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Abstract. Fixed pattern noise due to nonuniform amplifier gains and scintillator sensitivity should be alleviated in radiography imaging to acquire low-noise x-ray images from detectors. Here, the noise property of the detector is usually evaluated observing the noise power spectrum (NPS). A gain-correction scheme, in which uniformly illuminated images are averaged to design a gain map, can be applied to alleviate the fixed pattern noise problem. The normalized NPS (NNPS) of the gain-corrected image decreases as the number of images for the average increases and converges to an infimum, which can be achieved if the fixed pattern noise is completely removed. If we know the NNPS infimum of the detector, then we can determine the performance of the gain-corrected images compared with the achievable lower bound. We first construct an image-formation model considering the nonuniform gain and then consider two measurement methods based on subtraction and division to estimate the NNPS infimum of the detector. In order to obtain a high-precision NNPS infimum estimate, we consider a time-averaging method. For several flat-panel radiography detectors, we constructed the NNPS infimum measurements and compared them with NNPS values of the gain-corrected images. We observed that the NNPS values of the gain-corrected images approached the NNPS infimum as the number of images for the average increased. © 2018 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: 10.1117/1.JMI.5.1.013508]

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1 Introduction

Noise performance of radiography detectors can be characterized by multiple noise sources.¹ Among the sources, quantum noise from x-ray photons,² electronic readout noise,³ and fixed pattern noise from nonuniform amplifier gains and scintillator sensitivities^{4–8} have the most significant influence on the noise properties of the acquired image.^{9,10} The noise energy is usually measured by the image standard deviation or the noise power spectrum (NPS)¹¹ from the acquired images. From calculating the signal-to-noise ratio (SNR)¹² using the image standard deviation or a normalized NPS (NNPS)^{13,14} divided by the square of the image mean, we can evaluate the noise properties of radiography detectors.

Conducting a gain correction using an average of uniformly illuminated images as a gain map, we can efficiently reduce fixed pattern noise and consequently reduce NNPS values.^{3,5,10} However, because the image average contains noise from photons, applying the image average inflates the NNPS values of the gain-corrected images even when the fixed pattern noise is completely removed. In order to reduce the inflated noise, we should average as many images as possible in designing the gain map. Applying a moving average filter¹⁵ to the gain map could also alleviate the inflated noise problem.¹⁶ As the number of images increases, the NNPS value asymptotically decreases and approaches its infimum, which reflects the noise components excluding the fixed pattern noise. If we can completely remove the fixed pattern noise, then the infimum is an achievable NNPS performance for a given detector. Hence, measuring the NNPS infimum of a detector can provide us the best noise

performance that can be achieved from the detector and can be a guideline in designing a good gain-correction scheme.

Background trend removal schemes can be employed to reduce the fixed pattern noise influence on measuring the NNPS.^{17–19} Dobbins et al.¹⁴ used two-dimensional (2-D) first- and second-order polynomial fits, and Williams et al.²⁰ used an average of uniformly illuminated images to find the low-frequency background trends. The fixed pattern noise from the background trends can then be removed by subtracting the obtained trends. However, the fixed pattern noise generated from the nonuniform amplifier gains or the sensitivity variations in the scintillator is widely spread over the entire frequency range. Hence, this noise cannot be fully removed using detrending schemes.

Without searching for the background trends, Williams et al.²⁰ simply subtracted a single, uniformly illuminated exposure image from the input image and then divided the calculated NPS from the subtracted image by 2. Zhou et al.¹⁸ compared the abovementioned schemes and showed that the 2-D second-order polynomial fit was most appropriate. In fact, this subtraction scheme can estimate the NNPS infimum. Here, the subtraction can remove the fixed pattern noise even for high-frequency regions, and the compensation with the factor 2 can provide an unbiased estimate for the NNPS infimum. Dobbins et al.^{13,14} averaged 10 images to precisely describe the detector's fixed pattern noise and then subtracted the image average from the input image. They showed that the NNPS increases below $<0.15 \text{ mm}^{-1}$ was due to the fixed pattern noise and could be further suppressed by subtracting the image average compared

