Published in final edited form as:

Dent Mater. 2018 May ; 34(5): 726–736. doi:10.1016/j.dental.2018.01.020.

Approximate relative fatigue life estimation methods for thin-walled monolithic ceramic crowns

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Abstract

Objectives—The objective is to establish an approximate relative fatigue life estimation method under simulated mastication load for thin-walled monolithic restorations.

Methods—Experimentally measured fatigue parameters of fluormica, leucite, lithium disilicate and yttrium-stabilized zirconia in the existing literature were expressed in terms of the maximum cyclic stress and stress corresponding to initial crack size prior to N number of loading cycles to assess their differences. Assuming that failures mostly originate from high stress region, an approximate restoration life method was explored by ignoring the multi-axial nature of stress state. Experiments utilizing a simple trilayer restoration model with ceramic LD were performed to test the model validity.

Results—Ceramic fatigue was found to be similar for clinically relevant loading range and mastication frequency, resulting in the development of an approximate fatigue equation that is universally applicable to a wide range of dental ceramic materials. The equation was incorporated into the approximate restoration life estimation, leading to a simple expression in terms of fast fracture parameters, high stress area A , the high stress averaged over A and N . The developed method was preliminarily verified by the experiments. The impact of fast fracture parameters on the restoration life was separated from other factors, and the importance of surface preparation was manifested in the simplified equation. Both the maximum stress and the area of high stress region were also shown to play critical roles.

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Significance—While nothing can replace actual clinical studies, this method could provide a reasonable preliminary estimation of relative restoration life.

1. Introduction

Ceramic crowns are widely used due to their improved biocompatibility and esthetic appeal compared to the more traditional metal-based restorations. With the availability of high strength ceramic materials such as zirconia [1] and lithium disilicate [2], it has become possible for clinicians to make thin-walled monolithic crown similar to metal [3]. However, ceramics are brittle, and failure from fracture continues to be a major concern [4]. While it is understood that restoration geometry, loading conditions, surface preparation, environmental fatigue, and ceramic properties play a role in restoration survival, there is no clear understanding as to how each factor influences restoration lifetime. Currently, there is no easy way to estimate the relative fatigue survival rate of monolithic crowns.

A systematic review of all-ceramic crown survival suggest that many ceramic systems showed 5-year failure rates similar to metal ceramics [5]. Analysis of the complications indicated that porcelain chipping [6], especially with the high strength ceramics, was responsible for many of the reported failures and that substructure fractures were reported much less frequently. These clinical observations have moved the profession toward the use of monolithic ceramic crowns particularly for the restoration of posterior teeth.

The fractographic analyses of clinically failed crowns has improved our understanding of crown failure modes. Early all-ceramic monolithic materials such as fluoromica (FM) showed that most of the failures occur due to fracture initiated from the intaglio surface [7]. For higher strength ceramic materials such as lithium disilicate (LD), yttrium-stabilized zirconia (YZ), and alumina, failure initiation from the wall-margin area has also been reported [8, 9]. As the profession moves away from layered ceramic structures and toward the use of thinner-walled monolithic structures, it is anticipated that porcelain chipping will be less common failure mode and interface initiated fractures will be a more dominant fracture mode.

A number of 3D finite element (FE) models of complete restorations have been analyzed to identify high stress areas [10]. In addition, failure probability analysis software such as CARES [11] has been employed to predict fatigue life of dental restorations [12]. All these analyses were based on the maximum principal stress. More recently, focusing on realistic multi-axial stress state, Nasrin *et al.* [13] predicted fatigue life of actual restoration and verified the model against experimental data with a simple trilayer model restorations.

In this work, we propose a simplified approach to estimate fatigue life of thin-walled monolithic restoration. For this purpose, a wide range of representative dental ceramic materials, FM, LD, YZ and leucite reinforced glass-ceramic (LR), where material parameters were available in the existing literature, were chosen for analysis. A universal approximate fatigue equation applicable to all dental ceramic materials was developed and incorporated into crown life estimation. The fatigue survival rates of trilayer specimen with LD cemented to dentin/dentin substitute were experimentally determined and used to verify

the proposed approximation method. The effect of LD thickness on stress distribution and corresponding approximate fatigue life estimation were also examined.

2. Material and Method

2.1 Parameter conversion

2.1.1 Fast Fracture Parameter Conversion—To analyze the effect of multiaxial stress state on fracture initiation from surface flaws at the intaglio surface, statistical failure probability model by Batdorf and Cross [14] (Appendix A) was employed. Following the work of Chao and Shetty [15], surface flaws were characterized by the crack density function (the number of cracks with critical stress, σ_{cr} per unit area), N_{flaw} , with parameters m and $\bar{k}$ as follows,

$$N_{flaw} = \bar{k} \sigma_{cr}^m. \quad (1)$$

Weibull's scale and shape parameters, $\bar{\sigma}_0$ and m (Appendix A), were determined by four point bending tests for LD [16] and FM [17] and by cantilever tests for YZ [18]. Considering the uniaxial stress case based on Batdorf's formulation [14, 15] and comparing the resulting formulation with Weibull formulation, $\bar{k}$ in Batdorf's formulation was determined by

$$\frac{4m\bar{k}}{\pi} I_{A, uniaxial} A_{eff} = \left(\frac{1}{\bar{\sigma}_0} \right)^m, \quad (2)$$

$$\text{where} \quad I_{A, uniaxial} = \int_0^{\pi/2} \theta_{cr} \sin \theta_{cr} (\cos \theta_{cr})^{(2m-1)} d\theta_{cr}. \quad (3)$$

For each material, parameters $I_{A, uniaxial}$ and A_{eff} (Eqs. (A3), (A4), (A6) in Appendix A) were evaluated based on experimentally measured values of m and sample geometries.

