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Dynamic structure of lower limb joint angles during walking post-stroke

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Abstract

Background—Variability in joint kinematics is necessary for adaptability and response to everyday perturbations; however, intrinsic neuromotor changes secondary to stroke often cause abnormal movement patterns. How these abnormal movement patterns relate to joint kinematic variability and its influence on post-stroke walking impairments is not well understood.

Objective—The purpose of this study was to evaluate the movement variability at the individual joint level in the paretic and non-paretic limbs of individuals post-stroke.

Methods—Seven individuals with hemiparesis post-stroke walked on a treadmill for two minutes at their self-selected speed and the average speed of the six-minute walk test while kinematics were recorded using motion-capture. Variability in hip, knee, and ankle flexion/extension angles during walking were quantified with the Lyapunov exponent (LyE). Interlimb differences were evaluated.

Results—The paretic side LyE was higher than the non-paretic side at both self-selected speed (Hip: 50%; Knee: 74%), and the average speed of the 6-min walk test (Hip: 15%; Knee: 93%).

Conclusion—Differences in joint kinematic variability between limbs of persons post-stroke supports further study of the source of non-paretic limb deviations as well as the clinical implications of joint kinematic variability in persons post-stroke. The development of bilaterally-targeted post-stroke gait interventions to address variability in both limbs may promote improved outcomes.

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Conflict of interest statement

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Keywords

Gait; Variability; Stroke; Nonlinear analysis

1. Introduction

Nearly 7 million people in the United States are stroke survivors (Mozaffarian et al., 2016). A common consequence of stroke is hemiparesis, which is characterized by weakness and impaired control of one side of the body. Post-stroke hemiparesis affects gait and impairs nearly 80% of stroke survivors (Weiss, 2010). Indeed, individuals post-stroke exhibit reduced paretic knee flexion angle (Chen et al., 2005; Kao et al., 2014), decreased ankle dorsiflexion (Chen et al., 2005), and an increase in foot lateral displacement during paretic swing (Kao et al., 2014). These impairments ultimately result in slow gait speeds (Bonnyaud et al., 2015). While changes in the magnitudes (peaks, means, etc.) of kinematic and kinetic variables post-stroke have been well explored, there are few studies that explore how variability of gait is affected post-stroke, specifically at the joint level. Variability refers to the lack of repeatability. In the context of this paper, variability refers to the differences in joint angle trajectories from one gait cycle to the next (Lewek et al., 2006). A minimal level of variability in joint kinematics is necessary for adaptability and response to everyday perturbations, such as walking on uneven terrain. Stergiou and Decker (2011) describe this typical variability by saying that if a person “tried to repeat the same movement twice, the two actions will never be identical.” However, too much variability may be reflective of impaired motor control and can lead to irregular movement patterns (Stergiou and Decker, 2011). Linear measures of variability such as standard deviation and coefficient of variance are often used to describe movement in clinical populations, and provide valuable insight into errors or impairment of motor performance. These measures are limited, however, in that they describe the amount of variability, but not the evolution of the movement over time. To assess the evolution of movement and its implications in clinical populations, other techniques such as the Lyapunov Exponent (LyE) can be used (Harbourne and Stergiou, 2009).

LyE is a nonlinear measure that has been used to analyze time series data for different kinematic parameters by evaluating the amount of divergence or convergence present in the continuous movement trajectories (Ghayoumi Zadeh et al., 2016; Kaimakamis et al., 2016; Smith et al., 2010). For example, the LyE of completely repeatable data such as a sinusoid is zero, as the structure of the signal does not change over time. Conversely, random, noisy data demonstrates greater divergence over time (and greater LyE), as the signal is not perfectly repeated from one cycle to the next (Harbourne and Stergiou, 2009). In gait, LyE can demonstrate the stability of a movement over time by quantifying stride-to-stride changes in patterns. It is important to note, however, that this does not refer to stability of the center of mass over the base of support.

