Quantifying undesired parallel components in Thévenin-equivalent acoustic source parameters

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The calibration of an ear probe to determine its Thévenin-equivalent acoustic source parameters facilitates the measurement of ear-canal impedance and reflectance. Existing calibration error metrics, used to evaluate the quality of a calibration, are unable to reveal undesired parallel components in the source parameters. Such parallel components can result from, e.g., a leak in the ear tip or improperly accounting for evanescent modes, and introduce errors into subsequent measurements of impedance and reflectance. This paper proposes a set of additional error metrics that are capable of detecting such parallel components by examining the causality of the source admittance in the frequency domain and estimating the source pressure in the time domain. The proposed and existing error metrics are applied to four different calibrations using two existing calibration methods, representing typical use cases and introducing deliberate parallel components. The results demonstrate the capability of the proposed error metrics in identifying various undesired components in the source parameters that might otherwise go undetected. © 2018 Acoustical Society of America.

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I. INTRODUCTION

Measurements of ear-canal acoustic impedance and reflectance have been proposed for diagnoses of conductive hearing disorders (Piskorski *et al.*, 1999; Keefe *et al.*, 2000; Feeney and Keefe, 2001; Keefe *et al.*, 2012; Ellison *et al.*, 2012; Merchant *et al.*, 2014) and for calibrating stimulus levels *in situ* (Scheperle *et al.*, 2008; McCreery *et al.*, 2009; Lewis *et al.*, 2009; Scheperle *et al.*, 2011; Withnell *et al.*, 2009; Souza *et al.*, 2014). To facilitate such probe-based measurements, the acoustic source parameters of an ear probe, typically the Thévenin-equivalent source parameters, must be determined in a preliminary calibration procedure. The quality, or accuracy, of the source parameters is traditionally assessed by examining the calibration error, defined as the ability of the source parameters to estimate the known reference impedances of the waveguides or the measured probe pressures. Evanescent modes occur in the discontinuity between the probe tube and load waveguide and affect probe-based measurements in the ear canal (Brass and Locke, 1997) as higher-order, non-propagating modes are excited. However, they also affect the calibration procedure, which typically constitutes measurements using the probe in a set of reference loads with known acoustic input impedances. Evanescent modes can be approximated as an acoustic inertance in series with the plane-wave acoustic input impedance and essentially translates the position of impedance

minima in frequency (Keefe and Benade, 1981; Fletcher *et al.*, 2005).

Siegel and Neely (2017) demonstrated that the presence of evanescent modes during a widely used calibration methodology, proposed by Allen (1985), introduces an apparent non-causality into the source impedance. Siegel and Neely (2017) also demonstrated how a physical modification to the probe tube reduced the evanescent modes in front of the probe, and removed a large part of this undesired non-causal behavior in the source impedance.

Nørgaard *et al.* (2017b) further quantified the effect of evanescent modes on the source parameters and showed that it can be approximated by a phantom acoustic compliance in parallel with the source parameters. The compliance is phantom in the sense that it is not physically present during the calibration procedure, but is rather introduced as a consequence of indirectly estimating the waveguide model lengths from probe pressures that are affected by evanescent modes. They also proposed a method to account for evanescent modes during the calibration procedure and indirectly demonstrated how this approach appeared to eliminate the parallel compliance from the source parameters. However, they noted that the calibration error, typically used to assess the outcome of an acoustic Thévenin calibration, is incapable of identifying such undesired parallel component as such component is inherited into the source parameters.

Thus, there is a need for alternative error metrics that can reveal any undesired parallel components in the source parameters. These error metrics should provide a direct, absolute, and objective assessment of the physical validity of the source parameters produced by such acoustic Thévenin

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calibration procedure. This paper proposes a set of error metrics that can be utilized in combination with existing and during development of new calibration procedures to assess the physical validity of the source parameters and quantify parallel components. The proposed error metrics are based partly on the study of causality in the source admittance using the Hilbert transform, and a time-limited estimation of the source pressure from the source volume flow under in a resistive load. In addition, this paper will further explain the concept of parallel components in the source parameters, limitations in representing transfer functions in the time domain, and the seemingly non-causality associated with lumped acoustic elements that arises as a result of finite frequency spectra.

II. BACKGROUND

A. Existing error metrics

In the existing, widely used methodology (e.g., Allen, 1985; Keefe *et al.*, 1992; Voss and Allen, 1994), the probe pressures P_i in N short, rigidly terminated waveguides are measured. By modeling the input impedances Z_i of each waveguide as an acoustic transmission line, including thermo-viscous wall losses (Keefe, 1984), the Thévenin-equivalent source parameters (the source pressure P_s and source impedance Z_s) are obtained by solving an overdetermined system of N equations using the least-squares method,

$$\begin{bmatrix} Z_1 & -P_1 \\ Z_2 & -P_2 \\ \vdots & \vdots \\ Z_N & -P_N \end{bmatrix} \begin{bmatrix} P_s \\ Z_s \end{bmatrix} = \begin{bmatrix} P_1 Z_1 \\ P_2 Z_2 \\ \vdots \\ P_N Z_N \end{bmatrix}. \quad (1)$$

The lengths l_i of the calibration waveguides, used to calculate the analytical reference impedances Z_i , are estimated, and the final evaluation of the accuracy of the obtained source parameters can be assessed by the relative error in estimating the analytically modeled reference impedances,

$$\epsilon_i = \left| 1 - \frac{\hat{Z}_i}{Z_i} \right|, \quad (2)$$

where the estimated impedances

$$\hat{Z}_i = Z_s \frac{P_i}{P_s - P_i}. \quad (3)$$

It is possible to investigate such calibration error, absolute or relative, in terms of any variable in the system Z_i , P_i , P_s , Z_s , or other derived quantities. What all these possible different errors have in common is that they, in the same way, merely characterize the capability of the linear model in representing the data points used to derive the model, though with differences in scaling. These errors can reveal certain inaccuracies in the calibration, but do not provide a complete objective assessment of the accuracy of the source parameters. Schepeler *et al.* (2011) used, similar to Eq. (3), the source parameters to estimate the probe pressures

$$\hat{P}_i = P_s \frac{Z_i}{Z_s + Z_i}, \quad (4)$$

and compared them to the measured probe pressures P_i using the relative averaged error across frequency ω and waveguides i ,

$$\epsilon_{P_i} = \frac{\sum_i \sum_\omega |P_i - \hat{P}_i|^2}{\sum_i \sum_\omega |P_i|^2}. \quad (5)$$

One single number is thus obtained to represent the quality of the calibration. Such averaging has the disadvantage that it cannot identify large errors confined within small frequency ranges. Notice that, due to the order of the operations (the summations precede the division) in Eq. (5), this summed probe-pressure error ϵ_{P_i} is sensitive to the frequency contents of the stimulus. Other authors have also employed similar single-number error metrics (e.g., Keefe *et al.*, 1992).