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with the 2-D second-order polynomial fit case; however, the resultant estimate is a biased estimate of the NNPS infimum. Kim²¹ generalized the subtraction scheme from explicitly deriving the NNPS under a nonuniform-gain image-formation model and provided the compensation constant $(1 + 1/n)$ for an unbiased estimate. Here, we average n images and use the difference from the input image to measure the NNPS values. The single-image subtraction method of Williams et al.²⁰ contained the compensation constant 2. However, Dobbins et al.^{13,14} did not conduct any compensation while using 10 images for averaging and thus their result is a biased estimate of the NNPS infimum.

In this paper, we first formulate a nonuniform-gain image-formation model to describe the fixed pattern noise and then consider a compensated subtraction-based scheme²¹ to measure the NNPS infimum. For a fixed number of images for averaging, the subtraction-based scheme employs a compensation step for accurate measurement.²² For a given input image, only one additional image is sufficient to accurately measure the NNPS infimum due to the compensation constant 2 as in the single-image subtraction method.²⁰ Hence, instead of averaging the given images, calculating the NNPS using as many image pairs as possible for a given number of images and then time averaging those NNPS values can provide a high-precision estimate of the NNPS infimum. Based on this notion, using subtraction to obtain a high-precision estimate of the NNPS infimum is suggested for a given number of acquired images. We also consider division for estimating the NNPS infimum and compare that with the subtraction-based method. We measure and compare the detector NNPS infima for various detectors in this paper. We then compare the NNPS values of the gain-corrected images, in which we conduct the correction using the gain map⁵ to remove the fixed pattern noise with the corresponding NNPS infima to observe their differences.

This paper is organized in the following way. In Sec. 2, we formulate the image-formation model considering the nonuniform gain for acquired x-ray images and then formulate NNPS. In Sec. 3, we observe the NNPS of the gain-corrected image. In Sec. 4, we introduce two measurement methods for the NNPS infimum. We present the experimental results for the flat-panel general radiography detectors and the mammography detectors with discussions in Sec. 5, and the paper is concluded in the last section.

2 Nonuniform-Gain Image Formation Model and the Noise Power Spectrum

In this section, in preparation for observing the NNPS values of radiography detectors, we formulate an image-formation model including the nonuniform gain and we define NNPS.

During a specified exposure interval, electrons are generated from the incident x-ray photons and are collected at the pixels. Let a nonnegative integer $h(\mathbf{x})$ denote the number of collected electrons from a pixel, and suppose that h is a weak stationary sequence with the mean $\mu := E\{h\}$, in which $E\{h\}$ is the expectation of h . Here, a set $\Gamma(\subset \mathbb{R}^2)$ is finite with size m and $\mathbf{x}(\in \Gamma)$ denotes the discrete-spatial position of pixels in a flat panel. Multiple gate drivers control the flow of the collected electric charge through the thin film transistor (TFT) panel and multiplexers; the multiple charge amplifiers then convert and amplify the electric charge to the voltage signal. Since the gate drivers and amplifiers are not identical, the gains are nonuniform. The voltage signal is then converted to the digital value (DV) by an

analog-to-digital converter. We first formulate the nonuniform-gain image-formation model.^{8,21} Let random variables $g(\mathbf{x})(> 0)$, for $\mathbf{x} \in \Gamma$, denote the nonuniform gains. The discrete-spatial image f , which contains the nonuniform gain, is then defined as

$$f(\mathbf{x}) := g(\mathbf{x})h(\mathbf{x}), \quad \text{for } \mathbf{x} \in \Gamma, \quad (1)$$

in DV, where $g(\mathbf{x})$ (DV/e⁻) is a weak stationary sequence with mean $\bar{g}$ and variance σ_g^2 , and is independent of h . The nonuniform gain property is caused from the TFT panel with photodiodes and readout circuits, and can be independent of h , which represents the photon noise from x rays. Hence, the assumption of independence makes sense.

