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EXPLORATION AND DATA REFINEMENT VIA MULTIPLE MOBILE SENSORS BASED ON GAUSSIAN PROCESSES

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Abstract

We consider configuration of multiple mobile sensors to explore and refine knowledge in an unknown field. After some initial discovery, it is desired to collect data from the regions that are far away from the current sensor trajectories in order to favor the exploration purposes, while simultaneously, exploring the vicinity of known interesting phenomena to refine the measurements. Since the collected data only provide us with local information, there is no optimal solution to be sought for the next trajectory of sensors. Using Gaussian process regression, we provide a simple framework that accounts for both the conflicting data refinement and exploration goals, and to make reasonable decisions for the trajectories of mobile sensors.

1. INTRODUCTION

We consider the exploration problem using multiple mobile sensors. The initial problem is to locate regions where interesting phenomena occur then, having found interesting regions, to expend sensor capability on refining data in interesting regions while also continuing to search for other possibly existing interesting regions. There is thus after an initial discovery an intrinsic tradeoff between increasing knowledge by taking more measurements in regions known to be of interest, and increasing knowledge by exploring in regions where phenomena of interest (PoI) might possibly exist, but are not known to be present. Due to the uncertainties of exploration, the problem is not posed as one of optimal path planning, but a solution which balances the competing imperatives of refining measurements while exploring new territory.

The paradigm developed in this paper is applicable to general exploration problems, but to illustrate how the principles are used we consider a problem of space exploration in the vicinity of a planet. One of the main design steps in any space mission is to specify the trajectory of a satellite or a constellation of satellites [1–6]. The recent advances in small satellites have made it possible to perform missions with a launch of group of small satellites (swarm of satellites) but with an adaptive constellation [7]. For example, consider an exploration problem in which a swarm of low-cost small satellites are deployed and then the goal is to change their orbital planes in order to find, capture, and track the most interesting features of the PoI. The exploration problem with multiple mobile sensors that we investigate in this paper has application in such area. We assume here that measurements from the PoI are obtained directly from the current trajectories of mobile sensors. This differs from other sensor array configuration methods in remote sensing. For example, in [8] the configuration of sensors array in remote image formation was sought.

In this paper we devise a general exploratory framework for specifying the trajectory of sensor-bearing mobile sensors (e.g., satellites) in order to find and characterize the important features of phenomena in a region of interest. The approach adaptively makes decisions on the trajectories of mobile sensors based on balancing between refinement of the measurements and the desire of exploration. Our proposed framework consists of two main stages. The first stage is the prediction stage, where a Gaussian process regression (GPR) model is applied. Once some initial measurements are collected, the GPR model predicts the behavior of the PoI at the other unseen locations. GPs have been used as a powerful supervised learning tool for the regression and prediction problems [9]. (This is also referred to as kriging in the geo-statistics literature [10]). GPR models are applicable to a wide variety of problems such as the prediction and estimation of temperature, precipitation, missing pixel and un-mixing of pixels in hyper-spectral imaging (HSI), human head pose estimation, concentration of carbon dioxide in the atmosphere etc [9,11–14]. The second stage of the proposed framework is the decision stage. Suppose that some measurements from the region have already been collected. Also, suppose the GPR model (first stage) has already predicted the PoI at the unseen locations. Now the question is how to decide on the next set of trajectories based on the information obtained from the previous stage. There is a fundamental tradeoff between repeating measurements in the same (or nearby) locations to obtain more informative measurements where it is known that interesting things are happening *vs.* making measurements in new locations (exploration) with the possibility of discovering additional interesting information. There is not sufficient information to obtain a solution *a priori*, since the locality of the available information precludes information about the unexplored areas. Here, we set up an optimization problem, using the estimates obtained from GPR, to make reasonable decisions for the trajectory of mobile sensors. Our approach provides a parameter to be tuned in either greater emphasis on refining data in known-interesting regions, or exploring new regions. It is also possible to adjust this parameter in accordance with the cost of maneuvering between different trajectories.

