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Joint reconstruction of the initial pressure and speed of sound distributions from combined photoacoustic and ultrasound tomography measurements

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Abstract

The initial pressure and speed of sound (SOS) distributions cannot both be stably recovered from photoacoustic computed tomography (PACT) measurements alone. Adjunct ultrasound computed tomography (USCT) measurements can be employed to estimate the SOS distribution. Under the conventional image reconstruction approach for combined PACT/USCT systems, the SOS is estimated from the USCT measurements alone and the initial pressure is estimated from the PACT measurements by use of the previously estimated SOS. This approach ignores the acoustic information in the PACT measurements and may require many USCT measurements to accurately reconstruct the SOS. In this work, a joint reconstruction method where the SOS and initial pressure distributions are simultaneously estimated from combined PACT/USCT measurements is proposed. This approach allows accurate estimation of both the initial pressure distribution and the SOS distribution while requiring few USCT measurements.

Keywords

joint reconstruction; ultrasound tomography; photoacoustic tomography

1. Introduction

Photoacoustic computed tomography (PACT) is a hybrid imaging modality that combines optical excitation with acoustic detection [1, 2, 3]. It can provide both functional and structural information for pre-clinical and clinical applications, including human breast imaging [4, 5, 6, 7, 8, 9]. In PACT, the image contrast arises from the absorption of light, typically in the form of a short laser pulse, by molecules within the sample. One of the strongest endogenous absorbers in the human body over the visible and near-infrared wavelength ranges is hemoglobin [10]. As a result, blood vessels may be readily visualized without the introduction of contrast agents. The absorbed optical energy gives rise to pressure waves that are then recorded by ultrasonic transducers.

Most image reconstruction methods for PACT assume that the medium is acoustically homogeneous. This is not true in many biological imaging applications [11, 12, 13]. In particular, unaccounted for variations in the speed of sound (SOS) distribution can give rise to artifacts in the reconstructed initial pressure distribution. Several methods have been proposed to account for SOS variations, including time-reversal [14, 15, 16], iterative full-

wave inversion [17, 18], and the Neumann-series or iterative time reversal approach [19, 20]. However, each of these methods requires some knowledge of the SOS distribution in order to accurately reconstruct the initial pressure distribution. In practice, the SOS distribution is typically unknown. Further, it has been shown that, in general, the SOS and initial pressure distributions cannot both be stably recovered from PACT measurements alone [21, 22].

One alternative is try to estimate the SOS distribution by use of adjunct imaging data. Ultrasound computed tomography (USCT) has previously been shown to allow accurate reconstruction of the SOS distribution for breast imaging [23, 24, 25, 26, 27, 28, 29]. In USCT, the acoustic properties of the object are estimated by sending a series of ultrasonic pulses through the object and measuring the resulting wavefield with an array of ultrasonic transducers. As such, the detection hardware is similar to that commonly employed for PACT.

Several groups have begun investigation of combined USCT/PACT imaging systems [30, 31, 32]. These systems offer the potential to obtain automatically co-registered images with both optical and acoustic contrasts. These complementary contrasts could aid in a variety of medical imaging tasks, including breast cancer detection. In addition, the similar detection hardware should make integration of these modalities comparatively simple and inexpensive. Finally, the estimated SOS distribution could be employed to more accurately estimate the initial pressure distribution in PACT.

In cases where combined USCT/PACT systems have been employed, the SOS and initial pressure distributions were reconstructed in a sequential manner. First, the USCT data were employed to estimate the SOS distribution. Then, the estimated SOS distribution was employed when estimating the initial pressure distribution from the PACT data. As will be shown, this approach is not optimal. Since the photoacoustic waves propagate according to the acoustic properties of the medium, the measured PACT data contain information on the SOS distribution. This information is ignored under the sequential approach, which estimates the SOS distribution from USCT measurements alone. Here, a novel approach that jointly estimates the initial pressure distribution and the SOS distribution from combined PACT and USCT measurements is proposed. This synergistic approach effectively utilizes the acoustic information in the PACT data, allowing the SOS distribution to be accurately estimated from few-view USCT measurements. Further, it allows the initial pressure distribution to be accurately estimated by providing a more accurate estimate of the SOS distribution than could be obtained from PACT measurements alone.

The remainder of the paper is organized as follows. In Section 2, optimization-based image reconstruction methods for PACT and USCT are reviewed. There, the PACT and USCT image reconstruction problems are considered separately. Section 3 describes joint reconstruction of the SOS and initial pressure distributions from combined PACT and USCT data. Descriptions of the computer-simulation studies are presented in Section 4, and the results of these studies are presented in Section 5. The paper is concluded with a summary in Section 6.

2. Background

2.1. Imaging physics

In PACT, a short laser pulse is employed to illuminate a sample of interest. This light is absorbed by the tissue, giving rise to an initial pressure distribution via the photoacoustic effect. In this work, uniform illumination of the sample is assumed. Under these conditions, strong optical absorbers, such as blood vessels, will have large initial pressure values. The pressure waves then propagate outward according to the acoustic properties of the medium and the pressure is recorded by a collection of ultrasonic transducers surrounding the object. Assuming a lossless medium and unit density, this process is described by the coupled first-order differential equations [33],

$$\frac{\partial \mathbf{u}(\mathbf{r}, t)}{\partial t} + \nabla p(\mathbf{r}, t) = \mathbf{0} \quad (1a)$$

$$\frac{1}{c(\mathbf{r})^2} \frac{\partial p(\mathbf{r}, t)}{\partial t} + \nabla \cdot \mathbf{u}(\mathbf{r}, t) = 0 \quad (1b)$$

$$p(\mathbf{r}, 0) = p_0(\mathbf{r}) \quad (1c)$$

$$\mathbf{u}(\mathbf{r}, 0) = \mathbf{0}, \quad (1d)$$

where $\mathbf{u}(\mathbf{r}, t)$ is the acoustic particle velocity, $p(\mathbf{r}, t)$ is the acoustic pressure, $c(\mathbf{r})$ is the SOS, and $p_0(\mathbf{r})$ is initial pressure distribution. A schematic of this process is shown in Fig. 1a. In this case, the ultrasonic transducers are arranged in a circle surrounding the sample.