Similarly, using Weibull parameters, $\bar{\sigma}_0$ and m determined by Myers *et al.* [19] for LR based on biaxial tests, $\bar{k}$ in Batdorf's formulation was determined by

$$4\bar{k}mr_1^2 I_{A, biaxial} = \left(\frac{1}{\bar{\sigma}_0} \right)^m, \quad (4)$$

$$\text{where } I_{A, \text{biaxial}} = \int_0^1 \int_0^{\frac{\sigma_{cr}}{\sigma_{1, \max}}} \theta_{cr} \left(\frac{r}{r_1} \right) \left(\frac{\sigma_{cr}}{\sigma_{1, \max}} \right)^{(m-1)} d \left(\frac{\sigma_{cr}}{\sigma_{1, \max}} \right) d \left(\frac{r}{r_1} \right), \quad (5)$$

$$\text{and } \theta_{cr} = \frac{1}{2} \cos^{-1} \left[\frac{2\sigma_{cr} - \sigma_{1, \max} - \sigma_{2, \max}}{\sigma_{1, \max} - \sigma_{2, \max}} \right]. \quad (6)$$

The radius of support circle in bi-axial tests was denoted by r_1 , and the maximum tangential and radial normal stress at the bottom surface of the specimen at position r , were respectively represented by $\sigma_{1, \max}$ and $\sigma_{2, \max}$. To evaluate $I_{A, \text{biaxial}}$ the stress distribution at the bottom surface of a disk sample was determined by axisymmetric finite element analysis (FEA) model (disk radius: 6 [mm]; thickness: 1 [mm]; radius of support ring: 4.74 [mm]; indenter radius: 0.3 [mm]) based on ABAQUS [20] following ASTM F394-78 standard employed by Myers *et al.* [19].

2.1.2 Fatigue Parameter Conversion—To analyze fatigue crack growth (crack size a , time t), the power law with parameter n and A was employed.

$$\frac{da}{dt} = AK^n, \quad \sigma = \frac{K}{Y\sqrt{a}} \quad (7)_{1,2}$$

where stress, crack shape constant and stress intensity factor were respectively represented by σ , Y and K . For cyclic loading, Eq. (7)₁ was integrated from initial time ($t=0$) with crack size a_i to the final time ($t=t_f$) with crack size a_f . The resulting equation with the use of Eq. (7)₂ [21] lead to

$$\sigma_i = \left[\frac{2^{-n} h N T (\sigma_{\max})^n}{B} + (\sigma_{\max})^{(n-2)} \right]^{\frac{1}{n-2}}, \quad (8)$$

$$\text{where } B = \frac{2}{A Y^2 (n-2) K_{IC}^{(n-2)}}, h = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \left[\frac{\sigma_{\text{mean}}}{\sigma_{\text{amplitude}}} + \sin \theta \right]^n d\theta. \quad (9)_{1,2}$$

The maximum cyclic stress, the critical stress intensity factor, number of cycles, mean stress, amplitude of stress, and time period were respectively denoted by $\sigma_{\max}$, K_{IC} , N , σ_{mean} , $\sigma_{\text{amplitude}}$ and T in Eq. (8) and Eqs. (9). For mastication, the mean stress is equal to the amplitude of stress ($\sigma_{\text{mean}}/\sigma_{\text{amplitude}} = 1$).

For LD, Paris-law-based (Appendix B) fatigue parameters $\bar{A}(7.39 \times 10^{-6}[(\text{m/sec})(\text{MPa}\sqrt{m})^{-n}])$ and n (19.1) (Eq. (B1) in Appendix B) were determined by Joshi *et al.* [16] based on cyclic four point bending tests ($\sigma_{\text{amplitude}}=20.25$ [MPa]; $\sigma_{\text{mean}}=24.75$ [MPa]). Similarly, for YZ, Studart *et al.* [22] determined Paris-law-based fatigue parameters $\bar{A}^*(2.64 \times 10^{-22}[(\text{m/sec})(\text{MPa}\sqrt{m})^{-n}])$ and n (24.9) (Eq. (B2) in Appendix B) with zero mean stress based on cyclic cantilever tests. Both experiments were performed at $T=0.1$ [s] (10 [Hz]). Paris-law-based fatigue parameters or $\bar{A}^*$ (Eqs. (B1) and (B2) in Appendix B) were converted to power-law-based fatigue parameters using the following conversion equations [23].

$$A = \frac{2^n \bar{A}}{hT} \quad (\text{Non - zero mean stress}), \quad (10)$$

$$A = \frac{\bar{A}^*}{hT} \quad (\text{zero mean stress}). \quad (11)$$

For LR, Myers *et al.* [19] determined power-law-based fatigue parameters σ_{f0} (83.3 [MPa]) and n (31) using different stress rates (dynamic loading) in biaxial tests. Under dynamic tests, parameter B was determined from the experimentally determined parameter, σ_{f0} , inert strength σ_{in} (131.8 [MPa]), parameter n and normalized constant stress rate $\dot{\sigma}_0$ (1MPa/s) as follows [24].

$$\sigma_{f0} = \left[(n+1) \dot{\sigma}_0 B \sigma_{in}^{(n-2)} \right]^{\frac{1}{n+1}} \quad (12)$$

Since fatigue investigations for FM were not available in the literature, fatigue parameters determined for MACOR [25], were employed. Both MACOR® and DICOR® are mica type glass-ceramics with similar crystal phases [26]. Based on Eq. (9)₁ and the values of Y and K_{IC} (LR [27] and FM [28]), fatigue parameter A was also evaluated.