The LyE has been used to quantify the structure of physiological signals such as kinematic trajectories in a variety of populations including, but not limited to, subjects who have ruptured and reconstructed their anterior cruciate ligament (ACL) (Moraiti et al., 2010),

subjects with Down syndrome (Smith et al., 2010), subjects with focal cerebellar lesions (Hoogkamer et al., 2015), and subjects with lower extremity amputations (Wurdeman et al., 2013). In subjects who have ruptured and reconstructed their ACL, it was found that the intact knee had higher knee flexion/extension LyE values than the contralateral ACL-reconstructed knee, which is believed to be a compensatory mechanism to obtain more symmetrical gait cycles (Moraiti et al., 2010). Another study examined interlimb differences for subjects who have had transtibial amputation (Wurdeman et al., 2013). While there was no significant difference between the limbs for the knees and hips, it was found that the sound leg ankle had significantly lower LyE, and therefore less divergence in the dynamic structure of the ankle angle, compared to the prosthetic ankle (Wurdeman et al., 2013). Importantly, Wurdeman et al. demonstrated that LyE was the first objective measure that was able to identify how a patient perceived their prosthesis, showing a strong relationship between LyE and a patient's preference in prosthesis design (Wurdeman et al., 2013). Other metrics have been used to quantify dynamic structure of gait including phase portraits (DiBerardino et al., 2010), detrended fluctuation analysis (Peng et al., 1995; Rhea et al., 2012), and sample entropy (Balasubramanian et al., 2009; Lamoth et al., 2011; Rhea et al., 2012); however, the LyE is particularly effective in quantifying stride-to-stride fluctuations in gait patterns because it describes the overall consistency of a system by describing how similar gait kinematics are from one stride to the next (Wurdeman et al., 2013). In addition to measuring movement during gait, LyE is also used for diagnosing pathologies such as breast cancer (Ghayoumi Zadeh et al., 2016), depression (Acharya et al., 2015), and obstructive sleep apnea (Kaimakamis et al., 2016) by measuring the divergence of segmented thermal infrared images of breast tissue, variability of electrical activity in the brain using a continuous electroencephalogram signal, and divergence of respiratory signals measuring nasal cannula flow and thoracic belt movement during sleep, respectively.

Kinematic and spatiotemporal asymmetries are hallmark features of post-stroke hemiparetic gait (Balasubramanian et al., 2009), but bilateral differences in the dynamic structure of gait are not as commonly reported. In post-stroke gait research, the non-paretic limb is less often discussed as it is affected less directly by the stroke. The non-paretic limb is important, however, and requires more energy than that of a neurologically healthy individual to compensate for the weakness in the paretic limb (Chen et al., 2005). The objective of this study was thus to evaluate the dynamic structure of kinematic variability in the paretic and non-paretic limbs of individuals post-stroke. We hypothesized that the paretic limb will exhibit greater stride-to-stride fluctuations, quantified by the LyE, than the non-paretic limb due to the impaired neuromuscular control commonly observed in post-stroke hemiparesis.

2. Methods

2.1. Participants

Eight individuals with hemiparesis post-stroke were recruited for this study. One subject was excluded from analysis due to technical difficulties during data collection. Seven individuals (age 62 ± 5.4 , years post-stroke 3.5 ± 4 , 5 males) were able to complete all of the trials and thus were included for analysis (Table 1). Inclusion criteria included (1) greater than 6 months post-stroke, (2) first (single) lesion, (3) ambulatory but with residual gait deficits,

and (4) able to walk multiple bouts of 6 min with no orthotic support. Exclusion criteria included (1) evidence of moderate to severe chronic white matter disease on MRI, (2) cerebellar stroke, (3) history of lower extremity joint replacement, (4) pain that limits walking ability, (5) inability to communicate with investigators, (6) neglect and hemianopia, (7) unexplained dizziness, and (8) orthopedic conditions that change walking ability. All participants signed an informed consent that was approved by the Institutional Review Board.

2.2. Procedures

Subjects post-stroke performed a 10 m walk test to determine their self-selected (SS) speeds (Awad et al., 2015) and a 6-min walk test (6MWT) to determine their long distance walking speeds. Twenty-five single reflective markers and 19 markers on shells were placed on the subjects' bony landmarks and tracking segments, respectively, to allow the 8-camera motion capture system (Motion Analysis Corp.) to locate the subject in 3-dimensional space to record kinematic and kinetic data. Subjects wore athletic shoes during all walking trials. Each subject walked on an instrumented split-belt treadmill (Bertec Corp.) for two minute trials at their SS speed and 6MWT speed (Table 1) while kinematic data were analyzed at a sampling frequency of 60 Hz.

