An examination of Alternative Multidimensional Scaling Techniques

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Sofia Papazoglou¹ and Kostas Mylonas¹

Abstract

The purpose of this study is to compare alternative multidimensional scaling (MDS) methods for constraining the stimuli on the circumference of a circle and on the surface of a sphere. Specifically, the existing MDS-T method for plotting the stimuli on the circumference of a circle is applied, and its extension is proposed for constraining the stimuli on the surface of a sphere. The data analyzed come from previous research and concerns Maslach and Jackson's burnout syndrome and Holland's vocational personality types. The configurations for the same data on the circle and the sphere shared similarities but also had differences, that is, the general item-groupings were the same but most of the differences across the two methods resulted in more meaningful interpretations for the three-dimensional configuration. Furthermore, in most cases, items and/or scales could be better discriminated from each other on the sphere.

Keywords

constrained multidimensional scaling, circular and spherical constraints on MDS solutions, extension of MDS-T on the sphere, quadrant-specific arctangent transformations, burnout syndrome, Self-Directed Search

Introduction

Multidimensional scaling (MDS) aims to uncover the underlying structure in a proximity matrix by producing a simple geometrical model resembling a map, such that

¹National and Kapodistrian University of Athens, Greece

Corresponding Author:

Sofia Papazoglou, Department of Psychology, National and Kapodistrian University of Athens, Panepistimiopolis, Ilisia, Athens 15784, Greece.
Email: sofia.p@ppp.uoa.gr

the distances in the resulting configuration fit the raw data as well as possible (Dillon & Goldstein, 1984). The overall purpose of this article is to apply alternative MDS methods in order to depict the same data on a circular continuum and on a spherical surface and describe them in comparison to each other.

Plotting the stimuli on the circumference of a circle and on the surface of a sphere through MDS are types of constrained MDS analysis. The terms “constrained MDS” and “confirmatory MDS” (Borg & Groenen, 2005; Cox & Cox, 2001; Heiser & Meulman, 1983) refer to situations when additional constraints are imposed on the configuration, except for general constraints like the dimensionality and the type of analysis (metric or nonmetric; Borg & Groenen, 2005; Borg & Lingoes, 1980). These constraints refer to structural hypotheses or conditions that the derived configuration should satisfy so that the researcher may be able to test for these hypotheses. These can be tested by incorporating them as additional constraints in the MDS analysis. For example, in a study by Sidiropoulou-Dimakakou, Mylonas, and Argyropoulou (2008), Holland’s (1985) RIASEC types were constrained to circular arrangement. If the configuration with additional structural constraints is almost as good or better in terms of fit in comparison to a configuration without such constraints, then the hypotheses can be considered compatible with the data (Bentler & Weeks, 1978; Borg & Groenen, 2005; Borg & Lingoes, 1980). External constraints on the MDS configuration can be also useful in a more exploratory context. The derived configuration from an unrestricted MDS analysis may have some rather unattractive properties (Borg & Lingoes, 1980). For example, when the stimuli subjected to MDS analysis are designed to differ in terms of certain attributes and the axes of the unconstrained configuration do not correspond exactly to these attributes, the researcher cannot be certain whether these discrepancies are due to random error or due to some nonrandom effect (Bloxom, 1978). Before interpreting the solution, one may prefer to rescale the data with a constrained method (Borg & Lingoes, 1980). The types of constraints addressed in different MDS methods include equality restrictions on coordinates or distances, fixing parameters to a priori values, or estimating the parameters of an MDS solution so as to yield a specific geometrical structure (Bentler & Weeks, 1978; Bloxom, 1978; Borg & Groenen, 2005; Borg & Lingoes, 1980; de Leeuw & Mair, 2009; Lee, 1984; Lee & Bentler, 1980).

Constraining the MDS solution on a circle or on a sphere can be theoretically meaningful and aid the researcher in further stages of his analyses. In some domains the theoretical structure is supposed to be circular or spherical and discrepancies from it may be considered as “error.” Bimler and Kirkland (2005) refer to the different distances of items from the origin in an MDS solution as “specificity,” which they claim is approximately constant in a well-designed set of items. The theoretically expected structure is a circular or spherical arrangement, for example, when analyzing color-similarity data (the color circle describing perception of colors with differing wavelength) or similarities between nations (the globe is spherical; Borg & Lingoes, 1980; Cox & Cox, 1991, 2001; de Leeuw & Mair, 2009; Lee & Bentler, 1980). Further examples include models related to Prediger’s (1982) hypothesis

