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A new Mumford–Shah total variation minimization based model for sparse-view x-ray computed tomography image reconstruction

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Abstract

Total variation (TV) minimization for the sparse-view x-ray computer tomography (CT) reconstruction has been widely explored to reduce radiation dose. However, due to the piecewise constant assumption for the TV model, the reconstructed images often suffer from over-smoothness on the image edges. To mitigate this drawback of TV minimization, we present a Mumford-Shah total variation (MSTV) minimization algorithm in this paper. The presented MSTV model is derived by integrating TV minimization and Mumford-Shah segmentation. Subsequently, a penalized weighted least-squares (PWLS) scheme with MSTV is developed for the sparse-view CT reconstruction. For simplicity, the proposed algorithm is named as ‘PWLS-MSTV.’ To evaluate the performance of the present PWLS-MSTV algorithm, both qualitative and quantitative studies were conducted by using a digital XCAT phantom and a physical phantom. Experimental results show that the present PWLS-MSTV algorithm has noticeable gains over the existing algorithms in terms of noise reduction, contrast-to-ratio measure and edge-preservation.

Keywords

computer tomography; Mumford-Shah total variation; sparse-view; image reconstruction

1. Introduction

X-ray computed tomography (CT) has been widely used for clinical applications over the past decades. However, excessive x-ray radiation exposure during clinical examination has been reported to increase lifetime risk of cancerous, genetic, and other diseases [1-4].

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Therefore, minimizing the radiation risks is strongly desirable in clinical practices. To reduce the radiation dose, two major strategies have been widely discussed, including reducing the milliamperes-seconds per projection view or decreasing the required number of projection views (sparse-view) per rotation around the body [5-7]. In this work, we are focusing on low-dose CT image reconstruction from the sparse-view projection data. In the sparse-view CT scans, less projection views will unavoidably lead the acquired sinograms insufficient and increase the data inconsistency associated with the sparsity. And this data inconsistency would cause image artifacts. Therefore, the diagnostic quality of the CT images would be degraded if appropriate methods are not applied during image reconstruction.

To address the issues of sparse-view CT image reconstruction, various image reconstruction methods by incorporating adequate prior information of the desired image have been proposed [8-14]. A typical example is total variation (TV) based projection onto convex sets (POCS) reconstruction strategy, based on the piecewise constant assumption of the desired image, has shown its effectiveness for dealing with the data insufficiency from sparse-view sampling [8-10]. Furthermore, to address the limitations of the original TV constraint with isotropic edge property, different weighted-TVs were proposed recently [11-16].

In this work, aiming to improve the performance of TV minimization based algorithm, we introduce an edge-preserving TV-type minimization, named as “Mumford-Shah TV (MSTV), for CT image reconstruction under the penalized weighted least-squares (PWLS) criteria from sparse-view acquisitions. The present MSTV not only considers the TV norm of to-be-estimated image, but also considers the corresponding edge of the to-be-estimated image. For simplicity, the present algorithm is termed as “PWLS-MSTV”. In the implementation, the PWLS-MSTV is performed by integrating CT image reconstruction and segmentation, aiming to yield a continuous edge map and less noise-induced artifacts. To evaluate the present PWLS-MSTV algorithm, qualitative and quantitative studies were performed on both digital and physical phantoms in terms of noise reduction, image-similarity metric, and convergence analysis.

The remaining part of this paper is organized as follows. Section 2 first describes the MSTV model in brief, and then describes the proposed PWLS-MSTV image reconstruction algorithm in detail and the associative optimization algorithm. In section 3, experimental setting for both digital and physical phantoms is described. Section 4 is the results and discussion. Finally, the conclusion is given in sections 5.