In USCT, an acoustic pulse is emitted from a known location outside the object. The acoustic wave travels through the object and the resulting pressure wavefield is recorded by a collection of ultrasonic transducers surrounding the object. This process is repeated for several emitter locations corresponding to different views around the object. The acoustic wave propagation for the m -th view is described by [34]

$$\frac{\partial \mathbf{u}_m(\mathbf{r}, t)}{\partial t} + \nabla p_m(\mathbf{r}, t) = \mathbf{0} \quad (2a)$$

$$\frac{1}{c(\mathbf{r})^2} \frac{\partial p_m(\mathbf{r}, t)}{\partial t} + \nabla \cdot \mathbf{u}_m(\mathbf{r}, t) = 4\pi \int_0^t dt' s_m(\mathbf{r}, t') \quad (2b)$$

$$p_m(\mathbf{r}, 0) = \mathbf{0} \quad (2c)$$

$$\mathbf{u}_m(\mathbf{r}, 0) = \mathbf{0}, \quad (2d)$$

where $s(\mathbf{r}, t)$ is the emitted acoustic pulse, which is assumed to be known. These differential equations differ from those for PACT only in the specification of the acoustic source and the initial conditions. A schematic of this process is shown in Fig. 1b. From these schematics, it is clear that the detection hardware could be shared by the two modalities.

2.2. Forward models in discrete form

In order to facilitate the use of iterative image reconstruction methods, discrete-to-discrete (D-D) imaging models are introduced for PACT and USCT.

For PACT, the D-D model is given by [17]

$$\mathbf{g}_{pa} = \mathbf{M}\mathbf{H}_{pa}(\mathbf{c})\mathbf{p}_0, \quad (3)$$

where $\mathbf{p}_0 \in \mathbb{R}^N$ is a finite-dimensional approximation of the initial pressure distribution $p_0(\mathbf{r})$, $\mathbf{c} \in \mathbb{R}^N$ is a finite-dimensional approximation of the SOS distribution $c(\mathbf{r})$, and $\mathbf{H}_{pa}(\mathbf{c}) \in \mathbb{R}^{NL \times N}$ is an operator that gives a discrete approximation of the solution of the acoustic wave equation specified by Eqns. (1a)–(1d). To allow efficient implementation of the numerical wave solver, described in Section 4.1, all quantities were discretized according to a pixel basis. Thus, here, $N \equiv N_x N_y$ is the total number of pixels in the simulation grid (N_x and N_y are the number of pixels along the x- and y-directions, respectively), and L is the number of time points employed by the numerical wave solver. Additionally, $\mathbf{M} \in \mathbb{R}^{N^{rec}L \times NL}$ is a sampling matrix that computes the pressure at the transducer locations from the discrete representation of the pressure over the whole domain, $\mathbf{g}_{pa} \in \mathbb{R}^{N^{rec}L}$ is the measured pressure data, and N^{rec} is the number of transducers acting as receivers. The notation $\mathbf{H}_{pa}(\mathbf{c})$ is employed to emphasize the dependence of $\mathbf{H}_{pa}$ on the SOS $\mathbf{c}$.

The D-D model for USCT, for the m -th view, can be denoted [34, 35]

$$\mathbf{g}_m = \mathbf{M}\mathbf{H}_{us}(\mathbf{c})\mathbf{s}_m, \quad (4)$$

where $\mathbf{s}_m \in \mathbb{R}^{NL}$ is a finite-dimensional approximation of the acoustic excitation pulse for the m -th view $s_m(\mathbf{r}, t)$, $\mathbf{H}_{us}(\mathbf{c}) \in \mathbb{R}^{NL \times NL}$ is an operator that computes a discrete approximation of the solution to the wave equation given by Eqns. (2a)–(2d), and $\mathbf{g}_m \in \mathbb{R}^{N^{rec}L}$ is the data recorded at the transducers for the m -th view.

2.3. Image reconstruction methods

While there are many possible image reconstruction methods for both PACT and USCT, this paper will focus on use of optimization-based image reconstruction methods [36, 37]. These methods provide a flexible framework for image reconstruction that can accurately model the underlying physics of the imaging system.

2.3.1. Reconstruction of the initial pressure from PACT data—While it is possible to recover the underlying optical properties of the medium (see [38] and the references contained within), this work focuses on recovery of the initial pressure distribution.

An estimate of the initial pressure distribution can be obtained by solving the penalized least-squares optimization problem [17, 22, 37],

$$\hat{\mathbf{p}}_0 = \arg \min_{\mathbf{p}_0 \geq 0} F_p(\mathbf{p}_0, \mathbf{c}) + \lambda \mathcal{R}_p(\mathbf{p}_0), \quad (5)$$

where $F_p(\mathbf{p}_0, \mathbf{c})$ is a data fidelity term, $\mathcal{R}_p(\mathbf{p}_0)$ is a regularization function, and λ is a regularization parameter, which controls the relative weight of the $\mathcal{R}_p(\mathbf{p}_0)$ term. Here, $F_p(\mathbf{p}_0, \mathbf{c})$ is given by

$$F_p(\mathbf{p}_0, \mathbf{c}) \equiv \frac{1}{2} \|\underline{\mathbf{g}}_{pa} - \mathbf{M}\mathbf{H}_{pa}(\mathbf{c})\mathbf{p}_0\|_2^2, \quad (6)$$

where $\underline{\mathbf{g}}_{pa} \in \mathbb{R}^{N^{rec}L}$ is the measured data. For this problem, the data fidelity term is differentiable and convex with respect to $\mathbf{p}_0$. When the regularization function is chosen to be convex, Eqn. (5) represents a convex optimization problem since the constraint $\mathbf{p}_0 \geq 0$ defines a convex set. In this work, $\mathcal{R}_p(\mathbf{p}_0)$ is chosen to be the total variation (TV) semi-norm, given by

$$\mathcal{R}_p(\mathbf{p}_0) = \|\nabla \mathbf{p}_0\|_1, \quad (7)$$

which is convex, but non-smooth.

2.3.2. Reconstruction of the SOS from PACT data—When solving Eqn. (5), it is assumed that the SOS distribution is known. In practice, this is often not the case. One method to estimate the SOS distribution, in the absence of adjunct imaging data (e.g. USCT measurements), is to jointly estimate the SOS and initial pressure distributions from the PACT data alone [22, 39, 40]. This can be performed by solving the following optimization problem [22],

$$\hat{\mathbf{p}}_0, \hat{\mathbf{c}} = \arg \min_{\mathbf{p}_0 \geq 0, \mathbf{c}} F_p(\mathbf{p}_0, \mathbf{c}) + \lambda_p \mathcal{R}_p(\mathbf{p}_0) + \lambda_c \mathcal{R}_c(\mathbf{c}), \quad (8)$$

where $\mathcal{R}_c(\mathbf{c})$ is a regularization function for the SOS and λ_c is its corresponding regularization parameter value. This optimization problem is non-convex with respect to the SOS. It has been demonstrated analytically in the continuous case for the linearized wave equation [21] and numerically through computer simulation studies [41] that, in general, the SOS and initial pressure distributions cannot both be stably recovered.