about the structure of Holland's (1985) vocational personality types (e.g., Rounds & Tracey, 1993), where the theoretical presupposition is that the points lie on the circumference of a circle, the spherical model of vocational interests (Tracey & Rounds, 1996) and circular models of the interpersonal domain (Gurtman & Balakrishnan, 1998). Analysis of circumplex models (e.g., the *circulant model*, the *geometric circulant model*, and the *quasi-circumplex model* as described by Tracey, 2000) is an issue related to circular and spherical MDS structures. Such models can be studied through different methods including structural equation modeling and constrained MDS (e.g., Darcy & Tracey, 2007; Rounds & Tracey, 1993). The gain of constraining the MDS configuration on a circle or on a sphere in cases like the ones just mentioned is that the final solution is closer to the original theory, as the hypothesis of circular or spherical arrangement of points is met, and the similarities and differences of the final solution to the theoretically expected can be meaningfully described. In their paradigm of this, Sidiropoulou-Dimakakou et al. (2008) analyzed data with respect to Holland's hexagonal vocational personality model through MDS. In the unconstrained two-dimensional solution, the arrangement of the RIASEC types was approximately circular, but the different radial distances of the points did not aid interpretation, as the theory presupposes that the six types lie on a circular continuum. Consequently, the axes of this configuration could not be easily interpreted as dimensions. The constrained circular solution was closer to the original theory (the circular property was met), and as a result its similarities and differences to Holland's equilateral hexagon could be described for the specific sample (e.g., the main finding was that the Realistic and Investigative types had much smaller distance than expected).

The circular or spherical constraints can also be used as a way to obtain homogeneous groups of items, stimuli, individuals, and so on, for purposes of better interpreting the solution. Kruskal and Wish (1978) refer to the process of interpreting the derived MDS configuration by grouping stimuli that are close to each other, as neighborhood interpretation (they use cluster analysis applied to the proximity matrix in order to find the item-groups). Another example of a technique for obtaining groups of similar stimuli is Latent Class MDS (Vera, Macías, & Heiser, 2009), where the stimuli are partitioned into classes and the cluster centers are represented in a low-dimensional space. Such grouping of stimuli can also be achieved by locating the nearby points on the circumference of the circle (or on the surface of a sphere) in a constrained circular (or spherical) configuration, and describing their common characteristics that differentiate them from other such groups.

According to Guilford's (1954) homogeneity hypothesis, analyzing homogeneous groups of individuals can lead to bringing out the structure of the data more clearly than when these groups are heterogeneous. This kind of homogeneous groups are useful for bias reduction purposes in comparisons between groups. Thus, when homogeneous groups are compared to each other instead of single units, any similarity or difference that exists between these groups can become apparent because of the reduction of error within the homogeneous groups. For example, in Mylonas et al.

(2011), constrained MDS on a circle is used in order to form homogeneous groups of countries, where the countries comprising each group on the circle had similar factor structures with respect to some construct (work values). These country groups were then tested for factor structure equivalence with respect to another (correlate) construct (Person-Job Fit; Brkich, Jeffs, & Carless, 2002), and this country-group comparison resulted in reduction of “bias in terms of culture,” as opposed to comparison of separate countries where bias was present.

The points of an MDS configuration can be constrained to lie on a circular continuum or on a spherical surface as a special case of imposing constraints (Borg & Lingoes, 1980; Cox & Cox, 1991; de Leeuw & Mair, 2009; Lee, 1984; Lee & Bentler, 1980). The alternative MDS methods for imposing circular and spherical constraints, which are employed in this article, are described in the following sections.

Constraining the MDS Configuration to a Circular Structure and a Spherical Structure

The MDS-T method for constraining a two-dimensional configuration on the circumference of a circle has been proposed by the second author (Mylonas, 2009; Mylonas et al., 2011). Information with respect to the distance of each stimulus from the origin is not used, in order to simplify the patterns present on the configuration and facilitate interpretation. Discrete homogeneous groups of stimuli can become apparent through the grouping of points located on the same or neighboring positions on the circumference. In the MDS-T method, an unconstrained MDS solution in two dimensions is first calculated for a given data set (e.g., with the ALSCAL algorithm; MacCallum, 1981; Takane, Young, & de Leeuw, 1977). The symbols x and y are used to denote the first and second coordinates of a stimulus, respectively. A trigonometric transformation is applied to these coordinates in order to determine the position of the stimuli on the circumference of the circle. This position is determined by computing for each stimulus the angle (clockwise or counterclockwise) defined by the line connecting the respective stimulus with the axes-origin and the positive x -axis. The angles are calculated by employing the quadrant-specific arctangent function for every set of x - and y -coordinates. This trigonometric transformation yields angles expressed in radians with a range of $-\pi$ to π , which are subsequently converted to degrees with a range of -180 to 180 and plotted on the circumference of a circle. The computational procedures as presented in Mylonas (2009) are as follows:

Given that

$$\text{degrees} = \text{radians} \frac{180}{\pi}, \quad (1)$$

$$\text{sgn}(a) = \frac{a}{|a|}, \alpha \in \Re \quad (2)$$

and the formula for computing the tangent of an angle ϕ

$$\tan(\varphi) = \frac{y}{x}, \quad (3)$$

where x and y the coordinates of a point in two dimensions, the quadrant-specific arctangent value with range $(-\pi, \pi)$ for a stimulus is calculated using the following equations:

$$\text{for } y \neq 0 \text{ and } x > 0, \quad \text{radians}(x, y) = \tan^{-1} \left(\left| \frac{y}{x} \right| \right) [\text{sgn}(y)], \quad (4)^1$$

$$\text{for } y \neq 0 \text{ and } x = 0, \quad \text{radians}(x, y) = \left(\frac{\pi}{2} \right) [\text{sgn}(y)], \quad (5)$$

$$\text{for } y \neq 0 \text{ and } x < 0, \quad \text{radians}(x, y) = \left\{ \pi - \left[\tan^{-1} \left(\left| \frac{y}{x} \right| \right) \right] \right\} [\text{sgn}(y)] \quad (6)$$

$$\begin{aligned} &\text{and for } y = 0 \text{ and } x \neq 0, \text{ if } x > 0, \quad \text{radians}(x, y) = 0, \\ &\text{or if } x < 0, \quad \text{radians}(x, y) = \pi. \end{aligned} \quad (7)$$

The derived angles in radians are transformed to degrees and are then plotted on the circumference of a circle. A simple example of the computations involved in the two-dimensional MDS-T method is presented in the appendix.