2. Methods

2.1. Mumford-Shah Total Variation

The MSTV was first proposed by Shah in the image segmentation and image restoration [17-23]. The widely used MSTV is an approximation proposed by Alicandro [23]:

$$MSTV_{\varepsilon}(u, v) = \int_{\Omega} v^2 |\nabla u| dx + \alpha \int_{\Omega} \left(\varepsilon |v|^2 + \frac{(v-1)^2}{4\varepsilon} \right) dx \quad (1)$$

where Ω is a bounded domain, ∇u is the gradient of image u , v is the edge function of image u , which is approximate to zero in the edge of image u while it is approximate to one in other region of image u , ε is a small positive constant and α is a positive weight which needs to be tuned manually. Alicandro et al [23] also proved the Γ -convergence of this functional to

$$MSTV(u) = \int_{\Omega \setminus K} |\nabla u| dx + \alpha \int_K \frac{|u^+ - u^-|}{1 + |u^+ - u^-|} dH^1 + |D^c u|(\Omega) \quad (2)$$

where u^+ and u^- denote the image values on two sides of the edge set K , H^1 is the one-dimensional Hausdorff measure and $D^c u$ is the Cantor part of the measure-valued derivative Du . Through the definition of MSTV as shown in (2), it is obvious that MSTV not only considered the TV norm of image u in the image domain except for the edge, but also considered the measure of edge set K . Therefore, MSTV regularization brings more powerful regularity of solution than TV regularization.

2.2. PWLS-MSTV for CT Image Reconstruction

In this study, we propose the following cost function for CT image reconstruction with *MSTV* regularization:

$$\min_{u \geq 0, v} (y - Hu)^T G^{-1} (y - Hu) + \beta_2 MSTV_{\varepsilon}(u, v) \quad (3)$$

where $G = \frac{1}{\beta_1} HH^T + \Sigma$, β_1 and β_2 are two hyper-parameters to balance these two terms,

namely, the fidelity term and the regularization term. u is the vector of attenuation coefficients to be reconstructed, symbol T denotes the matrix transpose. The operator H represents the system or projection matrix with the size of $M \times N$. The element of h_{ij} is the length of the intersection of projection ray i with pixel j . Σ is a diagonal matrix with the i th element of σ_i^2 which is the variance of sinogram data y_i . In this work, the variance σ_i^2 is determined by the following mean-variance relationship based on our previous works [21-27].

Additionally, by introducing a new vector f , we have

$$\min_f (y - Hf)^T \sum^{-1} (y - Hf) + \beta_1 \|f - u\|^2 = (y - Hu)^T G^{-1} (y - Hu) \quad (4)$$

Hence, solving formula (4) is equal to solve the below formula:

$$\min_{u \geq 0, f, v} (y - Hf)^T \sum^{-1} (y - Hf) + \beta_1 \|f - u\|^2 + \beta_2 MSTV_\epsilon(u, v) \quad (5)$$

Namely,

$$\begin{aligned} \min_{u \geq 0, f, v} & (y - Hf)^T \sum^{-1} (y - Hf) + \beta_1 \|f - u\|^2 + \beta_2 \int_{\Omega} v^2 |\nabla u| dx + \beta_2 \alpha \int_{\Omega} \frac{(v-1)^2}{4\epsilon} \\ & + \epsilon |\nabla v|^2 dx \end{aligned} \quad (6)$$

For simplifying the redundant parameter, we replace $\beta_2 \alpha$ by l_1 without losing the previous meaning. The above formula is equal to:

$$\begin{aligned} \min_{u \geq 0, f, v} & (y - Hf)^T \sum^{-1} (y - Hf) + \beta_1 \|f - u\|^2 + \beta_2 \int_{\Omega} v^2 |\nabla u| dx + \alpha \int_{\Omega} \frac{(v-1)^2}{4\epsilon} \\ & + \epsilon |\nabla v|^2 dx. \end{aligned} \quad (7)$$

2.3. Optimization Scheme

In order to solve the cost function in (7), a modified alternating optimization method with three minimizing steps, which can be formulated as follows:

$$\begin{aligned} (P1): f &= \arg \min_f (y - Hf)^T \sum^{-1} (y - Hf) + \beta_1 \|f - u\|^2 \\ (P2): u &= \arg \min_{u \geq 0} \beta_1 \|f - u\|^2 + \beta_2 \int_{\Omega} v^2 |\nabla u| dx \\ (P3): v &= \arg \min_v \beta_2 \int_{\Omega} v^2 |\nabla u| dx + \alpha \int_{\Omega} \frac{(v-1)^2}{4\epsilon} + \epsilon |\nabla v|^2 dx \end{aligned} \quad (8)$$