2.2 Experimental determination of survival probability under cyclic indentation load

Following the work by Hernandez et al. [29], a group of 10 trilayer specimens with 1 mm thick LD ceramic plates bonded to flattened extracted human 3rd molars were fabricated. Additionally, 20 trilayer specimens with 1mm thick LD ceramic plates were bonded to cylinders of a simulated dentin substitute (G10, McMaster Carr) were fabricated. The ceramic plates were bonded to the dentin and simulated dentin using a two-step acid etch technique (Optibond Solo Plus, Kerr) and a dual cure resin cement (Nexus, Kerr). The intaglio surface of the ceramic plates were surface finished with a 600 grit SiC slurry on a glass slab, ultrasonically cleaned in water for 1 min, etched for 20 sec with a 5% HF acid gel (IPS Ceramic Etch, Ivoclar Vivadent), rinsed with a water syringe and air dried. Silane was applied to the etched surface (Silane Primer, Kerr) and air thinned prior to cementation. Specimens were subjected to maximum cyclic load of 300N for up to 6 million cycles.

Cyclic load was applied through 12.5 mm diameter brass ball coated with a vacuum formed 0.5 mm thick layer of polyethylene to avoid surface contact damage. The specimens were evaluated for evidence of internal crack initiation from interface after each 1 million cycle intervals under 10× magnification using a stereomicroscope (Model SMZ-1, Nikon) and transillumination with a fiber optic light (Model FO-150, Fostec). When a specimen was observed to have a radial crack, the number of cycles occurring prior to the observation period was recorded as the failure time, and survival probability with 95% confidence interval were evaluated using Kaplan-Meier analysis [30]. Following the work by Wang et al. [31], the fast fracture parameters of LD plates with the same surface finish employed for the trilayer specimens were experimentally determined from biaxial load-up-to-failure tests, and the resulting Weibull parameters $\bar{k} = 2.41 \times 10^{-17}$ [MPa^{-m}mm⁻²] and $m = 7.52$ were used for this evaluation.

2.3 Lifetime Approximation

2.3.1 Approximate fatigue equation for dental ceramic materials—Since the stress corresponding to initial critical crack size (prior to N cyclic loading with the maximum cyclic stress, σ_{max}), σ_i , is significantly large compared to σ_{max} , Eq. (8) could be rewritten as

$$\sigma_i = (C)^{\frac{1}{n-2}} (N)^{\frac{1}{n-2}} \sigma_{max}^{\frac{n}{n-2}}, \quad C = \frac{2^{-n} h T}{B}. \quad (13)_{1,2}$$

Experimentally determined values of parameter n for YZ, LD, LR and FM available in the literature were somewhat similar. If the order of magnitude of variation of $(C)^{\frac{1}{n-2}}$ value among YZ, LD, LR and FM is found to be small, then fatigue parameters, n and $(C)^{\frac{1}{n-2}}$, averaged over four dental ceramic materials could be utilized as approximation. For this purpose, parameter h in Eq. (9)₂ was evaluated for mastication frequency of 1.5 [Hz] ($T = 0.667$ [s]) with $\sigma_{mean}/\sigma_{amplitude} = 1$. Given a specific number of cycles (1, 2, 3, 4, 5, 6, 7 millions), σ_i was plotted as a function of σ_{max} with experimentally measured/converted fatigue parameters n and B based on Eq. (8). For each specified number of cycles, the resulting fatigue plots for YZ, LD, LR and FM were compared with Eq. (13) with average parameters n and $(C)^{\frac{1}{n-2}}$.

2.3.2 Approximate fatigue life estimation of monolithic ceramic restorations—

Based on the fact that failures mostly originate from high stress regions, an approximate method for fatigue life estimation was explored by considering only the maximum cyclic stress and neglecting the multi-axial nature of stress state. Since Eq. (1) provides the probability of finding critical flaws per unit area, failure probability might be approximated by $N_{flaw} A$, where A represents the area of high stress region. By combining Eq. (1) and the universal fatigue approximation (Eq. (13)) with replacement of σ_{cr} by σ_i in Eq. (1) and σ_{max} by $\sigma_{max,avg}$ (the maximum cyclic stress averaged over area A) in Eq. (13), respectively, the fatigue survival rate of restorations can be approximated. This approximate

method was applied to estimate fatigue survival rate of a trilayer model restoration using ceramic LD, and the resulting estimation was compared against experimentally obtained survival data described in 2.2. For A , the area with stress above 90% and 80% of the maximum cyclic stress, σ_{max} , were chosen based on FEA and the corresponding maximum stress averaged over A , $\sigma_{max,avg}$, were chosen respectively to be 95% and 90% of σ_{max} .

For ceramics YZ, LR and FM, the converted fast fracture parameters were employed to compare the approximate life estimation with different A against the multi-axial detailed numerical life prediction outlined in the section 2.4.