Several works have addressed the joint reconstruction problem in Eqn. (8) under restrictive conditions. Zhang, et al., showed via computer-simulation studies that the initial pressure and SOS distributions could both be accurately recovered when the SOS distribution had a known low-dimensional parameterization [39]. Kirsch and Scherzer proposed a joint reconstruction method that relied on acquisition of a large number of measurements employing optical sectioning, which may not be appropriate for large or highly scattering samples such as the breast [42]. Liu and Uhlmann demonstrated that the initial pressure and SOS could both be uniquely recovered from a single set of measurements when the SOS is constant (and certain other conditions are met) [43]. Huang, et al., provided a set of heuristic conditions, evaluated through computer-simulation studies, under which the joint reconstruction problem is more stable [22]. A related problem in quantitative PACT of jointly estimating the SOS and the optical absorption coefficient distributions was considered by Ding, et al. [44]. Under this approach, multiple optical illuminations were employed. It was assumed that the diffusion coefficient and Grüneisen coefficient distributions were known. Perhaps, because of these restrictions, as well as the difficulty of solving Eqn. (8) in a general setting, the SOS is not commonly estimated from PACT measurements alone.

In this work, to investigate the sensitivity of the estimated SOS distribution to the assumed initial pressure distribution, the following optimization problem is considered

$$\hat{\mathbf{c}} = \arg \min_{\mathbf{c}} F_p(\mathbf{p}_0, \mathbf{c}) + \lambda_c \mathcal{R}_c(\mathbf{c}). \quad (9)$$

Errors in the estimated SOS distribution induced by an inaccurate assumed initial pressure distribution may provide some insight into the stability of the joint reconstruction problem (see [22] for a detailed investigation). In order to provide flexibility in the type of optimization method employed, a smoothed version of the TV penalty is utilized,

$$\mathcal{R}_c(\mathbf{c}) \sum_{i,j} \sqrt{([\mathbf{c}]_{i,j} - [\mathbf{c}]_{i-1,j})^2 + ([\mathbf{c}]_{i,j} - [\mathbf{c}]_{i,j-1})^2 + \varepsilon}, \quad (10)$$

where $[\mathbf{c}]_{i,j}$ denotes the $(iN_x + j)$ -th element of $\mathbf{c}$ and $\varepsilon > 0$ is a small smoothing parameter used to prevent division by zero when calculating the gradient. Here, we choose $\varepsilon = 10^{-12}$.

2.3.3. Reconstruction of the SOS from USCT data—For USCT, the SOS distribution can be estimated by full-waveform inversion as

$$\hat{\mathbf{c}} = \arg \min_{\mathbf{c}} \frac{1}{2} \sum_{m=0}^{M-1} \|\underline{\mathbf{g}}_m - \mathbf{M}\mathbf{H}_{us}(\mathbf{c})\mathbf{s}_m\|_2^2 + \lambda_c \mathcal{R}_c(\mathbf{c}), \quad (11)$$

where $\underline{\mathbf{g}}_m \in \mathbb{R}^{N_{rec}L}$ is the measured data for m -th view. This approach can produce more accurate and higher-resolution estimated SOS distributions than ray-based methods [34], but can be computationally expensive as evaluating the cost function requires the wave equation to be solved M times. Recently, a source encoding technique has been employed to efficiently estimate the SOS distribution in USCT [34, 45, 46]. Using this technique, the SOS distribution can be estimated as

$$\hat{\mathbf{c}} = \arg \min_{\mathbf{c}} \mathbf{E}_{\mathbf{w}} \left\{ \frac{1}{2} \|\underline{\mathbf{g}}_w - \mathbf{M}\mathbf{H}_{us}(\mathbf{c})\mathbf{s}_w\|_2^2 \right\} + \lambda_c \mathcal{R}_c(\mathbf{c}), \quad (12)$$

where $\mathbf{w}$ is a random encoding vector, chosen according to a Rademacher distribution [46, 47], $\mathbf{E}_{\mathbf{w}}$ is the expectation with respect to $\mathbf{w}$, and

$$\underline{\mathbf{g}}_w = \sum_{m=0}^{M-1} [\mathbf{w}]_m \underline{\mathbf{g}}_m \quad \text{and} \quad \mathbf{s}_w = \sum_{m=0}^{M-1} [\mathbf{w}]_m \mathbf{s}_m \quad (13)$$

are the encoded measurement data and the encoded source term, respectively. Under this approach, rather than simulating the pressure wavefield for each emitter separately, the pressure is simulated for the case where all emitters are fired simultaneously. For each emitter, the excitation pulse is scaled by the corresponding element of $\mathbf{w}$. Since the acoustic wave equation is linear with respect to the source term, the measured data can be similarly encoded. When $\mathbf{w}$ satisfies certain properties, the optimization problems given in Eqns. (11) and (12) are equivalent [47]. For brevity, this new data fidelity term will be denoted as

$$F_u(\mathbf{c}) \equiv \mathbf{E}_{\mathbf{w}} \left\{ \frac{1}{2} \|\underline{\mathbf{g}}_w - \mathbf{M}\mathbf{H}_{us}(\mathbf{c})\mathbf{s}_w\|_2^2 \right\}. \quad (14)$$

The optimization problem given in Eqn. (12) was solved by use of stochastic gradient descent as described in [34]. This approach involves choosing a single realization of $\mathbf{w}$ for each iteration of the stochastic gradient descent method, greatly reducing the number of times that the acoustic wave equation must be solved. Additionally, an initial guess for the SOS distribution must be specified. As the optimization problem is non-convex, this initial guess can have a large impact on the estimated SOS distribution.

