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Accelerating acquisition strategies for low-frequency conductivity imaging using MREIT

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Abstract

We sought to improve efficiency of Magnetic Resonance Electrical Impedance Tomography data acquisition so that fast conductivity changes or electric field variations could be monitored. Undersampling of k-space was used to decrease acquisition times in spin-echo-based sequences by a factor of two. Full MREIT data were reconstructed using continuity assumptions and preliminary scans gathered without current. We found that phase data were reconstructed faithfully from undersampled data. Conductivity reconstructions of phantom data were also possible. Therefore, undersampled k-space methods can potentially be used to accelerate MREIT acquisition. This method could be an advantage in imaging real-time conductivity changes with MREIT.

1. Introduction

Magnetic Resonance Electrical Impedance Tomography (MREIT) (Seo and Woo 2011, Seo and Woo 2013a, Woo and Seo 2008) is an imaging modality that can be used to reconstruct the conductivity distribution of the human body from internal measurements of magnetic flux density. MREIT is based on methods established in the late 1980s and early 1990s by Joy *et al.* (Joy *et al.* 1989, Scott *et al.* 1991, Scott *et al.* 1992) as part of Magnetic Resonance Current Density Imaging (MRCDI). The methods described in (Joy *et al.* 1989, Scott *et al.* 1991, Scott *et al.* 1992) required the subject to be rotated in the MR scanner to recover different components of magnetic flux density caused by currents. MREIT techniques (Seo and Woo 2011) can be used to reconstruct electromagnetic parameters from only one component of the magnetic flux density, B_z . In MREIT, small electric currents are injected into the imaged object via electrodes placed on the surface of an object such that these current pulses are synchronized with an MR pulse sequence. A magnetic flux density distribution $\mathbf{B} = (B_x, B_y, B_z)$ will be induced by the injected current. B_z , the component of the magnetic flux density in the direction of the MRI system's main magnetic field $\mathbf{B}_0$, can

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be recovered from MR phase images and processed to calculate conductivity(Seo *et al.* 2003), current density (Nam and Kwon 2008) or electric field images(Kranjc *et al.* 2015). The low frequency electrical property information derived from conductivity contrast images can provide valuable and distinct information from T1, T2 or proton density contrasts (Sadleir *et al.* 2006).

MREIT imaging has typically used spin- or gradient-echo pulse sequences for acquisition of phase data (Kim *et al.* 2007, Woo and Seo 2008, Kwon *et al.* 2014). Spin-echo sequences have been preferred because there is less artifact created by T2* processes. Spin-echo pulse sequences typically encode a single line of k-space per slice in each repetition time (TR), with the number of phase encoding steps (PE) determining the image resolution (Bernstein *et al.* 2004). The number of frequency encoding steps selected within each line of k-space is usually selected to match PE, creating a square image. A typical TR for in-vitro samples may be 300 ms, which results in an image acquisition time of 38 s for a single image with PE = 128. This strategy can be used to produce high quality images of static conductivity distributions. However, it cannot be used to image fast changes in conductivity. Long imaging times are also generally undesirable for *in vivo* applications.

Faster MREIT methods have been proposed. Park *et al.* (Park *et al.* 2011) reconstructed B_z images using partially measured k-space data based on conjugate symmetry properties of k-space. An initial estimate of k-space data and an iteration scheme were required to reconstruct B_z in using this method, and reasonable B_z distributions were reconstructed. Their method resulted in an improvement in acquisition time of almost a factor of two. Echo-planar imaging strategies, where multiple phase and frequency encoding steps are performed within one TR, have been implemented for MREIT and have been used to reconstruct conductivity images after application of corrections for timing and geometric distortions (Chauhan *et al.* 2016, Hamamura and Muftuler 2008). Balanced ssfp methods have also been tested to accelerate MREIT acquisitions (Minhas *et al.* 2009). However, to date, systematic k-space undersampling methods have not been applied to improve MREIT data acquisition times.

One way to accelerate data acquisition is to skip some phase encoding lines. However, this risks violating the Nyquist sampling criterion (Seo and Woo 2013a), and could cause aliasing artifacts (Song *et al.* 2016, Song *et al.* 2017). In this paper, we sought to reconstruct high-quality MREIT data using only half the number of phase encoding steps. Simulated and phantom MREIT data collected using spin-echo sequences were used to validate the methods in this instance. However, we can combine the method in (Park *et al.* 2011) and our method to accelerate the data acquisition by a factor of around four. Further, it may also be possible to combine this method with fast MREIT-EPI measurement strategies (Chauhan *et al.* 2016) to improve data quality by mitigating EPI-related artifacts. Finally, this method could potentially also be combined with parallel acquisition strategies (Larkman and Nunes 2007).