2.3.3 Effect of ceramic thickness on approximate fatigue life estimation—In order to explore the effect of thickness on fatigue survival rates, approximate fatigue life of model trilayer restorations using ceramic LD with 1.0 mm, 0.7 mm and 0.6 mm thickness under 300N cyclic load were calculated based on the method described in section 2.3.2. For this analysis, the stress distribution as a function of radius at the bottom of the ceramic plates for different thickness (1mm, 0.7 mm and 0.6 mm) were also analyzed.

2.4. Numerical survival probability with multi-axial stress consideration for simple trilayer model restoration

In order to examine the validity of the proposed fatigue life approximation prediction (section 2.3.2) for the various dental ceramic materials and ceramic thicknesses, the more traditional multiaxial FE analysis approach was employed. For this purpose, fatigue was introduced in the axisymmetric failure probability model developed by Wang *et al.* [31]. Using this technique, the survival probability of the model trilayer restorations with different ceramic materials and properties as listed in Table 1 [28, 31–35] were numerically simulated under conditions of 300 N cyclic loading with a frequency of 1.5Hz (time period $T=0.667\text{sec}$). As in Wang *et al.* [31], the survival probability of an infinitesimal concentric ring bounded by radii r and $(r+\Delta r)$ at the bottom surface of the ceramic, denoted by S , was given by

$$S = \exp[-2\pi r \Delta r k], \quad (14)$$

$$k = \frac{m\bar{k}}{2\pi} \int_0^{\sigma_{1i}} 2 \cos^{-1} \left[\frac{2\sigma_{cr} - \sigma_{1i} - \sigma_{2i}}{\sigma_{1i} - \sigma_{2i}} \right] (\sigma_{cr})^{(m-1)} d\sigma_{cr}. \quad (15)$$

Critical principal stress components prior to fatigue, denoted by σ_{1i} and σ_{2i} , were evaluated by replacing σ_{max} with the maximum and the minimum principal cyclic stress $\sigma_{1,max}$ and $\sigma_{2,max}$ respectively in Eq. (8). For each concentric ring, the critical principal stress component (σ_{1i}) was divided into 100 number of critical stress increments, and the integration was numerically calculated based on Matlab code to evaluate the survival probability. The bottom surface of the ceramic was discretized into 100,647 numbers of concentric rings along the radius. This method was repeated for each concentric ring, and

the summation of the survival probability of all the concentric rings provided the overall survival probability.

3. Result

3.1. Fast fracture and fatigue parameters

Experimentally measured Weibull parameters $\bar{\sigma}_0$ and m , fatigue parameter n , parameter h from frequency used for experiments (10 [Hz]) for YZ and LD, and parameters necessary for conversion such as critical stress intensity factor K_{IC} , effective area A_{eff} , parameter I_A , and crack shape constant Y for YZ, LD, LR and FM are summarized in Table 2. Batdorf-based fast fracture parameter $\bar{k}$, power-law-based fatigue parameters A and B and parameter h for mastication at 1.5 [Hz] frequency for the four materials are shown in Table 2. A substantial difference in the order of magnitude in $\bar{k}$ values among four materials was observed.

3.2. Approximation

The difference in the values of $(C)^{\frac{1}{n-2}}$ in Eq. (13) for four dental ceramic materials was found to be small as listed in Table 2. Therefore, utilizing average values of $(C)^{\frac{1}{n-2}}$ and n in Eq. (13), we obtained

$$\sigma_i = 0.838N^{.0415}\sigma_{max}^{1.08}. \quad (16)$$

Given a specific number of cycles (1, 2, 3, 4, 5, 6, 7 millions), all four curves for YZ, LD, LR and FM evaluated from Eq. (8) were found to be close in values despite the fact there exists substantial differences in the value of experimentally measured/converted fatigue parameter B for the different ceramics. In Figure 1, comparison between approximate universal fatigue equation (Eq. (16)) and material dependent fatigue equation (Eq. (8)) were shown for 1 and 7 million cycles. As demonstrated in Figure 1, four curves could be reasonably approximated by the average universal fatigue Eq. (16).

Replacing σ_{max} by $\sigma_{max,avg}$ in Eq. (16) and combining the resulting equation with Eq. (1) with replacement of σ_{cr} by σ_i , the following approximate fatigue survival equation was obtained.

$$1 - N_{flaw}\Delta A = 1 - \bar{k}\sigma_i^m\Delta A = 1 - \bar{k}(0.838N^{.0415}\sigma_{max,avg}^{1.08})^m\Delta A \quad (17)$$

3.3. Verification of fatigue life estimation

Comparison of approximate fatigue survival rates of 1 mm a trilayer with experimental data and multi-axial detailed simulation—In Figure 2 (a), experimentally obtained survival curves for two sets of trilayer data utilizing material LD were shown. The shaded gray area represents the 95 % confidence intervals (CI) for the

larger simulated dentin group. Based on FEA, σ_{max} was determined to be 47.0 [MPa] and A , the area with stress above 90% and 80% of σ_{max} , were determined to be 0.126 [mm²] and 0.196 [mm²] respectively. The corresponding $\sigma_{max,avg}$ were 44.7 [MPa] and 42.3 [MPa] respectively. Utilizing these values, the approximate fatigue survival rates based on Eq. (17), were demonstrated to be within 95% confidence interval of experimental data. The approximation was also compared against the multi-axial detailed simulation which was also within 95% 6CI of experimental data.