2.3. Analysis

Continuous hip, knee, and ankle flexion/extension angles, defined as the angle between the proximal and distal segments of a joint in the sagittal plane, were determined using Visual 3D. The continuous 2 min trials with the joint angles were then run through a custom MATLAB code that utilized the Wolf algorithm (Wolf et al., 1985) to calculate the maximum Lyapunov exponent. The Lyapunov exponent is a measure of how infinitesimally close trajectories of dynamical system vary or diverge in a certain dimension of phase space (Fig. 1) (Cignetti et al., 2012; Wolf et al., 1985). The Wolf algorithm was selected because of its sensitivity to smaller data sets in older adults (Cignetti et al., 2012). In order to calculate the Lyapunov exponent, a long, continuous selection of data is required. It has been successfully demonstrated that a 2-min trial is sufficient for accurate calculation of the LyE (Moraiti et al., 2010).

A 2×2 (speed by limb) repeated measures ANOVA was run to determine the statistical significance between the LyE of paretic and non-paretic limbs at SS and 6MWT speeds for hip, knee and ankle joint angles of the stroke subjects. Cohen's *d* effect size was calculated for pairwise comparisons to determine the strength of the differences observed, with $d > 0.8$ defining a large effect size.

3. Results

3.1. Speed by limb comparison

The overground speeds for SS and 6MWT for the subjects post-stroke were 0.97 ± 0.27 m/s and 1.11 ± 0.36 m/s, respectively, with no main effect of speed observed for all three joints. Overall, the mean LyE for all sagittal plane angles was greater on the paretic side than the non-paretic side for the subjects post-stroke at both SS speed and 6MWT speed. Analysis of

the main effect of limb revealed no significant difference between limbs at the ankle, a trend toward difference at the hip ($p = .067$) and a significant difference between paretic and non-paretic limbs at the knee ($p = .017$) (Fig. 2). Pairwise comparisons at the hip revealed a larger difference between limbs at SS speed (paretic $1.52 \pm 0.6 >$ non-paretic 1.01 ± 0.4 , $d = 1.03$) when compared to the 6MWT speed (paretic $1.23 \pm 0.35 >$ non-paretic 1.07 ± 0.42 , $d = 0.44$). Paretic knee divergence significantly exceeded non-paretic knee divergence at both SS (paretic $2.02 \pm 0.6 >$ non-paretic 1.16 ± 0.2 , $p = .014$, $d = 1.99$) and 6MWT speeds (paretic $2.33 \pm 0.9 >$ non-paretic 1.21 ± 0.3 , $p = .041$, $d = 1.81$). Individually, 6 of the 7 subjects had greater paretic LyE for knee angles compared to the non-paretic limb for both SS speed and 6MWT speed, with the seventh subject exhibiting greater non-paretic LyE for knee flexion/extension (Fig. 2).

4. Discussion

The LyE for knee flexion/extension angles were greater on the paretic side than the non-paretic side for the stroke subjects, suggesting that the paretic limb has significantly higher divergence than the non-paretic limb at the joint level, supporting our hypothesis. Greater paretic-side divergence may indicate difficulty with controlling the paretic limb. Conversely, lower divergence on the non-paretic limb may indicate a focus on controlling their movement with the non-paretic limb. This agrees with observed divergence in individuals with unilateral lower limb prostheses, where sound leg ankle had a significantly lower divergence compared to the prosthetic ankle (Wurdeman et al., 2013). Additionally, greater standard deviations of swing time and pre-swing time were observed on the paretic side in persons post-stroke (Balasubramanian et al., 2009).