2. Methods

Let the imaged object occupy a three dimensional bounded domain Ω with the conductivity distribution σ . We attach a pair of surface electrodes $\mathfrak{E}^+$ and $\mathfrak{E}^-$ on the boundary Ω and place the object into an MRI scanner. We assume that the z -axis is the direction of the main field of the MR scanner. Without current injection, the measured complex k-space data for each slice imaged $\Omega_{z_0} = \Omega \cap \{z = z_0\}$ is

$$S(k_x, k_y) = \int_{\Omega_{z_0}} M(x, y) e^{i\delta(x, y)} e^{i2\pi(k_x x + k_y y)} dx dy, \quad (1)$$

where $M(x, y) > 0$ is an MR magnitude image on the slice Ω_{z_0} , and δ is any residual phase, referred to as systematic phase artifact. Applying a two dimensional inverse Fourier transform (Seo and Woo 2013a) on both sides of (1), we obtain the following complex MR image of the slice Ω_{z_0}

$$\mathcal{M}(x, y) := M(x, y) e^{i\delta(x, y)}. \quad (2)$$

The data $\mathcal{M}$ in (2) contains information about the distributions of M and δ , which can be computed using the formulae

$$M(x, y) = |\mathcal{M}(x, y)| \quad (3)$$

$$\text{and } \delta(x, y) = \frac{1}{i} \ln \frac{\mathcal{M}(x, y)}{M(x, y)}, \quad (4)$$

respectively. In general, δ may require unwrapping. Now, we are ready to inject currents of I mA during the MREIT data collection process. The injection current induces an internal magnetic flux density $\mathbf{B} = (B_x, B_y, B_z)$ inside the object. The measured complex k-space data corresponding to the injection current I contains B_z data on each slice Ω_{z_0} in the following way:

$$S^I(k_x, k_y) = \int_{\Omega_{z_0}} M(x, y) e^{i[\gamma T_c B_z(x, y) + \delta(x, y)]} e^{i2\pi(k_x x + k_y y)} dx dy, \quad (5)$$

where $\gamma = 26.75 \times 10^7 \text{ rad/T}\cdot\text{s}$ is the gyromagnetic ratio of hydrogen and T_c is the current pulse width in seconds.

Applying a two dimensional inverse Fourier transform on both sides of (5), we obtain

$$\mathcal{M}^I(x, y) := M(x, y)e^{i[\gamma T_c B_z(x, y) + \delta(x, y)]}. \quad (6)$$

Using M and δ obtained from (3) and (4), we obtain that on each slice Ω_{z_0} , a wrapped B_z may be calculated as follows

$$B_z = \frac{1}{i\gamma T_c} \ln \frac{\mathcal{M}^I(x, y)}{M(x, y)e^{i\delta(x, y)}}. \quad (7)$$

We note that the Laplacian of B_z data probes changes of $\ln \sigma$ along the direction of $(0, 0, 1) \times (\nabla \times \mathbf{B})$, whereas B_z can not probe changes of $\ln \sigma$ in the direction $\nabla \times \mathbf{B}$ (Seo and Woo

2014). To probe $(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}) \ln \sigma$ (the full transverse direction), we attach two pairs of electrodes, $\mathcal{E}_1^\pm$ and $\mathcal{E}_2^\pm$, to produce two linearly independent magnetic flux densities $\mathbf{B}_1 = (B_{x,1}, B_{y,1}, B_{z,1})$ and $\mathbf{B}_2 = (B_{x,2}, B_{y,2}, B_{z,2})$. In previous MREIT strategies (Seo and Woo 2014), images of current flow have been obtained via each pair of electrodes ($\mathcal{E}_1^\pm$ or $\mathcal{E}_2^\pm$), twice, with the current flow direction reversed in the second image. Results of current flow in each direction are then complex divided to obtain $B_{z,1}$ or $B_{z,2}$ data that are independent of δ , and that have twice the original amplitude.

It is crucial to observe that the data acquisition time to produce the data B_z can be significantly reduced by using prior knowledge of M and δ , which are independent of current injection. To be precise, if these are known, only one image of undersampled current flow in each direction need be obtained, with the result that only one quarter of the original k-space data is enough to produce the data $B_{z,1}$ and $B_{z,2}$. We explain the details below.