Extension of the MDS-T Method on a Sphere

In this article, based on the Mylonas' MDS-T method, the first author proposes its extension to three dimensions. In order to plot a specific data set on the surface of a sphere, an unconstrained MDS solution in three dimensions is first computed (e.g., through ALSCAL). If x , y , and z are used to denote the resulting coordinates of a stimulus on the first, the second, and the third dimensions, respectively, these coordinates can be transformed into two angles that determine the location of each stimulus on the spherical surface. The first one of these angles expresses the distance of the stimulus (clockwise or counterclockwise) from the positive x -axis as measured on the circumference of the "equator" (similar to longitude). This angle is computed from the x and y coordinates with the MDS-T method for a circle. The second angle is the latitude of the stimulus and expresses the height in which the point is located above or below the "equator," that is, the parallel circle on the circumference of which the stimulus will be plotted. It is the angle defined by the line connecting the stimulus with the center of the sphere and the xy -plane. In order to find the latitude of a point given the Cartesian coordinates x , y , and z , the hemisphere-specific arctangent in radians is computed for the coordinates $(x^2 + y^2)^{1/2}$ and z (Hapgood, 1992). The quantity $(x^2 + y^2)^{1/2}$ is the distance of the projection of a stimulus on the xy -plane from the origin, that is, the distance of the stimulus from the origin if there were only two dimensions. The angle that expresses the latitude of the point ranges from $-1/2\pi$ to $1/2\pi$ radians and can be converted to degrees (with range -90 to 90) when

Table 1. Hypothetical Example for MDS-T in Three Dimensions.

Item	Equations applied	x	y	z	$(x^2 + y^2)^{1/2}$	Radians, xy-plane	Degrees, xy-plane	Radians, height	Degrees, height
a	(4), (8)	1.1	2.4	2.0	2.64	1.14	65	0.65	37
b	(4), (8)	1.4	-2.3	-2.0	2.693	-1.02	-59	-0.64	-37
c	(5), (8)	0.0	1.2	1.5	1.2	1.57	90	0.90	51
d	(9)	0.0	0.0	-1.5	0.0	N/A	N/A	-1.57	-90
e	(9)	0.0	0.0	1.6	0.0	N/A	N/A	1.57	90

multiplied by $(180/\pi)$. Based on Hapgood (1992) the computations for the latitude of a stimulus are as follows:

$$\text{for } (x^2 + y^2)^{1/2} \neq 0 \quad \text{radians}(x, y, z) = \tan^{-1} \left(\left| \frac{z}{(x^2 + y^2)^{1/2}} \right| \right) [\text{sgn}(z)] \quad (8)$$

$$\text{and for } (x^2 + y^2)^{1/2} = 0 \quad \text{radians}(x, y, z) = \left(\frac{\pi}{2} \right) [\text{sgn}(z)]. \quad (9)$$

The angles computed in the MDS-T method for a circle and its extension for a sphere are the result of a mathematical transformation of the unconstrained solution on the circular continuum or sphere. The property preserved in the transformed solution is the relative distances of the points irrespective of the distance from the origin in order to simplify the structure. They are not estimates of population parameters and their statistical properties in terms of use with psychological data are not known. A word of caution though is that MDS solutions may vary depending on rescaling methods initially imposed on the raw data; the constrained configuration (MDS-T) is obviously dependent on this rescaling research decision as well. It is also true that by constraining the MDS configuration on a circle or sphere some information may be lost with respect to the relative positions of stimuli along dimensions, but in exchange for this loss it is much easier to decide which group each stimulus belongs to.

The calculations involved in MDS-T for sphere are illustrated using a set of hypothetical three-dimensional MDS coordinates (unconstrained) for five stimuli-items (“a” to “e”), as presented in Table 1. Item “a” has positive *x*- and *y*-coordinates and applying Equation (4) results in 65° for the “longitude.” The quantity $(x^2 + y^2)^{1/2}$ is positive, so Equation (8) is used to compute its latitude. From Equation (8), we get

$$\begin{aligned} \text{Latitude in radians} &= \tan^{-1} \left(\left| \frac{2}{(1.1^2 + 2.4^2)^{1/2}} \right| \right) [\text{sgn}(2)] = \tan^{-1} \left(\left| \frac{2}{2.64} \right| \right) * \left(\frac{2}{|2|} \right) = \\ &\tan^{-1}(.76) * (+1) = .65 * (+1) = .65 \text{ radians,} \end{aligned}$$

that is, $.65 * (180/\pi) \cong 37^\circ$ above the equator.