Traditionally, increased variability in gait corresponds to decreased mechanical stability and cooperation of the system whereas decreased variability corresponds to a more stable, cooperative system (Stergiou et al., 2006). However, given the complexity and chaotic behavior inherent in gait, nonlinear measures, such as LyE, can help determine the dynamic structure of variability. For example, when you stand “still,” you are constantly shifting and swaying around a fixed point. This constant movement that occurs while performing a seemingly static action indicates that, while variability is not equivalent to mechanical stability, it enables, to a certain extent, stability because it “reflects multiple options for movement, providing for flexible adaptive strategies” that are not present in rigid, fixed systems (Harbourne and Stergiou, 2009). The dynamic structure of variability, quantified by LyE, refers to the fluctuations of the entire gait cycle away from its typical periodic behavior (Wurdeman et al., 2013). In other words, LyE quantifies the structure of the variability by evaluating the differences in gait cycle trajectories (e.g., how knee angle across the gait cycle differs from one gait cycle to the next) as opposed to linear measures such as standard deviation or coefficient of variation that quantify the magnitude of the variability of a specific event within a gait cycle (e.g., peak knee flexion angle during swing phase) independent of the variability of other specific events in the gait cycle (e.g., knee flexion angle at heel strike) (Smith et al., 2010; Stergiou and Decker, 2011). Wurdeman et al. (2013) describes LyE as the variation within an attractor, which is often defined as the stability of the attractor in mathematics. However, clinically based research often “avoid[s] the use of

the term stability to avoid confusion with the more common meaning associated with balance as these are markedly different.”

The higher LyE of the paretic knee is observed during stance phase. Because there are higher metabolic costs to move the hip than the knee (Farris and Sawicki, 2012) and the ankle is limited in motion due to its proximity to the planted foot, it is understandable that the knee is more able to tolerate this “error” and more likely to have these variations of movement and a higher LyE value.

It is likely that paretic LyE may be higher than non-paretic LyE because the hemiparesis results in a lack of control on that side of the body. Due to impaired sensorimotor function (Lower Extremity Fugl Meyer 26 ± 5) we expect higher variability in the paretic limb. Similarly, this lack of control is observed in previous studies where the standard deviation for the mediolateral margin of stability is significantly higher on the paretic limb compared to the non-paretic limb (Kao et al., 2014). For the measure of LyE, increased variability in individual joint angles, which may be indicative of a lack of control, may have limited the subjects’ ability to have a consistent, cyclic, and repeatable gait pattern.

Every subject, with the exception of subject 7, had higher LyE values on the paretic side than the non-paretic side for knee angles at both speeds. Subject 7 had higher LyE values on the non-paretic side. Interestingly, subject 7 was the only subject to use both handrails while walking on the treadmill. This restricted motion could have restricted the divergence that may have occurred in the knees if the subject had walked without the handrails. Further research into the impact of assistive devices on the dynamic structure of variability is warranted.

Previous studies have found that five out of the eight recorded muscles indicated increased muscle contraction in the non-paretic limb compared to a healthy control (Raja et al., 2012). Increased co-contraction of the non-paretic limb would likely result in more rigid and ‘robotic’ movements, as evidenced in decreased divergence in the non-paretic limb joint angles compared to the paretic limb during gait.

The ratio of LyE values calculated for the paretic and non-paretic knee angles of individuals post-stroke are comparable to those of other individuals with gait impairments. Calculating LyE depends on sampling frequency and the number of data points selected, making direct comparisons of LyE values difficult. To compare our findings with others, we computed the ratios of the limb with higher variability to the limb with lower variability and compared these with studies of three affected populations (Moraiti et al., 2010; Wurdeman et al., 2013). For individuals with post-stroke hemiparesis, the ratio between their paretic and non-paretic knee LyE values was 1.7 and 1.9 for SS and 6MWT speed, respectively. For subjects that have a reconstructed ACL, the ratio between their intact knee LyE and the reconstructed knee LyE was 1.1 for both reconstruction methods studied (Moraiti et al., 2010). In lower limb amputees, the ratio between sound and prosthetic knee LyE was 1.1 (Wurdeman et al., 2013). This demonstrates that the LyE ratio for post-stroke individuals is in the same order of magnitude as these other affected populations.

This study has a limited sample size; while statistical significance and large effect sizes were found particularly at the knee, it is possible that differences in hip and ankle joints may be detected with increased statistical power. Given the heterogeneity of the post-stroke population, increased sample size would also allow for a more diverse sample and would improve the generalizability of our findings. In order to collect the large amount of continuous data required to evaluate the dynamic structure of gait variability in this study, treadmill walking was evaluated. While treadmill walking has been shown to generalize to over ground walking (Matsas et al., 2000), it is possible that variability may be decreased due to the consistency of treadmill speed. Given that all subjects walked on the same treadmill under the same conditions, the relative comparisons drawn from this study are justified. Additionally, as the treadmill is an important rehabilitation tool post-stroke, characterization of treadmill gait post-stroke is warranted. Finally, although overall functional ability varied among post-stroke subjects, the current study was performed with generally higher functioning stroke subjects. Further research is merited to explore differences in variability between functional groups post-stroke (i.e. household vs. community ambulation).