3. Joint reconstruction from PACT/USCT measurements

In the previous section, several existing methods for estimating the initial pressure and SOS distributions were reviewed. Here, a novel method for jointly estimating the initial pressure

and SOS distributions from combined PACT and USCT measurements is introduced. Instead of estimating the SOS distribution from USCT measurements alone, as described in Section 2.3.3, the SOS distribution can be estimated from combined PACT and USCT measurements in order to leverage the acoustic information present in the PACT measurements. This can be accomplished by jointly estimating the SOS and initial pressure distributions as

$$\hat{\mathbf{p}}_0, \hat{\mathbf{c}} = \arg \min_{\mathbf{p}_0 \geq 0, \mathbf{c}} F_p(\mathbf{p}_0, \mathbf{c}) + \beta F_u(\mathbf{c}) + \lambda_p \mathcal{R}_p(\mathbf{p}_0) + \lambda_c \mathcal{R}_c(\mathbf{c}). \quad (15)$$

This optimization problem is solved using the alternating minimization approach described in Algorithm 1. A similar approach was successfully employed in [22] for joint reconstruction from PACT measurements alone. Under this approach, a subproblem is solved to estimate the initial pressure distribution from PACT measurements alone. This subproblem is identical to the one given in Eqn. (5). Then, the SOS distribution is estimated from combined USCT and PACT measurements as

$$\hat{\mathbf{c}} = \arg \min_{\mathbf{c}} F_p(\mathbf{p}_0, \mathbf{c}) + \beta F_u(\mathbf{c}) + \lambda_c \mathcal{R}_c(\mathbf{c}), \quad (16)$$

where $\beta > 0$ is a scalar that controls the relative weight of the two data fidelity terms. Like the joint reconstruction problem from the PACT data alone, this optimization problem is non-convex.

In addition to providing acoustic information to stabilize the joint reconstruction approach, the USCT measurements can be employed to generate a more accurate initial guess for the SOS distribution. This initial guess is important as proposed approach for estimating the SOS distribution involves solving a non-convex optimization problem. Such problems may have many local minima or saddle points. When the initial guess is far from the true SOS distribution, the approach could converge to a local minimum and the estimated SOS distribution may be very inaccurate.

Algorithm 1

Alternating joint reconstruction method

Input: $\mathbf{c}^{(0)}, \mathbf{p}_0^{(0)}, \lambda_c, \lambda_p, \beta$

Output: $\hat{\mathbf{c}}, \hat{\mathbf{p}}_0$

- 1: $k \leftarrow 0$ { k is the algorithm iteration number. }
- 2: **while** stopping criterion is not satisfied **do**
- 3: $\mathbf{p}_0^{(k+1)} \leftarrow \arg \min_{\mathbf{p}_0} F_p(\mathbf{p}_0, \mathbf{c}^{(k)}) + \lambda_p \mathcal{R}_p(\mathbf{p}_0)$
- 4: $\mathbf{c}^{(k+1)} \leftarrow \arg \min_{\mathbf{c}} F_p(\mathbf{p}_0^{(k+1)}, \mathbf{c}) + \beta F_u(\mathbf{c}) + \lambda_c \mathcal{R}_c(\mathbf{c})$
- 5: $k \leftarrow k + 1$

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6:   end while
7:    $\hat{\mathbf{c}} \leftarrow \mathbf{c}^{(k)}$ 
8:    $\hat{\mathbf{p}}_0 \leftarrow \mathbf{p}_0^{(k)}$ 

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4. Description of computer-simulation studies

Proof-of-concept 2-D computer-simulation studies were performed to evaluate the proposed joint reconstruction approach.

4.1. Methods

The numerical phantoms for the initial pressure and SOS distributions, shown in Fig. 2, were generated from segmented clinical MRI images of a human breast [48]. The SOS phantom shows fibroglandular tissue in white and fat in black. The initial pressure distribution shows the cross-section of a number of blood vessels in white. While not visually obvious in Fig. 2, the parenchymal breast tissue also has a small, non-zero value for the initial pressure.

The measurement system consisted of a circular transducer array with 512 evenly-distributed elements and a radius of 110 mm. This geometry was chosen to match an existing USCT breast imaging system [25]. Similar ring array systems have been employed in PACT, e.g. [49]. A diagram of the imaging geometry is shown in Fig. 3. Each transducer was treated as a perfect point emitter and detector and was placed on the simulation grid by use of nearest neighbor interpolation.

The excitation pulse for USCT was described by a windowed sine wave as

$$s(t) = \exp\left(-\frac{(t-t_c)^2}{2\sigma^2}\right) \sin(2\pi f_c t), \quad (17)$$

where $f_c = 0.8$ MHz is the central frequency, and $t_c = 3.2$ μ s and $\sigma = 0.75$ μ s are the center and width of a Gaussian window, respectively. This corresponds to an excitation pulse containing approximately 3 cycles. Since nearest neighbor interpolation was employed to place the transducers on the simulation grid, the source term for the m -th view is given by

$$s_m(\mathbf{r}, t) = s(t)\delta(\mathbf{r} - \mathbf{r}_m), \quad (18)$$

where $\mathbf{r}_m$ is the location of the pixel nearest to the emitter for the m -th view. When the number of emitters was less than the number of transducers, the emitters were chosen to be evenly-distributed over the circular transducer array.

In order to simulate the pressure data recorded by the transducers, the acoustic wave equations for PACT and USCT, given by Eqns. (1a)–(1d) and Eqns. (2a)–(2d), respectively,

were solved by use of a first-order k-space pseudo-spectral method [33]. In both cases, the corresponding discrete operators, $\mathbf{H}_{pa}(\mathbf{c})$ and $\mathbf{H}_{us}(\mathbf{c})$, were implemented using NVIDIA's CUDA platform [50].

In order to avoid inverse crime [51], the measured pressure data were generated by use of different spatial and temporal sampling rates than were employed for image reconstruction. When the measured data were being generated, a simulation grid of size $256 \times 256 \text{ mm}^2$, sampled on a 2048×2048 uniform Cartesian grid with a spacing of 0.125 mm, was employed. The temporal sampling rate was 40 MHz, and the pressure data was recorded for 7000 time steps, corresponding to a simulation time of 175 μs . For the reconstruction, a simulation grid of size $256 \times 256 \text{ mm}^2$, sampled on a 1024×1024 uniform Cartesian grid with a spacing of 0.25 mm, was employed. The temporal sampling rate was 20 MHz, and the pressure data was recorded for 3500 time steps.

Results were generated for both noiseless and noisy data. For the noisy data, additive Gaussian white noise was added to the measurement data. For the PACT case, this noise had zero mean and a standard deviation equal to 1% of the maximum pressure recorded by the transducers. For the USCT case, the noise had zero mean and a standard deviation equal to 5% of the maximum pressure recorded by the transducer opposite the emitter for a homogeneous medium.

4.2. Reconstruction of the initial pressure from PACT data

Most image reconstruction methods in PACT for estimating the initial pressure distribution assume a constant SOS. In order to investigate the impact of ignoring variations in the SOS, the initial pressure distribution was reconstructed from noiseless PACT data assuming two different fixed SOS distributions: (1) the true SOS distribution and (2) a constant SOS equal to the background SOS. In both cases, the images were reconstructed from noiseless data by solving Eqn. (5) for $\lambda_p = 0$. The optimization problem was solved by use of FISTA [52] with a backtracking line search [53] and adaptive restart [54]. A method for computing the gradient of the data fidelity term is discussed in [17]. The initial guess for the initial pressure distribution was the vector of all zeros.