In Figure 2(b), for 1 [mm] thick YZ, LR and FM trilayers, the comparison between the multi-axial detailed numerical simulation and the approximate fatigue survival rates (Eq. (17)) were shown. Based on FEA, σ_{max} for YZ, LR and FM were determined to be 56.2 [MPa], 37.0 [MPa] and 37.0 [MPa], respectively. The average stress, $\sigma_{max,avg}$, and the area corresponding to stress above 90% of the maximum stress were 53.4[MPa], 0.053 [mm²] for YZ, 35.2[MPa], 7.07 [mm²] for LR and 35.2[MPa], 7.07 [mm²] for FM, respectively. The corresponding values for stress above 80% approximation were 50.6[MPa], 0.102 [mm²] for YZ, 33.3[MPa], 10.2 [mm²] for LR and 33.3[MPa], 10.2 [mm²] for FM, respectively. The maximum errors of the approximate method compared to the detailed numerical simulation after 6 million cycles for LD, LR and FM were 3%, 12% and 16%, respectively, while for YZ curves of detailed simulation and approximation with A values overlapped and were indistinguishable in the graph presented.

Comparison between approximate fatigue survival rates for 0.7 and 0.6 mm LD trilayers and multi-axial detailed simulation—In Figure 3, for LD trilayer under 300N load, significant differences in stress distribution due to the thickness differences were shown. As LD trilayer thickness decreased from 1 mm to 0.6 mm, not only the maximum stress increased from 47.0 [MPa] to 68.8 [MPa] but also the area with stress level above 90% of the maximum stress significantly increased from 0.126 [mm²] (radius 0.2 mm) to 1.17 [mm²] (radius 0.61 mm). In Figure 4, comparison between approximate fatigue survival rates and multi-axial detailed simulation were shown for 1 mm, 0.7 mm and 0.6 mm LD trilayers. In each of the analysis, ceramic thickness has a significant effect on the survival rate. The approximation errors compared to multi-axial detailed simulation were 11.4% (90% approximation) and 9.2% (80% approximation) for 0.7 mm thick trilayer. The corresponding errors for 0.6 mm trilayer were 22.7% (90% approximation) and 17.1% (80% approximation).

4. Discussion

In this study, an approximate universal fatigue growth equation for dental ceramic materials (Eq. (16)) was developed. The stress corresponding to initial critical crack size (prior to N cyclic loading with maximum cyclic stress, σ_{max}), $\sigma_{i,c}$, was expressed in terms of σ_{max} and the number of cycle N . The experimentally measures fatigue parameters of a representative range of dental ceramics (YZ, LR, LD and FM) were obtained from various published works. There were substantial differences in the experimental methods used to obtain the parameters and the resulting values of the experimentally measured/converted fatigue parameter B among the four ceramic materials. In addition, the material properties such as Young's modulus and critical stress intensity factor differ significantly as shown in Table 1

and 2. Despite these differences, the propagation phase of crack growth under simulated mastication loading was similar for the range of dental ceramic materials examined in this work. This includes fatigue parameters of YZ experimentally measured by Studart et al. [22]. In general, transformation toughening occurs in YZ and this effect can significantly impact crack growth phenomena. For clinically relevant surface flaw fatigue crack growth, this approximation of crack growth with averaged fatigue parameters (n and $(C)^{\frac{1}{n-2}}$) was demonstrated to be a reasonable approximation. While the proposed parameters are not meant to substitute for precise experimental determination, this approximate universal fatigue equation could be utilized as a preliminary estimation of the fatigue behavior of dental ceramics.

Focusing our attention on failures initiating from surface flaws in high stress region at the ceramic intaglio surface, the approximate fatigue survival rate (Eq. (17)) as a function of the number of cycles for simple trilayer ceramic restoration was developed. This approximation requires experimental determination of fast fracture parameters m and $\bar{K}$ of ceramic materials and FEA determination of $\sigma_{max,avg}$ and A for a specific restoration geometry under a specified load. The survival approximation for ceramic LD trilayer specimen produced an estimation which compared favorably with the more detailed numerical simulation as well as with two different experimental data sets (Figure 2(a)) from actual fatigued trilayer specimens when A was chosen to be the area with stress above 80% or 90% of the maximum stress.

Using this approximation method, estimations of the fatigue life for 1 mm thick ceramic trilayer restorations fabricated with ceramics YZ, LR and FM were compared to multi-axial detailed simulation as shown in Fig. 2b. In each case the detailed fatigue simulations resulted in slightly higher failure estimates (4–10% over 6 million cycles) than the proposed approximation method. However, when comparing the range of different ceramic materials the relative difference in the fatigue estimates were in good agreement with the detailed simulations.

Additionally, the fatigue life estimations for thinner ceramic trilayers (1.0, 0.7 and 0.6 mm thickness) utilizing ceramic material LD were compared to their comparable multi-axial detailed simulation as shown in Fig. 4. Because the thickness of the ceramic layer can greatly influence the stress distribution on the intaglio surface, it is necessary to consider this in the estimations. Figure 3 shows how the maximum stress area changes as a function of ceramic thickness. The 1mm thick ceramic plates have a much smaller area of maximum stress compared to the thinner ceramic plates as well as a much lower stress level for the same given loads. These two factors greatly influence the survival probability for a given ceramic material as shown in Figure 4. Thinner ceramic specimens exhibit higher stresses over a greater area and thus experience significantly greater probabilities of failure for a given load.