B. Parallel components

Nørgaard *et al.* (2017b) noted that existing error metrics are incapable of identifying undesired parallel components in the source parameters. Such parallel component can be introduced due to, for example, not accounting for evanescent modes when waveguide lengths are estimated or a leak in the ear tip. Figure 1(a) shows a Thévenin-equivalent circuit of a probe and its source parameters P_s and Z_s , the measured i th probe pressure P_i in a given reference load impedance Z_i , and an additional impedance Z_{par} in parallel with the load. The distinction between physical and phantom parallel components is important. If a parallel component is physically present during the calibration procedure, such as a leak in the ear tip, and not accounted

FIG. 1. (a) Thévenin- and (b) Norton-equivalent circuit diagrams of the probe and load, including the source parameters (source pressure P_s or source volume flow U_s , and source impedance Z_s), an undesired parallel impedance Z_{par} , the ideal impedance of the i th calibration waveguide Z_i , and probe pressure P_i .

for in the modeled reference impedances Z_i , its effect will be represented in the measured probe pressures P_i and thus inherited into the source parameters. Conversely, a phantom parallel component is not physically present during the measurement, and the probe pressures P_i consequently unaffected by it, but rather, unintentionally, included in the modeled reference impedances Z_i . It thus changes its sign of operation when it is inherited into the source parameters.¹ An alternative set of source parameters P'_s and Z'_s can be defined that include such parallel component, physical or phantom. The alternative source pressure P'_s is given by a voltage division of the true source pressure P_s between the true source impedance Z_s and the impedance of the parallel component Z_{par} ,

$$P'_s = P_s \frac{Z_{\text{par}}}{Z_s + Z_{\text{par}}}. \quad (6)$$

Similarly, the alternative source impedance Z'_s is given by the parallel coupling of the true source impedance Z_s and the impedance of the parallel component Z_{par} ,

$$Z'_s = \frac{Z_s Z_{\text{par}}}{Z_s + Z_{\text{par}}}. \quad (7)$$

Interestingly, these alternative Thévenin source parameters are, on a relative scale, equivalently related to their true counterparts, as they are multiplied by the same complex factor $Z_{\text{par}}/(Z_s + Z_{\text{par}})$. Figure 1(b) shows the Norton-equivalent circuit of the acoustical system under consideration. The source volume flow U_s now replaces the source pressure P_s and the source impedance Z_s appears as a parallel component. Thus,

$$U_s = \frac{P_s}{Z_s} = \frac{P'_s}{Z'_s}, \quad (8)$$

and the source volume flow U_s remains unaffected by the parallel component Z_{par} , while Eq. (7) still applies for the alternative source impedance Z'_s .

C. Limitations in discrete-time representation of acoustic transfer functions

Acoustic response functions, such as the probe pressure, are typically calculated from a time-domain waveform using the Fourier transform. Conversely, acoustic transfer functions, such as impedance and reflectance, are well-defined, measured, and modeled in the frequency domain. As the term “transfer function” suggests, such functions consist of the ratio of two frequency-domain response functions. Under certain assumptions, it is possible to transform such transfer functions into the time domain using the inverse Fourier transform. The continuous-time impedance $z(t)$ can be thought of as the resulting sound pressure in response to a Dirac-delta volume-flow impulse, and the continuous-time reflectance $r(t)$ as the pressure response following an incident Dirac-delta pressure impulse. However, such transformation imposes certain constraints and assumptions onto

the quantities in the practical case of a finite frequency spectrum and, thus, discrete-time representation. Without further modification, the band-limited, Hermitian-symmetric, discrete spectrum supplied to the inverse Fourier transform is implicitly multiplied by an ideal, zero-phase, low-pass filter. Such filter has a non-causal impulse response of a sinc function, which the time-domain pressure and volume-flow response functions are implicitly convolved by. With the continuous-time volume-flow Dirac-delta impulse occurring at $t=0$, its discrete-time representation becomes

$$u(n) = \text{sinc } \pi n, \quad (9)$$

where n is the integer sample number. The volume-flow response function thus yields $u(0) = 1$, $u(n \neq 0) = 0$, and does as such not pose a limitation. However, the pressure-response function of an arbitrary load does not necessarily arrive back to the input plane at integer samples. The discrete-time representation of any pressure pulse arriving back after time τ can be represented by

$$p(n) = \text{sinc } \pi(n - \tau f_s), \quad (10)$$

where f_s is the sampling frequency. If τ is not an integer multiple of the sampling interval $1/f_s$, it results in $p(n \neq 0) \neq 0$ and this severely limits the interpretation of time-domain transfer functions. Similar considerations apply to the Hilbert transform, as it consists of intermediate steps of the inverse and forward Fourier transforms, and the residuals from the sinc function introduces non-zero components into negative time.

Different approaches can be employed to circumvent the effects of these constraints. Rasetshwane and Neely (2011) and Rasetshwane *et al.* (2012) multiplied the frequency-domain reflectance by a zero-phase frequency-domain Blackman window prior to calculating the inverse Fourier transform. The time-domain transfer functions are thus convolved by the impulse response of this window rather than the sinc function and are smeared in time, leading to a possible overlap between effects that were otherwise limited in time. Nørgaard *et al.* (2017a) used a frequency-domain truncation methodology that discarded part of the high-frequency spectrum to restore differentiability at the Nyquist frequency between the original and complex-conjugate part of the replicated spectrum. This approach effectively changed the sampling rate of the synthesized time-domain transfer function, ensuring that response components occur at integer sample numbers. While this approach circumvents most of the limitations, it is still dependent on the capability of restoring such differentiability. Zero-phase, band-limited filters are inherently non-causal and thus, using either approach, it is not possible to study the time-domain behavior of an acoustic transfer function quantity without applying a non-causal filter to the transfer function. Applying instead a causal filter shifts the response functions in time and thus alters the real and imaginary parts of the transfer function, which complicates the assessment of causality.