4.3. Reconstruction of the SOS from PACT data

To move beyond assuming a constant SOS, the SOS distribution could be estimated from PACT measurements alone. However, as mentioned earlier, the SOS and initial pressure distributions cannot both be stably recovered from PACT measurements alone. This was previously demonstrated theoretically [21] and through computer-simulation studies [22]. To investigate the feasibility of extracting useful information about the SOS distribution from PACT measurements, the SOS distribution was reconstructed assuming two different, fixed initial pressure distributions: (1) the true initial pressure distribution and (2) the true initial pressure distribution shifted by 1 mm along the x-direction. This was done to assess the sensitivity of the estimated SOS distribution to errors in the assumed initial pressure distribution. In both cases, the images were reconstructed from noiseless data by solving Eqn. (9) for $\lambda_c = 10^{-6}$. This problem was solved by use of the L-BFGS method [55, 56]. A method of computing the gradient of the data fidelity term with respect to the SOS can be

found in [22]. The initial guess for the SOS was homogeneous and equal to the background SOS.

4.4. Reconstruction of the SOS from USCT data

Alternatively, the SOS distribution can be accurately estimated from USCT measurements [23, 24, 25, 26, 27, 28, 29]. While there are many factors that affect the accuracy of the reconstructed SOS distribution in USCT, one key factor is the number of views employed during the data acquisition process. More views may allow the SOS distribution to be estimated more readily [57]. Using less views may decrease image acquisition times and allow for less complex and less expensive imaging hardware. Images were reconstructed from both noiseless and noisy USCT data for 512, 32, 16, and 8 views by solving Eqn. (12). For each case, a range of values for the regularization parameter λ_c was considered. The optimization problem given in Eqn. (12) was solved by use of stochastic gradient descent as described in [34]. The initial guess for the SOS was chosen to be the image reconstructed by use of a bent-ray method [58].

4.5. Reconstruction of initial pressure and SOS distributions using a sequential approach

The conventional approach when both USCT and PACT measurements are available is to reconstruct the SOS and initial pressure distributions sequentially. First, the SOS distribution is estimated from USCT measurements alone. Then, the initial pressure distribution is estimated from the PACT measurements by use of the previously estimated SOS distribution. The estimated initial pressure distributions will depend on the accuracy of the SOS distributions estimated from the USCT measurements. When the number of views for the USCT measurements is large, the estimated SOS distributions may be accurate enough for the desired task, even without utilizing the acoustic information present in the PACT measurements. As a result, this work focuses on the case where the number of views is small. In this case, the differences between the sequential approach and the joint reconstruction approach may be more pronounced. Thus, for this study, the SOS distribution was reconstructed from 8 noiseless USCT views, as detailed in Section 4.4. The initial pressure distribution was reconstructed from noiseless PACT data by solving Eqn. (5) for $\lambda_p = 0$ by use of this estimated SOS distribution. The optimization problem was solved as discussed in Section 4.2.

4.6. Joint reconstruction from combined PACT/USCT data

Under the proposed method, the initial pressure and SOS distributions are jointly estimated from combined PACT and USCT measurements by solving Eqn. (15) by use of the alternating minimization approach given in Algorithm 1. This approach involves iteratively solving two subproblems, one to estimate the initial pressure distribution and one to estimate the SOS distribution. The subproblem for estimating the initial pressure distribution was solved by use of the approach described in Section 4.2. The subproblem for estimating the SOS distribution was solved by use of stochastic gradient descent [34]. The gradients for the two data fidelity terms were calculated as described in Sections 4.3 and 4.4. Both subproblems require specification of stopping criteria. In this work, for the initial pressure subproblem, iteration was stopped when the ℓ_2 -norm of the change in the object between successive iterations was less than 10^{-4} or the number of iterations exceeded 50. For the

SOS subproblem, iteration was stopped when the ℓ_2 -norm of the change in the object between successive iterations was less than 10^{-4} or the number of iterations exceeded 200. Other stopping criteria could potentially accelerate the alternating joint reconstruction method. This general procedure was employed to generate all of the joint reconstruction results. Different studies differ only in their choice for the initial guess for the SOS distribution. Using a single NVIDIA Tesla K20 GPU, the computational time for 4 outer iterations of the joint reconstruction method was about 2 days.

The proposed approach introduces a tunable parameter β (see Eqn. (15)) that controls the relative weight of the PACT data fidelity term and the USCT data fidelity term. As the value of this parameter is increased, the estimated SOS distribution will depend more on the information present in the USCT measurements. When β is too small, the estimated SOS distribution will depend largely on the acoustic information present in the PACT measurements. This is similar to the case where the SOS and initial pressure distributions are jointly estimated from PACT measurements alone. As a result, it may be subject to the same limitations. When β is too large, the acoustic information in the PACT measurements is largely ignored. To investigate the impact of β , the initial pressure and SOS distributions were reconstructed from combined noiseless PACT and USCT data for 512 USCT views for different values of β . Since the number of USCT views is large for this case, the regularization terms may be ignored ($\lambda_p = \lambda_c = 0$) without having a significant impact on the reconstructed images. The initial guess for the SOS was chosen to be the image reconstructed by use of a bent-ray method [58].

The convergence of the alternating minimization approach was investigated for the case of 8 noiseless USCT views and parameter values of $\lambda_c = 1 \times 10^{-3}$, $\lambda_p = 0$, and $\beta = 10^4$. The convergence rate was evaluated in terms of the cost function values for each subproblem of the alternating minimization approach (i.e. the subproblem for estimating the initial pressure and the subproblem for estimating the SOS) and in terms of the root-mean-square errors (RMSEs) of the two estimated distributions. The RMSEs were calculated by computing the pixel-wise difference between the reconstructed image and the true numerical phantom for all pixels within a radius of 63 mm, which is slightly larger than the size of the object. The true phantoms were downsampled by a factor of 2 in order to match the spatial sampling rate of the reconstructed images. The initial guess for the SOS distribution was taken to be the image reconstructed from USCT measurements alone by use of the WISE method for $\lambda_c = 10^{-7}$ [34].