In the implementation, we utilized a separable paraboloidal surrogates (SPS) algorithm [28] to solve (P1), let

$$\Phi(u; f) \triangleq (y - Hf)^T \Sigma^{-1} (y - Hf) + \beta_1 \|f - u\|^2 \quad (9)$$

Since the surrogate is a separable paraboloid, it can be easily minimized by zeroing the first derivative. This leads to the following simultaneous update algorithm:

$$f_j^{k+1} = f_j^k - \frac{\sum_{i=1}^M ((1/\sigma_i^2) h_{ij} ([Hf^k]_i - y_i))}{\sum_{i=1}^M ((1/\sigma_i^2) h_{ij} \sum_{i=1}^N h_{it}) + \beta_1} - \frac{\sum_{i=1}^M \beta_1 (f_j^k - u_j^k)}{\sum_{i=1}^M ((1/\sigma_i^2) h_{ij} \sum_{i=1}^M h_{it}) + \beta_1} \quad (10)$$

where the superscript $k=1,2,L,K$ denotes the iteration index. The first and second derivatives of the surrogate are easily shown to be

$$\left. \frac{\partial \Phi(u; f)}{\partial u_j} \right|_{f=u} = \sum_{i=1}^M ((1/\sigma_i^2) h_{ij} ([Hf^k]_i - y_i)) + \beta_1 (f_j^k - u_j^k) \quad (11)$$

$$\left. \frac{\partial^2 \Phi(u; f)}{\partial u_j^2} \right|_{f=u} = \sum_{i=1}^M ((1/\sigma_i^2) h_{ij} \sum_{i=1}^N h_{ij}) + \beta_1 \quad (12)$$

Then focusing on (P2), the solution of (P2) can be obtained by calculate the derivate of the cost functional in (P2), which can be written as follow:

$$2\beta_1(u - f) - 2\beta_2 \text{Div}(v^2 \nabla u) = 0 \quad (13)$$

where Div is the symbol of divergence and assuming $L(v)$ denote the differential operator

$$L(v) = -\beta_2 \text{Div}(v^2 \nabla u). \quad (14)$$

Then the above equation (14) equals to

$$\beta_1(u - f) + \beta_2 L(v) = 0 \quad (15)$$

Let $A(v)u = \beta_1 u + P_2 L(v)$, by rearranging (15), we can obtain

$$A(v)u = \beta_1 f. \quad (16)$$

As the literature [21], it is shown that the operator $A(v)$ is self-adjoint and positive definite. Therefore, the conjugate gradient algorithm is employed to solve equation (1). Similar to solving (P2), calculating the derivate of the cost function in (P3) is as follow:

$$2\beta_2 v |\nabla u| + \alpha \left(\frac{(v-1)}{2\varepsilon} - 2\varepsilon \text{Div}(\nabla v) \right) = 0 \quad (17)$$

Following discretization of eq (17), we obtain a linear system of equations

$$2\beta_2 v_{ij} [(\Delta_+^x u_{ij})^2 + (\Delta_+^y u_{ij})^2] + \alpha \left(\frac{(v_{ij}-1)}{2\varepsilon} - 2\varepsilon (\Delta_-^x \Delta_+^y v_{ij} + \Delta_-^y \Delta_+^x v_{ij}) \right) = 0. \quad (18)$$

where $\Delta_{\pm}^x v_{ij} = \pm (v_{i \pm 1, j} - v_{i, j})$ and $\Delta_{\pm}^y v_{ij} = \pm (v_{i, j \pm 1} - v_{i, j})$.

With column-stack ordering of $\{v_{ij}\}$, this system is of the form $Pv = q$, where the matrix P is symmetric and sparse. It is solved using the minimal residual algorithm [29].

2.4. Parameter Selections

Two hyper-parameters β_1 and β_2 were used to balance these two terms, namely, the fidelity term and the regularization term. In our studies, we select β_1 range from 1×10^{-4} to 1×10^{-3} that the results of (P1) were less sensitive to the value of β_1 . For the other parameter value of β_2 controls the smoothing strength of the reconstructed image, selection of this variable was optimized case by case on the basis of the noise level of image and sparse-view projections.