D. Non-causality and discrete-time representation of acoustic lumped components

Acoustic impedance is inherently a causal quantity. A pressure response cannot occur prior to a volume-flow stimulation. However, Siegel and Neely (2017) observed that the source impedance, obtained from an acoustic Thévenin calibration procedure, was non-causal when evanescent modes were improperly accounted for. This non-causality can likely be accredited to the presence of the phantom parallel compliance in the source impedance (Nørgaard *et al.*, 2017b). Similarly, Rasetshwane and Neely (2011) and Nørgaard *et al.* (2017a) noted that probe-based measurements of acoustic impedance and reflectance were non-causal when affected by evanescent modes, and thus an approximate series inductance. The continuous-time relationships between the sound pressure $p(t)$ and volume flow $u(t)$ for an acoustic inductance L and compliance C are given by

$$p(t) = L \frac{du(t)}{dt}, \quad (11)$$

$$u(t) = C \frac{dp(t)}{dt}, \quad (12)$$

respectively, and are both causal components. In order to interpret $p(t)$ as the time-domain impedance of the inductance $z_L(t)$ in Eq. (11), the Fourier transform of the volume flow must satisfy $\mathcal{F}[u(t)] = 1$ for all frequencies. Similarly for $u(t)$ and the time-domain admittance of the compliance $y_C(t)$ in Eq. (12), the Fourier transform of the pressure must satisfy $\mathcal{F}[p(t)] = 1$. For the continuous-time representation, this condition is only satisfied with the Dirac-delta function $\mathcal{F}[\delta(t)] = 1$, and the time-domain impedance of an acoustic inductance $z_L(t)$ and admittance of an acoustic compliance $y_C(t)$ are given by the derivative of the Dirac-delta function,

$$z_L(t) = L \frac{d\delta(t)}{dt}, \quad (13)$$

$$y_C(t) = C \frac{d\delta(t)}{dt}. \quad (14)$$

When these components are investigated using the inverse Fourier transform of a band-limited spectrum, a signal non-causality is introduced due to the incapability of properly representing the Dirac-delta function derivative of the physical system in the discrete-time representation. With the term “signal non-causality” is understood a degree of non-causality resulting from a signal-processing artifact that occurs as a result of the underlying assumptions in the inverse Fourier transform. It is thus not the physical system under consideration that is non-causal. By truncating the continuous-time spectrum, the Dirac-delta function is represented by the sinc function, which has the derivative

$$\frac{d}{dt} \text{sinc } \pi t = \frac{\cos \pi t}{t} - \frac{\sin \pi t}{\pi t^2}. \quad (15)$$

The respective discrete-time representations of $z_L(n)$ and $y_C(n)$, derived from the inverse Fourier transform, thus result in

$$z_L(n) = f_s L \left(\frac{\cos \pi n}{n} - \frac{\sin \pi n}{\pi n^2} \right), \quad (16)$$

$$y_C(n) = f_s C \left(\frac{\cos \pi n}{n} - \frac{\sin \pi n}{\pi n^2} \right). \quad (17)$$

While the integer samples n coincided with the zeros in Eq. (9) for $n \neq 0$, they now occur close to the extrema in the sinc-function derivative. It is interesting to note that, even though $z_L(n)$ and $y_C(n)$ are physically causal, the signal non-causality appears due to these remaining residuals of the sinc-function derivative in the discrete-time representation. Figure 2 shows the continuous- and discrete-time behavior of the sinc function and its derivative. Similar considerations hold for the inverse functions, the time-domain admittance of an inductance $y_L(t)$ and impedance of a compliance $z_C(t)$, which are both represented by a Heaviside step function in continuous time.

Although inductances and compliances are inherently physically causal components, it is useful to consider the hypothetical nonphysical case of negative L and C . This is a relevant consideration as the evanescent-modes inductance L is not a physically present acoustic inductance, but rather an approximation of the low-frequency behavior of higher-order modes between the speaker and microphone apertures in the probe tube. Although this inductance tends to be positive, it can also be negative in waveguides that are narrow relative to the probe-tube geometry (Fletcher *et al.*, 2005). In addition, a negative parallel compliance is inherited into the source impedance in the presence of a positive evanescent-modes inductance during calibration (Nørgaard *et al.*, 2017b). In general, inverting the sign of operation of the imaginary part of a transfer function effectively reverses its time-domain representation. The effect of such negative inductance or compliance thus does not change the physical causality of Eqs. (13) and (14), as the polarity of the Dirac-delta function derivative is simply reversed. However, the Heaviside step function, representing the inverse functions $y_L(t)$ and $z_C(t)$, is reversed in time in the case of negative quantities. This is an anti-causal, non-physical case where the behavior, for a

FIG. 2. (Color online) Continuous-time t and discrete-time n behavior of the sinc function and the sinc-function derivative sinc' . The discrete-time functions are plotted for integer n .

negative compliance, can be interpreted as the sound pressure stored in the compliance discharging in reverse time, prior to the volume-flow excitation. For this case of a negative inertance or compliance, $z_L(t)$ and $y_C(t)$ are thus causal, but not minimum phase, since their inverse functions $y_L(t)$ and $z_C(t)$ are anti-causal (Oppenheim and Schaffer, 1975).

E. Lumped waveguide extension

Nørgaard *et al.* (2017b) showed that, if evanescent modes are not accounted for during calibration, an impedance measurement in a waveguide of radius identical to the calibration waveguides results in an apparent plane-wave impedance. This is a result of an evanescent-modes inertance and parallel compliance effectively translating the position of impedance minima and maxima equivalently. They also noted the approximate relationship between the evanescent-modes inertance L , the parallel compliance C , and the characteristic impedance of the calibration waveguides $Z_{0,cal}$,

$$C \simeq \frac{L}{Z_{0,cal}^2}. \quad (18)$$

Figure 3 shows the schematic of such evanescent-modes inertance and parallel compliance in front of a transmission line with the characteristic impedance Z_0 . The combination of an infinitesimally small inertance and compliance satisfying Eq. (18) represents an elemental length of a lossless transmission line and thus an extension of the transmission line in Fig. 3. This transmission-line extension can thus also be characterized by the time delay τ ,

$$\tau = CZ_0 = \frac{L}{Z_0}. \quad (19)$$

However, the transmission-line extension is lumped since it only consists of one non-infinitesimal L - C coupling. This is the reason for Eq. (18) being approximate, and the transmission-line extension is only well approximated when the wavelength $\lambda \gg c\tau$, where c is the speed of sound. $c\tau$ represents the effective length of the extension. It is interesting to consider the consequence of such inertance and compliance on the signal causality of impedance. As long as the approximation is valid, the impedance of a transmission line with such extension appears completely causal as it simply represents a delay in the system. This potentially limits the methodology of Nørgaard *et al.* (2017a) in estimating the evanescent modes inertance in presence of a parallel compliance. Notice, that for a negative L , the corresponding C and

FIG. 3. Coupling of a series inertance L and a parallel compliance C to a uniform transmission line with characteristic impedance Z_0 .

