers, the sensitivity is given as a ratio to an excitation voltage V_{ex} , $S(\omega) = V(\omega)/(V_{\text{ex}}F(\omega))$. The capabilities of this system are shown by calibrating two commercial shear-web design strain-gauge-based force transducers.

We evaluate all suspected contributors of significant uncertainty to the calibration, including a detailed consideration of the uncertainty arising from misalignment and rocking. This uncertainty analysis may be applied to other dynamic force measurement systems, wherein the displacement (or velocity) of the calibration mass is measured using laser vibrometry. The remainder of the paper is organized as follows: in Section 2 we describe the design of the calibration system; in Section 3 we discuss contributions to the uncertainty of calibration measurements and present the results of evaluations of these contributions; in Section 4 we present sample calibration measurements for two force transducers; in Section 5 we give a short recapitulation and outlook. In the Appendix we present an analysis of the uncertainty due to rocking. Some measurement results used in uncertainty evaluation are not included with the text but provided as supporting material online.

2. System Description

A detailed schematic of the experimental arrangement is shown in Fig. 1, while a photograph of one portion of the experimental arrangement is provided in Fig. 2. The system described here is the current realization of the system earlier presented in Ref. [16]. The transducer under test is secured to a steel mounting plate by a bolt matching the central mounting threads on the transducer base, and this mounting plate is bolted to the table of an electrodynamic shaker using 12 size M8 bolts. A calibration mass is attached to the sensing or "live" end (for transducers that have such a distinction) of the transducer. The calibration mass has a threaded hole in the center of its base and is attached to the transducer either by directly screwing the mass onto a threaded mounting stud that is part of the transducer, or by using a thread adapter. Some calibration masses are hexagonal to facilitate tightening them with a standard size socket and torque wrench. The calibration masses have epoxy-attached mirrors, such as all the masses used for the measurements reported in Section 4, or polished top surfaces, to reflect a laser beam for interferometric displacement measurement (described below). A sinusoidal signal is provided to an amplifier by a signal generator, which in turn drives the shaker at a prescribed frequency and amplitude.

The electrodynamic shaker can produce a maximum force of 2 kN over the range of 10 Hz to 4 kHz. As the shaker table oscillates, the transducer-mass assembly accelerates, applying a force to the transducer. This applied force is then indicated by the transducer either as a digital signal (i.e. integrally incorporating analog-to-digital conversion), or as an analog output such as a voltage, charge, or current. For strain-gauge-based transducers, the output voltage is due to an imbalance in a strain-gauge bridge circuit, the output voltage being proportional to the strain-induced change in resistance of one or more strain gauges and to the bridge excitation voltage. The sensitivity of strain-gauge-bridge transducers is typically expressed as a ratio of the bridge voltage to the excitation voltage (units of V/V or more commonly mV/V). These two voltages are measured separately in the calibration system presented here, but certain voltmeters are capable of performing a ratiometric measurement, in which the voltmeter measures the ratio of the two signals. We measure the output voltage from the transducer using an 8.5-digit § fast-sampling DC multimeter (Agilent 3458A, hereafter referred to as DVM1), and the bridge excitation voltage with a 6.5-digit multimeter (Agilent 34411A, hereafter referred to as DVM2) ||. Transducers that produce a digital output do not require an output signal measurement (but merely a recording of the output indication) for their calibration.

The displacement of the calibration mass as a function of time is measured by means of laser vibrometry, and the acceleration is calculated from the displacement. This calculation is performed in the frequency domain, that is, $A_p = Y_p(2\pi f)^2$, where A_p and Y_p are the acceleration and displacement amplitudes respectively, and f is the excitation frequency. The quantities A_p and Y_p are determined as the magnitudes of the complex Fourier amplitudes of the force and the transducer response at the driving frequency. The sensitivity magnitude is determined as the ratio of the response and force magnitudes, while the sensitivity phase is determined as the phase difference between the two complex amplitudes. As described below, different instruments are used for measuring the sensitivity amplitude and phase. As shown in the appendix, Fourier filtering helps to reduce the uncertainty due to tilting of the calibration mass. A schematic of the interferometer used for the vibrometry measurements is provided in Figure 3. The interferometer is a custom-assembled heterodyne interferometer, in which the heterodyne generation is performed using two acousto-optic modulators (AOMs) that modulate the laser at frequencies $f_1 = 70$ MHz and $f_2 = 70.25$ MHz. Using two modulators allows generation of a carrier signal within the 1 MHz bandwidth of our detection and demodulation electronics, using the available AOMs. Since these AOMs work in a double-pass configuration, the generated carrier beat frequency is $2(f_2 - f_1) = 500$ kHz. The interferometer can be used in two configurations, measuring either the absolute displacement of the calibration mass or the relative displacement between the mass and the shaker table. In the absolute displacement configuration, the reference beam of the interferometer reflects from a stationary mirror. In the relative displacement configuration, the reference beam reflects from the shaker. The entire optical measurement system is

§ Voltmeter “digits” are characteristic of DC measurements using relatively slow sampling rates and long integration apertures. The actual accuracy for AC amplitudes measured by fast DC sampling, the mode in which we use the voltmeter, is much lower, as will be presented in Section 3.

|| Certain commercial equipment, instruments, or materials are identified in this article in order to describe the experimental procedure adequately. Such identification is not intended to imply recommendation or endorsement by the National Institute of Standards and Technology, nor is it intended to imply that the materials or equipment identified are necessarily the best available for the purpose.

mounted on a table with both active and passive isolation components to minimize mechanical vibrations of the optical platform. The heterodyne demodulation and phase-detection for this interferometer is done with a lock-in amplifier (Signal Recovery 7280). The resolution of the interferometer is less than 1 nanometer for frequencies greater than 100 Hz and a collection time of 1 second (see supporting material online). An accelerometer is mounted directly below the transducer on the mounting plate and measures the acceleration of the transducer base as shown in Figure 1. In primary calibration measurements such as those reported in Section 4, the accelerometer is used only in determining the uncertainty contribution due to vibrations of the interferometer platform, which will be discussed in Section 3.2.3. The lock-in quadrature phase output voltages, transducer output voltage and the accelerometer output voltage are simultaneously sampled and recorded using a 12-bit digitizing oscilloscope (LeCroy Waverunner 66Zi). The oscilloscope-measured transducer output voltage is used in determining the phase of the sensitivity, whereas the voltmeter measurements are used in determining the amplitude of the sensitivity.