2.1. Generating B_z from partial k-space data

Now, we are ready to explain how to generate B_z by using only half of the k-space data S^I . We assume that phase encoding gradients are applied along the y -direction, and frequency encoding gradients are applied in the x -direction. To reduce the time to produce $B_{z,1}$ or $B_{z,2}$, we sample only every second line of k-space. For notational simplicity, we set the field of

view to 1 and M is defined in $[-\frac{1}{2}, \frac{1}{2}] \times [-\frac{1}{2}, \frac{1}{2}]$. We extend M in the y -direction periodically so that $M(x, y+1) = M(x, y)$. It follows from the Poisson summation formula (Seo and Woo 2013a), that the inverse Fourier transform of this ‘2-skipped’ k-space data, with zero-padding on alternate rows, produces the following 2-folded image:

$$\mathcal{M}_2(x, y) = \frac{1}{2} \sum_{j=0}^1 M(x, y + \frac{j}{2}) e^{i[\nu B_z(x, y + \frac{j}{2}) + \delta(x, y + \frac{j}{2})]} \quad (8)$$

for $(x, y) \in \Omega_{z_0}$, where $\nu = \gamma T_c$. With knowledge of M and $e^{i\delta}$, both of which are independent of the current, B_z can be reconstructed from the corresponding folded data $\mathcal{M}_2$

in (8). Hence, the k-space data time for measuring N lines of B_z data is reduced to around

$$\frac{1}{2N} + \frac{1}{4}.$$

Denoting $\mathcal{T}_{1/2}$ as a translation operator, such that $\mathcal{T}_{1/2}f(x, y) = f(x, y + \frac{1}{2})$, it follows from the formula (8) that B_z satisfies

$$M \cos(\nu B_z + \delta) + \mathcal{T}_{1/2}[M \cos(\nu B_z + \delta)] = 2\Re(\mathcal{M}_2)$$

$$M \sin(\nu B_z + \delta) + \mathcal{T}_{1/2}[M \sin(\nu B_z + \delta)] = 2\Im(\mathcal{M}_2) \quad (9)$$

$$\cos^2(\nu B_z + \delta) + \sin^2(\nu B_z + \delta) = 1,$$

where $\Re(\mathcal{M}_2)$ and $\Im(\mathcal{M}_2)$ are respectively the real and imaginary parts of $\mathcal{M}_2$.

We solve the system (9) in terms of $\cos(\nu B_z + \delta)$ and $\sin(\nu B_z + \delta)$ to obtain

$$\sin(\nu B_z + \delta) = \frac{F\Im(\mathcal{M}_2) \pm \sqrt{[\Re(\mathcal{M}_2)]^2[|\mathcal{M}_2|^2 - F^2]}}{|\mathcal{M}_2|^2}, \quad (10)$$

$$\cos(\nu B_z + \delta) = \frac{F\Re(\mathcal{M}_2) \pm \sqrt{[\Im(\mathcal{M}_2)]^2[|\mathcal{M}_2|^2 - F^2]}}{|\mathcal{M}_2|^2},$$

where

$$F = \frac{4|\mathcal{M}_2|^2 + (M^2 - \mathcal{T}_{1/2}M^2)}{4M}. \quad (11)$$

See Appendix A for the detailed derivation of (10).

From (10),

$$\sin(\nu B_z) = \xi_1^\pm \cos \delta - \xi_2^\pm \sin \delta, \quad (12)$$

where

$$\xi_1^\pm = \frac{F\Im(\mathcal{M}_2) \pm \sqrt{[\Re(\mathcal{M}_2)]^2[|\mathcal{M}_2|^2 - F^2]}}{|\mathcal{M}_2|^2} \quad (13)$$

$$\xi_2^\pm = \frac{F\Re(\mathcal{M}_2) \pm \sqrt{[\Im(\mathcal{M}_2)]^2[|\mathcal{M}_2|^2 - F^2]}}{|\mathcal{M}_2|^2}.$$

Hence, for each point $(x, y) \in \Omega_{\mathcal{M}}$, the true (wrapped) $B_z(x, y)$ should be one of four candidates:

$$B_z(x, y) = B_z^j(x, y) \quad \text{for some } j \in \{a, b, c, d\}, \quad (14)$$

where

$$B_z^a = \frac{1}{\nu} \arcsin[\xi_1^+ \cos \delta - \xi_2^+ \sin \delta],$$

$$B_z^b = \frac{1}{\nu} \arcsin[\xi_1^- \cos \delta - \xi_2^+ \sin \delta], \quad (15)$$

$$B_z^c = \frac{1}{\nu} \arcsin[\xi_1^+ \cos \delta - \xi_2^- \sin \delta],$$

$$B_z^d = \frac{1}{\nu} \arcsin[\xi_1^- \cos \delta - \xi_2^- \sin \delta].$$

Determination of the appropriate B_z can be achieved using the continuity property of B_z (Seo *et al.* 2013b).