τ would also become negative and thereby result in an effective decrease in length of the transmission-line. This could violate the physical causality of the system.

III. METHODS

Two error metrics are proposed that can be used to assess the quality of the source parameters and reveal undesired physical and phantom parallel components. For the first metric, a Hilbert-transform-based approach will be used to quantify the signal causality of the source admittance. The second metric employs the source pressure and volume flow, whereby it is not subject to the limitations and assumptions of the discrete inverse Fourier transform and Hilbert transform of transfer functions.

A. Source-admittance error metric

The source impedance Z_s can be physically interpreted as the input impedance of the probe tube and is therefore an inherent minimum-phase quantity. Minimum phase is equivalent to a quantity being causal, stable, and having a causal inverse (Oppenheim and Schaffer, 1975). With the primary motivation for proposing additional error metrics in revealing parallel components, it is convenient to investigate the behavior in the inverse function of the source impedance Z_s , namely, the source admittance $Y_s = 1/Z_s$, as this quantity must also conform to causality. The parallel component in Eq. (7) now simply results in the alternative source admittance

$$Y'_s = Y_s + Y_{par}, \quad (20)$$

which is advantageous in characterizing this parallel component.

Nørgaard *et al.* (2017a) employed the Hilbert transform to identify and compensate for the signal non-causality introduced into waveguide acoustic impedance measurements as a result of an evanescent-modes inertance. Here, a similar approach will be employed to assess the causality of the source admittance that can reveal and quantify physical and phantom parallel components, such as a compliance, that introduce a signal or physical non-causality. Considering the assumed causal function, the time-domain source admittance $y_s(t < 0) = 0$, the Hilbert transform relates the real and imaginary parts of the Fourier transform of that function, the frequency-domain admittance Y_s , and the behavior in $\lim_{\omega \rightarrow \infty} Y_s(\omega) = Y_0$ (Papoulis, 1962; Hsu, 1967),

$$\text{Re}\{Y_s\} = Y_0 + \mathcal{H}[\text{Im}\{Y_s\}], \quad (21)$$

$$\text{Im}\{Y_s\} = \mathcal{H}^{-1}[\text{Re}\{Y_s\}]. \quad (22)$$

$\mathcal{H}[\cdot]$ is the Hilbert-transform operator, $\text{Re}\{\cdot\}$ and $\text{Im}\{\cdot\}$ denote the real and imaginary parts, respectively, and $Y_0 = A/(\rho c)$ is the characteristic admittance of the probe tube with geometrical cross-sectional area A , air density ρ , and speed of sound c . The source admittance can now be estimated,

$$\hat{Y}_s = Y_0 + \mathcal{H}[\text{Im}\{Y_s\}] + j\mathcal{H}^{-1}[\text{Re}\{Y_s\}], \quad (23)$$

and the source-admittance estimation error defined,

$$\epsilon_{Y_s} = Y_s - \hat{Y}_s. \quad (24)$$

Similar to Nørgaard *et al.* (2017a), the non-causality will be quantified from the real and imaginary source-admittance estimation errors

$$\text{Re}\{\epsilon_{Y_s}\} = \text{Re}\{Y_s\} - Y_0 - \mathcal{H}[\text{Im}\{Y_s\}], \quad (25)$$

$$\text{Im}\{\epsilon_{Y_s}\} = \text{Im}\{Y_s\} - \mathcal{H}^{-1}[\text{Re}\{Y_s\}]. \quad (26)$$

For a source admittance containing a purely non-causal parallel component such as Eq. (20), the real and imaginary source admittance estimation errors yield

$$\text{Re}\{\epsilon_{Y_s}\} = \text{Re}\{Y_{\text{par}}\} + \mathcal{H}[\text{Im}\{Y_{\text{par}}\}], \quad (27)$$

$$\text{Im}\{\epsilon_{Y_s}\} = \text{Im}\{Y_{\text{par}}\} + \mathcal{H}^{-1}[\text{Re}\{Y_{\text{par}}\}]. \quad (28)$$

Nørgaard *et al.* (2017a) evaluated the Hilbert transform on the reflectance due to the non-smooth behavior of the impedance and admittance of a rigidly terminated waveguide, and transformed the reflectance estimation error ϵ_R into an impedance estimation error ϵ_Z . For the investigated cases in this study, the source admittance was generally a smooth function with frequency, rendering such transformation unnecessary. Frequency truncation is employed where possible in order to restore differentiability of the spectrum by adjusting the truncation frequency to coincide with a local extreme of the real part and an ordinate zero-crossing of the imaginary part. For a band-limited transfer function, the source-admittance estimation error characterizes the signal non-causality, and thus not the non-causality of the physical system. The Hilbert transform therefore reveals any lumped acoustic components, such as a compliance, in the source admittance, although they do not introduce any physical non-causality into the system.

B. Source-pressure error metric

Equations (6)–(8) showed that any parallel component, regardless of causality, scales the source impedance and source pressure equivalently, while leaving the source volume flow unchanged. It might therefore be unnecessary to assess the causality properties of the source pressure. However, the source pressure and volume flow differ from the source admittance in that they are response functions rather than transfer functions, and are unaffected by the inherent limitations regarding the inverse Fourier transform and Hilbert transform of transfer functions, as described in Sec. II C. They can therefore readily be transformed into the time domain without applying any windowing, truncation, or further precautions. An alternative metric to assess the causality of the source pressure is proposed that is based only on the inverse Fourier transform of response functions.