The proposed approach was compared against the conventional sequential approach for noiseless and noisy data and different numbers of USCT views. Recall that in the sequential approach, detailed in Section 4.5, the SOS distribution is first estimated from USCT measurements alone. Then, the initial pressure distribution is estimated from the PACT measurements alone by use of this estimated SOS distribution. Thus, the sequential approach fails to utilize the acoustic information present in the PACT measurements.

Both approaches require specification of one or more tunable parameters. Ideally, the values of these parameters would be chosen to maximize performance on a specified task [59]. In practice, performing studies to compute task-based measures of image quality can be time-

consuming and require considerable effort. As a result, they are outside the scope of the current work. Other advanced evaluation methods, such as bias-variance analysis, are similarly difficult to conduct due to the computational complexity of the proposed approach. Instead, for these computer-simulation studies, the reconstructed images will be evaluated based on their accuracy, as quantified by the RMSE. In particular, for the joint reconstruction approach, the values of the parameters were chosen to minimize the RMSE of the estimated SOS distribution. It was observed that this measure also resulted in accurate recovery of the initial pressure distribution.

Images were reconstructed from both noiseless and noisy data. For noiseless data, use of the regularization function $\mathcal{R}_p(\mathbf{p}_0)$ did not lead to meaningful improvement in the accuracy of the reconstructed images. Thus, in this case, only the values of λ_c and β were swept. Images were reconstructed from combined data for 16 and 8 USCT views. For noisy data, the value of the regularization parameter λ_p has a larger impact on the accuracy of the reconstructed images. In this case, images were reconstructed for various values of λ_p and λ_c for $\beta = 10^3$ and for $\beta = 10^4$ for 8 USCT views. The initial guesses for the SOS distributions were taken to be the images reconstructed from USCT measurements alone by use of the WISE method.

5. Results of computer-simulation studies

5.1. Reconstruction of the initial pressure from PACT data

The initial pressure distribution was reconstructed from noiseless data assuming two different SOS distributions: (1) the true SOS distribution and (2) a constant SOS equal to the background SOS. The reconstructed images and profiles through the reconstructed images are shown in Fig. 4. From these results, it can be seen that ignoring the heterogeneities in the SOS distribution results in obvious errors in the reconstructed initial pressure distribution. In the reconstructed image assuming a constant SOS distribution, the blood vessels are noticeably blurred and the maximum amplitude of the initial pressure distribution is significantly underestimated. This is consistent with prior work demonstrating the impact of ignoring SOS variations during reconstruction of the initial pressure distribution [11, 14, 17, 60].

5.2. Reconstruction of the SOS from PACT data

The SOS distribution was reconstructed assuming two different, fixed initial pressure distributions (see Fig. 5). When the SOS is estimated assuming the true initial pressure distribution, which is typically unknown, the reconstructed SOS distribution is accurate. However, when there are small errors in the assumed initial pressure distribution, the reconstructed SOS distribution may be much worse. For example, Fig. 5(b) shows the reconstructed SOS distribution when the assumed initial pressure distribution is shifted by 1 mm. The fact that the SOS distribution can be accurately reconstructed when the true initial pressure distribution is known suggests that the PACT measurements contain important information on the acoustic properties of the medium. However, in practice, the true initial pressure distribution is never known. Errors in the estimated initial pressure distribution can have a significant impact on the accuracy of the reconstructed SOS distribution. As a result,

in many cases, it may not be practical to jointly estimate both the SOS and initial pressure distributions from PACT measurements alone.

5.3. Reconstruction of the SOS from USCT data

In Fig. 6, the SOS distribution is reconstructed from noisy USCT measurements for different numbers of views. The RMSE is shown in the lower left of each image in Fig. 6. The value of the regularization parameter was tuned in order to minimize RMSE. Additional reconstructed images for a range of regularization parameter values are shown in the supplementary material. The accuracies of the reconstructed images decrease as the number of views is decreased. For 16 views, the apparent visual quality is noticeably degraded, even when the regularization parameter value is tuned. For 8 views, the image quality is reduced even further. Images were also reconstructed from noiseless USCT measurements. For brevity, these results are shown in the supplementary material.

5.4. Reconstruction of initial pressure and SOS distributions using a sequential approach

The conventional approach when both USCT and PACT measurements are available is to reconstruct the SOS and initial pressure distributions sequentially. First, the SOS distribution is estimated from USCT measurements alone. Then, the initial pressure distribution is estimated from the PACT measurements by use of the previously estimated SOS distribution. In this case, the estimated SOS distributions are simply those shown in Section 5.3 and in the supplementary material. In Fig. 7, the reconstructed initial pressure distribution, obtained from noiseless data, assuming a SOS distribution equal to that obtained from noiseless USCT measurements for 8 views, is shown. From these results, it can be seen that the initial pressure distribution can be relatively accurately estimated, even when there are small errors in the assumed SOS distribution. As a result, the largest differences between the sequential and joint reconstruction approaches may be seen in the estimated SOS distributions. In this case, the acoustic information present in the PACT measurements has the most direct and immediate impact.

5.5. Joint reconstruction from combined PACT/USCT data

5.5.1. Impact of the parameter β —The proposed joint reconstruction approach introduces a tunable parameter β (see Eqn. (15)) that controls the relative weight of the PACT data fidelity term and the USCT data fidelity term. Figure 8 shows reconstructed SOS distributions for different values of β . When β is too small, the estimated SOS distribution will depend largely on the acoustic information present in the PACT measurements. The artifacts seen in the reconstructed image when $\beta = 10^{-1}$ are qualitatively similar to those found in Fig. 5(b), which was reconstructed from PACT data alone. When β is too large, the acoustic information present in the PACT data may be largely ignored. In this case, since the number of USCT views is large, the SOS distribution can be accurately estimated from USCT measurements alone, and so there is only a minor increase in the RMSE when β is large.

5.5.2. Convergence of the alternating minimization approach—The convergence of the proposed alternating minimization approach can be seen in Fig. 9. For the chosen stopping criteria for the two subproblems, only a few outer iterations are needed to achieve

convergence. The estimated initial pressure distribution, in particular, changes little after only a couple of outer iterations. This is consistent with the prior observation that the initial pressure can be relatively accurately estimated even when there are small errors in the assumed SOS distribution [22, 61].

5.5.3. Comparison of sequential and joint reconstruction approaches—The proposed approach was compared against the conventional sequential approach for noiseless and noisy data and different numbers of USCT views. Recall that in the sequential approach, detailed in Section 4.5, the SOS distribution is first estimated from USCT measurements alone. Then, the initial pressure distribution is estimated from the PACT measurements alone by use of this estimated SOS distribution. Thus, the sequential approach fails to utilize the acoustic information present in the PACT measurements.