2. REVIEW OF GAUSSIAN PROCESSES

Suppose that there exist an unknown (and probably nonlinear) function representing the behavior of PoI. A common method for estimating such a function is to construct a parametric model that provides a good match with the observations. In contrast, GP is a non-parametric model where a probability distribution function can be defined as a prior over the set of unknown functions. In other words, GP defines distributions over functions in the function space and the inference is performed directly in the space of functions [9]. In GP it is assumed that the behavior of phenomena is governed by a stochastic process, in which each observation is an outcome of jointly distributed Gaussian random variables. GP provides a posterior distribution over the unknown function $f(\cdot)$ once some data are observed. The arguments of the function are collected into a variable set $\mathbf{u}$, referred to as the input data. For a set of N observations with the input data set $\{\mathbf{u}_1, \dots, \mathbf{u}_N\}$, GP assumes that the distribution $p(f(\mathbf{u}_1), \dots, f(\mathbf{u}_N))$ is jointly Gaussian with mean $\boldsymbol{\mu}(U)$ and covariance matrix $K(U)$ given by $[K]_{ij} = \kappa(\mathbf{u}_i, \mathbf{u}_j)$, $\forall i, j=1, \dots, N$, where $\kappa(\cdot, \cdot)$ is a kernel function and $U := [\mathbf{u}_1, \dots, \mathbf{u}_N]$. The kernel function specifies the covariance between pairs of random

variables at the corresponding data points. A GP model is denoted as $\mathbf{f}(U) \sim \mathcal{G}(\boldsymbol{\mu}(U), K(U))$, where $\mathbf{f}(U) := [f(\mathbf{u}_1), \dots, f(\mathbf{u}_N)]^T$ and $\boldsymbol{\mu}(U) := [\mu(f(\mathbf{u}_1)), \dots, \mu(f(\mathbf{u}_N))]^T$.

GP can be used as a regression tool, referred to as GPR. GPR treats the available data as the training data and predicts the behavior of the phenomena at the unseen data points. Let $\mathbf{y} = \mathbf{f}(U) + \boldsymbol{\varepsilon}$ be the observation model, where $\boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \sigma_n^2 I)$ and $y_n = f(\mathbf{u}_n) + \varepsilon_n, \forall n = 1, \dots, N$. We refer the pair $(\mathbf{u}_n, f(\mathbf{u}_n))$ to as the n th training data. The goal is to predict the underlying function f evaluated at other input data $U_\star$ i.e., inferring $\mathbf{f}(U_\star)$, where $U_\star := [\mathbf{u}_{\star,1}, \dots, \mathbf{u}_{\star,M}]^T$ and $\mathbf{f}(U_\star) := [f(\mathbf{u}_{\star,1}), \dots, f(\mathbf{u}_{\star,M})]^T$. We refer to $U_\star$ as the input test data set. Based on GP modeling, the prior joint distribution between the training and test data can be expressed as [15]

$$\begin{bmatrix} \mathbf{y} \\ \mathbf{f}_\star \end{bmatrix} = \mathcal{N}\left(\begin{bmatrix} \boldsymbol{\mu}_y \\ \boldsymbol{\mu}_{f_\star} \end{bmatrix}, \begin{bmatrix} K & K_\star^T \\ K_\star & K_{\star\star} \end{bmatrix}\right), \quad (1)$$

where $\mathbf{f}_\star$ denotes $\mathbf{f}(U_\star)$. Also $K := K(U, U)$, $K_\star := K(U, U_\star)$, and $K_{\star\star} := K(U_\star, U_\star)$. Therefore, the predictive distribution over the test data can be expressed as follows

$$\mathbf{f}_\star | U, \mathbf{y}, U_\star \sim \mathcal{N}(\boldsymbol{\mu}_{f_\star}, \Sigma_{f_\star}), \quad (2)$$

where

$$\begin{aligned} \boldsymbol{\mu}_{f_\star} &= \boldsymbol{\mu}_y + K_\star^T (K + \sigma_n^2 I)^{-1} (\mathbf{y} - \boldsymbol{\mu}_y) \\ \Sigma_{f_\star} &= K_{\star\star} - K_\star^T (K + \sigma_n^2 I)^{-1} K_\star. \end{aligned} \quad (3)$$