Consider the incident source pressure $P_{s,0}$ resulting from the source volume flow U_s into the probe-tube characteristic impedance Z_0 . In the frequency domain, this quantity is inherently different from the source pressure P_s due to the non-resistive components in the source impedance Z_s ,

$$P_{s,0} = U_s Z_0 \neq P_s. \quad (29)$$

Evaluating the inverse Fourier transform, the convolution $u_s(n) * z_0(n) = u_s(n)Z_0$, as Z_0 is a constant. With regards to the typical length of a probe tube (1–2 cm), the time-domain source impedance $z_s(n)$ can be assumed to behave purely resistive with no reflections until a certain point in time $\tau_{Z_0} = n_{Z_0}/f_s$, and therefore $z_s(n < n_{Z_0}) = z_0(n)$, if the characteristic impedance of the probe tube remains constant along the probe tube. Thus,

$$p_{s,0}(n) = u_s(n)Z_0 = p_s(n), \quad \text{for } n < n_{Z_0}. \quad (30)$$

While the time-domain source pressure $p_s(n)$ is affected by a parallel component, the source volume flow $u_s(n)$ is unaffected [Eq. (8)]. The probe-tube characteristic impedance $Z_0 = \rho c/A$ does not contain this parallel component, as it is calculated from the known cross-sectional area A of the probe tube. Besides visually assessing the similarity of the functions $p_s(n)$ and $p_{s,0}(n)$ until the sample number n_{Z_0} , the following normalized source-pressure error metric is proposed:

$$\epsilon_{p_s} = \frac{\sum_{n=-M/2+1}^{n_{Z_0}} |p_s(n) - p_{s,0}(n)|^2}{\sum_{n=-M/2+1}^{n_{Z_0}} |p_s(n)|^2}. \quad (31)$$

M is the fast Fourier transform (FFT)-length and n_{Z_0} must be chosen such that only the resistive behavior in $z_s(n)$ is represented in the error. In this study, n_{Z_0} was chosen to simply coincide with the first peak in $u_s(n)$, and it might thus not contain the entire resistive part of the source impedance. In other cases it might be necessary to adopt different ways to determine n_{Z_0} .

C. Predicted incident sound power

Although it does not immediately serve as an error metric, it is useful to observe how physical and phantom parallel components contribute to the predicted sound power delivered into a non-reflecting resistive load, corresponding to the probe-tube characteristic impedance Z_0 . To do this, the predicted incident pressure P_0 and volume flow U_0 into this load are first calculated,

$$P_0 = P_s \frac{Z_0}{Z_s + Z_0}, \quad (32)$$

$$U_0 = \frac{P_s}{Z_s + Z_0}. \quad (33)$$

Using the inverse Fourier transform, the corresponding time-domain quantities $p_0(n)$ and $u_0(n)$ are obtained and the instantaneous incident sound power $w_0(n)$ can be calculated,

$$w_0(n) = p_0(n)u_0(n). \quad (34)$$

By summing the sound power, the predicted total-incident dissipated acoustic energy E_0 can be calculated,

$$E_0 = \frac{1}{f_s} \sum_{n=-M/2+1}^{M/2} w_0(n). \quad (35)$$

D. Measurements and equipment

The measurements reported in this study were carried out using a FireFace UC sound card (RME Audio, Haimhausen, Germany) controlled through custom-written MATLAB (The MathWorks, Inc., Natick, MA) software and the third-party utility Playrec.² Both an ER-10X ear probe (Etymotic Research, Inc., Elk Grove Village, IL) and a Titan-based ear probe (Interacoustics A/S, Middelfart, Denmark), but modified to improve the high-frequency performance via internal cross-talk reduction, were used.

Probe pressures were obtained by applying frequency-equalized wideband chirps to the probes so as to provide a flat incident probe pressure in a non-reflecting load of radius $a=4$ mm, similar to an adult ear canal. The chirps were played back in the probes in 128 phase-locked blocks with a length of $M=2048$ samples at a sampling rate of $f_s=44.1$ kHz, which were each recorded using the probe microphones and averaged to reduce the noise in the measurements. The derived source pressures and source volume flows were divided by the complex frequency response of the electrically supplied chirp to transform them into their respective impulse responses.

The hilbert function, as part of the MATLAB Signal Processing Toolbox, was used to calculate discrete Hilbert transforms from the Hermitian-symmetric source-admittance spectra. When possible, the source-admittance spectra were truncated to restore differentiability (Nørgaard *et al.*, 2017a). Frequency-domain impedances and admittances were multiplied by a frequency-domain Blackman window (Rasetshwane and Neely, 2011; Rasetshwane *et al.*, 2012) before calculating their time-domain equivalents using the ifft function. Time-domain source pressures and volume flows were calculated from the entire spectrum using only the ifft function. Due to the anti-aliasing filters in the sound card, the maximum considered calibration frequency was 20 kHz. Thus, the obtained source impedances and admittances beyond this frequency and up to the Nyquist frequency $f_s/2$ are neither considered valid nor included in calculating the inverse Fourier transform and Hilbert transform (Nørgaard *et al.*, 2017a).

Two existing methodologies were used for calibrating each probe type and four calibrations were conducted in total:

- (1) Calibration of the ER-10X ear probe using the original methodology of Allen (1985), Keefe *et al.* (1992), and

Voss and Allen (1994) of estimating waveguide lengths and not accounting for evanescent modes. The lengths were estimated by minimizing the relative impedance calibration error ϵ_i [Eq. (2)] in a region averaged around the first probe-pressure minima. The ER-10X built-in calibration waveguide of radius $a=4$ mm was used with software-selected lengths $l_i=1.4, 1.7, 2$, and 2.3 cm. The probe was inserted into the waveguide using a standard, mushroom-shaped, green, 9 mm, rubber ear tip (Sanibel Supply, Middelfart, Denmark).

- (2) Identical to calibration (1), but cutting off an annular sector of the mushroom part of the ear tip, thus, deliberately introducing a leak into the ear tip, representing a physically present, undesired parallel component.
- (3) Identical to calibration (1), but using the beveled probe tube proposed by Siegel and Neely (2017) to reduce the effect of evanescent modes, and thus proposedly eliminating the phantom parallel compliance from the source parameters.
- (4) Calibration of the Titan probe using the waveguides and methodology proposed by Nørgaard *et al.* (2017b) of accounting for evanescent modes and flow losses during calibration, and thus proposedly eliminating the phantom parallel compliance from the source parameters. These calibration waveguides were of lengths $l_i=1.2, 1.45, 1.75$, and 2 cm and radius $a=2$ mm. The probe was inserted into the waveguides using their same utilized hard-rubber part.

IV. EXPERIMENTAL RESULTS

A. Calibration error and source admittance

Figures 4(a)–4(d) show the relative impedance calibration errors ϵ_i [Eq. (2)] of calibrations (1)–(4), respectively, and Figs. 4(e) and 4(f) show the real and imaginary parts of the obtained source admittances, respectively, normalized by the probe-tube characteristic admittances, Y_s/Y_0 of calibrations (1)–(4). The summed probe-pressure errors ϵ_{P_i} [Eq. (5)] (0.25–18 kHz, Scheperle *et al.*, 2011) are listed in Table I. Since frequency-equalized chirps were used for each ear probe, ϵ_{P_i} should be comparable.