The voltmeter DVM1 can make measurements at up to 100 kS/s, although for the measurements reported here we used a sampling rate of 10 kS/s. The interferometer photodetector output signals are sampled at 7.5 MS/s by the lock-in amplifier. The oscilloscope can sample at up to 1 GS/s; we use it at 1 MS/s for the measurements reported here. An anti-aliasing filter (Stanford Research Systems SIM965) is used to prevent aliasing errors in the sampling of the transducer output voltage by the DVM. In order to accurately determine the transducer voltage, the transfer function of the filter must be measured. The distortion of the measured voltage amplitude and phase can then be corrected. The filter is discussed further in Section 3.4. The lock-in amplifier also implements an anti-aliasing filter (elliptic type) at its input with a 2 MHz bandwidth, and implements a low pass filter (set at approximately 100 kHz for the measurements reported in this paper) on its outputs. The voltmeter measurements are corrected for aperture error.

Data are collected from the calibration system at discrete frequencies, where at each frequency, the shaker amplitude is adjusted to keep the acceleration of the calibration mass a constant (to within approximately $\pm 5\%$ of the nominal value). Keeping the mass acceleration amplitude constant prevents attenuation of the mass acceleration (and thus the applied force) at frequencies above the calibration stack resonance frequency; this attenuation has the effect of increasing the uncertainty at high frequencies if the shaker table acceleration is kept constant.

2.1. Traceability

Traceability of the applied force is achieved through traceability of the measured calibration mass and acceleration amplitude. The mass artifacts used in the dynamic calibrations have been calibrated at NIST against the U.S. national mass standard, which in turn has been calibrated against the International Prototype Kilogram. Additional couplings used for some calibrations have been weighed using an electronic balance which is calibrated with test masses that are traceable (through intermediate comparisons) to the international prototype kilogram. The acceleration of the mass is comprised of a displacement measurement and a

time interval measurement. The displacement is measured interferometrically relative to our laser wavelength (in air), which has been calibrated against an iodine-stabilized He-Ne laser and is thereby traceable to the SI definition of the meter [17]. The laser calibration consisted of a traceable measurement of its beat frequency against the reference laser. The timing of sample acquisition is controlled by the instruments described above, the clock signals of which are compared to a master clock, which in turn has been compared to the Global Positioning System (GPS) one pulse-per-second signal and is thereby traceable to the SI definition of the second. The transducer output and excitation voltage measurements are traceable via the calibration chain of the DVMs, which are linked to Josephson voltage standards via one or more intermediate DVMs, and thereby to the Josephson junction representation of the Volt.

3. Uncertainty Analysis

Each suspected source of possible significant uncertainty is evaluated by measurement or estimated by calculation. The results are summarized in Tables 1 – 3. In detail, the uncertainty depends on the particular operating point (frequency, force, calibration mass) and the properties of the transducer under calibration. The two largest uncertainty contributions come from the DVM measurement of the voltage output of the transducer and from tilting (rocking) of the shaker-transducer-mass assembly.

The measurement equation for the sensitivity at one frequency can be expanded in terms of measurands as

$$S = \frac{V}{FV_{\text{ex}}} \quad (1)$$

The uncertainty of the sensitivity is determined as

$$u_S^2 = u_V^2 \left(\frac{\partial S}{\partial V} \right)^2 + u_{V_{\text{ex}}}^2 \left(\frac{\partial S}{\partial V_{\text{ex}}} \right)^2 + u_F^2 \left(\frac{\partial S}{\partial F} \right)^2 + u_{\text{rep}}^2, \quad (2)$$

where u_y denotes the standard uncertainty associated with measurand y . The additional term u_{rep} accounts for lack of repeatability in the measurement, when the calibration is performed multiple times. In this paper relative uncertainties are distinguished by over-tildes, i.e. $\tilde{u}_y \equiv u_y/y$. The uncertainty contribution due to the excitation voltage is considerably smaller than the other terms in equation (2) and has been neglected. Given that the forcing is harmonic, the force amplitude acting on the transducer is calculated from

$$F = m_p Y_p (2\pi f)^2, \quad (3)$$

which has an uncertainty of

$$u_F^2 = u_{m_p}^2 \left(\frac{\partial F}{\partial m_p} \right) + u_{Y_p}^2 \left(\frac{\partial F}{\partial Y_p} \right)^2 + u_f^2 \left(\frac{\partial F}{\partial f} \right)^2 + u_{F_d}^2. \quad (4)$$

Here m_p is the total mass of the calibration mass and any additional threaded adapters that are used to mount the mass to the transducer. The quantity Y_p is the amplitude of the mass displacement, which is determined by taking the amplitude of the Fourier transform in the frequency bin corresponding to frequency f . The voltage amplitude V is determined in the same way. The term u_{F_d} is the uncertainty due to air drag force acting on the calibration mass. We note that covariance terms corresponding to dependence of the drag force on m_p , Y_p , and f are not included in equation (2). We do not evaluate these in the uncertainty analysis, as the overall drag force contribution is small relative to other terms. The frequency uncertainty u_f is negligible in comparison to the other uncertainty components and is also not explicitly evaluated. Measurements and estimates of the uncertainties in equation (2) and equation (4) are provided in the following subsections.