How well this approximation method can be utilized for actual restoration geometries is not known and further work would be needed. However, based on our prior work [13, 36], the high stress region was found to be relatively small in both the simple trilayer restoration and

in an actual crown geometry even if the wall thickness is quite thin. Despite the differences in geometry, the maximum stress (47.0 MPa) for 1 mm thick LD trilayer under 300N load was somewhat similar to the maximum stress (57.7 MPa) for an actual thin-walled crown restoration under 500N load [13]. The stress distribution in actual restorations is substantially more complicated than that in flat trilayers examined in this work. Failure initiation sites in actual restorations, however, are likely to be in the high stress regions. Therefore, this approximation technique with high stress area (80~90% above the maximum stress) might be utilized as a preliminary relative restoration life estimation.

Both critical stress intensity factor and surface flaw size distribution impact fast fracture parameters. Other factors such as restoration geometry, cyclic load, and Young's modulus significantly impact the area A (with stress above 80~90% of the maximum cyclic stress) and the corresponding maximum stress averaged over A , $\sigma_{max,avg}$. In this approximation, therefore, the impact of fast fracture parameters on restoration life was separated from other factors and was shown to play a critical role in restoration life. Based on fractographic analysis, Thompson *et al.* [7] stated that the fracture initiation sites were controlled primarily by the location and size of the critical flaw in intaglio surface. The developed approximate restoration life estimation (Eq. (17)) also demonstrated the importance of surface flaws through fast fracture parameters m and $\bar{k}$. Clinically employed surface preparation technique for the intaglio surface is different from the surface preparation technique employed for flat ceramic samples, and it is anticipated that this difference can significantly impact restoration life. Based on our analysis and existing fractographic analysis, experimental determination of fast fracture parameters for ceramic materials with more clinically relevant surface preparation technique appears to be most important.

5. Conclusion

An approximate universal fatigue growth equation applicable to all dental ceramic materials was developed based on the existing experimental data in literature. A simple approximate survival rate prediction equation was developed to estimate fatigue survival rates of monolithic ceramic crowns focusing on failures initiating from the ceramic intaglio surface. The universal fatigue growth equation was incorporated into the equation.

While the method was preliminary verified against experimental failure data of a single ceramic trilayer model, the method will require careful experimental verifications utilizing several ceramic materials and actual dental restorations to substantiate its practical use. With the increasing amount of restoration designs that are being managed digitally, the application of this simple life estimation formula could be combined with simple material dependent fast fracture data and standard FEA methods to quickly assess restoration failure probabilities. This could be a more meaningful way to optimizing monolithic ceramic crown designs in real time.

Acknowledgments

This research was partially supported by the National Institute of Dental and Craniofacial Research (grant R56DE 021470). The authors declare no potential conflicts of interest with respect to the authorship and/or publication of this article.

Appendix A

Weibull's Formulation

Phenomenological failure model by Weibull [37], widely used in dental literature, with parameters m and σ_0 to predict failure of ceramics is summarized as follows.

$$F_{weibull} = 1 - \exp\left[-\int_A \left(\frac{\sigma}{\sigma_0}\right)^m dA\right] = 1 - \exp\left[-\left(\frac{\sigma_{max}}{\sigma_0}\right)^m\right], \quad (A1)$$

$$\text{where } \frac{1}{\sigma_0} = \frac{1}{\sigma_0} A_{eff}, A_{eff} = \int_A \left(\frac{\sigma}{\sigma_{max}}\right)^m dA. \quad (A2)$$

By curve fitting experimental failure probability $F_{weibull}$ and the maximum stress at the time of failure, σ_{max} , two parameters m and σ_0 are experimentally determined. Standard equations for A_{eff} for specific test method and specimen geometry are readily available [15] and are listed as follows.

$$A_{eff} = \frac{[BS(1 + \frac{L}{S}m)(1 + m) + TS(1 + \frac{L}{S}m)]}{(1 + m)^2} \quad (\text{four point bend}), \quad (A3)$$

where parameters S , B , and T respectively represent the length, width and thickness of the specimen, and L and m represent the loading span and Weibull parameter respectively.

$$A_{eff} = \left(\frac{2L}{m+1}\right)\left(b + \frac{h}{m+1}\right) \quad (\text{cantilever test, both side}), \quad (A4)$$

where parameters L , b and h is the length, width and thickness of the specimen respectively and m is Weibull parameter. The effective area was evaluated considering all surfaces of sample due to the fact that for cantilever tests, the fast fracture parameter σ_0 was normalized for all surfaces [18].

Batdorf's Formulation

Based on Batdorf and Crose [14], the failure probability under multi-axial stress states for surface area A with crack density distribution, $N_{flaw}(\sigma_{cr})$, is given by

$$F_{Batdorf} = 1 - \exp\left[-\int_A \int_0^{\sigma_{cr}^{max}} \frac{\Omega}{2\pi} \left(\frac{dN_{flaw}(\sigma_{cr})}{d\sigma_{cr}}\right) d\sigma_{cr} dA\right], \quad (A5)$$

$$\text{where } \Omega = 4\theta_{cr}, \theta_{cr} = \frac{1}{2}\cos^{-1}\left[\frac{2\sigma_{cr} - \sigma_1 - \sigma_2}{\sigma_1 - \sigma_2}\right]. \quad (\text{A6})$$

The maximum and minimum normal stress tangential to the interface is respectively denoted by σ_1 and σ_2 .