A series of reconstructed SOS images for different values of λ_c and β , obtained by joint reconstruction of combined noiseless PACT/USCT measurements for 16 USCT views, can be found in the supplementary material. The best result, in terms of the RMSE of the SOS, is compared with the corresponding best result obtained by use of the sequential approach in Fig. 10. From this, it can be seen that the joint reconstruction approach is able to produce a more accurate reconstructed image for the SOS compared with the sequential approach, which only utilizes the USCT measurements to estimate the SOS. The profiles demonstrate that the reconstructed image for the sequential approach exhibits lower contrast and has more oscillations in the background. There is little difference in the reconstructed initial pressure distributions.

The sequential and joint reconstruction approaches were also compared for noiseless data corresponding to 8 USCT views. The reconstructed SOS images for this case for different parameter values can be found in the supplementary material. The reconstructed images for the joint reconstruction approach are compared against the reconstructed images obtained by use of the sequential approach in Fig. 11. With fewer USCT views, the differences between the joint reconstruction and sequential approaches are more pronounced. Again, the reconstructed SOS image for the sequential approach exhibits lower contrast and has more oscillations in the background. There is only a minor difference in the estimated initial pressure distributions. The joint reconstruction approach slightly lowered the RMSE, but there is little obvious visual difference between the reconstructed initial pressure images.

Joint reconstruction was also performed from noisy USCT and PACT measurements. Since the largest differences between the joint reconstruction and sequential approaches were seen for the 8 USCT view case, only this case is considered here. Reconstructed SOS images for various values of λ_p and λ_c for $\beta = 10^3$ and for $\beta = 10^4$ can be found in the supplementary material. The best reconstructed images for the joint reconstruction and sequential approaches are compared in Fig. 12. The results are comparable to those observed in the noiseless case.

6. Conclusions

This work represents the first time that joint reconstruction was applied to combined PACT/USCT measurements, allowing the acoustic information in the PACT data to be effectively exploited. Joint reconstruction of the SOS and initial pressure distributions offers several advantages over conventional approaches. First, compared with use of PACT measurements alone, the approach allows the SOS distribution to be more accurately estimated. This can result in a more accurate reconstructed initial pressure distribution compared with assuming a constant SOS. Second, compared with use of the sequential approach, which estimates the SOS distribution from USCT measurements alone, the joint reconstruction approach allows the SOS distribution to be more accurately estimated from few-view USCT measurement data. This could allow for construction of simpler and lower cost combined PACT/USCT imaging systems. In this way, a synergistic improvement in the reconstructed image quality by the use of joint reconstruction from combined measurements is observed. The initial pressure distribution is more accurately estimated by the use of USCT measurements and the SOS distribution is more accurately estimated by the use of PACT measurements.

The proposed approach, however, does have some limitations. First, the method requires the user to specify a number of tunable parameters. Choosing these parameters to maximize image quality can be a time-consuming process requiring some trial and error. Second, the method involves some computational cost. Each outer iteration requires solving a subproblem to estimate the SOS distribution and a subproblem to estimate the initial pressure distribution. The subproblem for estimating the SOS, in particular, is very computationally demanding. This cost is mitigated by the fact that few outer iterations are needed to reach convergence. Further optimization of the stopping criteria for each of the subproblems could help reduce the computational cost. Third, the imaging models employed for these initial proof-of-concept studies ignore several aspects of experimental imaging systems. Notably, the assumed models simulate 2-D acoustic wave propagation rather than the 3-D propagation that occurs experimentally. Density variations and the acoustic attenuation are also ignored. Further, the transducers are modeled as ideal point detectors and emitters, ignoring their spatial and electro-acoustic impulse responses. These limitations will be addressed through future work. In addition, other imaging geometries, such as limited-view acquisition geometries, could be considered.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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Appendix A

Tuning of joint reconstruction parameters

The proposed approach, as outlined in Algorithm 1, requires specification of several tunable parameters: β , which controls the relative weight of the USCT data fidelity term, λ_p , which controls the relative weight of the regularization term for $\mathbf{p}_0$, and λ_c , which controls the relative weight of the regularization term for $\mathbf{c}$. The value of λ_p most directly affects the estimated initial pressure distribution. It affects the estimated SOS distribution only so far as the accuracy of the estimated initial pressure distribution affects how easily useful acoustic information can be extracted from the PACT data. In the case of noiseless measurements, the initial pressure distribution may be accurately recovered even when $\lambda_p = 0$. Thus, for noiseless data, only β and λ_c are tuned. These two parameters must be tuned jointly. Note that, for example, as β is increased, the relative weight of the USCT data fidelity term is increased relative to the PACT data fidelity term, but the relative weight of the USCT data fidelity term is also increased relative to the SOS regularization term. Thus, the optimal value of λ_c according to some measure will differ for different values of β . For noisy measurements, all three parameters must be jointly tuned. This can create a large search space to investigate.

The RMSEs of the reconstructed SOS images obtained by tuning β and λ_c for noiseless measurements for 16 USCT views are summarized in Table A1. The RMSEs of the reconstructed images are reduced relative to the RMSEs obtained for images reconstructed from USCT measurements alone for a wide range of values of β and λ_c . This suggests that some improvement can be achieved without fine-tuning the values of β and λ_c too precisely. The RMSEs of the reconstructed SOS images for tuning β and λ_c for noiseless measurements for 8 USCT views are summarized in Table A2. The results similarly show an improvement in RMSE compared with those obtained from USCT measurements alone for a wide range of values of β and λ_c .

For noisy measurements, all three parameters must be tuned. For the case of noisy measurements for 8 USCT views, λ_p and λ_c are jointly tuned for two values of β . The RMSEs for the reconstructed SOS images for $\beta = 10^3$ and $\beta = 10^4$ are summarized in Table A3. Changing the value of λ_p does impact the accuracy of the reconstructed SOS. However, this impact is predictably less dramatic than that of λ_c . As in noiseless case, improvement over the images reconstructed from USCT measurements alone is achieved across a wide range of values of the parameters.

Reconstructed images for each of these cases can be found in the supplementary material.

Table A1

RMSEs ($\times 10^{-3}$) of the reconstructed SOS images for joint reconstruction from noiseless data for 16 USCT views for different parameter values. The lowest RMSE value is shown in bold.