Item “b” has a positive x -coordinate and a negative y -coordinate, and applying Equation (4) results in -59° for the longitude dimension. The quantity $(x^2 + y^2)^{1/2}$ is positive, so the latitude is computed applying Equation (8), as follows:

$$\text{Latitude in radians} = \tan^{-1} \left(\left| \frac{-2}{[1.4^2 + (-2.3)^2]^{1/2}} \right| \right) [\text{sgn}(-2)] = \tan^{-1} \left(\left| \frac{-2}{2.69} \right| \right) * \left(\frac{-2}{|-2|} \right) = \tan^{-1}(.74) * (-1) = .64 * (-1) = -.64 \text{ radians,}$$

that is, $-.64 * (180/\pi) \cong -37^\circ$, that is, 37° below the equator.

Item “c” has an x -coordinate equal to zero and positive y - and z -coordinates. Its longitude, based on Equation (5) is 90° . The latitude is computed in the same way as for the previous items and equals 51° above the equator.

Item “d” in the fourth row of Table 1 has an x -coordinate equal to zero, a y -coordinate equal to zero, and a negative z -coordinate. Item “e” in the fifth row of Table 1 has identical x - and y -coordinates and a positive z -coordinate. Since Items “d” and “e” are on the z -axis ($x = y = 0$), the longitude dimension is not considered (its projection is in the center of the sphere). Also, the quantity $(x^2 + y^2)^{1/2}$ is zero for both items. The latitude for Item “d,” applying Equation (9), is computed as follows:

$$\text{Latitude in radians} = \left(\frac{\pi}{2} \right) [\text{sgn}(-1.5)] = \left(\frac{\pi}{2} \right) * \left(\frac{-1.5}{|-1.5|} \right) = (1.57) * (-1) = -1.57 \text{ radians,}$$

that is, $-1.57 * (180/\pi) \cong -90^\circ$ (the item is on the “south pole”).

For Item “e,” the latitude in radians is

$$\left(\frac{\pi}{2} \right) [\text{sgn}(1.6)] = \left(\frac{\pi}{2} \right) * \left(\frac{1.6}{|1.6|} \right) = 1.57 * (+1) = 1.57 \text{ radians,}$$

that is, $1.57 * (180/\pi) \cong 90^\circ$ (the item is on the “north pole”).

An alternative method for graphically representing three-dimensional MDS configurations has been proposed by Bimler, Kirkland, and their colleagues (Bimler & Kirkland, 2001, 2003, 2005; Harvey, Bimler, Evans, Kirkland, & Pechtel, 2012; Kirkland, Bimler, Drawneek, McKim, & Schölmerich, 2004; Stewart, Hill, Stewart, Bimler, & Kirkland, 2005). The MDS coordinates are first transformed on the surface of a sphere and the resulting outcomes are displayed in separate two-dimensional plots for each hemisphere, created through stereographic projection (e.g., Snyder, 1987). In this type of graphic projection, the relative distances of the stimuli on the sphere are not preserved exactly, as pairs of stimuli with identical distances on the spherical surface have somewhat different distances on the two-dimensional projection, depending on their location with respect to the poles. However, the focus of this method is the graphic depiction of the relationships between items that lie on the

surface of a sphere, in two dimensions. In another alternative way to graphically present the MDS solution transformed on the sphere (Bimler, Kirkland, Yuhara, Kurosaki, & Coxhead, 2005), the longitude and latitude of the stimuli are depicted as x - and y -axes on a two-dimensional plane. The relative distances of the stimuli as graphically presented on the sphere are not preserved on the plane, since the parallel circles, with different radii, are represented as parallel lines with the same length on the plane. In this approach also, the emphasis is in the depiction of a spherical plot as a two-dimensional configuration.

MDS-T Applications

The MDS-T method for a circle and its extension for a sphere can be used by a researcher for many purposes. As noted earlier in the introduction, the theoretical structure in some content areas is expected to be circular or spherical, so the transformation of the configuration on the circumference of a circle or on the surface of a sphere is a constraint in line with the original theory. An example of this, as also mentioned previously, is the study by Sidiropoulou-Dimakakou et al. (2008) concerning Holland's vocational personality types, constrained to lie on a circle.

Another use of the MDS-T methods is obtaining homogeneous groups of items, stimuli, individuals, and so on, for descriptive and interpretative purposes. These homogeneous groups consist of the stimuli that are located in nearby positions on the circle or the sphere. Examples here are the study by Veligekas, Mylonas, and Zervas (2007); the study by Mylonas (2009); and the study by Gari, Mylonas, and Portešová (2015). In the first one of these, MDS-T on the circle was applied in order to find possible homogeneous groups of composite scores in the field of goal orientation and beliefs about the causes of sport success. In the Mylonas (2009) study, homogeneous groups of university departments were identified on the circular continuum. The input data for MDS concerned the pairwise dissimilarities between departments, expressing how similar each pair of departments was with respect to the factor structures of their students' religious values. In the third study (Gari et al., 2015), MDS-T was used in order to identify homogeneous groups of teachers' attitudes toward gifted students with learning difficulties in two countries (Greece and Czech Republic). In all three studies, the homogeneous groups in which the MDS-T method resulted were interesting and theoretically meaningful.