2.5. PWLS-MSTV Algorithm

In summarize the implementation of the present PWLS-MSTV algorithm for CT image reconstruction from sparse-view acquisitions can be written as follows:

PWLS-MSTV algorithm

Initialization: $\beta, u^0, v^0, \beta_1, \beta_2, k=0$

While stop criterion is not met

 Compute f^k by solving (P1) with SPS algorithm;

 Compute u^k by solving (P2) with conjugate gradient algorithm;

 Compute v^k by solving (P3) with minimal residual algorithm;

$k = k + 1$;

End

3. Experimental Setting

In this work, to validate and evaluate the performance of the PWLS-MSTV algorithm in CT image reconstruction from sparse-view projection measurements, a digital XCAT phantom [30] and a physical phantom [8, 9] were used in the experiments. For comparison, TV and TGV regularizations were also conducted under the PWLS criteria, referred to as the PWLS-TV and PWLS-TGV algorithms.

3.1. Digital XCAT Phantom

Figure 1(a) shows a slice of the digital XCAT phantom, which contains head anatomy structures with a tumor lesion. In the numerical simulation, a representative geometry of a monoenergetic fan-beam CT scanner setup was used for the CT projection generation. The imaging parameters of the CT scanner were as follows: (1) each rotation includes 1160 projection views that were evenly spaced on a circular orbit; (2) the number of channels per view was 672; (3) the distance from the detector arrays to the x-ray source was 1040.00 mm; (4) the distance from the rotation center to the x-ray source was 570.00 mm; and (5) the space of each detector bin was 1.407 mm. All the reconstructed images were composed of 512×512 square pixels with the pixel size of 0.625 mm × 0.625 mm. Each projection datum along an x-ray through the phantom image was calculated based on the known densities and intersection areas of the ray with the geometric shapes of the objects in the phantom image.

As our previous studies mentioned [21], we first simulated the noise-free sinogram data $\hat{y}$, and then generated the noisy transmission measurement I according to the statistical model of the pre-logarithm projection data as follow:

$$I = \text{Poisson}(I_0 \exp(-\hat{y})) + \text{Normal}(0, \sigma_e^2) \quad (19)$$

where I_0 is the incident x-ray intensity and σ_e^2 is the background electronic noise variance. In the simulation, I_0 and σ_e^2 were set to 1.0×10^6 and 11.0, respectively. Finally, the noisy sinogram data y were calculated by performing the logarithm transformation on the transmission data I .

3.2. Physical Phantom

To demonstrate the efficacy of the proposed PWLS-MSTV algorithm in more realistic situations, an anthropomorphic torso phantom was used for the experimental data acquisition. The physical phantom was scanned by a clinical CT scanner (Siemens SOMATOM Sensation 16 CT) at the protocol of 100 mAs and 120 kVp. Figure 1(b) is a slice of the physical phantom image reconstructed by the FBP method with ramp filter based on the averaged sinogram data of scans repeated 150 times at 100 mAs and 120 kVp. The physical phantom was scanned in cine mode at a fixed bed position. The associated imaging parameters of the CT scanner were same as those listed in section 3.1. The CT image was also reconstructed as 512×512 array with 1.2 mm pixel size. We evenly extracted 116-view

projection from the sinogram data acquired at 100 mAs to ensure the same noise level for each projection.

4. Results and Discussions

4.1. Digital XCAT Phantom Study

4.1.1. Visualization-based evaluation—Figure 2 shows the results reconstructed by different algorithms from 30-, 40-, and 60- view projections. Serious noise and streak artifacts can be observed in the FBP reconstructed images for all three cases. Especially in the 30-view case, the lesion in the FBP result becomes difficult to identify because of serious artifacts. PWLS-based algorithms could suppress the noise and artifacts well. Comparing the results of three PWLS-based algorithms to the true phantom image, PWLS-MSTV yields more gains than PWLS-TV and PWLS-TGV in terms of structure details preserving.

To further illustrate the gains of the present PWLS-MSTV algorithm, figure 3 draws the vertical profiles (indicated by the blue solid line in figure 1(a)) of the reconstructed images shown in figure 2. The PWLS-MSTV algorithm can produce more closely matching results compared with the ground truth. We can notice that the image edge of the PWLS-MSTV result is sharper than those of the PWLS-TV and PWLS-TGV results via comparing the profiles in the corners. The profile comparison indicates that the PWLS-MSTV algorithm achieves more noticeable gains than other two PWLS-based algorithms in terms of edge-preservation.