We now discuss the NPS of f as a function of g and h . Let $\psi_\eta(\mathbf{x})$ denote the autocovariance function of a weakly stationary sequence $\eta(\mathbf{x})$, for $\mathbf{x} \in \Gamma$, and $\Psi_\eta(\boldsymbol{\omega})$, for $\boldsymbol{\omega} \in \Omega$, denote the discrete-spatial Fourier transform²³ of $\psi_\eta(\mathbf{x})$, where $\Omega := (-\pi, \pi]^2$. Ψ_η implies the power density spectrum of η from the Wiener-Khinchin theorem.^{11,24} Here, if η contains no object, then Ψ_η implies the NPS of η . Instead of NPS, a NNPS,¹³ which is denoted as NNPS, is further generally used. Letting NNPS $_\eta$ denote the NNPS of η , NNPS $_\eta$ is defined as

$$\text{NNPS}_\eta(\boldsymbol{\omega}) := \frac{\Psi_\eta(\boldsymbol{\omega})}{E^2\{\eta\}}, \quad \text{for } \boldsymbol{\omega} \in \Omega. \quad (2)$$

The NPS of η , $\Psi_\eta(\boldsymbol{\omega})$, is usually estimated from the time average of periodograms, which is known as Bartlett's method,²⁵ with the estimate precision of $1/\sqrt{M}$ for $\boldsymbol{\omega} \neq 0$, where M is the number of nonoverlapped periodograms.^{26,27} A discrete-frequency NPS of Ψ_η can be calculated using the N -point discrete Fourier transform with the spectral resolution of N .²⁸ Here, because the estimate is asymptotically unbiased as N increases,^{27,28} N should be relatively large to achieve a high-accuracy estimate and our experimental result showed 2.5% accuracy for $N \geq 64$. For the general radiography and mammography detectors, NPS measurement methods are described in the documents from IEC62220-1-1¹⁹ and IEC62220-1-2,²⁹ respectively, based on the overlapped periodograms as described by Welch.³⁰ Here, for the continuous-spatial spectrum, the square of the pixel pitches are multiplied. The NNPS of f can then be expanded as follows:⁸

$$\text{NNPS}_f(\boldsymbol{\omega}) = \text{NNPS}_h(\boldsymbol{\omega}) + \text{NNPS}_g(\boldsymbol{\omega}) + \text{NNPS}_h(\boldsymbol{\omega}) * \text{NNPS}_g(\boldsymbol{\omega}) \quad (3)$$

from the assumption that g is independent of h , where NNPS $_g$ is the NNPS of the fixed pattern noise⁸ and "*" implies the convolution. From Eq. (3), we can notice that NNPS $_h$, which is independent of the fixed pattern noise, is a lower bound of the NNPS, i.e., $\text{NNPS}_f \geq \text{NNPS}_h$. If we can completely correct the nonuniform gain or remove the fixed pattern noise, then the measured NNPS is equal to the lower bound. In other words, NNPS $_h$ is an achievable lower bound of NNPS values.

3 Noise Power Spectra of Gain-Corrected Images

Using an average of several uniformly illuminated images as a gain map to find the fixed pattern noise, we can efficiently reduce the fixed pattern noise from the acquired x-ray images.^{5,10,31}

Suppose that $h_1(\mathbf{x}), \dots, h_n(\mathbf{x})$ are independent and identically distributed random variables with the mean $\mu_0 := E\{h_1\}$. Letting $f_1, \dots, f_n$ denote the uniformly illuminated images for designing the gain map, f_i is defined as

$$f_i(\mathbf{x}) := g(\mathbf{x})h_i(\mathbf{x}), \quad (4)$$

for $\mathbf{x} \in \Gamma$ and $i = 1, \dots, n$. Here, we call $\alpha := \mu/\mu_0$ the particle ratio for the mean numbers of particles. In Eqs. (1) and (4), the same gain g is used for all f and f_i to describe the fixed pattern noise.⁹ The fixed pattern noise g can be obtained from the image average $n^{-1} \sum_{i=1}^n f_i(\mathbf{x})$ for relatively large n because it converges to $g(\mathbf{x})E\{h_1\}$, almost surely.³²