The homogeneous groups from an MDS-T analysis can also be used for bias reduction purposes (see the respective discussion earlier on circular and spherical constraints). In the studies by Gari, Mylonas, and Panagiotopoulou (2009) on social axioms and by Mylonas et al. (2011) on work-related variables, MDS-T on a circle was applied in order to obtain homogeneous groups of countries. In both of the studies, this grouping of countries resulted in reduced "bias in terms of culture" in further comparisons of the country groups. That is, comparisons of the country groups showed equivalent factor structures for these groups concerning the constructs of interest (reduced bias), when the factor structures were not equivalent (bias present) for individual countries compared to each other.

Research Illustration

According to this study's aims, the MDS-T techniques will be applied in order to plot the same data from previous research on the circumference of a circle and on the surface of a sphere. These constrained configurations will be examined in terms of fit with the data and in terms of interpretability. Regarding goodness of fit, the circular or spherical configuration can be justified if its fit with the data is acceptable and not substantially worse than the fit of the unconstrained configuration with the same number of dimensions. In addition to goodness of fit, interpretability is also important for the decision of which configuration the researcher should choose for a particular dataset (Borg & Groenen, 2005; Everitt, 1996). With respect to this aspect of a solution, a first aim is to examine whether the circular or the spherical configuration for the same data is more interpretable in terms of psychological meaning. Another aim is to examine whether plotting the stimuli on the sphere can provide new information about the underlying structure in addition to the information provided by the circular configuration.

Some steps are first necessary to decide when to choose the constrained (MDS-T) over the unconstrained solution, and also when the two- or three-dimensional solution is preferable, as researcher guidelines. One needs to (1) compute an unconstrained configuration in two dimensions, and examine it in terms of goodness-of-fit and interpretability; (2) transform the coordinates on the circle through MDS-T, and examine goodness-of fit and interpretability; (3) repeat Steps 1 and 2 for three dimensions, that is, compute an unconstrained and an MDS-T configuration, and examine them in terms of fit and interpretability; (4) if the MDS-T solution (two- or three-dimensional) is acceptable in terms of fit (the stress measure has an acceptable value, not substantially worse than for the respective unconstrained solution) and if this constrained configuration is theoretically more meaningful, it may be preferable over the unconstrained one; and (5) finally, when choosing between the two- or three-dimensional MDS-T solution, fit and interpretability should also be considered.

Concerning the issue of interpretability, the third dimension is insightful (1) when the differences with respect to the two-dimensional solution are substantively meaningful and this new information is not apparent on the two-dimensional solution or (2) when the positions of some points/items are difficult to interpret on the two-dimensional configuration, and are theoretically meaningful on the three-dimensional configuration. The gain in MDS-T as compared to the unconstrained traditional methods is that the patterns are simpler, as all the points lie on the same continuum/surface. In the three-dimensional solution before MDS-T (unconstrained) we have to examine the positions of the points along all three dimensions, in order to decide whether the third dimension adds something to understanding the structure of the data.

Data Sets Employed

The first data set concerns the burnout syndrome (Koliadis, Mylonas, Koubias, Varfi, & Valsami, 2000) assessed through the Maslach Burnout Inventory (MBI;

Maslach & Jackson, 1986). This 22-item questionnaire consists of three subscales, labeled, Emotional Exhaustion, Depersonalization, and Personal Accomplishment. Each item is answered using a 7-point scale, expressing how often the respondent experiences its content. Maslach and Jackson (1986) reported internal consistency reliability coefficients that reached .90 for the Emotional Exhaustion subscale, .79 for the Depersonalization subscale, and .71 for the Personal Accomplishment subscale.

The sample in the Koliadis et al. (2000) study consisted of 415 participants, 111 (26.7%) males and 304 (73.3%) females. The mean age of the participants was 35.4 years ($SD = 7.03$, ranging from 20 to 65 years). For males, the mean age was 38.2 years ($SD = 8.08$, range = 24-59 years) and females had a mean age of 34.4 years ($SD = 6.31$, range = 20-65 years).

The second data set comes from a study by Sidiropoulou-Dimakakou et al. (2008) concerning Holland's hexagonal personality model. The instrument administered was the Greek version of Holland's Self-Directed Search (SDS; Sidiropoulou-Dimakakou, 2010), which is designed to assess the six RIASEC vocational personality types. The analyses of the present article were conducted on data for the Activities subscale, containing 66 items measuring all six personality types, answered on a binary basis. In the Sidiropoulou-Dimakakou et al. (2008) study internal consistency reliability coefficients ranged from .67 to .76 for this subscale for the six types separately. This study's sample consists of 152 students, 93 (61.2%) females and 59 (38.8%) males. Concerning the level of their studies, 80 students (52.6%) were at the undergraduate level and 72 students (47.4%) were at the post-graduate level. The mean age of the respondents ($N = 150$) was 24.5 years ($SD = 5.52$, range = 19-50 years).