4.1.2. Noise reduction measure—To evaluate the noise reduction performance of the PWLS-MSTV algorithm, the peak signal to noise ratio (PSNR) metric was utilized for quantitative comparisons, defined as follows:

$$\text{PSNR} = 10 \log_{10} \left(\frac{\text{MAX}^2(\mu_{\text{true}})}{\sum_{n=1}^Q (\mu(n) - \mu_{\text{true}}(n))^2 / (Q - 1)} \right) \quad (20)$$

where μ denotes the to-be-reconstructed image, μ_{true} denotes the true phantom image, $\text{MAX}(\mu_{\text{true}})$ represents the associated maximum intensity value of μ_{true} , and μ_{true} denotes the associated average pixel value of μ_{true} , wherein n indexes the pixels in the image, and Q is the number of image pixels. Table 1 lists the PSNRs of the reconstructed images by three PWLS-based algorithms from 30-, 40-, and 60- view projections. It can be seen that the present PWLS-MSTV algorithm achieves the higher PSNR than other two PWLS-based algorithms in all three views. In other words, the PWLS-MSTV algorithm can yield better noise reduction than the PWLS-TV and PWLS-TGV algorithms for sparse-view CT image reconstruction.

4.1.3. Contrast-to-noise ratio—The contrast-to-noise ratio (CNR) metric is commonly used for the evaluation of image contrast and noise properties within a selected region of interest (ROI) relative to its background region. In order to calculate CNR, we selected two

ROIs of the same size as indicated by the solid-line boxes in figure 1(a), in which the red solid-line box embedded in the lesion was the ROI and the blue solid-line box near the ROI was the background region (BG). The CNR is defined as

$$CNR = \frac{2|\bar{\mu}_{ROI} - \bar{\mu}_{BG}|}{\sigma_{ROI} + \sigma_{BG}} \quad (21)$$

where $\bar{\mu}_{ROI}$ and $\bar{\mu}_{BG}$ denote the means of the pixels inside the ROI and BG, respectively. The terms σ_{ROI} and σ_{BG} are the standard deviations over the ROI and BG, respectively. Table 2 shows the CNRs of the reconstructed results by three PWLS-based algorithms in the cases of 30-, 40- and 60- views. The PWLS-MSTV algorithm has higher CNR values than the PWLS-TV and PWLS-TGV algorithms.

4.1.4. Image-similarity metric—To explore the performance of the various algorithms at the local detail level, the universal quality index (UQI) [24] was used in the ROI-based analysis, which was performed by evaluating the ROI similarity of the reconstructed images and the ground true images (i.e., ideal phantom). Given the aligned ROI within the reconstructed image μ and the ground true image μ_{true} , the associative covariance and UQI over the selected ROI can be calculated as follows:

$$\text{Cov}\{\mu, \mu_{true}\} = \frac{1}{Q-1} \sum_{n=1}^Q (\mu_{true}(n) - \bar{\mu}_{true})(\mu(n) - \bar{\mu}) \quad (22)$$

$$UQI = \frac{4\text{Cov}\{\mu, \mu_{true}\}}{\sigma^2 + \sigma_{true}^2} \cdot \frac{\bar{\mu} \cdot \bar{\mu}_{true}}{\bar{\mu}^2 + \bar{\mu}_{true}^2} \quad (23)$$

where $\bar{\mu}$ and $\bar{\mu}_{true}$ denote the associative means of the reconstructed image μ and the ground true image μ_{true} over the selected ROI, σ^2 and σ_{true}^2 denote the associative variances of the reconstructed image μ and the ground true image μ_{true} over the selected ROI, n indexes the pixels in the ROI, and Q denotes the number of pixels within the ROI. The UQI measures the similarity degree between the reconstructed image and the ground true image, and its value ranges from 0 to 1. A UQI value closer to 1 suggests better similarity to the ground true image. In this study, we selected a ROI of the lesion indicated by the red dashed-line box in figure 1(a). Figure 4 shows the UQI comparison among the FBP, PWLS-TV, PWLS-TGV, and the proposed PWLS-MSTV algorithms. The result demonstrates that the reconstructed image by the PWLS-MSTV algorithm has higher similarity than the reconstructed images by other three algorithms in all cases of 30-, 40-, and 60- views.