Let the normalized flat-field correction image⁵ or the gain map,¹⁶ which is denoted as G_n , be defined as

$$G_n(\mathbf{x}) := E\{f_1(\mathbf{x})\} \left[\frac{1}{n} \sum_{i=1}^n f_i(\mathbf{x}) \right]^{-1}, \quad (5)$$

for $\mathbf{x} \in \Gamma$. The gain-corrected image is then given by $G_n f$ and $\text{NNPS}_{G_n f}$, which is the NNPS of $G_n f$, satisfies the following approximation:

$$\text{NNPS}_{G_n f}(\omega) \approx \text{NNPS}_h(\omega) + \frac{1}{n} \text{NNPS}_{h_1}(\omega), \quad (6)$$

if we consider high SNR images.²¹ From Eq. (6), because $\inf_n \text{NNPS}_{G_n f} = \text{NNPS}_h$, we call NNPS_h the NNPS infimum. The second term on the right side of Eq. (6) implies the inflated noise produced from the x-ray images for designing the gain map. Hence, we can see that the NNPS of the gain-corrected image is always greater than the NNPS infimum and approaches the NNPS infimum as n increases. If $\alpha = 1$, then Eq. (6) can be rewritten as

$$\text{NNPS}_{G_n f}(\omega) \approx \left(1 + \frac{1}{n}\right) \text{NNPS}_h(\omega). \quad (7)$$

For a linear, time-invariant system model for the power density spectrum,²³ we can assume that the shape of the NPS is independent of the incident exposure and that the NPS values are inversely proportional to the incident exposure for the photon-limited exposures. We then obtain a relationship $\text{NNPS}_h \approx \alpha \cdot \text{NNPS}_{h_1}$. Hence, from Eq. (6), the NNPS of the gain-corrected image can be rewritten as

$$\text{NNPS}_{G_n f}(\omega) \approx \left(1 + \frac{\alpha}{n}\right) \text{NNPS}_h(\omega). \quad (8)$$

The total averages of $\text{NNPS}_{G_n f}$ and NNPS_h over ω are given by s_n^{-2} and s^{-2} , respectively, where s_n and s are the SNRs of the gain-corrected image and h , respectively. Hence, the average of Eq. (8) can be rewritten as

$$\frac{1}{s_n^2} \approx \left(1 + \frac{\alpha}{n}\right) \frac{1}{s^2}, \quad (9)$$

which was also observed by Kim²¹ from an asymptotic equivalent as μ increases for a fixed α .

From Eq. (8), we can see that the NNPS of the gain-corrected image can be quite far from the true NNPS infimum for practical values of n . For example, even though we use $n = 10$ images for the gain-map design, the resultant NNPS is 10% higher than the NNPS infimum when $\alpha = 1$. In order to conduct a good gain

correction for a fixed n , we can acquire x-ray images for high incident exposures to increase μ_0 or decrease α in Eq. (8). Using the measured NNPS of the gain-corrected image and the relationship of Eq. (8), we can obtain the NNPS infimum, NNPS_h , which can provide an achievable lower bound of NNPS.

Once we obtain the NNPS infimum, we can acquire the following information from the NNPS infimum.

- Achievable lower bound of the NNPS for a given detector.
- NNPS of the gain-corrected image from Eq. (8) for given α and n .

Here, α can be the exposure ratio to that of the gain-map images under the proportional assumption on the pixel values.^{21,33}

4 Infimum of the Noise Power Spectrum

In this section, we introduce two methods for estimating a high-precision NNPS infimum of NNPS_h based on the image subtraction and division, respectively.