Comparing first calibrations (1) and (2), it is worth noticing that ϵ_i in Figs. 4(a) and 4(b) remain almost unchanged across the frequency spectrum, despite the introduction of a leak into the ear tip, although ϵ_{P_i} appears to have increased slightly. Despite these consistent ϵ_i , $\text{Re}\{Y_s\}$ and $\text{Im}\{Y_s\}$ in Figs. 4(e) and 4(f) undergo quite a significant change below 2 kHz. The leak appears to be similar to the combination of a pressure-release condition, a resistance, and an inertance. Such inertance has an admittance $Y_L = 1/(j\omega L)$, which is indeed the behavior observed between $\text{Im}\{Y_s\}$ below 2 kHz in Fig. 4(f). In addition, it should be noted that, while $\text{Re}\{Y_s\}$ are positive, $\text{Im}\{Y_s\}$ have a negative gradient, representing the phantom parallel compliance, and are thus not minimum phase in its literal interpretation.

Comparing now calibrations (1) and (3) the observation is made, that ϵ_i appear to increase slightly between Figs. 4(a)

FIG. 4. (Color online) The resulting relative impedance calibration errors ϵ_i [Eq. (2)] for each waveguide i of calibrations (a) (1) ER-10X, (b) (2) ER-10X leak, (c) (3) ER-10X bevel, and (d) (4) Titan (see Sec. III D for details). The resulting (e) real and (f) imaginary parts of the source admittances, normalized by the characteristic admittance of the probe tubes, Y_s/Y_0 for each calibration (1)–(4).

and 4(c), which is further indicated by the slightly larger ϵ_{P_i} . However, the parallel compliance in $\text{Im}\{Y_s\}$ in Fig. 4(f) has been reduced to roughly one-third of its value with the standard probe tube, though not entirely removed. $\text{Re}\{Y_s\}$ in Fig. 4(e) have changed slightly, however, this is to be expected as the beveled probe tube constitutes a physical modification and thus a potential change in resonances, etc. It is not immediately possible to observe this effect in $\text{Im}\{Y_s\}$ due to the dominating parallel compliance. Calibrations (1)–(3) clearly demonstrate the incapability of the existing error metrics in identifying undesired parallel components.

Finally, calibration (4), which was conducted using a different probe and methodology that incorporates

evanescent modes into the model rather than attempting to physically reduce them, shows much lower ϵ_i and ϵ_{P_i} than the previous calibrations (1)–(3). This can presumably be accredited to the methodology also accounting for flow losses in the transition between probe tube and calibration waveguide. It has the effect that ϵ_i are further reduced in impedance minima, which are the points that are most distinct in the other calibrations (1)–(3). This approach could be potentially risky as the methodology cannot distinguish between including evanescent modes and flow losses, or maybe an entirely different undesired phenomenon in the model, such as cross talk. Despite this concern, $\text{Im}\{Y_s\}$ appears to be centered around the abscissa indicating that there is no, or at least only a small, parallel compliance and that Y_s could be minimum phase.

B. Hilbert-transform assessment of source-admittance causality

Figures 5(a)–5(d) show the normalized time-domain source impedances $z_s(t)/Z_0$ and admittances $y_s(t)/Y_0$ of calibrations (1)–(4), respectively, and Figs. 5(e) and 5(f) show the real and imaginary parts of the source-admittance estimation errors, respectively, normalized by the probe-tube characteristic admittances, ϵ_{Y_s}/Y_0 [Eq. (24)] of calibrations

TABLE I. The obtained probe-pressure summation errors ϵ_{P_i} [Eq. (5)], source-pressure error metrics ϵ_{P_s} [Eq. (31)], estimated time limits τ_{Z_0} for the resistive behavior in the source impedance, and predicted incident acoustic energy E_0 [Eq. (35)] for calibrations (1)–(4) (see Sec. III D for details).

Calibration	(1) ER-10X	(2) ER-10X leak	(3) ER-10X bevel	(4) Titan
ϵ_{P_i}	1.87×10^{-4}	2.55×10^{-4}	3.14×10^{-4}	1.87×10^{-6}
ϵ_{P_s}	1.38	6.45	0.40	0.016
τ_{Z_0} (ms)	0.136	0.136	0.136	0.113
E_0 (nJ)	0.054	0.020	0.092	0.12

FIG. 5. (Color online) Time-domain impedances $z_s(t)$ and admittances $y_s(t)$ of calibrations (a) (1) ER-10X, (b) (2) ER-10X leak, (c) (3) ER-10X bevel, and (d) (4) Titan (see Sec. III D for details). The resulting (e) real and (f) imaginary parts of the source-admittance estimation errors ϵ_Y , [Eq. (24)] for each calibration (1)–(4). Quantities are normalized by either the characteristic impedance Z_0 or admittance Y_0 of the probe tubes.

(1)–(4). Due to the large magnitude of the parallel compliances, it is only possible to apply the frequency-truncation process (Nørgaard *et al.*, 2017a) to calibration (4), and thus the Blackman window was applied to all time-domain impedance and admittance measures for better comparison.

Comparing first calibrations (1) and (2), $z_s(t)$ and $y_s(t)$ in Figs. 5(a) and 5(b) are non-causal. While $y_s(t)$ mainly show the double-peak effect originating from the sinc-function derivative discussed in Sec. II D, $z_s(t)$ extend much further into negative time, especially evident for calibration (1). This is presumably due to the non-physical anti-causal property of the parallel compliance being negative and thus discharging into the resistive part of the source impedance in reverse time. In calibration (2), $z_s(t)$ seems to be of much smaller magnitude, which can presumably be accredited to the pressure-release properties of the ear-tip leak. $\text{Im}\{\epsilon_Y\}$ in Fig. 5(f) appear to have accurately identified the parallel compliance from the source impedance in calibration (1). On the other hand, calibration (2) is affected by the additional parallel inductance, seen from the deviation from frequency proportionality below 2 kHz. This also introduces instability into the Hilbert transform at low frequencies due to the increased noise and diverging admittance resulting from the

pressure-release condition. As the non-causality is mostly represented by an imaginary component in calibration (1), $\text{Re}\{\epsilon_Y\}$ in Fig. 5(e) essentially represent the Hilbert transform of the imaginary part of the parallel component $\mathcal{H}[\text{Im}\{Y_{\text{par}}\}]$ [see Eq. (27)]. However, this also means that $\text{Im}\{\epsilon_Y\}$ is largely unaffected by any $\mathcal{H}^{-1}[\text{Re}\{Y_{\text{par}}\}]$.