3.1. Calibration Mass Uncertainty: u_{m_p}

Each calibration mass is calibrated with NIST mass standards that are traceable to the U.S. mass standard and the International Prototype Kilogram. When securing the calibration mass to the transducer, a small amount of wax or semisolid lubricant is used to ensure good coupling at the transducer-mass and transducer-mounting plate interfaces. This added mass varies with each reattachment, but was estimated to be 5 mg to 20 mg by weighing several small amounts. This additional mass is the largest contributing uncertainty, which gives a total relative uncertainty of the calibration mass $\tilde{u}_{m_p} = 1.5 \times 10^{-5}$.

3.2. Calibration Mass Displacement Uncertainty: u_{Y_p}

All of the uncertainty components presented in this subsection contribute to the uncertainty associated with the displacement u_{Y_p} in equation (2) as

$$u_{Y_p}^2 = u_{\text{misalign}}^2 + u_{\text{dyn}}^2 + u_{\text{plat}}^2 + u_{\text{int}}^2 + u_{\text{pn}}^2, \quad (5)$$

where u_{misalign} is the uncertainty due to angular motions of the calibration mass (rocking or tilting motions) referred to as misalignments, u_{dyn} is the uncertainty originating from calibration mass dynamics, u_{plat} is the uncertainty due to platform vibrations, u_{int} is the uncertainty due to the interferometer resolution, and u_{pn} is the uncertainty in the displacement from optical and phasemeter nonlinearity in the interferometer measurement.

3.2.1. Misalignment and Angular Motion: u_{misalign} —The largest source of uncertainty in the displacement (and the applied force) is due to tilting of the calibration assembly, and this is in general a function of frequency. This has also been observed by other workers [15,18]. The error in the measured displacement arising from misalignment and angular motion is analyzed in Appendix A. The magnitude of this angular motion is determined from the strength of the interferometer interference signal. A calibration of the

variation of this signal strength with angular deflection was performed, in which the calibration mass mirror was replaced by a mirror on a precision mount, and the angle of this mirror was varied and the corresponding change in interferometer signal strength recorded. This information was used to determine the angular deflections of the mass during force calibration measurements, with the resulting relative displacement uncertainty (coverage factor $k = 1$) being 1.1×10^{-5} to 1.2×10^{-2} for the calibration measurements reported in Section 4, using the expression derived in Appendix A.

3.2.2. Calibration Mass Internal Dynamics: u_{dyn} —The position of the calibration mass is determined by measuring the position of one point on its surface by laser vibrometry. However the acceleration of this material point is not exactly equal to the acceleration of the entire mass (i.e., the body-averaged acceleration). The actual force acting on the sensor is the sum of the acceleration-mass product of every material point in the mass, $F = \int_{\mathcal{V}} a(\mathbf{r})\rho(\mathbf{r}) d\mathcal{V}$, where $a(\mathbf{r})$ and $\rho(\mathbf{r})$ are acceleration and density at the material point $\mathbf{r}$ and the integration is performed over the volume $\mathcal{V}$ [19]. The correction factor which accounts for the internal dynamics of the calibration mass was determined using finite-element analysis [20] and was found to lie between 0.3 % and 2 % at the maximum calibration frequency of 2 kHz, for the range of calibration masses used. The uncertainty of this correction is near 10 % of the correction, dominated by uncertainty in the material properties, which were taken from tabulated data. A uniform distribution is assumed for this uncertainty component. The calibration mass dynamics also contribute to the phase uncertainty discussed below in Section 3.6.

3.2.3. Vibration of the Interferometer Platform: u_{plat} —The interferometer measures the differential change of optical path length between the target arm and the reference arm. Vertical motions of the interferometer platform affect the target arm length in exactly the same way as motion of the target, and will erroneously be interpreted as motions of the calibration mass ¶. A vibration isolation system is used to minimize platform vibrations. The relative displacement error $\tilde{e}_{\text{plat}}$ due to interferometer platform vibrations is a function of frequency and is given by

$$\tilde{e}_{\text{plat}} = \frac{Y_b T_{\text{plat}}}{Y_p}, \quad (6)$$

where Y_b is the displacement amplitude of the shaker table, Y_p is the displacement amplitude of the calibration mass, and T_{plat} is the experimentally measured transmission coefficient between the shaker table and the isolation platform, and is a function of frequency. The transfer function from the shaker table to the interferometer platform was measured using accelerometers mounted at both locations, and is provided in supporting materials online. For most frequencies, the contribution of the optical platform vibrations to the measured displacement uncertainty is less than 0.1 %. Due to resonances of the support frame of the interferometer platform, at certain frequencies within the calibration range, the

¶When the interferometer is configured to measure the relative motion between two points on the calibration assembly, such motion of the platform is common to both arms and causes no error.

relative amplitude is as large as 0.5 %. This is taken to be the one-sigma width of a Gaussian probability distribution ⁺, so that $\tilde{u}_{\text{plat}} = \tilde{\epsilon}_{\text{plat}}$. Although not implemented here, it is possible to measure the platform vibrations in real time during calibration measurements and to correct the acceleration assigned to the calibration mass accordingly.

3.2.4. Interferometer Resolution: u_{int} —The noise floor of the interferometer was measured to be less than $1 \text{ nm}/\sqrt{\text{Hz}}$ throughout the calibration frequency range. For an integration time of 1 s, corresponding to a Fourier bin width of 1 Hz, this gives an interferometer resolution $a_{\text{int}} < 1 \text{ nm}$. This measurement was done by reorienting an aiming mirror that ordinarily directs the target beam towards the shaker table, so that the beam is instead reflected straight back to the interferometer beam-splitter. This measurement captures all sources of interferometer noise except refractive index fluctuations in the section of the target beam-path between the aiming mirror and the calibration mass top surface, and any changes in the mechanical vibration of the aiming mirror caused by its reorientation. The measured interferometer noise floor is provided in the supporting materials online. The uncertainty associated with the interferometer resolution is taken to have a Gaussian distribution with a 67 % probability (1-sigma) so that $\tilde{u}_{\text{int}} = \tilde{a}_{\text{int}}$.