4.1 Subtraction Method for a High-Precision NNPS Infimum Estimate

The NNPS infimum can be measured from subtracting the image average as mentioned by Williams et al.²⁰ and Dobbins et al.^{13,14} We first introduce the subtraction method. Assume that the images of f_i are acquired at $\alpha = 1$. Letting d_n denote an image difference⁸ as

$$d_n(\mathbf{x}) := f(\mathbf{x}) - \frac{1}{n} \sum_{i=1}^n f_i(\mathbf{x}), \quad (10)$$

for a given n , the NNPS infimum approximately satisfies

$$\text{NNPS}_h(\omega) \approx \left(1 + \frac{1}{n}\right)^{-1} \frac{\Psi_{d_n}(\omega)}{E^2\{f\}}, \quad (11)$$

under a weak gain assumption.²¹ Here, $(1 + 1/n)$ is a compensation constant to obtain the NNPS infimum. From Eq. (11), we can observe that the NPS of Ψ_{d_n} is independent of g . In other words, the nonuniform gain of g can be removed from the image difference of Eq. (10) under an appropriate assumption.

A single-image subtraction scheme, which was used by Williams et al.²⁰ and Zhou et al.,¹⁸ is a special case of $n = 1$ in Eq. (11). These authors derived the NPS of the image difference $d_1(\mathbf{x}) = f(\mathbf{x}) - f_1(\mathbf{x})$ and calculated the NNPS using $E\{f\}$, in which the resultant NNPS is divided by 2. It is clear that the compensation constant 2 is from $(1 + 1/1) = 2$ in Eq. (11). Dobbins et al.^{13,14} used $n = 10$ images to obtain the image difference d_{10} , and then calculated the NNPS without any compensation as $\Psi_{d_n}(\omega)/E^2\{f\}$. However, the resultant NNPS has 10% higher values than the true NNPS infimum from $(1 + 1/10) = 1.1$. Hence, compensation based on Eq. (11) is important even for the $n = 10$ case. It should be noted that the purpose of using the image difference of Eq. (10) is to remove the fixed pattern noise from the acquired image in calculating the NNPS infimum and is different from that of the detrending methods, such as polynomial fitting and moving averaging. The purpose of detrending is to alleviate the low-frequency fluctuation problem or reduce the low-frequency increases in NNPS.

We now introduce a method to obtain a high-precision estimate of the NNPS infimum based on the subtraction scheme Eq. (11). Suppose that M nonoverlapped periodograms can be calculated for an acquired image and L images are given for measuring the NNPS. The estimate precision is then given as $1/\sqrt{LM}$ and hence increasing L can provide a high-precision estimate as addressed in the document from IEC62220-1-1. For the subtraction in Eq. (11) to calculate the NNPS infimum, we can use $n = L - 1$ images for the image average. The estimate precision of Eq. (11) is then given as $1/\sqrt{M}$ independently of n even though $L = n + 1$ images are used. We can also use the difference between two image averages, which are calculated using n and n' images. Then, the NNPS infimum can be calculated using the image difference with the compensation constant of $(1/n + 1/n')$ as shown by Kim and Lee.³⁴ However, this subtraction scheme also has the estimate precision of $1/\sqrt{M}$ independent of n and n' .

In order to obtain a high-precision estimate of the NNPS infimum for a given set of images, it is preferable to consider the $n = 1$ case of Eq. (11) because the estimate precision is independent of n . For the image difference $f - f'$, the NNPS infimum can be rewritten as

$$\text{NNPS}_h(\omega) \approx \frac{1}{2} \frac{\Psi_{f-f'}(\omega)}{E^2\{f\}}, \quad (12)$$

which is a special case of the subtraction in Eq. (11) when $n = 1$. If the total number of x-ray images, L , is an even number, then we can obtain $L/2$ image pairs to calculate the NNPS from Eq. (12). Hence, the estimate precision can be improved to $\sqrt{2/LM}$ from averaging $LM/2$ periodograms. In order to further improve the precision, we first calculate $\binom{L}{2}$ NNPS values

from Eq. (12) using $\binom{L}{2}$ image pairs among L images and then average them. Lee and Kim³⁵ experimentally observed the estimate accuracy of this approach for real x-ray images. We call this approach, which uses $\binom{L}{2}$ calculations of Eq. (12) and then time averages them, the subtraction method for a given L images. The subtraction method is summarized as follows:

Subtraction method:

1. For a given exposure condition and fixed L , acquire L images.
2. Construct $\binom{L}{2}$ image pairs from choosing two images among L images.
3. Calculate $\binom{L}{2}$ NNPS values from Eq. (11) using the image pairs, where the NPS, $\Psi_{f-f'}$, is calculated based on the periodogram methods, such as described in IEC62220-1-1 or 1-2.
4. Conduct the time average of the $\binom{L}{2}$ NNPS values to obtain a high-precision NNPS infimum.