Appendix B

Paris Law

Paris law in combination with Weibull distribution to evaluate fatigue parameters (n , or $*$) of ceramics are summarized as follows.

$$\frac{da}{dN} = \bar{A}(\Delta K)^n \quad \text{with non-zero mean stress,} \quad (\text{B1})$$

$$\text{or } \frac{da}{dN} = \bar{A}^*(K_{max})^n \quad \text{with zero mean stress,} \quad (\text{B2})$$

where $\frac{da}{dN}$ is crack growth per cycle and K and K_{max} respectively represent the amplitude of stress intensity factor and the maximum stress intensity factor.

Considering the case with non-zero mean stress, and integrating from initial time (when $N=0$) to final time to failure (when $N=N_f$), Eq. (B1) can be written as

$$N_f = \frac{2}{\bar{A}Y^2\Delta\sigma^n(n-2)K_{IC}^{(n-2)}\sigma_i^{n-2}}, \quad (\text{B3})$$

where crack geometric constant, critical stress intensity factor, the stress corresponding to initial critical crack size prior to fatigue and stress amplitude are respectively denoted by Y , K_{IC} , σ_i and σ .

Following Munz's work [23], Weibull distribution of cycles to failure N_f (Eq. (B3)) can be related to Weibull distribution of strength as follows.

$$F_{weibull}(N_f) = 1 - \exp\left[-\left(\frac{N_f}{N_{f0}}\right)^{m*}\right], \quad (\text{B4})$$

$$\text{where } N_{f0} = \frac{2\overline{\sigma}_0^{(n-2)}}{\bar{A}Y^2\Delta\sigma^n(n-2)K_{IC}^{(n-2)}}, \quad m^* = \frac{m}{n-2}. \quad (\text{B5})$$

Parameters, m and $\overline{\sigma}_0$, in Eq. (B5) are the Weibull parameters in Eq. (A1). Paris law parameters with zero mean stress, n and m^* , can be experimentally determined by curve fitting the graph $\ln[\ln(\frac{1}{1-F_{\text{weibull}}})]$ vs $\ln(N_f)$. Similarly, for the case with non-zero mean stress, n and m^* can be experimentally determined following the same procedure.

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Figure 1.

Comparison between approximate fatigue Eq. (16) with averaged fatigue parameters

($n=26.1$, $C^{\frac{1}{n-2}} = 0.838$) and experimentally measured fatigue equations in the form of Eq. (8) (FM: $n=29.4$, $h=73.5 \times 10^6$, $B=5.95 \times 10^{-2} [\text{MPa}^2\text{s}]$; LR: $n=31$, $h=217 \times 10^6$, $B=3.005 \times 10^{-2} [\text{MPa}^2\text{s}]$; YZ: $n=24.9$, $h=3.5 \times 10^6$, $B=3839.8 [\text{MPa}^2\text{s}]$; LD: $n=19.1$, $h=72.1 \times 10^3$, $B=10.8 [\text{MPa}^2\text{s}]$). Fatigue expressed in terms of the maximum cyclic stress, σ_{max} and the stress corresponding to initial crack size, σ_i , given specific number of cycles (1 million

and 7 million cycles shown as examples). Note that YZ and approximate fatigue curves overlapped and were indistinguishable in the graph.

Figure 2. Resulting Survival rate comparisons a) Comparison of approximate fatigue survival rate (Eq. (17)) utilizing $\sigma_{\max, \text{avg}}$ as 80% and 90% of $\sigma_{\max}$ (dot dashed line) and the multi-axial detailed numerical simulation Eq.(14) utilizing fatigue parameters in Table 2 (solid line) for material LD. The solid gray line (simulated dentin) and the dashed line (dentin) represent experimentally obtained survival curves for two sets of trilayer data utilizing 1mm thick ceramic plates (material LD). The shaded gray area represents the $\pm 95\%$ confidence intervals for the larger simulated dentin group. b) Comparison between the approximated fatigue survival rates (dotted line 90%, dashed line 80%, dashed -dotted line 80% and 90%) and the multi-axial detailed numerical simulation Eq.(14) utilizing fatigue parameters in Table 2 (solid line) for material LD.

overlapped) and the corresponding detailed numerical simulations (solid lines) for trilayer specimens with ceramics YZ, LR and FM. For material YZ the approximations overlap with the detailed simulation and appear as one solid line.

Figure 3.

Comparison of the maximum normal stress distribution as a function of radius for ceramic LD trilayer with thickness of 1mm, 0.7mm and 0.6 mm. The horizontal and vertical solid lines respectively indicate stress level at 90% of the maximum stress and the corresponding radius. The maximum stress, 90% of the maximum stress, the corresponding radius and area

A for 1 mm thick trilayer are 47.0 [MPa], 42.3 [MPa], 0.2 [mm] and 0.126 [mm^2]. These corresponding values for the 0.7 mm thick trilayer are 59.2 [MPa], 53.3 [MPa], 0.5 [mm] and 0.785 [mm^2] respectively and for the 0.6 mm thick trilayer, the corresponding values are 68.8 [MPa], 61.9 [MPa], 0.61 [mm] and 1.17 [mm^2].