3. TRAJECTORY DETERMINATION OF MOBILE SENSORS

In the prediction setting, the collected data are treated as the training set, where the locations at which the data are collected are used as the input-training data and the corresponding measurements are referred to as the output-training data. The test data are then the other unseen locations over the region of under study and the output-test data are unknown and are required to be predicted using GPR. We assume that the locations of the training data (previously observed data) are collected into the set $U_s = \{\mathbf{u}_1, \dots, \mathbf{u}_{N_1}\}$. The output-training data are accumulated in $\mathbf{y} = [y_1, \dots, y_{N_1}]^T$, where y_n is the output corresponding to the input $\mathbf{u}_n$. Furthermore, we collect the locations of the test data into the set $U_\star = \{\mathbf{u}_{\star,1}, \dots, \mathbf{u}_{\star,N_2}\}$. The unknown outputs evaluated at the input-test data are defined as $\mathbf{f}_\star = [f(\mathbf{u}_{\star,1}), \dots, f(\mathbf{u}_{\star,N_2})]^T$. As prior knowledge, we assume that the joint density function between the training and test data is zero-mean Gaussian, meaning that on the average we expect

interesting phenomena occur very rarely. Suppose that the measurement noise for location on the field of interest is $\mathcal{N}(0, \sigma_n^2)$. We further assume that the PoI has a smooth behavior, as often occurs in nature for distributed phenomena. We define the squared exponential covariance function to promote smoothness over the nearby regions as

$$K(\mathbf{u}_i, \mathbf{u}_j) = \exp \left[-\frac{\|\mathbf{u}_i - \mathbf{u}_j\|_2^2}{2l} \right], \quad (4)$$

where l is a scale factor, and $\mathbf{u}_i$ and $\mathbf{u}_j$ are the coordinates of any two arbitrary locations in the region under investigation. As the available prior knowledge changes, one may define a different kernel function than (4). Let $\mathcal{S} = \{s^{(1)}, s^{(2)}, \dots, s^{(K)}\}$ denote the set of all feasible trajectories of the sensors over the PoI. We denote the set of locations along the trajectory $s^{(k)}$ at which measurements are collected by the set $U_{s^{(k)}} = \{\mathbf{u}_{s^{(k)}}^1, \dots, \mathbf{u}_{s^{(k)}}^P\}$. The locations in the set $U_{s^{(k)}}$ belong to the set of test data i.e., $U_{s^{(k)}} \in U_\star$. Using the GPR model (2) and the kernel function (4), we obtain two set of information over the region of under study. One is the estimate of the interesting stuff and the second is the amount of uncertainty of our predictions. The uncertainty can be evaluated via the kernel function in (4). We denote $\hat{\mathbf{f}}(U_{s^{(k)}})$ and $\hat{\Sigma}(U_{s^{(k)}})$ as the estimate of the PoI and the measure of uncertainty (variance) along the trajectory $s^{(k)}$, respectively. This process is performed in the first stage of the proposed framework, the prediction-refinement stage.

The goal is to decide on the next trajectory of the sensors based on the available and estimated data and also our measure of uncertainty over the region in order to explore the interesting phenomena. The further away the estimates of the PoI are from the past measurements, the higher the corresponding variance becomes, resulting in less confidence in the estimates at those locations. Therefore, if we put emphasis on only the measurements, there is little impetus to continue exploration. In contrast, if we put more emphasis on the evaluated amount of uncertainty then the sensors are encouraged to choose the trajectories which are far away from the previous trajectories to fulfill only the exploration objective of the mission. In this case, even when interesting phenomena are found on previous trajectories, the sensors have preference to return, in the interest of exploring more. In order to incorporate this tradeoff, we set up an optimization problem. We define the functions