Instead of the acquired raw image f , we can use the gain-corrected images, which employ the same gain map in the subtraction method. Even though the input images are multiplied by the gain map, the effect of the gain map is also canceled from the image subtraction in Eq. (10), and thus the resultant NNPS is equal to the NNPS infimum, NNPS_h .

4.2 Division Method for a High-Precision NNPS Infimum Estimate

Because the gain map, which is proportional to the inverse of the nonuniform gain g , is multiplied to the input image, the fixed pattern noise from the nonuniform gain can be easily removed for a further general condition compared with the subtraction method case.²¹ Hence, based on the gain-correction approach, we can also calculate the NNPS infimum. We now introduce the division method, where the gain correction and then compensation are conducted.²¹ Let r_n denote an image ratio defined as

$$r_n(\mathbf{x}) := \frac{f(\mathbf{x})}{\frac{1}{n} \sum_{i=1}^n f_i(\mathbf{x})}. \quad (13)$$

From Eqs. (5) and (13), the gain-corrected image can be rewritten as $G_n f = E\{f(\mathbf{x})\} \cdot r_n$. Hence from Eq. (7) with $\alpha = 1$, the NNPS infimum approximately satisfies

$$\text{NNPS}_h(\omega) \approx \left(1 + \frac{1}{n}\right)^{-1} \Psi_{r_n}(\omega), \quad (14)$$

where $\Psi_{r_n}(\omega)$ is the NPS of the ratio r_n .

The division scheme of Eq. (14) is equivalent to the NNPS of the gain-corrected images if $\alpha = 1$, i.e., n images, $f_1, \dots, f_n$, are acquired at the same condition of the f case. We can also use the image average ratio to another image average, where the averages use n and n' images, respectively.³⁴ Here, the compensation constant changes to $(1/n + 1/n')$. In a similar manner to the subtraction case, we can also measure the NNPS infimum from

$$\text{NNPS}_h(\omega) \approx \frac{1}{2} \Psi_{f/f'}(\omega), \quad (15)$$

which is a special case of $n = 1$ in the division scheme of Eq. (14). For a given L images, calculating $\binom{L}{2}$ NNPS values of Eq. (15) and time averaging them can provide a high-precision estimate. We can also use the gain-corrected images instead of the raw images of f to calculate the NNPS infimum from the division scheme. Therefore, the division method can be summarized as follows:

Division method:

1. For a given exposure condition and fixed L , acquire L images.
2. Construct $\binom{L}{2}$ image pairs from choosing two images among L images.
3. Calculate $\binom{L}{2}$ NNPS values from Eq. (15) using the image pairs, where the NPS, $\Psi_{f/f'}$, is calculated based on the periodogram methods, such as described in IEC62220-1-1 or 1-2.
4. Conduct the time average of the $\binom{L}{2}$ NNPS values to obtain a high-precision NNPS infimum.

Even though the calculation schemes of the subtraction and division methods use different quantities, the image difference and ratio, respectively, both methods can approximately provide the same NNPS infima under appropriate assumptions.²¹ However with division, very small values of the denominator f' may produce large numbers, which can dramatically increase

Table 1 Development prototypes of the general radiography detectors (DRTECH Co. LTD.).

Detector/ RQA 5	Type	Collection element	Scintillator or photoconductor/ thickness (μm)	Pixel pitch ($\mu\text{m}/\text{pixel}$)	Image size (pixels)
A	Indirect	TFT/photodiode	Columnar CsI(Tl)/380	140	2560 × 3072
B	Indirect	TFT/photodiode	Columnar CsI(Tl)/400	143	3008 × 3072
C	Direct	TFT	a-Se/500	139	2560 × 3072

the NNPS values. Hence, the outliers of $1/f'$ should be carefully discarded to obtain an accurate NNPS. In contrast, subtraction does not suffer from this outlier problem and can thus provide a stable estimate. Therefore, subtraction is preferable to division.