4.1.5. Convergence analysis—To analyze the convergence of the present PWLS-MSTV algorithm, the relative root mean square error (rRMSE) metric was used in this study. Also, the rRMSE metric which measures the accuracy of the reconstructed image was used as the stop criterion to find a stable convergent solution. The rRMSE is defined as follows:

$$\text{rRMSE} = \sqrt{\frac{\sum_{n=1}^Q (\mu(n) - \mu_{\text{true}}(n))^2}{\sum_{n=1}^Q (\mu_{\text{true}}(n))^2}} \quad (24)$$

where $\mu(n)$ and $\mu_{\text{true}}(n)$ denote the reconstructed and true attenuation coefficient at pixel n , respectively. Q is the total number of image pixels of the desired image. A small rRMSE value indicates a small difference value between two images and vice versa. Figure 5 shows the plots of the rRMSE versus the iteration step for the proposed PWLS-MSTV algorithms from different sparse-views (30-, 40-, and 60- view). It can be seen that the PWLS-MSTV algorithm can converge to a steady solution after enough iterations in terms of the rRMSE metric in all three cases of 30-, 40-, and 60- views. The results also illustrate that the PWLS-MSTV algorithm can successfully minimize the objective function with a satisfactory solution in all three cases.

4.2. Physical Phantom Study

4.2.1. Visualization-based evaluation—Figure 6 shows the images reconstructed by different algorithms from 116-view projection at 100 mAs. It can be seen that serious artifacts and noise existed in the FBP result, while the PWLS-based algorithms achieved better artifact suppression. Comparing three PWLS-based algorithms, PWLS-TGV and PWLS-MSTV show noticeable gains in suppressing patchy artifact appeared in the PWLS-TV result. Moreover, PWLS-MSTV yielded better edge-preservation than PWLS-TGV. The results suggest that PWLS-MSTV achieves best performance among three PWLS-based algorithms in terms of both noise and artifacts reduction and edge-preservation.

4.2.2. Normal vector flow evaluation—To further evaluate the performance of the PWLS-MSTV algorithm, a small ROI from the results displayed in figure 6 were selected and used for the illustration of the normal vector flow (NVF) images, as shown in figure 7. The NVF image of the FBP reconstruction from the averaged full-view (1160-view) sinogram data of the scans repeated 150 times was drawn as gold standard in figure 7(a). Figures 7(b-e) were the NVF images of reconstructed results by the FBP, PWLS-TV, PWLS-TGV and PWLS-MSTV algorithms, respectively. The gradual changes in the intensities of the desired image are often shown as ordered arrows in NVF images, whereas the noise in the image is often shown as disordered arrows, as shown in figures 7(a) and 7(b). Comparing the NVF images of different algorithms to the gold standard, we can observe that PWLS-MSTV shows the remarkable recovery from the disordered arrows, which indicates PWLS-MSTV has the best preservation of small textures of the reconstructed image.

5. Conclusion

To improve the performance of TV regularization for the sparse-view CT reconstruction, we introduced an improved TV-based minimization algorithm, namely PWLS-MSTV algorithm, which integrates TV minimization and Mumford–Shah segmentation. The proposed PWLS-MSTV algorithm jointly solves the CT image reconstruction and segmentation simultaneously. The Mumford–Shah segmentation information could make good the loss resulted by TV regularization. And these reconstruction and segmentation results mutually promote. Experimental results demonstrated that the proposed PWLS-MSTV algorithm can attain notable gains over the PWLS-TV and PWLS-TGV algorithms in terms of noise reduction, CNR measure and edge-preservation.