$$\hat{f}_n(U_{s^{(k)}}) = \frac{1}{P} \sum_{p=1}^P \hat{f}(\mathbf{u}_{s^{(k)}}^p) \quad (5)$$

as a measure of the data-refinement, and

$$\widehat{\Sigma}_n(U_{s(k)}) = \frac{1}{P} \sum_{p=1}^P \widehat{\Sigma}(\mathbf{u}_{s(k)}^p). \quad (6)$$

as a measure of the desire for data exploration. These are combined in the decision functional

$$k^\star = \arg \max_k \widehat{f}_n(U_{s(k)}) + \lambda(t) \widehat{\Sigma}_n(U_{s(k)}). \quad (7)$$

Here, $k^\star$ denotes the best trajectory of the sensor from the set of all possible practical trajectories: The factor λ balances between data-refinement and exploration. Selecting large values for λ places more emphasis on the desire for exploration, while smaller values for λ emphasize refinement of knowledge. λ may depend on time. For example, with few measurements, the usual desire may be to put more emphasis on the exploration to get some sense of the PoI, while there is still available energy for unfocused exploration. That is, λ may be set to a large value depending on the affordable cost. Once more measurements over the PoI are collected, the value of λ may be decreased to focus more on data refinement compared to the exploration. We refer to the decided trajectory as $U_{s(k^\star)}$. Finally, the data obtained from $U_{s(k^\star)}$ is added to the training set U , and the whole process starts again.

4. SIMULATION RESULTS

In order to demonstrate how the framework works, we consider a simplified version of a space mission consisting of a constellation of two small steerable satellites for exploring a phenomena in some region of under study. Based on some prior knowledge the initial orbital planes for the constellation over the region of interest has been predetermined. The PoI remains essentially unchanged for the amount of time of the study. The sensors of the satellites take *in situ* measurement from the phenomena along the current trajectory of the satellites. Fig. 1(a) illustrates an example including the orbital planes of such satellites, where the rectangular shape shows the region of under study, and Fig. 1(b) shows a phenomena of interest (from [8]). In order to emphasize the PoI, we illustrate the PoI as shown in Fig. 1(c). For the simulation purposes the most and the least interesting stuff corresponding to the PoI are shown with yellow and blue, respectively. This image can be thought of as a discretized version of the region of interest defined by the pixel values. Here, without loss of generality and for the simplification purposes, we assume that the PoI has the same profile as shown in Fig. 1(c) along the z -axis over the rectangular region shown in Fig. 1(a). Based on the trajectories determined via the initial constellation, shown in Fig. 1, the corresponding set of obtained measurements is illustrated in Fig. 4. Once the measurements are obtained, we apply the GPR model defined in (2) and (4) to estimate the behavior of the PoI and the measure of uncertainty at the un-sampled locations. In the simulations, we set $L=10$ in the kernel function (4). Fig. 4 also illustrates the estimation of the PoI and the measure of variance over the region of under study, using the GPR interpolation. The

variance along the initial trajectories is set to zero, as shown by the dark blue color in the middle plot of Fig. 4. The variance increases with increasing distance away from the measured trajectories.

In order to make comparison, we define four different cases and the trajectories are determined using (2) and (7). In case (1), we set $\lambda = 0.25$, which places less emphasis on exploring the whole field. In Fig. 5, from left to right, we illustrate the measurements corresponding to the previous and current trajectories, the measure of amount of uncertainty based on the measurements, and the reconstruction of the field based on the estimation obtained from the available data. Fig. 6 illustrates the results obtained after making 13 successive decisions for the trajectory of satellites for the case where $\lambda = 0.25$. The legend Tr_i in our figures denotes the i th determined trajectory. We observe in Fig. 6 that all the selected trajectories are in the vicinity of the initial trajectories. The reason for these selected trajectories is because the initial trajectories had found some interesting phenomena and also λ was set to a small value.