Data Analyses

The data sets described above were analyzed by two different methods. First, non-metric MDS in two dimensions was performed using the ALSCAL algorithm. The resulting configuration was trigonometrically transformed on the circumference of a circle with the MDS-T method. Second, ALSCAL was used to obtain a nonmetric MDS solution in three dimensions and the resulting configuration was transformed on the surface of a sphere with the method proposed in this article. The outcomes were plotted using specifically designed computer software.² For both methods of analysis (MDS-T for circle, MDS-T for sphere) the Euclidean distance metric was used as a dissimilarity measure between pairs of items.

Results

Concerning the Burnout data, the unconstrained two-dimensional MDS solution fitted the dissimilarity data well (Young's S -stress = .01569, Kruskal's stress = .04399, $R^2 = .99473$). The S -stress and stress values for the constrained MDS-T (circle) solution

Figure 1. Configuration of the Burnout items on the circumference of a circle.

Note. EE = Emotional Exhaustion; DP = Depersonalization; PA = Personal Accomplishment.

are somewhat higher (.04166 and .05377, respectively) and the R^2 is a little lower (.99275), but they still indicate a good fit, so the circular configuration is acceptable.

The unconstrained three-dimensional MDS solution had also good fit with the Burnout data (Young's S -stress = .01257, Kruskal's stress = .03344, R^2 = .99653). The S -stress and stress indices (.06308 and .06318, respectively) are higher for the constrained MDS-T (sphere) solution and the R^2 = .98786 has a somewhat lower value, but they still indicate a good fit, so the sphere is an acceptable configuration. The relative positions of the MBI items are shown in Figure 1 (circle) and Figure 2 (sphere).

Two groups of items were apparent on both configurations, located opposite to each other. The first group is formed by the Personal Accomplishment items and one Emotional Exhaustion item (EE14). The second group consists of the Depersonalization items and most Emotional Exhaustion items. There are two items (EE8 and EE16) that are somewhat separated from the two groups. However, on the circle they seem to be a part of the second group, but on the sphere they are not included in the two homogeneous item groups due to their placement in terms of height.

For the SDS data, the two-dimensional unconstrained MDS solution had a marginal fit with the dissimilarities (Young's S -stress = .14716, Kruskal's stress = .14588 and R^2 = .91711). The constrained circular MDS-T solution for these items had higher S -stress and stress values (.20352 and .1722, respectively) and a lower R^2

Figure 2. Configuration of the Burnout items on the sphere.
Note. EE = Emotional Exhaustion; DP = Depersonalization; PA = Personal Accomplishment.

value of .88442, so the fit is not very good. The unconstrained three-dimensional MDS solution had adequate fit (Young’s *S-stress* = .11843, Kruskal’s *Stress* = .10652, R^2 = .94602) and the constrained MDS-T solution on the sphere had somewhat higher *S-stress* and *stress* values (.14963 and .11577) and a somewhat lower R^2 value (.93363), but the deterioration in fit was not very large, so the spherical configuration can be accepted. The angles for the 66 items on the circle and the sphere are presented in Table 2, and the respective plots are shown in Figure 3 (circle) and Figure 4 (sphere).

Examining the placement of the majority of items of each subscale on the circle, it is observed that the configuration resembles a contradistinction between two groups of personality types with the Realistic (R), Conventional (C), and Investigative (I) types forming the first group and the Enterprising (E), Social (S), and Artistic (A) types forming the second group with the R and S types at the opposite ends of a continuum. The types of the first group were very close to each other, and for the second group the S type is in the middle with the A and E types located to its right and the left side respectively.

For the sphere, there seems to be a contradistinction between the Social and Realistic types with most items of these scales located around opposite points. Most items of the C and I subscales lie close to the R type and most items of the A and E subscales are closer to the S type, so there are two groups of types (R-I-C and S-A-E). The placement of items in terms of height gives the impression that the R type is

Table 2. Coordinates in Degrees for SDS Items.

Item	MDS-T Circle	MDS-T for sphere		Item	MDS-T Circle	MDS-T for sphere	
		Height	xy-plane			Height	xy-plane
r1	-6	-5	-2	s1	-148	-21	-152
r2	-2	-4	-1	s2	-102	-72	-130
r3	5	0	8	s3	-155	-32	161
r4	-56	16	-58	s4	-179	0	-179
r5	4	-3	5	s5	-158	-23	-163
r6	-3	-7	-2	s6	175	3	174
r7	-8	-2	-6	s7	180	7	-180
r8	-4	3	-6	s8	178	6	-179
r9	-2	-2	-3	s9	173	9	168
r10	-36	9	-37	s10	146	19	139
r11	-4	-8	-1	s11	178	7	180
i1	-168	3	-165	e1	160	26	167
i2	95	85	110	e2	21	0	22
i3	153	11	152	e3	78	51	72
i4	1	-4	2	e4	116	6	114
i5	10	33	-2	e5	157	-1	151
i6	178	4	179	e6	100	32	99
i7	32	15	30	e7	145	12	144
i8	5	11	5	e8	127	-37	131
i9	10	12	9	e9	175	2	173
i10	-2	11	-4	e10	164	-12	161
i11	92	87	-90	e11	35	3	38
a1	-120	17	-116	c1	-179	-13	179
a2	-173	3	-170	c2	-75	-73	-56
a3	-13	17	-24	c3	8	-7	10
a4	-42	0	-44	c4	43	-54	33
a5	-153	3	-147	c5	17	-37	16
a6	-168	-1	-168	c6	-32	-44	-11
a7	-171	6	-168	c7	7	-4	9
a8	-109	-2	-107	c8	0	-4	2
a9	-146	12	-141	c9	4	-2	4
a10	-153	7	-149	c10	148	-39	152
a11	-150	16	-146	c11	-1	-7	0