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References

1. Brenner D, Hall E. Computed Tomography—an increasing source of radiation exposure. *The New England Journal of Medicine*. 2007; 357:2277–2284. [PubMed: 18046031]
2. Einstein A, Henzlova M, Rajagopalan S. Estimating risk of cancer associated with radiation exposure from 64-slice Computed Tomography coronary angiography. *The Journal of the American Medical Association*. 2007; 298(3):317–323. [PubMed: 17635892]
3. Zhang H, Park J, Chen Y. A novel method for 4D cone-beam computer-tomography reconstruction. *Spie Medical Imaging*. 2015; 9413(24):1–13.
4. Kudo H, Suzuki T, Rashed E. Image reconstruction for sparse-view CT and interior CT—introduction to compressed sensing and differentiated backprojection. *Quantitative Imaging in Medicine and Surgery*. 2013; 3(3):147–161. [PubMed: 23833728]
5. Kalra M, Maher M, Toth T, Hamberg L, et al. Strategies for CT radiation dose optimization. *Radiology*. 2004; 230:619–628. [PubMed: 14739312]
6. McCollough C, Bruesewitz M, Kofler J. CT dose reduction and dose management tools: Overview of available options. *Radiographics*. 2006; 26:503–512. [PubMed: 16549613]
7. Yu L. Radiation dose reduction in computed tomography: techniques and future perspective. *Imaging Med*. 2009; 1:65–84. [PubMed: 22308169]
8. Sidky E, Kao C, Pan X. Accurate image reconstruction from few-views and limited-angle data in divergent beam CT. *J X-Ray Sci Technol*. 2006; 14:119–39.
9. Sidky E, Pan X. Image reconstruction in circular cone-beam Computed Tomography by constrained, total-variation minimization. *Physics in Medicine and Biology*. 2008; 53(17):4777–4807. [PubMed: 18701771]
10. Chen B, Zou Q, Li Y. A New Image Segmentation Model with Local Statistical Characters Based on Variance Minimization. *Applied Mathematical Modeling*. 2015; 39(12):3227–3235.
11. Niu S, Gao Y, Bian Z, Huang J, et al. Sparse-view x-ray CT reconstruction via total generalized variation regularization. *Physics in Medicine and Biology*. 2014; 59(12):2997–3017. [PubMed: 24842150]

12. Gao Y, Bian Z, Huang J, et al. Low-dose X-ray computed tomography image reconstruction with a combined low-mAs and sparse-view protocol. *Optics Express*. 2014; 22(12):15190–15210. [PubMed: 24977611]
13. Liu Y, Ma J, Fan Y, et al. Adaptive-weighted total variation minimization for sparse data toward low-dose X-ray computed tomography image reconstruction. *Physics in Medicine and Biology*. 2012; 57(23):7923–7956. [PubMed: 23154621]
14. Liu Y, Liang Z, Ma J, et al. Total variation stokes strategy for sparse-view X-ray CT image reconstruction. *IEEE Transactions on Medical Imaging*. 2014; 33(3):749–763. [PubMed: 24595347]
15. Chen B, Zou Q, Chen W, et al. A Novel Adaptive Partial Differential Equation Model for Image Segmentation. *Applicable Analysis*. 2014; 93(11):2440–2450.
16. Chen B, Chen W. Noisy Image Segmentation Based on Wavelet Transform and Active Contour Model. *Applicable Analysis*. 2011; 90(8):1243–1255.
17. Shah J. A common framework for curve evolution, segmentation and anisotropic diffusion. *Proceedings of IEEE Conference on Computer Vision and Pattern Recognition*. 1996:136–142.
18. Cai X, Chan R, Zeng TA. Two-Stage Image Segmentation Method Using a Convex Variant of the Mumford--Shah Model and Thresholding. *Siam Journal on Imaging Sciences*. 2013:368–390.
19. Li F, Ng M, Li C. Variational Fuzzy Mumford--Shah Model for Image Segmentation. *Siam Journal on Applied Mathematics*. 2010; 70(7):2750–2770.
20. Bar L, Sochen N, Kiryati N. Semi-blind image restoration via Mumford-Shah regularization. *IEEE Transactions on Image Processing*. 2006; 15(2):483–493. [PubMed: 16479818]
21. Rudin L, Osher S, Fatemi E. Nonlinear total variation based noise removal algorithms. *Physica D: Nonlinear Phenomena*. 1992; 60(1-4):259–268.
22. Bar L, Brook A, Sochen N, Kiryati N. Deblurring of color images corrupted by impulsive noise. *IEEE Transactions on Image Processing*. 2007; 16(4):1101–1111. [PubMed: 17405440]
23. Alicandro R, Braides A, Shah J. Free discontinuity problems via functional involving the norm of the gradient and their approximation. *Interfaces Free Bound*. 1999; 1:17–37.
24. Ma J, Liang Z, Fan Y, et al. Variance estimation of X-ray CT sinogram in radon domain. *Proc SPIE*. 2012; 8313:83132G.
25. Ma J, Liang Z, Fan Y, et al. Variance analysis of x-ray CT sinograms in the presence of electronic noise background. *Med Phys*. 2012; 39:4051–4065. [PubMed: 22830738]
26. Wang J, Li T, Lu H, et al. Penalized weighted least-squares approach to sinogram Noise reduction and image reconstruction for low-dose X-ray computed tomography. *IEEE Transactions on Medical Imaging*. 2006; 25(10):1272–1283. [PubMed: 17024831]
27. Ma J, Zhang H, Gao Y, et al. Iterative image reconstruction for cerebral perfusion CT using a pre-contrast scan induced edge-preserving prior. *Phys Med Biol*. 2012; 57:7519–7542. [PubMed: 23104003]
28. Elbakri I, Fessler J. Statistical image reconstruction for polyenergetic X-ray computed tomography. *IEEE Transactions on Medical Imaging*. 2002; 21(2):89–99. [PubMed: 11929108]
29. Weisstein, E. Minimal residual method. *MathWorld—A Wolfram Web Resource*. [Online]. Available: <http://mathworld.wolfram.com/MinimalResidualMethod.html>
30. Segars W, Sturgeon G, Mendonca S, Grimes J, Tsui B. 4D XCAT phantom for multimodality imaging research. *Med Phys*. 2010; 37:4902–15. [PubMed: 20964209]
31. Wang Z, Bovik A. A universal image quality index. *IEEE Signal Processing Letters*. 2002; 9(3): 81–84.