β	λ_c								
	3E-6	1E-5	3E-5	1E-4	3E-4	1E-3	3E-3	1E-2	1E-2
10^2	1.18	1.14	1.15	1.52	–	–	–	–	–
10^3	–	0.87	0.84	0.86	1.14	–	–	–	–
10^4	–	–	0.59	0.57	0.50	0.64	–	–	–
10^5	–	–	–	–	–	0.56	0.52	0.60	1.09

Table A2

RMSEs ($\times 10^{-3}$) of the reconstructed SOS images for joint reconstruction from noiseless data for 8 USCT views for different parameter values. The lowest RMSE value is shown in bold.

β	λ_c							
	1E-5	3E-5	1E-4	3E-4	1E-3	3E-3	1E-2	3E-2
10^2	3.11	2.99	3.05	4.06	–	–	–	–
10^3	2.62	2.52	2.40	2.65	–	–	–	–
10^4	–	–	2.45	2.27	2.25	2.71	–	–
10^5	–	–	–	–	3.22	3.02	2.65	2.94

Table A3

RMSEs ($\times 10^{-3}$) of the reconstructed SOS images for joint reconstruction from noisy data for 8 USCT views for different parameter values. The lowest RMSE value is shown in bold.

λ_p	$\beta = 10^3$			$\beta = 10^4$			
	λ_c			λ_c			
	1E-5	1E-4	1E-3	λ_p	1E-4	1E-3	1E-2
3E-3	6.21	2.61	3.70	1E-2	5.56	2.92	4.32
1E-2	5.42	2.47	3.78	3E-2	5.53	2.87	4.31
3E-2	6.84	2.79	3.54	1E-1	5.63	2.92	4.33

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Figure 1.
Schematics of the data acquisition process for (a) PACT and (b) USCT.

Figure 2.
(a) The SOS distribution, given in units of mm/μs, and (b) the initial pressure distribution, given in arbitrary units, of the numerical breast phantom.

Simulation grid

Figure 3.

Diagram of imaging system geometry. The measurement system consists of a circular array of ultrasonic transducers. These transducers are located in a large rectangular simulation grid over which the acoustic wave equation is solved. Within the transducer ring array is a smaller rectangular region that represents the reconstructed image. The SOS and initial pressure distributions are estimated within a circular field-of-view, shown in gray.

Figure 4.

Initial pressure distributions reconstructed from noiseless data assuming (a) the true SOS distribution and (b) a constant SOS equal to the background SOS. (c) Profiles through the reconstructed images at $y = 14.25$ mm. Results are shown after 50 iterations. No regularization was employed.

Figure 5. Reconstructed SOS distribution from noiseless PACT data given (a) the true initial pressure distribution and (b) an initial pressure distribution shifted 1 mm in the x-direction from the true distribution.

Figure 6.

Reconstructed SOS distributions from noisy USCT measurements for (a) 512 views and $\lambda_c = 3 \times 10^{-6}$, for (b) 32 views and $\lambda_c = 3 \times 10^{-7}$, for (c) 16 views and $\lambda_c = 3 \times 10^{-7}$, and for (d) 8 views and $\lambda_c = 1 \times 10^{-7}$. All reconstructed images are shown after 400 iterations and in a grayscale window of [1.467, 1.520] mm/ μ s.

Figure 7.

(a) Reconstructed initial pressure distribution from noiseless data assuming the SOS distribution obtained from 8 noiseless USCT views. (b) Profile through the reconstructed image at $y = 14.25$ mm. The profile through the reconstructed initial pressure distribution assuming the true SOS distribution is shown as a reference.

Figure 8. SOS distributions reconstructed from combined noiseless PACT and USCT data for 512 USCT views for (a) $\beta = 10^{-1}$, (b) $\beta = 10^3$, and (c) $\beta = 10^6$. No regularization was employed. Results are shown after 2 outer iterations in a grayscale window of [1.467, 1.520] mm/ μ s.

Figure 9.

Convergence of (a) the cost function value for the initial pressure subproblem, (b) the cost function value for the SOS subproblem, (c) the initial pressure RMSE, and (d) the SOS RMSE for the case of combined noiseless PACT and USCT data for 8 USCT views. The parameters were $\lambda_c = 1 \times 10^{-3}$, $\lambda_p = 0$, and $\beta = 10^4$. For computational expediency, the value of F_u (c) was evaluated only for a single realization of the encoding vector $\mathbf{w}$.

Figure 10.

SOS distributions reconstructed from (a) noiseless USCT data and (b) noiseless PACT and USCT data for 16 USCT views. (c) Profiles through the reconstructed images at $y = 16.75$ mm. Initial pressure distributions reconstructed by (d) the sequential approach and (e) the joint reconstruction approach for noisy data and 16 USCT views. (f) Profiles through the reconstructed images at $y = 14.25$ mm. The sequential SOS distribution is shown for $\lambda_c = 1 \times 10^{-7}$ after 400 iterations. The sequential initial pressure distribution is shown for $\lambda_p = 0$ after 50 iterations. The joint reconstruction results correspond to 4 outer iterations for $\beta = 10^4$, $\lambda_c = 3 \times 10^{-4}$, and $\lambda_p = 0$.

Figure 11.

SOS distributions reconstructed from (a) noiseless USCT data and (b) noiseless PACT and USCT data for 8 USCT views. (c) Profiles through the reconstructed images at $y = 16.75$ mm. Initial pressure distributions reconstructed by (d) the sequential approach and (e) the joint reconstruction approach for noiseless data and 8 USCT views. (f) Profiles through the reconstructed images at $y = 14.25$ mm. The sequential SOS distribution is shown for $\lambda_c = 1 \times 10^{-7}$ after 400 iterations. The sequential initial pressure distribution is shown for $\lambda_p = 0$ after 50 iterations. The joint reconstruction results correspond to 4 outer iterations for $\beta = 10^4$, $\lambda_c = 1 \times 10^{-3}$, and $\lambda_p = 0$.

Figure 12.

SOS distributions reconstructed from (a) noisy USCT data and (b) noisy PACT and USCT data for 8 USCT views. (c) Profiles through the reconstructed images at $y = 16.75$ mm. Initial pressure distributions reconstructed by (d) the sequential approach and (e) the joint reconstruction approach for noisy data and 8 USCT views. (f) Profiles through the reconstructed images at $y = 14.25$ mm. The sequential SOS distribution is shown for $\lambda_c = 3 \times 10^{-7}$ after 400 iterations. The sequential initial pressure distribution is shown for $\lambda_p = 3 \times 10^{-3}$ after 50 iterations. The joint reconstruction results correspond to 4 outer iterations for $\beta = 10^3$, $\lambda_c = 1 \times 10^{-4}$, and $\lambda_p = 1 \times 10^{-2}$.