Figure 4.

Comparison of approximate fatigue survival rates (dotted line 90%, dashed line 80%) to the corresponding detailed simulations for LD trilayers with different ceramic thicknesses (1 mm, 0.7 mm and 0.6 mm). In each of the analysis, ceramic thickness has a significant effect on the survival rate. While in each case the proposed approximations tend to overestimate survival probability when compared to the detailed simulation, the differential assessments are in good agreement.

Table 1

Dimension and material property of flat trilayer models subjected to 1.5 Hz cyclic indentation load by a spherical indenter (20 mm radius and analytical rigid) used for fatigue survival simulation and experiment

	Young's modulus [GPa]	Poison's Ratio	Thickness [mm]
YZ	210 ⁽¹⁾	0.3 ⁽¹⁾	1
LD	95 ⁽²⁾	0.23 ⁽²⁾	1 [*] , 0.7, 0.6
LR	67 ⁽³⁾	0.21 ⁽⁴⁾	1
FM	66 ⁽⁵⁾	0.29 ⁽⁵⁾	1
cement	10.21 ⁽⁶⁾	0.3 ⁽⁶⁾	0.1
dentin	12.65 ⁽⁶⁾	0.33 ⁽⁶⁾	5

* Experiments conducted with the maximum cyclic load of 300N for LD specimens up to 6 million cycles

⁽¹⁾ Dejak *et al.* [32],

⁽²⁾ Ivoclar Vivadent [33],

⁽³⁾ Rekow and Thompson [28],

⁽⁴⁾ Gonzaga *et al.* [34],

⁽⁵⁾ Sun *et al.* [35],

⁽⁶⁾ Wang *et al.* [31]

Table 2

Fast fracture and fatigue parameters for four dental materials

$\bar{\sigma}_0$	m	A_{eff}	I_A	k	n	h	K_{IC}	Y	A	B	h	$C^{1/n-2}$
[Mpa]		[mm ²]		[Mpa ^{-m} mm ⁻²]		(experiment)	[MPa m ^{0.5}]		[(m/sec)(MPa ^{0.5}) ⁻ⁿ]		[MPa ² s] (1.5 Hz)	
			0.013 ⁽³⁾									
YZ	1021 ⁽¹⁾	36.6 ⁽²⁾		8×10^{-33} ⁽⁴⁾	24.9 ⁽⁵⁾	0.075 ⁽⁶⁾	5.6 ⁽⁵⁾	1.3 ⁽⁵⁾	3.2×10^{-20} ⁽⁷⁾	10.8 ⁽⁸⁾	3.5×10^6 ⁽⁶⁾	0.805 ⁽⁹⁾
LD	67.5 ⁽¹⁰⁾	56.5 ⁽¹¹⁾	0.016 ⁽³⁾	1.4×10^{-18} ⁽⁴⁾	19.1 ⁽¹⁰⁾	558.8 $\times 10^3$ ⁽⁶⁾	0.93 ⁽¹⁰⁾	1.19 ⁽¹⁰⁾	7.4×10^{-5} ⁽¹²⁾	3839.8 ⁽⁸⁾	72.1×10^3 ⁽⁶⁾	0.826 ⁽⁹⁾
LR	144.5 ⁽¹³⁾	7.2 ⁽¹³⁾	0.001 ⁽¹⁴⁾	4.3×10^{-16} ⁽¹⁵⁾	31 ⁽¹³⁾		1.3 ⁽¹⁶⁾	1.2 ⁽¹⁶⁾	7.1×10^{-4} ⁽⁸⁾	3.005×10^{-2} ⁽¹⁷⁾	217×10^6 ⁽⁶⁾	0.93 ⁽⁹⁾
FM	75.5 ⁽¹⁸⁾	55.9 ⁽¹¹⁾	0.034 ⁽³⁾	3.5×10^{-12} ⁽⁴⁾	29.4 ⁽¹⁹⁾		1.2 ⁽²⁰⁾	1.2 ⁽²⁰⁾	5.8×10^{-3} ⁽⁸⁾	5.95×10^{-2} ⁽¹⁹⁾	73.5×10^6 ⁽⁶⁾	0.796 ⁽⁹⁾
Average					26.1							0.838

⁽¹⁾ Studart *et al.* [18].

⁽²⁾ Calculated based on Eq. (A4).

⁽³⁾ Calculated based on Eq. (3).

⁽⁴⁾ Calculated based on Eq. (2).

⁽⁵⁾ Studart *et al.* [22].

⁽⁶⁾ Calculated based on Eq. (9)2.

⁽⁷⁾ Calculated based on Eq. (11).

⁽⁸⁾ Calculated based on (9)1.

⁽⁹⁾ Calculated based on Eq. (13)2.

⁽¹⁰⁾ Joshi *et al.* [16].

⁽¹¹⁾ Calculated based on Eq. (A3).

⁽¹²⁾ Calculated based on Eq. (10).

⁽¹³⁾ Myers *et al.* [19].

⁽¹⁴⁾ Calculated based on Eq. (5).

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- (15) Calculated based on Eq. (4),
- (16) Wolfram Holand and George Beall [27],
- (17) Calculated based on Eq.(12),
- (18) Tinscherta *et al.* [17],
- (19) Magida *et al.* [25],
- (20) Rekow and Thompson [28].