5 Experimental Results

In this section, we introduce experimental observations of the NNPS values of the gain-corrected images and the NNPS infima of various radiography detectors. In order to experimentally measure the NNPS values of the radiography detectors, we first summarize three types of general radiography detectors in Table 1; they were development prototypes of DRTECH Co. The x-ray images were acquired at the RQA 5 condition with the x-ray tube voltage of 70 kVp. The continuous-spatial NPS values were calculated using the scheme of IEC62220-1-1.¹⁹

The calculated NNPS infima from the subtraction and division methods are compared in Fig. 1. We can observe that the NNPS infimum calculated from subtraction is almost equal to that calculated from division even though the fixed pattern noise from detector A was the largest among the detectors in Table 1. Hence, we can expect similar equality relationships for other detectors and thus prefer to use the subtraction method as discussed in the previous section.

In Fig. 2, as introduced by Williams et al.,²⁰ the curve of “NNPS from d_{10} ” is calculated using the image difference d_{10} of Eq. (10) without the compensation term $(1 + 1/n)$. Thus, the curve is about 10% higher than the true NNPS infimum from $(1 + 1/10) = 1.1$. We can compensate this curve by multiplying $(1 + 1/n)^{-1}$ as shown in the subtraction scheme of “subtraction ($n = 10$)” to obtain the NNPS infimum from

Fig. 1 Comparison of the high-precision NNPS infima calculated from the subtraction method “Infimum (subtraction, $L = 10$)” and the division method “Infimum (division, $L = 10$)” for $L = 10$ (detector A).

Fig. 2 Comparison of NNPS measurements (detector A at the incident exposure of 33 μGy). “Subtraction ($d = 10$)” is obtained from the subtraction scheme of Eq. (11) at $n = 10$, and is equivalent to a compensated NNPS from “NNPS from d_{10} .” “Division ($n = 10$)” is obtained from the division scheme of Eq. (14) at $n = 10$, and is equivalent to a compensated NNPS from the gain-corrected image “Gain correction ($n = 10$).”

Eq. (11). The NNPS of the gain-corrected image at $n = 10$ also has a 10% higher curve than the true NNPS infimum from $(1 + 1/10) = 1.1$ of Eq. (7). This NNPS curve can be compensated as described in Eq. (14) of the division scheme to calculate the NNPS infimum. The compensated NNPS is shown as “division ($n = 10$)” in Fig. 2. The values of “subtraction ($n = 10$)” and “division ($n = 10$)” imply the NNPS infimum and appear to be very similar.

In Fig. 3, we compared the NNPS values of the gain-corrected images with the corresponding NNPS infima. For the gain-corrected image as “gain correction ($n = 1$)” in Fig. 3, even though the fixed pattern noise was removed, the inflated noise from the gain map design produces a spectrum of $\text{NNPS}_{G_{nf}}$, which shows a difference from the NNPS infimum, NNPS_h , as observed in Eq. (7): As n increases, the NNPS value, $\text{NNPS}_{G_{nf}}$, approached the NNPS infimum, NNPS_h , based on the relationship of Eq. (6) as “gain correction ($n = 10$).” For the low-exposure case in Fig. 3 (4.1 μGy), the NNPS of the input images, “input image,” were very close to those of the NNPS infimum especially for the high-frequency region. In contrast, for the high-exposure case in Fig. 3 (33 μGy), because the fixed pattern noise from detector A was relatively high due to the sensitivity variation problem with the scintillator,^{8,36} the NNPS of the input image was higher than the NNPS infimum. An example of the fixed pattern noise of detector A is shown in Fig. 4(a), where the blocky differences of the gray levels due to different readout circuits and high-frequency patterns can

Fig. 3 NNPS values of gain-corrected images for different n in the gain map design with $\alpha = 1$ (detector A).

be observed.^{8,21} Detrending with the 2-D second-order polynomial fit^{14,18,20} or the moving average filter^{15,20,23} can reduce the low-frequency increase.^{8,21} However, this detrending cannot remove the fixed pattern noise, especially for the high-frequency region as shown in Fig. 4(b), compared with the case of the gain map correction for detector A. Such a fixed pattern noise can be efficiently removed from the gain correction as n increases.