In case (2), we set $\lambda = 2$ in order to put more emphasis on the exploration rather than data refinement. Fig. 7 illustrates some of the results obtained. As we expected, we observe in Fig. 7 that since λ is set to a large value, the first specified trajectories are further away from the initial trajectories to satisfy the impetus to exploration. We also see that the trajectories provide a kind of quasi-uniform sampling over the region of under study — exploration occurs while at the same time reducing uncertainty over the region.

In case (3), we vary λ starting from 2 and the decrease rate of 0.7 ($\lambda^{[t+1]} = 0.7\lambda^{[t]}$), meaning that first the exploration desire is emphasized and then once we obtain some sense about the phenomena, λ decreases to emphasize refining information where information is already roughly known. Fig. 8 illustrates some of the results obtained.

In case (4), we make λ dependent on the cost to do the maneuvering and orbit change. The more maneuvering we can afford, the higher the desire to explore will be. This means that if the total remaining available thrust is high, we set λ to a large value and vice versa. In practice, changing the constellation and maneuvering depends on the delta-V budget, which is provided by the thrust of the rocket engine of the satellites. For simulation purposes, we create a scenario with synthetic data such that initially we set $\lambda(0) = 2$ corresponding to the maximum affordable cost for the maneuvering. As the trajectories are determined successively and the maneuvering is performed, the maximum affordable is reduced and as a result λ also decreases. Fig. 9 illustrates some of the results.

In Fig. 2, the first two plots show the value chosen for λ for solving (7) and the total affordable cost for changing the trajectories, respectively. Initially, since the affordable cost is large, we also have a large value for λ , meaning that we can afford to select the trajectories which satisfy the exploration desire over the region. The bottom plot in Fig. 2 shows the cost to perform a trajectory change. Comparing all the four cases, it turns out that for the PoI, the best performance belongs to case (1), where $\lambda=2$. Notice that such case neglected the constraint on the cost for performing the maneuvering. The reason for having the best performance in case (1) compared to the other cases is because of having some sort

of uniform sampling all over the region of under study, the contiguity behavior of the true PoI, and our modeling of the contiguity in the kernel function of the GPR. Finally, Fig. 3 provides a more detailed results for all the four cases. According to the scenarios we considered, it turns out that emphasis on exploration provides more information about the PoI. For example, in the left plot of Fig. 3 the most reduction on the measure of uncertainty over the region belongs to case (2), where $\lambda=2$ in case of having no constraint on the cost. But, once we constraint the decision making on the cost, the best performance belongs to case (4). This is due to the fact that for case (4) was initialized to $\lambda^{[0]}=2$ and then it was reduced more slowly than case (3). As a conclusion, in cases where the PoI has a very smooth behavior, solving the sensor trajectory problem for satisfying both the exploration and data refinement desires via the proposed framework seems to provide encouraging results.

5. CONCLUSION

The general problem of exploration using multiple mobile sensors was considered. We focused on the case where desire is to get as much information as possible from the phenomena of interest based on the local information in a sub-optimal fashion. Using GPR, we provided a framework that is able to balance between the desire of exploration and data refinement.

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Figure 1.

An example showing the satellites orbits and the region of interest.

Figure 2.

From top to bottom, showing the change of emphasizing factor on exploration, affordable cost, and cost of change in trajectories.

Figure 3.

Comparison of all the four case scenarios (1–4) in terms of variance, reconstruction error, and peak signal-to-noise ratio.

Figure 4.
From left to right: Initial measurement, measure of variance, and reconstruction of PoI.

Figure 5.

Measurements, measure of uncertainty, and reconstruction using the initial and the first two determined trajectories for case (1) i.e., when $\lambda = 0.25$.

Figure 6. Measurements, measure of uncertainty, and reconstruction after 13 successive trajectories for case (1) i.e., when $\lambda = 0.25$.

Figure 7.
Measurements, measure of uncertainty, and reconstruction after 13 successive trajectories
for case (2) i.e., when $\lambda = 2$.

Figure 8. Measurements, measure of uncertainty, and reconstruction based on the 13 successive trajectories for case (3).

Figure 9.
Measurements, measure of uncertainty, and reconstruction based on the 13 determined trajectories for case (4).