Comparing calibrations (1) and (3), the non-causality in $z_s(t)$ and $y_s(t)$ has been reduced between Figs. 5(a) and 5(c), which is evident from the decreased extension into negative time of $z_s(t)$ and reduced double peak of $y_s(t)$. Both obtained $z_s(t)$ align fairly well with those presented by Siegel and Neely (2017), and $\text{Re}\{\epsilon_Y\}$ and $\text{Im}\{\epsilon_Y\}$ in Figs. 5(e) and 5(f), respectively, indicate that the parallel compliance has indeed been much reduced as a consequence of the beveling of the probe tube.

Finally, calibration (4) indicates causal $z_s(t)$ and $y_s(t)$ in Fig. 5(d), aside from the remaining smearing of the Blackman window. In addition, it is worth noticing that the contribution at $t=0$ represents the characteristic resistive behavior in both quantities, which they are both normalized by, and are thus identical. Finally, $\text{Re}\{\epsilon_Y\}$ and $\text{Im}\{\epsilon_Y\}$ have been reduced to a very low level in Figs. 5(e) and 5(f), respectively, also indicating causality.

C. Source-pressure estimation from source volume flow

Figures 6(a)–6(d) show for calibrations (1)–(4), respectively, the time-domain source pressures $p_s(t)$ and the estimated incident source pressures $p_{s,0}(t)$ [Eq. (30)]. Table I further lists the source-pressure error metrics ϵ_{p_s} [Eq. (31)] and time limits τ_{Z_0} , obtained for calibrations (1)–(4).

Comparing calibrations (1) and (2) in Figs. 6(a) and 6(b), respectively, a similar behavior and extension into negative time of $p_s(t)$ is observed as for the source impedances $z_s(t)$ in Figs. 5(a) and 5(b). This is in accordance with Eqs. (6) and (7), that P'_s and Z'_s are scaled by the same factor. In addition, $p_{s,0}(t)$ are very similar between the two calibrations and do in either case not coincide with $p_s(t)$ over any intervals prior to τ_{Z_0} . ϵ_{p_s} in Table I indicate that the calibrations are quite poor.

Comparing calibrations (1) and (3) in Figs. 6(a) and 6(c), respectively, a behavior is also observed between $p_s(t)$ that is similar to $z_s(t)$ in Figs. 5(a) and 5(c). $p_{s,0}(t)$ are still very similar, and for calibration (3), $p_s(t)$ and $p_{s,0}(t)$ are to a larger degree aligned, but not completely. Also, ϵ_{p_s} in Table I is reduced accordingly.

Finally, calibration (4) in Fig. 6(d) shows well-aligned $p_s(t)$ and $p_{s,0}(t)$ until approximately 0.2 ms, representing the actual time-limit of the resistive behavior of the probe tube. This indicates that the source pressure is indeed unaffected by any non-causal, non-resistive parallel components. Finally, ϵ_{p_s} in Table I has been reduced to a very low value.

D. Incident sound power

Figures 7(a)–7(d) show, for calibrations (1)–(4), respectively, the predicted incident sound power $w_0(t)$ [Eq. (34)], and Table I further lists the predicted incident acoustic energy E_0 [Eq. (35)]. Comparing calibrations (1) and (2), it

is evident that the leak in the ear tip dramatically decreases $w_0(t)$ and E_0 , mainly due to the much lower source pressure. Comparing calibrations (1) and (3), the reduction of the parallel compliance in the source parameters increases $w_0(t)$ and E_0 . It is difficult to interpret the results of calibration (4) as there is nothing to compare with, but it is still reported for reference.

V. DISCUSSION

The incapability of identifying parallel components in the source parameters has been a significant limitation for the most widely used methodologies for acoustic Thévenin calibration of ear probes. A parallel component, such as an acoustic compliance resulting from improperly accounting for evanescent modes, can cause large errors in subsequent measurements of impedance and reflectance (Nørgaard *et al.*, 2017b). This paper has proposed a set of error metrics, in addition to the calibration error, that can effectively identify such parallel components in the source parameters. Nørgaard *et al.* (2017a) demonstrated that the Hilbert transform is an efficient method to characterize the signal non-causality in the measured waveguide reflectance. The results presented in this paper further support the viability of the methodology of Nørgaard *et al.* (2017a). An important property of the Hilbert transform, and thus the Hilbert-transform-based estimation error, of a band-limited spectrum, is that they quantify the signal causality of a transfer function. Any signal non-causality of a transfer function represents the superimposed effects of actual physical non-causality and non-causality resulting from signal-processing artifacts when observing the physical system through the time-domain transfer function using the inverse Fourier transform. A non-zero Hilbert-transform estimation error can therefore not be interpreted as a physically non-causal system.

FIG. 6. (Color online) Time-domain source pressures $p_s(t)$ and the predicted incident source pressures $p_{s,0}(t)$ [Eq. (30)] of calibrations (a) (1) ER-10X, (b) (2) ER-10X leak, (c) (3) ER-10X bevel, and (d) (4) Titan (see Sec. III D for details).

FIG. 7. (Color online) Predicted incident sound power $w_0(t)$ [Eq. (34)] of calibrations (a) (1) ER-10X, (b) (2) ER-10X leak, (c) (3) ER-10X bevel, and (d) (4) Titan (see Sec. III D for details).

However, a low Hilbert-transform estimation error does give an indication that the physical system is causal.