3.2.5. Interferometer Nonlinearity: u_{pn} —The interferometer has an uncertainty contribution due to electronic nonlinearity of the phasemeter [23] as well as optical nonlinearity (see e.g. [24]). The former was tested by applying electronic signals (of similar amplitude to the interferometer outputs), of the same frequency and varying relative phase, to the lock-in amplifier reference and measurement inputs. Comparing the phase difference indicated by the lock-in amplifier to that indicated by an oscilloscope bounded the phasemeter error at 0.1 nm. The first and second order periodic optical nonlinearities were measured to be 0.11 nm and 0.08 nm, respectively. However, the optical nonlinearity errors fall almost entirely outside the relevant Fourier frequency bin. The total estimated displacement uncertainty contribution (95 % confidence interval) due to interferometer nonlinearity is estimated as 0.1 nm.

3.3. Air Drag: u_{Fd}

As the calibration mass moves, a parasitic drag force acts on its surface. This force can be estimated from the calibration mass geometry and the experimental operating conditions. At the highest velocities (approximately 1 m/s), the total drag force is estimated to be below 0.1 mN based on empirical data [25]. For a calibration force range down to 1 N, the drag force is thus expected to produce a relative error no greater than 1×10^{-4} . The drag force is not corrected for, but rather is treated as an uncertainty of magnitude equal to the maximum expected drag of 0.1 mN. A uniform distribution is assumed for this.

3.4. Voltage Measurement Uncertainties: u_v

The main components of the voltage measurement uncertainty are the uncertainty of the voltmeter measurement, uncertainty of the anti-aliasing filter gain, and possible parasitic voltages due to angular and transverse motions of the experimental assembly. As previously

⁺The shape of the distribution is required for treating uncertainties in accordance with [21, 22].

mentioned, the transducer output voltage is measured with a fast-sampling digital voltmeter (DVM1), while the excitation voltage of the transducer is measured using a separate voltmeter (DVM2), also in (a slower) DC sampling mode. The uncertainties are combined in a root sum of squares fashion as

$$u_V^2 = u_{\text{DVM}}^2 + u_{\text{trans}}^2 + u_{\text{filt}}^2. \quad (7)$$

3.4.1. Digital Voltmeter: u_{DVM} —The uncertainty in the DVM voltage measurement is a function of the signal amplitude, sampling frequency, and additional DVM settings. For the settings used for the calibration measurements reported in the next section, the manufacturer-specified relative uncertainty using DVM1 is $\tilde{a}_{\text{DVM}} = 6.8 \times 10^{-5}$ for a 10 mV signal and $\tilde{a}_{\text{DVM}} = 4.2 \times 10^{-3}$ for a 0.1 mV signal. The voltage uncertainty is computed using this range assuming a uniform distribution; that is $\tilde{u}_{\text{DVM}} = \tilde{a}_{\text{DVM}}/\sqrt{3}$.

3.4.2. Misalignment and Angular Motion: u_{trans} —As described in Appendix A, misalignment between the sensing axis of the force transducer and the principal axis of motion, as well as transverse and angular motions of the calibration assembly, causes moments and transverse forces to be applied to the transducer. This results in a frequency-dependent voltage uncertainty due to small moment and transverse-load sensitivities of the transducer. For the calibration measurements described below in Section 4, the transducers' transverse-load and moment sensitivity coefficients resulted in a maximum voltage uncertainty of $\tilde{a}_{\text{trans}} = 2.5 \times 10^{-4}$ for the transducer calibrations reported in this paper.

3.4.3. Anti-aliasing Filter Response: u_{filt} —To prevent aliasing errors in the sampling of the transducer voltage, an 8th-order Bessel filter with a selected cut-off frequency of 3.0 kHz, which corresponds to a -3 dB amplitude attenuation at 1.553 kHz, is inserted between the transducer and DVM1. The cut-off frequency was selected as a compromise between maximizing attenuation at the 5 kHz Nyquist frequency corresponding to the used voltmeter sampling rate of 10 kS/s, and minimizing attenuation at the maximum test frequency of 2 kHz. A Bessel filter was chosen because of its linear phase response. The applied filter provided -24 dB attenuation at the Nyquist frequency. Thus to characterize the transducer under test, the frequency-dependent amplitude and phase response of the filter also needs to be determined and corrected for. The experimentally measured amplitude and phase response of the filter is shown in Figure 4. In determining the response of the filter, eight runs were performed taking data at discrete steps from 10 Hz to 2.5 kHz in 5 Hz steps (Figure 4 shows data only up to 2.0 kHz). At each frequency, the eight data points were averaged and plotted in Figure 4, along with the standard deviation at each frequency. The mean of all the standard deviations over all frequencies was 2.1×10^{-4} . A Gaussian distribution is assumed at each frequency with a 1-sigma uncertainty.

3.5. Lack of Repeatability: u_{rep}

As mentioned earlier, the calibration is performed multiple times for different calibration mass values. There is non-repeatability between the different runs that may arise due to a number of factors including among others noise, dependence of the transducer response on

variations in its mounting, varying off-center alignment of the laser spot and possible unaccounted sources of measurement error. Some of these possible causes of potential non-repeatability (for example voltmeter noise, interferometer noise, and laser spot off-center alignment) have already been accounted for in the evaluation of uncertainty components above. However, for the calibration measurements reported in Section 4 (see Figs. 5 and 7), the observed non-repeatability was larger than the estimated uncertainty at many data points. This indicates that not all sources of non-repeatability are already captured in the uncertainty analysis. Therefore we have included the measured non-repeatability as u_{rep} in equation (2). In general the non-repeatability is expected to be transducer-dependent, and was approximately 1×10^{-2} ($k = 2$) for the two transducers calibrated in this paper.