Note. r = Realistic; i = Investigative; a = Artistic; s = Social; e = Enterprising; c = Conventional.

located in the middle with the C and I types tending to be located below and above the R type respectively.

Discussion, Limitations, and Overall Conclusions

Concerning the Burnout data set, there are both similarities and differences between the configuration of the MBI items on the circumference of a circle and on the

Figure 3. Configuration of SDS items on the circumference of a circle.

Note. r = Realistic; i = Investigative; a = Artistic; s = Social; e = Enterprising; c = Conventional.

surface of a sphere. Two groups of items were formed in both configurations, consisting of almost the same questions. Based on the items belonging to each group, the first one can be labeled "Personal Accomplishment" and the second group can be labeled "Emotional Exhaustion and Depersonalization." They are located opposite to each other with their points concentrated around two poles of a single dimension. Maslach and Jackson (1986), based on factor structures and correlations, state that the Personal Accomplishment subscale is independent from the other two MBI subscales. In our analysis, which is based on dissimilarities between items, Personal Accomplishment is marking one end of the continuum, showing that this subscale may have some relation to the Emotional Exhaustion and Depersonalization subscales, which are located at the other end of the same continuum. However, the aim of the present study is not theory testing, as our methods are descriptive and the purpose is just to describe the results with reference to the original theory.

Some differences are also observed between the two- and three-dimensional MDS-T configurations of the Burnout data. First, if the groupings of items were used in order to form aggregate scores, the "Emotional Exhaustion and Depersonalization" score would be different for the two configurations, as according to the circular one, items EE8 and EE16 would be considered a part of it. Based on the spherical configuration these items would not be included in this

Figure 4. Configuration of SDS items on the sphere.

Note. r = Realistic; i = Investigative; a = Artistic; s = Social; e = Enterprising; c = Conventional.

composite score. A second difference related to the “Emotional Exhaustion and Depersonalization” group is that, in the case of the sphere its two component subscales do not show any overlap, being on the right side (EE items) and the left side (DP items) of this group, respectively; however, there is a small overlap between the two subscales on the circle, since Item EE1 is placed in the “depersonalization group.” A third difference between the two MDS-T configurations related to the same item group is that on the sphere the height dimension tends to discriminate items concerning people in the upper part of the group, and items dealing with feelings of disappointment, exhaustion, being tired, and so on, due to work, in the lower part of the group. A similar distinction is not apparent on the circumference of the circle.

Concerning the configurations of the SDS items, circular and spherical, it is observed that they have some differences from Holland's (1985) theoretical model. It was expected according to theory to have six homogeneous groups of items, each

group consisting of the 11 items of one vocational personality type. On the circle, these groups were expected to lie on the vertices of an equilateral hexagon with the RIASEC order. The contrast between two groups of personality types (R-I-C and S-A-E), which emerged here, resembles the dimension “people–things” of Prediger’s (1982) model, which aims at interpreting the relationships between the six RIASEC types. A necessary condition for Prediger’s model to hold is the hexagonal arrangement of the personality types (Prediger, 1982; Rounds & Tracey, 1993), and as a consequence, the results of the present article do not correspond exactly to this model. The present analyses concern a specific subscale of the SDS (Activities) and this fact is a possible reason for the differences from the theoretical hexagonal model. In analyses involving total scores on the six personality types (e.g., Sidiropoulou-Dimakakou et al., 2008) the results may be more stable compared to analyzing a single subscale.

Compared to the circular plot, some of the subscales seem to be better distinguished from each other on the sphere. For the R-I-C group of types, the tendency of the I and C types to be placed to a greater and lower heights in relation to the R type, respectively, results in better discrimination among these subscales. The A subscale is also better distinguished from the S scale on the sphere, as the two types do not have any overlap due to the placement of some S items in terms of height. The E subscale is better depicted on the circumference of the circle, as on the sphere its items seem more scattered.

In summary, the MDS-T configurations of the same data on the circumference of a circle and on the surface of a sphere had both similarities and differences. In some cases the third dimension added interesting details to the general picture and resulted in more meaningful positions for some items. According to Everitt (1996), two-dimensional solutions may be of most practical importance due to their simplicity. In this respect the circular configuration is more parsimonious compared to the spherical one and leads to similar conclusions. However, most of the additional features on the sphere could be substantively interpreted and resulted in better discrimination among items or scales.