One of the main advantages of the Hilbert transform is that it is able to provide information on inaccuracies in a calibration that are confined to small frequency ranges. Such effect is visible around 4 kHz in Figs. 5(e) and 5(f) for calibrations (1)–(3) where small variations in the source admittance estimation error can be observed. Proposedly, this is caused by a combination of the source parameters mainly being determined from impedance minima in this region and not accounting for flow losses. Despite the effectiveness of the Hilbert transform, it might not be possible to utilize it in certain cases due to potential unknown problems encountered with frequency truncation, besides those resulting from a component in parallel with the source impedance, and noise. It should be noted that the source-admittance estimation error ϵ_{Y_s} relies on that no physical lumped elements are actually present in the true source parameters, e.g., that the effect of the material properties of the ear tip are negligible in the source admittance and that the probe tube is uniform at the tip of the probe. While the assessment of the probe-pressure estimation error ϵ_{p_s} might be less direct than quantifying the non-causality of the source admittance, it is a more robust methodology in its basic formulation because it does not rely on the inverse Fourier transform of a transfer function. The limitations of this probe-pressure estimation error are mainly constituted in determining a representative time limitation τ_{z_0} . This quantity will vary depending on the length of the probe tube, and the source-pressure error might not be immediately comparable between different probe types. An additional fundamental difference between the two proposed error metrics is worth emphasizing. While the Hilbert-transform methodology relies on the undesired parallel component comprising either a physical or signal non-causality, the probe-pressure estimation error only relies on

the time-limited resistive behavior of the probe tube. In addition, neither of the proposed error metrics are applicable in the case of a protruded microphone tube in the probe (e.g., Keefe *et al.*, 1992; Voss and Allen, 1994), as this protrusion results in an actual lumped compliance in parallel with the source parameters. In principle, this condition of resistive behavior in the source admittance for both error metrics is not satisfied with the beveling of the probe tube on the ER-10X probe either. However, the phantom parallel compliance appears to be much more dominant than potential errors due to this effect.

A fundamental prerequisite in the methodology proposed by Nørgaard *et al.* (2017b) and utilized for calibration (4) is that the waveguide lengths are known quantities, which was also the case for the waveguides used for this calibration. The same assumption cannot be made for the calibration in the built-in ER-10X calibration waveguide due to the uncertainty associated with the insertion of the probe using an ear tip and the movable piston termination. The results of this study indicate that properly accounting for evanescent modes during calibration is crucial for obtaining a set of causal source parameters. The proposed error metrics could potentially enable such calibration procedure without preliminary knowledge of the waveguide lengths, though this has not been further investigated. However, it is interesting to note that introducing a signal non-causal component into the reference impedances, in the form of an inertance to model evanescent modes, during calibration results in causal source parameters. Conversely, failing to account for evanescent modes by assuming causal, plane-wave reference impedances results in non-causal source parameters.

Although the presented results are each based on one single calibration, the standard calibrations of the ER-10X and Titan probes, i.e., calibrations (1) and (4), respectively, yielded reproducible results with negligible deviations over

the course of several calibrations, utilizing different probe tubes and ear tips. However, the physical modifications performed on the probes in calibrations (2) and (3) are less reproducible. The ear-tip modification in calibration (2) involved manually cutting away an annular sector of the mushroom part of the ear tip. This introduces a different leak when performing a similar modification to another ear tip. In addition, the leak in the ear tip is shaped slightly different with different insertions into the calibration waveguide. The beveling of the probe tube in calibration (3) also involves a manual modification of the probe tube, resulting in an inevitable variation between different beveled probe tubes, unless the beveling can be incorporated as part of the manufacturing process. In the light of these inherent variations in deliberately induced parallel components, calibrations (2) and (3) should merely be considered as examples of such errors and not an assessment of, for example, the general accuracy in the methodology in beveling the probe tube of the ER-10X probe.

The present study has proposed a set of error metrics that may be used in combination with the existing calibration error to further assess the validity of an acoustic Thévenin calibration procedure. However, no error thresholds have been defined that the source parameters should satisfy to be considered valid, although the presented examples in calibrations (1)–(4) can be used as a set of guidelines for what could be considered acceptable and poor calibrations. Defining such thresholds would not only require additional data from different probes, but should also include a systematic assessment of acceptable errors in subsequent measurements under similar conditions to the expected use case, e.g., adult or infant ears. The error metrics considered in this study were normalized by probe-tube characteristic impedance or admittance, mainly to yield similar values for the source impedance and admittance and facilitate the comparison of data between the two utilized probes. To assess the potential influence of a given parallel component on a subsequent measurement, normalization should be carried out with respect to the characteristic impedance of the considered load. While the proposed error metrics can be a valuable tool in improving acoustic Thévenin-equivalent calibration methods, they could also be utilized in routine calibrations in clinical settings. However, depending on the stability of the utilized calibration methodology, this may not be necessary. The authors strongly encourage other researchers in the field to familiarize themselves with and apply the proposed or similar alternative error metrics in their research that can identify undesired parallel components and objectively assess the validity of the source parameters. However, it cannot be precluded that the proposed error metrics may not be able to reveal other unknown types of calibration errors.

The authors would like to emphasize that the objective of the present study is not to compare the performance of the ER-10X and Titan ear probes, but rather to explore underlying calibration methodologies. Ideally, methods for measuring impedance and reflectance should be independent of the utilized probe. The beveling of the probe tube is only possible for the ER-10X probe due to the axisymmetrical

construction of the probe tube. In addition, an appropriate coupling mechanism of the ER-10X probe to the waveguides proposed by Nørgaard *et al.* (2017b) does not currently exist, and thus the Titan probe was utilized instead. However, this study does point toward the need for a calibration methodology that can be used in combination with the built-in ER-10X waveguide that takes evanescent modes into account. While a set of causal source parameters should generally be desired, it has the effect that subsequent measurements of impedance can be interpreted as a transfer impedance, the ratio of sound pressure on the probe microphone to volume flow injected by the probe speaker. These measurements thus remain affected by evanescent modes, and additional compensation, such as proposed by Fletcher *et al.* (2005) or Nørgaard *et al.* (2017a) must be applied to obtain an estimate of the plane-wave impedance or reflectance.

VI. CONCLUSION

This paper has proposed a set of error metrics that can be used to assess the physical validity of the source parameters resulting from an acoustic Thévenin calibration procedure. As opposed to existing metrics, the proposed metrics are capable of identifying undesired physical or phantom parallel components in the source parameters. Four different calibrations using two existing calibration methodologies were compared to assess the effects of different physical and phantom components on the error metrics. The results showed that the proposed error metrics were accurately able to identify and quantify both the physical and phantom parallel components. Furthermore, the calibration methodology proposed by Nørgaard *et al.* (2017b) yielded a set of causal source parameters. The proposed additional error metrics could potentially pave the way for more reliable and accurate measurements of ear-canal impedance and reflectance, and increase the precision in *in situ* calibration of stimulus levels.

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¹An alternative interpretation is that the sign change occurs when a series component is represented as an equivalent inverse parallel component prior to being incorporated into the source parameters.

²www.playrec.co.uk (Last viewed 12 February 2018).

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