Figure 1.

Digital and physical phantoms used in the studies: (a) a slice of digital XCAT phantom that contains head anatomy structures; (b) the physical phantom image reconstructed by the FBP method with ramp filter based on the averaged sinogram data of scans repeated 150 times at a protocol of 100 mAs and 120 kVp.

Figure 2.

The XCAT phantom images reconstructed by the FBP with ramp filter, PWLS-TV, PWLS-TGV, and PWLS-MSTV algorithms from 30-, 40-, and 60- view projections, respectively. The zoomed region was indicated by the red dashed square in figure 1(a). All images are displayed in the same window.

Figure 3.

Vertical profiles (indicated by the blue solid line in figure 1(a)) of the images reconstructed by the PWLS-TV, PWLS-TGV and PWLS-MSTV algorithms from different projections: (a) 30-view projection, (b) 40-view projection, (c) 60-view projection.

Figure 4.

The UQIs of lesion ROI indicated by the red dashed-line box in figure 1(a) from the reconstructed image by the FBP, PWLS-TV, PWLS-TGV, and PWLS-MSTV algorithms in the case of 30-, 40-, and 60- views.

Figure 5.

Plots of the rRMSE versus the iteration step for the PWLS-MSTV algorithm from different sparse-views: (a) is from 30-view projection; (b) is from 40-view projection; (c) is from 60-view projection.

Figure 6. Images reconstructed from 116-view projection at 100 mAs by different algorithms: (a) FBP with ramp filter, (b) PWLS-TV, (c) PWLS-TGV and (d) PWLS-MSTV. All images are displayed in the same window.

Figure 7.

NVF images of the reconstructed results by the FBP, PWLS-TV, PWLS-TGV and PWLS-MSTV algorithms from 116-view projection at 100 mAs: (a) the gold standard, (b) FBP, (c) PWLS-TV, (d) PWLS-TGV, and (e) PWLS-MSTV.

Table 1

PSNRs (dB) of the reconstructed images by three different algorithms.

Views	PWLS-TV	PWLS-TGV	PWLS-MSTV
30-view	34.8248	36.0283	36.5586
40-view	37.6215	38.5319	38.8995
60-view	39.1314	40.1056	40.5901

Table 2

CNRs of the reconstructed results by three different algorithms.

Views	PWLS-TV	PWLS-TGV	PWLS-MSTV
30-view	23.19	24.17	24.85
40-view	28.73	29.85	30.47
60-view	43.79	45.23	45.86