3.6. Phase Measurement Uncertainties

The uncertainty u_{phase} in the measured transducer sensitivity phase was calculated according to

$$u_{\text{phase}}^2 = u_{\text{phase,instr}}^2 + u_{\text{phase,mass}}^2 + u_{\text{phase,rep}}^2. \quad (8)$$

The quantity $u_{\text{phase,instr}}$ is uncertainty in the phase delay introduced by instrumentation, such as filters and amplifiers. This value was experimentally determined by generating a phase-modulated sine wave at a known frequency and performing the demodulation with the lock-in amplifier, while simultaneously sampling the modulating signal. Thereby the overall phase response of the instrumentation was determined as a function of frequency, and was found to correspond to a constant time delay of $(18.3 \pm 0.61) \mu\text{s}$. The uncertainty $u_{\text{phase,mass}}$ accounts for phase uncertainty due to the dynamics of the calibration mass. This phase uncertainty was determined using finite element analysis on a calibration mass with a length to diameter ratio of 1.5 and a damping coefficient of 0.01. The absolute difference was found not to exceed 2×10^{-4} radians. The uncertainty associated with this difference is modeled as uniformly-distributed, such that $u_{\text{phase,mass}} = 2 \times 10^{-4} / \sqrt{3}$. The uncertainty $u_{\text{phase,rep}}$ describes non-repeatability in the phase measurement and is determined at each frequency by $u_{\text{phase,rep}} = \sigma$, where σ is the standard deviation. We include this non-repeatability in the measurement uncertainty as we believe that it arises from the measurement system, rather than non-repeatability of the transducer response. This phase repeatability measurement was performed four times (using four different calibration masses) for one force transducer and three times for another (see Figures 6 and 8 in Section 4). The non-repeatability was found to be quite large at low frequencies (below 500 Hz), and may originate from angular motions of the transducer-mass assembly. We note that the low-frequency range is less important than the higher-frequency region, where significant deviation of the dynamic response from the static sensitivity is more likely. A summary of the phase measurement uncertainties is presented in Table 3.

3.7. Summary of Uncertainties

The estimated maximum relative uncertainty over a range of frequencies and displacements, for each significant contributor is summarized in Tables 1 to 3. The standard (relative)

uncertainty of the amplitude of a given sensitivity measurement is the root-sum-of-squares of the force and voltage standard (relative) uncertainties. Other potential sources of uncertainty investigated and found to be insignificant for our measurements so far include changing the connecting cable and possible transducer magnetic susceptibility. It is important to emphasize that the uncertainties achieved with the calibration system are transducer-dependent. First, transducers with small sensitivities, resulting in relatively-low output voltages, will have higher voltage measurement uncertainties. Second, transducers with large off-axis sensitivities will have increased voltage uncertainties, as described in Section 3.4.2. In addition, the non-repeatability of a calibration is expected to be transducer-dependent; a transducer with a large non-repeatability will then have an increased calibration uncertainty.

4. Example Calibration Measurements

Dynamic calibration measurements were performed on two strain-gauge-based force transducers, transducer 1 with a rated capacity of 2220 N and transducer 2 with a rated capacity of 1330 N. A list of both transducers' performance parameters and selected properties is provided in Table 4. A 10 V DC bridge excitation voltage was used. Calibration measurements were made on these transducers at a set of discrete frequencies from 100 Hz to 2 kHz in 50 Hz increments, while maintaining a constant nominal calibration mass acceleration amplitude of 50 m/s² and collecting one second of data at each frequency. The data were obtained this way for three to four different calibration masses. The interferometer quadrature phase outputs and the transducer output voltage were sampled by the oscilloscope at 1 MS/s, while the transducer output voltage was sampled by DVM1 at 10 kS/s.

The measured sensitivity amplitudes for transducer 1 and transducer 2 are shown in Figure 5 and Figure 7, respectively. The sensitivity phase is shown in Figure 6 for transducer 1 and Figure 8 for transducer 2. The error bars in Figure 5 and Figure 7 are shown for every third data point for clarity. At the lowest frequencies 50 m/s² acceleration could not be achieved due to interferometer limitations or shaker limitations, so lower acceleration levels were used. Similarly, for the sensitivity data for transducer 2, shown in Figure 7, a constant 50 m/s² was not maintained at the higher frequencies with the larger mass due to amplitude limitations of the accelerometer mounted below the shaker plate. As is evident from the data shown in Figure 5(a)(b) and Figure 7(a)(b), the sensitivity is a function of the calibration mass connected to the transducer; this is due to the force offset due to acceleration of the internal mass in the transducer. In other words, the mass being applied to the transducer is a sum of the internal mass and calibration mass (including any mass of threaded adapters), $m_e = m_p + m_s$, where m_e is the effective dynamic mass that is applied to the transducer. As previously mentioned, m_p is the mass of the calibration mass and connector. The mass m_s is an effective lumped mass inherent to the transducer that accounts for the portion of the transducer that is attached to the calibration mass and accelerated together with it; it is not the mass of the entire transducer itself. As the connected calibration mass is increased, the sensitivity approaches a limiting curve. For both transducers studied, as $\omega \rightarrow 0$ in the high-calibration-mass limit, the sensitivity was found to agree with the manufacturer's static sensitivity to within the measurement uncertainty. Scaling the sensitivity data shown in

Figure 5(a) for the first five frequencies (corresponding to 100 Hz to 300 Hz), to agree with the static sensitivity, yields an internal mass of $m_s = 0.257 \pm 0.0089$ kg ($k = 2$) for transducer 1. Similarly scaling the sensitivity for transducer 2 yields an internal mass value of $m_s = 0.015 \pm 0.014$ kg. A roll-off with increasing frequency was observed in the transducer response, particularly for transducer 1. Possible sources of this roll-off include the dynamics of the connected lumped masses in the calibration assembly and damping in the system. Such roll-off has been previously observed [11].