This study shows a significant difference in the dynamic structure of flexion/extension angle variability, quantified by the Lyapunov exponent, for the knee between the paretic and non-paretic limbs post-stroke. This is especially important because it shows not only how the paretic leg is affected by hemiparesis, but also how the non-paretic leg is affected. As gait is a bilateral task, understanding the interlimb relationship post-stroke is crucial for improving overall walking function. The interlimb relationship for joint angle variability during gait has never been explored to our knowledge and could allow for a variety of treatment options for subjects post-stroke. A shift towards a focus on controlling movement with the non-paretic limb, evidenced through decreased variability, may be a compensatory strategy for impaired stroke survivors who cannot rely on their paretic limb to produce a consistent motion.

It is unclear whether this asymmetrical shift in variability is beneficial or detrimental to the stroke survivor and how this relates to clinically relevant performance such as risk of falls or response to perturbations. Further research should explore how the dynamic structure of interlimb variability is modulated with training protocols which promote symmetry, such as split-belt training (Reisman et al., 2013), and active assistive technology, such as robotic exosuits (Awad and Bae, 2017). It is possible that lowering the LyE on the paretic limb may simultaneously increase the LyE on the non-paretic limb enabling a more symmetric or optimal variability in both limbs of all joints in response to perturbation in the environment.

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References

- Acharya UR, Sudarshan VK, Adeli H, Santhosh J, Koh JEW, Puthankatti SD, Adeli A. A novel depression diagnosis index using nonlinear features in EEG signals. *Eur Neurol.* 2015; 74:79–83. <https://doi.org/10.1159/000438457>. [PubMed: 26303033]