The present study has some limitations. First, only two data sets were employed to compare the MDS methods. The conclusions might vary if these techniques were applied in more data sets. MDS-T is an exploratory–descriptive technique aiming to depict the structure in a specific sample and generalization to other samples is not the goal. The MDS-T method for a circle has already been applied repeatedly in previous studies and its results have been shown useful and in accordance to theoretical expectations. The purpose of this study is a first presentation of the MDS-T (sphere) method and the generalization of the results/conclusions can be tested in further studies. In addition, the results were not tested with other methods (e.g., confirmatory factor analysis) but the aims of the present study are descriptive and not theory testing or generalization.

The main gain of the trigonometric transformations on the circle and the sphere (MDS-T) with respect to the initial unconstrained solution is that we no longer

describe the meaning of each dimension separately and the positions of the points along each one of them, but we speak of one continuum or surface on which all points are comparable in terms of relative position. In future research, these methods can be compared using other data from different samples and content areas in order to study whether the present results can be replicated. Also, the MDS-T (sphere) method could be compared to other statistical techniques, to gain a more full understanding of its potential. In the present study, the dissimilarity measure used as input for the analyses is the Euclidean distance metric. Other dissimilarity measures could be used in future studies in comparison to Euclidean distances, in order to investigate their impact on the MDS-T configuration. Furthermore, the MDS-T method has been applied to unconstrained solutions computed with the ALSCAL algorithm, as it is widely used and has many applications (Cox & Cox, 2001). The effect of different MDS algorithms for computing the unconstrained configuration on MDS-T solutions can also be investigated in future research.

Appendix

To illustrate the MDS-T on the circle calculations, the following simple example with the five hypothetical stimuli-items (a to e) is presented (Table A). The unconstrained MDS coordinates are given in the third and fourth columns, labelled “x” and “y” and correspond to the initial ALSCAL solution (two-dimensional coordinates). The first two items (a and b) have a positive x-coordinate and a y-coordinate different than zero, so Equation (4) is applied in order to plot them on the circle. For Item “a,” the inverse tangent is $\tan^{-1}\left(\left|\frac{2.4}{1.1}\right|\right) * \text{sgn}(2.4) = \tan^{-1}(2.18) * \left(\frac{2.4}{2.4}\right) = 1.14 * (+1) = 1.14$ radians, that is, $1.14 * (180/\pi) = 65^\circ$.

For Item “b,” applying Equation (4), $\tan^{-1}\left(\left|\frac{-2}{1.4}\right|\right) * \text{sgn}(-2) = \tan^{-1}\left(\frac{2}{1.4}\right) * \left(\frac{-2}{-2}\right) = \tan^{-1}(1.43) * (-1) = .96 * (-1) = -.96$ radians, that is, $(-.96) * (180/\pi) \cong -55^\circ$.

Item “c” in the third row of the table has an x-coordinate equal to zero and a y-coordinate greater than zero, so Equation (5) is applied. The position on the circle is calculated as follows:

$$\text{Angle in radians} = \left(\frac{\pi}{2}\right) * [\text{sgn}(1.2)] = 1.57 * \left(\frac{1.2}{1.2}\right) = 1.57 * (+1) = 1.57 \text{ radians,}$$

that is $1.57 * (180/\pi) \cong 90^\circ$.

Item “d” in the fourth row of Table A has both coordinates negative, so its position on the circle is determined by applying Equation (6). Consequently,

$$\begin{aligned} \text{Angle in radians} &= \{\pi - [\tan^{-1}\left(\left|\frac{-2.1}{1.6}\right|\right)]\} [\text{sgn}(-2.1)] = \{\pi - [\tan^{-1}(1.31)]\} * \\ &\left(\frac{-2.1}{-2.1}\right) = (3.14 - .92) * (-1) = 2.22 * (-1) = -2.22 \text{ radians,} \end{aligned}$$

that is $(-2.22) * (180/\pi) \cong -127^\circ$.

Finally, Item “e” has a positive x -coordinate and a y -coordinate equal to zero, so according to Equation (7) it corresponds to zero radians (or zero degrees).

Table A. Hypothetical Example for MDS-T in Two Dimensions.

Item	Equation applied	x	y	Radians	Degrees
a	Equation (4)	1.1	2.4	1.14	65
b	Equation (4)	1.4	−2.0	−0.96	−55
c	Equation (5)	0.0	1.2	1.57	90
d	Equation (6)	−1.6	−2.1	−2.22	−127
e	Equation (7)	1.3	0.0	0.0	0

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Notes

1. For calculation of the trigonometric number $\tan^{-1}(\frac{y}{x}) = \tan^{-1}(z)$ rapid convergence is achieved through Euler’s function

$$\tan^{-1}(z) = \sum_{n=0}^{\infty} \frac{2^{2n}(n!)^2}{(2n+1)!} \frac{z^{2n+1}}{(1+z^2)^{n+1}}.$$

2. We would like to thank Giorgos Santipantakis, PhD, postdoctoral researcher at the University of Piraeus, for developing the software program used for plotting the outcomes of our methods. The software for calculating MDS-T 2-dimensional and 3-dimensional solutions and for graphically presenting them on the circle and the sphere can be accessed at psychlabuoa.psych.uoa.gr

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