A table of the overall relative expanded uncertainty (coverage factor $k = 2$) for the calibration data presented in Section 4 is provided in Table 5. These include the measured non-repeatability for each transducer. This non-repeatability was determined by calculating the relative deviation of each sensitivity measurement from the mean sensitivity of the given transducer at that frequency, and then averaging over all data points for the given transducer. This assumes that the non-repeatability is not a function of frequency; we consider this to be more accurate than estimating the non-repeatability at each frequency from a small sample size (three or four data points). The resulting non-repeatabilities ($k = 2$) were 1.0×10^{-2} and 9.8×10^{-3} for transducer 1 and transducer 2 respectively. The non-repeatability ($k = 2$) as a function of frequency ranged from 9.6×10^{-4} to 3.4×10^{-2} for transducer 1 and, 2.4×10^{-3} to 2.6×10^{-2} for transducer 2. We ascribe this variation to the very small sample sizes.

The achieved overall uncertainties depend on the calibration mass used, and at each calibration frequency an expanded uncertainty of 1.2 % or below is demonstrated for both transducers. The operating points where relatively high levels of angular motion or vibration noise occur, resulting in relatively high overall uncertainty levels, are known or detected during the calibration and in practice can be omitted in many calibrations. At the majority of calibration points in Table 5, the dominant contributor to the uncertainty is the non-repeatability. A question that remains is how much of the excess non-repeatability is attributable to the transducers, and how much is due to unaccounted uncertainty in the measurements of force and voltage by the calibration system. The agreement between the non-repeatabilities determined for the two transducers tested suggests attribution to unaccounted uncertainty in the force and voltage measurements, although the sample size of two is small.

5. Summary

We have described a calibration system used to determine the dynamic sensitivity of force transducers, along with the sources of significant uncertainty. Sample calibrations were performed on two shear-web-type strain-gauge-based transducers over the range 100 Hz to 2 kHz, with an expanded uncertainty (coverage factor $k = 2$) of 1.2 % or below, achieved by calibrating using multiple calibration masses. At most frequencies, the largest uncertainty contributor in the calibrations so far is non-repeatability. The largest identified sources of uncertainty in the force measurements are the angular motion of the calibration stack and mechanical vibrations of the interferometer platform; we are working to reduce these contributions. The uncertainties in the transducer output measurement are small in comparison to the force measurement uncertainties, as long as the transducer output voltage amplitude is greater than approximately 100 μ V. The sample calibrations demonstrated that

significant error can occur in measurements of dynamic forces if only static calibration data is used to interpret the transducer output.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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References

1. Kümme R, Dixon MJ. The results of comparisons between two different dynamic force measurement systems. *Measurement*. 1992; 10(3):140–144.
2. Ueda, K., Umeda, A. Evaluation of force transducer characteristic by impact; Proceedings of the 13th IMEKO World Congress; 1994.
3. Dixon MJ. A traceable dynamic force transducer. *Experimental Mechanics*. 1990; 30(2):152–157.
4. Kümme, R. Wirtschaftsverl. NW: Verlag für Neue Wiss; 1996. Untersuchung eines direkten Verfahrens zur dynamischen Kalibrierung von Kraftmessgeräten: ein Beitrag zur Verringerung der Messunsicherheit.
5. Knight WA, Sadek MM. The correction for dynamic errors in machine tool dynamometers. *Annals of the C.I.R.P.* 1971; 19:237–245.
6. Fujii Y. Toward Dynamic Force Calibration. *Measurement*. 2009; 42:1039–1044.
7. Bruns, Th, Kümme, R., Kobusch, M., Peters, M. From oscillation to impact: the design of a new force calibration device at PTB. *Measurement*. 2002; 32:85–92.
8. Hjelmgren J. Dynamic measurement of force - a literature survey. Swedish National Testing and Research Institute. 2002; 2002:27. Report.
9. Usuda, T., Bruns, T., Wright, J., Filtz, J-R., Esward, T., Arrhen, F., Fletcher, N. Report on the BIPM workshop on challenges in metrology for dynamic measurement. Rapport BIPM-2013/01. 2013. http://www.bipm.org/en/conference-centre/bipm-workshops/dynamic_measurement
10. Kümme R. Investigation of the comparison method for the dynamic calibration of force transducers. *Measurement*. 1998; 23:239–245.
11. Schlegel, Ch, Kieckenap, G., Glöckner, B., Buss, A., Kümme, R. Traceable periodic force calibration. *Metrologia*. 2012; 49:224–235.
12. Link, A., Glöckner, B., Schlegel, C., Kümme, R., Elster, C. System Identification of Force Transducers for Dynamic Measurements; Proceedings of the 19th IMEKO World Congress; 2009.
13. Kobusch, M., Link, A., Buss, A., Bruns, T. Comparison of Shock and Sine Force Calibration Methods; Proceedings of the 20th IMEKO World Congress; 2007.
14. Bartoli C, Beug MF, Bruns T, Elster C, Esward T, Klaus L, Knott A, Kobusch M, Saxholm S, Schlegel C. Traceable dynamic measurement of mechanical quantities: Objectives and first results of this European project. *Int. J. Metrol. Qual. Eng.* 2012; 3:127–135.
15. Medina N, de Vicente J. Force Sensor Characterization Under Sinusoidal Excitations. *Sensors*. 2014; 14(10):18454–18473. [PubMed: 25290287]
16. Chijioke, A., Seifarth, RL., Kubarych, ZJ. NIST 1-Kilonewton Sine Force Calibration System; Proceedings of the 22nd IMEKO TC3 International Conference; 2014.
17. Quinn TJ. Practical Realization of the Definition of the Metre, Including Recommended Radiations of Other Optical Frequency Standards (2001). *Metrologia*. 2003; 40(2):103–133.
18. Schlegel C, Kieckenap G, Kahmann H, Kümme R. Mechanical Influences in Sinusoidal Force Measurement. *Acta Imeko*. 2015; 4(2):57–61.