- Awad LN, Bae J, O'Donnell K, De Rossi S, Hendron K, Sloom L, Kudzia S, Allen S, Holt K, Ellis T, Walsh C. A soft robotic exosuit improves walking in patients after stroke. *Sci Transl Med*. 2017; 26:eaa9084. <https://doi.org/10.1126/scitranslmed.aai9084>.
- Awad LN, Palmer JA, Pohlig RT, Binder-Macleod SA, Reisman DS. Walking speed and step length asymmetry modify the energy cost of walking after stroke. *Neurorehabil Neural Repair*. 2015; 29:416–423. <https://doi.org/10.1177/1545968314552528>. [PubMed: 25288581]
- Balasubramanian CK, Neptune RR, Kautz SA. Variability in spatiotemporal step characteristics and its relationship to walking performance post-stroke. *Gait Posture*. 2009; 29:408–414. <https://doi.org/10.1016/j.gaitpost.2008.10.061>. [PubMed: 19056272]
- Bonnyaud C, Pradon D, Vuillerme N, Bensmail D, Roche N. Spatiotemporal and kinematic parameters relating to oriented gait and turn performance in patients with chronic stroke. *PLoS One*. 2015; 10:e0129821. <https://doi.org/10.1371/journal.pone.0129821>. [PubMed: 26091555]
- Chen G, Patten C, Kothari DH, Zajac FE. Gait differences between individuals with post-stroke hemiparesis and non-disabled controls at matched speeds. *Gait Posture*. 2005; 22:51–56. <https://doi.org/10.1016/j.gaitpost.2004.06.009>. [PubMed: 15996592]
- Cignetti F, Decker LM, Stergiou N. Sensitivity of the Wolf's and Rosenstein's algorithms to evaluate local dynamic stability from small gait data sets. *Ann Biomed Eng*. 2012; 40:1122–1130. <https://doi.org/10.1007/s10439-011-0474-3>. [PubMed: 22116622]
- DiBerardino LA, Polk JD, Rosengren KS, Spencer-Smith JB, Hsiao-Wecksler ET. Quantifying complexity and variability in phase portraits of gait. *Clin Biomech*. 2010; 25:552–556. <https://doi.org/10.1016/j.clinbiomech.2010.03.007>.
- Farris DJ, Sawicki GS. The mechanics and energetics of human walking and running: a joint level perspective. *J R Soc Interface*. 2012; 9:110–118. <https://doi.org/10.1098/rsif.2011.0182>. [PubMed: 21613286]
- Ghayoumi Zadeh H, Haddadnia J, Montazeri A. A model for diagnosing breast cancerous tissue from thermal images using active contour and Lyapunov exponent. *Iran J Public Health*. 2016; 45:657–669. [PubMed: 27398339]
- Harbourne RT, Stergiou N. Movement variability and the use of nonlinear tools: principles to guide physical therapist practice. *Phys Ther*. 2009; 89:267–282. <https://doi.org/10.2522/ptj.20080130>. [PubMed: 19168711]
- Hoogkamer W, Bruijn SM, Sunaert S, Swinnen SP, Van Calenbergh F, Duysens J. Toward new sensitive measures to evaluate gait stability in focal cerebellar lesion patients. *Gait Posture*. 2015; 41:592–596. <https://doi.org/10.1016/j.gaitpost.2015.01.004>. [PubMed: 25618477]
- Kaimakamis E, Tsara V, Bratsas C, Sichletidis L, Karvounis C, Maglaveras N. Evaluation of a decision support system for obstructive sleep apnea with nonlinear analysis of respiratory signals. *PLoS One*. 2016; 11:e0150163. <https://doi.org/10.1371/journal.pone.0150163>. [PubMed: 26937681]
- Kao PC, Dingwell JB, Higginson JS, Binder-Macleod S. Dynamic instability during post-stroke hemiparetic walking. *Gait Posture*. 2014; 40:457–463. <https://doi.org/10.1016/j.gaitpost.2014.05.014>. [PubMed: 24931112]
- Lamoth CJ, van Deudekom FJ, van Campen JP, Appels BA, de Vries OJ, Pijnappels M. Gait stability and variability measures show effects of impaired cognition and dual tasking in frail people. *J Neuroeng Rehabil*. 2011; 8:2. <https://doi.org/10.1186/1743-0003-8-2>. [PubMed: 21241487]
- Lewek MD, Scholz J, Rudolph KS, Snyder-Mackler L. Stride-to-stride variability of knee motion in patients with knee osteoarthritis. *Gait Posture*. 2006; 23:505–511. <https://doi.org/10.1016/j.gaitpost.2005.06.003>. [PubMed: 16024250]
- Matsas A, Taylor N, McBurney H. Knee joint kinematics from familiarised treadmill walking can be generalised to overground walking in young unimpaired subjects. *Gait Posture*. 2000; 11:46–53. [https://doi.org/10.1016/S0966-6362\(99\)00048-X](https://doi.org/10.1016/S0966-6362(99)00048-X). [PubMed: 10664485]
- Moraiti CO, Stergiou N, Vasiladis HS, Mosis E, Georgoulis A. Anterior cruciate ligament reconstruction results in alterations in gait variability. *Gait Posture*. 2010; 32:169–175. <https://doi.org/10.1016/j.gaitpost.2010.04.008>. [PubMed: 20591671]
- Mozaffarian D, Benjamin EJ, Go AS, Arnett DK, Blaha MJ, Cushman M, Das SR, de Ferranti S, Després JP, Fullerton HJ, Howard VJ, Huffman MD, Isasi CR, Jiménez MC, Judd SE, Kissela BM, Lichtman JH, Lisabeth LD, Liu S, Mackey RH, Magid DJ, McGuire DK, Mohler ER, Moy CS,