For different values of α in designing gain maps, the gain-corrected results for $n = 2$ were observed in Fig. 5. For input image dose of $33 \mu\text{Gy}$ and gain map image dose of $4.1 \mu\text{Gy}$, the particle ratio was $\alpha = 33/4.1 \approx 8.05$. On the other hand, for input image dose of $4.1 \mu\text{Gy}$ and gain map image dose of $33 \mu\text{Gy}$, the particle ratio was $\alpha = 4.1/33 \approx 0.124$. As shown in Eq. (8), the gain map, which was designed with high-exposure images, showed better gain-correction performance as “gain correction ($4.1 \mu\text{Gy}$, $\alpha = 0.124$)” compared to the low-exposure case as “gain correction ($33 \mu\text{Gy}$, $\alpha = 8.05$)” in Fig. 5. Therefore, as discussed after Eq. (8), we should maintain the ratio α as low as possible for a good gain correction if the gain correction is conducted using the gain map. In other words, if the NNPS is not compensated with $(1 + 1/n)$, the

Fig. 5 NNPS values of gain-corrected images for different ratios of $\alpha = 0.124$ and 8.05 with $n = 2$ (detector A).

measured NNPS will be more accurate as the gain-correction map is created with higher exposure.

In Fig. 6, the measured NNPS infimum values from the subtraction method are illustrated for the different general radiography detectors in Table 1. The corresponding NNPS values of the gain-corrected images are also depicted for comparisons where $n = 7$ and $\alpha = 1$. We can notice that the NNPS infima showed different values depending on detectors. Analyzing the detector that shows the lowest NNPS infimum, we can obtain a guideline to design a low-noise detector. Furthermore, from comparing the NNPS infima and the NNPS values of the gain-corrected images, we can evaluate the performances of the gain-correction schemes. Because the NNPS values are inversely proportional to the incident exposure for the quantum-limited regions,^{14,37} the NNPS values of Fig. 6(a) should be similar to the NNPS values that are multiplied by $33/4.1 \approx 8.05$ to those of Fig. 6(b). However, the NNPS values from detector B are higher than the expected values from the proportionality as observed in Fig. 6(a). Hence, we can notice that the electronic noise from detector B is higher than that from detectors A and C even though detector B shows the lowest noise performance at the incident exposure of “ $33 \mu\text{Gy}$ ” in Fig. 6(b).

Fig. 4 Example of the fixed pattern noise of detector A at the incident exposure of $33 \mu\text{Gy}$. (a) Fixed pattern noise in the magnified 256×256 image. (b) Detrending result with the 2-D second-order polynomial fit. The overall gray values seem to be the same, but the high-frequency components still remain.

Fig. 6 NNPS infima from the subtraction method ($L = 10$) and the NNPS values of the gain-corrected images ($n = 7$) for the general radiography detectors in Table 1. (a) Incident exposure of $4.1 \mu\text{Gy}$. (b) Incident exposure of $33 \mu\text{Gy}$.

6 Conclusion

The NNPS of gain-corrected images is first observed in this paper, in which the correction is conducted by multiplying the gain map. As the number of images for designing the gain map increases, the NNPS decreases and converges to the NNPS infimum, which can be achieved if the fixed pattern noise is completely removed. We then formulate two measurement methods for the NNPS infimum with a high precision. Measuring the NNPS infimum of a detector, we could determine the noise performance of a given detector and the gain-correction performance for an employed correction algorithm. Using real x-ray images from multiple radiography detectors, the NNPS values of the gain-corrected images could be compared with the corresponding NNPS infima. The experimental results showed explicit performance differences from the NNPS infimum and could provide design guidelines for designing both a gain-correction algorithm and the detector hardware.

Disclosures

No conflicts of interest, financial or otherwise, are declared by the authors.

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