19. Kümme R. The determination of the effective dynamic force for the calibration of force transducers, with due regard to the distribution of mass and acceleration. 1996 Proceedings of the 15th IMEKO TC3 Conference.
20. Bankert, JC., Chijioke, A. Elasticity of mass artifacts in sinusoidal force calibration; Proceedings of the 21st IMEKO World Congress; 2015.
21. Taylor BN, Kuyatt CE. Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results. NIST Technical Note 1297. 1994
22. ISO, Guide to the Expression of Uncertainty in Measurement, prepared by ISO Technical Advisory Group 4 (TAG 4), Working Group 3 (WG 3). Oct. 1993
23. Oldham NM, Kramar JA, Hetrick PS, Teague EC. Electronic Limitations in Phase Meters for Heterodyne Interferometry. Precision Engineering. 1993; 15(3):173–179.
24. Bobroff N. Recent Advances in Displacement Measuring Interferometry. Measurement Science and Technology. 1993; 4(9):907–926.
25. Hoerner S. Fluid Dynamic Drag. Hoerner Fluid Dynamics. 1965

Figure 1.
A schematic of the test and calibration arrangement.

Figure 2.

Photograph of a force transducer (transducer 1 in Section 4) mounted onto the shaker plate with a 3 kg calibration mass secured to the top. In some cases (for example transducer 2 in Section 4) no connector is used between the transducer and the calibration mass.

Figure 3.
Schematic diagram of the interferometer.

Figure 4.
 (a) Amplitude (gain) and (b) phase response of the anti-aliasing filter used to filter the transducer output signal.

Figure 5.

Calibration data for the 2220 N force transducer: (a) Sensitivity not accounting for internal mass m_s , (b) sensitivity accounting for internal mass m_s , (c) force on top of the transducer, and (d) output voltage of the transducer. The data point at 400 Hz with the 1.5 kg mass is unreliable, as at this data point the interferometer lost alignment due to large rocking motion of the calibration stack. Error bars represent the $k = 2$ expanded uncertainty for each data point.

Figure 6.
Phase of the sensitivity for the 2220 N force transducer. Error bars represent the $k=2$ expanded uncertainty for each data point.

Figure 7.

Calibration data for the 1300 N force transducer: (a) Sensitivity not accounting for internal mass m_s , (b) sensitivity accounting for m_s , (c) force on top of the transducer, and (d) output voltage of the transducer. The data point at 450 Hz with a 1.5 kg mass was erroneously not recorded. Error bars represent the $k = 2$ expanded uncertainty for each data point.

Figure 8.
Phase of the sensitivity for the 1300 N force transducer. Error bars represent the $k=2$ expanded uncertainty for each data point.

Table 1

Summary of uncertainties associated with force measurement.

Origin of Uncertainty	Rel. Unc.	Unc. Type	Distr. Type	D.O.F.
Misalignment & angular motion [*] , $\tilde{u}_{\text{misalign}}$	$1.2 \times 10^{-2} - 1.1 \times 10^{-5}$	B	Uniform	∞
Vibration of the interferometer platform [*] , $\tilde{u}_{\text{plat}}$	$5.0 \times 10^{-3} - 7.3 \times 10^{-7}$	A	Gaussian	9 – 1999
Interferometer resolution [*] , $\tilde{u}_{\text{int}}$	$2.0 \times 10^{-3} - 3.5 \times 10^{-6}$	A	Gaussian	9 – 1999
Calibration mass internal dynamics [*] , $\tilde{u}_{\text{dyn}}$	$1.2 \times 10^{-3} - 1.7 \times 10^{-5}$	B	Uniform	∞
Interferometer nonlinearity [*] , $\tilde{u}_{\text{pn}}$	$3.2 \times 10^{-4} - 8.0 \times 10^{-7}$	B	Gaussian	∞
Calibration mass uncertainty, $\tilde{u}_{\text{mp}}$	1.5×10^{-5}	B	Uniform	∞
Air drag (1 N to 1 kN) [*] , $\tilde{u}_{\text{Fd}}$	$2.9 \times 10^{-5} - 2.9 \times 10^{-8}$	B	Uniform	∞
Combined Standard Uncertainty	$1.5 \times 10^{-2} - 5.0 \times 10^{-4}$			
Expanded Uncertainty ($k = 2$)	$3.0 \times 10^{-2} - 1.0 \times 10^{-3}$			

Quantities denoted with a superscript star (*) are a function of the calibration mass frequency and displacement.

The abbreviation D.O.F. is the assumed or known number of degrees of freedom.

Table 2

Summary of uncertainties associated with transducer voltage measurement. The four different uncertainty values correspond to transducer output voltage amplitudes of 0.1 mV, 0.5 mV, 1 mV and 10 mV.

Origin of Uncertainty	Rel. Unc.	Unc. Type	Distr. Type	D.O.F.
Digital voltmeter, DVM1 (0.1, 0.5, 1, 10) mV, $\bar{u}_{\text{DVM}}$	$(24.0, 4.9, 2.5, 0.39) \times 10^{-4}$	A, B	Uniform	∞
Misalignment & angular motion, $\bar{u}_{\text{trans}}$	2.5×10^{-4}	B	Gaussian	∞
Anti-aliasing filter response, $\bar{u}_{\text{filt}}$	2.1×10^{-4}	A	Gaussian	γ^*
<hr/>				
Combined Standard Uncertainty:	$(24.2, 5.9, 4.1, 3.3) \times 10^{-4}$			
Expanded Uncertainty ($k = 2$):	$(48.4, 1.2, 0.82, 0.66) \times 10^{-4}$			

* Eight measurements at each frequency.

The abbreviation D.O.F. is the assumed or known number of degrees of freedom.

Table 3

Summary of uncertainties associated with transducer phase measurement.