- Muntner P, Mussolino ME, Nasir K, Neumar RW, Nichol G, Palaniappan L, Pandey DK, Reeves MJ, Rodriguez CJ, Rosamond W, Sorlie PD, Stein J, Towfighi A, Turan TN, Virani SS, Woo D, Yeh RW, Turner MB. Heart disease and stroke statistics—2016 update. *Circulation*. 2016; 133:e38–e360. <https://doi.org/10.1161/CIR.0000000000000350>. [PubMed: 26673558]
- Peng CK, Havlin S, Stanley HE, Goldberger AL. Quantification of scaling exponents and crossover phenomena in nonstationary heartbeat time series. *Chaos Interdiscip J Nonlin Sci*. 1995; 5:82–87. <https://doi.org/10.1063/1.166141>.
- Raja B, Neptune RR, Kautz SA. Coordination of the non-paretic leg during hemiparetic gait: expected and novel compensatory patterns. *Clin Biomech (Bristol, Avon)*. 2012; 27:1023–1030. <https://doi.org/10.1016/j.clinbiomech.2012.08.005>.
- Reisman DS, McLean H, Keller J, Danks KA, Bastian AJ. Repeated split-belt treadmill training improves poststroke step length asymmetry. *Neurorehabil Neural Repair*. 2013; 27:460–468. <https://doi.org/10.1177/1545968312474118>. [PubMed: 23392918]
- Rhea CK, Wutzke CJ, Lewek MD. Gait dynamics following variable and constant speed gait training in individuals with chronic stroke. *Gait Posture*. 2012; 36:332–334. <https://doi.org/10.1016/j.gaitpost.2012.03.014>. [PubMed: 22503594]
- Smith BA, Stergiou N, Ulrich BD. Lyapunov exponent and surrogation analysis of patterns of variability: profiles in new walkers with and without down syndrome. *Motor Control*. 2010; 14:126–142. [PubMed: 20237407]
- Stergiou N, Decker LM. Human movement variability, nonlinear dynamics, and pathology: is there a connection? *Hum Mov Sci*. 2011; 30:869–888. <https://doi.org/10.1016/j.humov.2011.06.002>. [PubMed: 21802756]
- Stergiou N, Harbourne R, Cavanaugh J. Optimal movement variability: a new theoretical perspective for neurologic physical therapy. *J Neurol Phys Ther*. 2006; 30:120–129. <https://doi.org/10.1097/01.NPT.0000281949.48193.d9>. [PubMed: 17029655]
- Weiss, TC. Hemiparesis – Types, Treatment, Facts and Information – Disabled World [WWW Document]. *Disabl World*. 2010. URL <https://www.disabledworld.com/health/neurology/hemiparesis.php> (accessed 4.14.17)
- Wolf A, Swift JB, Swinney HL, Vastano JA. Determining Lyapunov exponents from a time series. *Phys D Nonlin Phenom*. 1985; 16:285–317. [https://doi.org/10.1016/0167-2789\(85\)90011-9](https://doi.org/10.1016/0167-2789(85)90011-9).
- Wurdeman SR, Myers SA, Jacobsen AL, Stergiou N. Prosthesis preference is related to stride-to-stride fluctuations at the prosthetic ankle. *J Rehabil Res Dev*. 2013; 50:671. <https://doi.org/10.1682/JRRD.2012.06.0104>. [PubMed: 24013915]

Fig. 1. Reconstructed attractors for (A) non-paretic and (B) paretic knee flexion/extension angle of subject 3 at 6MWT speed, shown in three dimensions for visualization purposes. The attractor is reconstructed in multiple dimensions using the measured knee flexion angle and time delayed copies of itself. The Lyapunov exponent enumerates the divergence of the attractor and is generally unaffected by the attractor shape. Greater disorganization, as seen in the paretic knee, is indicative of greater divergence and greater positive LyE.

Fig. 2.

(A) Paretic vs. non-paretic LyE of hip, knee, and ankle angles in flexion/extension of subjects post-stroke at 6MWT and SS speed. LyE was averaged across all seven subjects. (B) Paretic vs. non-paretic LyE of knee flexion/extension angles of stroke subjects at 6MWT speed. Every subject, with the exception of subject 7, had higher LyE on the paretic side than the non-paretic side. This trend was consistent across all three joints at both speeds.

Table 1

Stroke subjects' characteristics.

Subject	Age (years)	Gender	Time post-stroke (months)	Affected side	Lower extremity Fugl Meyer	SS speed (m/s)	6MWT speed (m/s)	SS Treadmill Speed (m/s)	6MWT Treadmill Speed (m/s)
1	62	F	15.5	L	29	0.95	1.22	0.95	1.22
2	61	M	14.2	R	30	0.95	1.14	0.95	1.14
3	55	M	21.7	R	22	1.38	1.52	0.90	1.10
4	65	M	17.2	L	23	0.86	1.03	0.50	0.65
5	60	M	21.6	R	18	0.59	0.60	0.35	0.45
6	61	M	148.1	L	25	1.26	1.55	1.00	1.20
7	74	F	57.8	R	33	0.83	0.73	0.60	0.35