Origin of Uncertainty	Abs. Unc. [rad]	Unc. Type	Distr. Type	D.O.F.
Instrumentation, $u_{\text{phase,instr}}$	2.29×10^{-3}	A	Gaussian	14
Calibration mass dynamics, $u_{\text{phase,mass}}$	1.15×10^{-4}	B	Uniform	∞
Repeatability, $u_{\text{phase,rep}}$	$2.27 \times 10^{-3} - 8.45 \times 10^{-2}$	A	Gaussian	2
Combined Standard Uncertainty:	$3.23 \times 10^{-3} - 8.45 \times 10^{-2}$			
Expanded Uncertainty ($k = 2$):	$6.48 \times 10^{-3} - 1.69 \times 10^{-1}$			

The abbreviation D.O.F. is the assumed or known number of degrees of freedom.

Table 4

Some properties of the two shear-web force transducers calibrated within this study.

Transducer Property	Transducer 1	Transducer 2	unit
Sensitivity (at maximum capacity)	2.135	2.098	mV/V
Maximum Capacity	2220	1330	N
Nonlinearity [*] (compression)	−0.0028	−0.004	%
Nonlinearity [*] (tension)	0.0094	0.026	%
Side Load Sensitivity	± 0.050	± 0.25	% [†]
Eccentric Load Sensitivity (moment)	± 0.020	± 0.098	% [†] /cm
Base Connector	1/2" – 20 (female)	1/4" – 28 (female)	-
Sense Connector	1/2" – 20 (male)	1/4" – 28 (male)	-
Mass	1.3	0.45	kg
Bridge Resistance	350	700	Ohm

^{*} *Nonlinearity* is defined as the maximum relative difference, over the working force range of the transducer, between the measured voltage output at an applied force point and the voltage output predicted by a linear fit of all the static calibration data.

[†] These units are a percentage of the static sensitivity.

Table 5

Summary of relative expanded uncertainties ($k = 2$) for transducer 1 and transducer 2 sensitivity amplitude. The interferometer did not maintain alignment for the data point at 400 Hz with a 1.5 kg mass with transducer 1, and the data point at 450 Hz with a 1.5 kg mass for transducer 2 was erroneously not recorded.

Frequency / Hz	Transducer 1				Transducer 2			
	1.5 kg	3 kg	Transducer	5 kg	8 kg	1.5 kg	3 kg	5 kg
100	0.0109			0.0102	0.0101	0.0104	0.0108	0.0099
150	0.0105		0.0103	0.0101	0.0101	0.0104	0.0100	0.0099
200	0.0103		0.0102	0.0101	0.0102	0.0101	0.0099	0.0099
250	0.0102		0.0102	0.0102	0.0102	0.0099	0.0101	0.0103
300	0.0103		0.0102	0.0102	0.0101	0.0100	0.0100	0.0099
350	0.0127		0.0102	0.0118	0.0112	0.0121	0.0113	0.0105
400	-		0.0123	0.0108	0.0104	0.0109	0.0104	0.0100
450	0.0102		0.0111	0.0101	0.0101	-	0.0099	0.0099
500	0.0102		0.0102	0.0101	0.0101	0.0099	0.0099	0.0099
550	0.0102		0.0101	0.0101	0.0101	0.0099	0.0099	0.0099
600	0.0102		0.0101	0.0101	0.0101	0.0099	0.0099	0.0099
650	0.0102		0.0101	0.0101	0.0102	0.0099	0.0099	0.0100
700	0.0102		0.0101	0.0102	0.0103	0.0099	0.0099	0.0100
750	0.0102		0.0101	0.0102	0.0102	0.0099	0.0100	0.0128
800	0.0102		0.0102	0.0102	0.0102	0.0099	0.0103	0.0115
850	0.0102		0.0102	0.0102	0.0102	0.0099	0.0187	0.0104
900	0.0102		0.0102	0.0102	0.0102	0.0099	0.0159	0.0102
950	0.0102		0.0115	0.0102	0.0102	0.0099	0.0113	0.0102
1000	0.0103		0.0103	0.0102	0.0103	0.0099	0.0109	0.0102
1050	0.0101		0.0102	0.0103	0.0103	0.0099	0.0108	0.0103
1100	0.0102		0.0102	0.0103	0.0103	0.0103	0.0107	0.0102
1150	0.0102		0.0102	0.0103	0.0103	0.0116	0.0107	0.0103
1200	0.0102		0.0102	0.0103	0.0104	0.0100	0.0107	0.0104
1250	0.0102		0.0102	0.0104	0.0105	0.0103	0.0108	0.0103
1300	0.0103		0.0103	0.0106	0.0108	0.0109	0.0119	0.0111

Frequency / Hz	Transducer 1			Transducer 2		
	1.5 kg	Transducer	5 kg	8 kg	1.5 kg	3 kg
1350	0.0103	0.0107	0.0104	0.0105	0.0107	0.0116
1400	0.0103	0.0104	0.0105	0.0105	0.0101	0.0116
1450	0.0103	0.0103	0.0105	0.0107	0.0101	0.0120
1500	0.0103	0.0104	0.0106	0.0107	0.0102	0.0126
1550	0.0103	0.0104	0.0107	0.0112	0.0102	0.0132
1600	0.0103	0.0104	0.0107	0.0111	0.0101	0.0136
1650	0.0103	0.0104	0.0107	0.0110	0.0102	0.0143
1700	0.0104	0.0104	0.0110	0.0115	0.0102	0.0156
1750	0.0104	0.0105	0.0125	0.0149	0.0108	0.0183
1800	0.0104	0.0110	0.0109	0.0114	0.0103	0.0190
1850	0.0104	0.0105	0.0112	0.0122	0.0106	0.0216
1900	0.0104	0.0107	0.0111	0.0126	0.0108	0.0252
1950	0.0104	0.0109	0.0115	0.0127	0.0118	0.0258
2000	0.0104	0.0112	0.0124	0.0124	0.0118	0.0259