To determine B_z from (14), we start by considering the portion of the image outside the region

$$D := \{(x, y) \in \Omega_{\mathcal{M}} : ||\mathcal{M}_2(x, y)|^2 - F^2(x, y)| \neq 0\}. \quad (16)$$

Noting that $|\mathcal{M}_2|^2 - F^2 = 0$ implies $\xi_1^+ = \xi_1^-$ and $\xi_2^+ = \xi_2^-$, it follows from (15) that

$$B_z = B_z^a = B_z^b = B_z^c = B_z^d \quad \text{in } \Omega_{z_0} \setminus D. \quad (17)$$

Now it remains to determine B_z in the region D . From (15), D can be decomposed into four subdomains

$$D = D^a \cup D^b \cup D^c \cup D^d \quad (18)$$

such that

$$B_z = B_z^a \chi_{D^a} + B_z^b \chi_{D^b} + B_z^c \chi_{D^c} + B_z^d \chi_{D^d}, \quad (19)$$

where χ_{D^a} is the characteristic function of D^a .

For the segmentation of D^j ($j \in \{a, b, c, d\}$), we take advantage of the fact that B_z is continuous and the following observation and remark.

Observation 2.1—(i) For each $j \in \{a, b, c, d\}$, there exists a map

$$(\Phi_j, \Psi_j): \Omega_{z_0} \times \Omega_{z_0} \rightarrow \{-2, 0, 2\} \times \{-2, 0, 2\}$$

such that

$$\sin(\nu B_z) = \sin(\nu B_z^j) + \Upsilon_j \quad \text{in } \Omega_{z_0}, \quad (20)$$

where

$$\Upsilon_j = \frac{\Phi_j |\Re(\mathcal{M}_2)| \cos \delta - \Psi_j |\Im(\mathcal{M}_2)| \sin \delta}{|\mathcal{M}_2|^2} \sqrt{\mathcal{T}_{1/2} M^2 - \frac{|\mathcal{M}_2|^2 - (M^2 + \mathcal{T}_{1/2} M^2)}{4M^2}}. \quad (21)$$

(ii) $\sin[\nu B_z(x, y)] = \sin[\nu B_z^j(x, y)]$ if and only if $\Phi_j^2(x, y) + \Psi_j^2(x, y) = 0$.

The proof of this observation is provided in Appendix B.

According to Observation 2.1, either $B_z = B_z^j$ for some $j \in \{a, b, c, d\}$ or the difference between $\sin(\nu B_z)$ and $\sin(\nu B_z^j)$ is a discontinuous function Υ_j . Hence, we can segment D^j using discontinuities in $\sin(\nu B_z^j)$. For the segmentation, see Figure 3 in Section 3.

Remark 2.1—Even if data are wrapped, use of trigonometric transformations as in (9) will ensure continuity of B_z distributions until a final step, after which normal unwrapping

procedures may be applied (Ghiglia and Pritt 1998). Discontinuities in these functions may then be used to identify undersampling artifacts.

In the next section, we provide an algorithm for B_z reconstruction from 2-skipped undersampled k-space data.

2.2. Algorithm for B_z reconstruction from 2-skipped undersampled k-space data

We summarize the procedures in previous sections as the following B_z reconstruction algorithm:

- i. Attach two pairs of electrodes $\mathcal{E}_j^\pm (j=1, 2)$ to the object to be imaged and place it into the MRI scanner.
- ii. Without current injection, measure fully sampled k-space data (1) and calculate M and δ using (3) and (4), respectively.
- iii. Inject a current I_1 into the object through electrodes $\mathcal{E}_1^\pm$ and obtain 2-skipped undersampled k-space data.
- iv. Compute $\mathcal{M}_2$ by performing a two dimensional inverse Fourier transform on the 2-skipped undersampled k-space data obtained in (iii).
- v. Solve equation (9) and compute $B_{z,1}^a, B_{z,1}^b, B_{z,1}^c$ and $B_{z,1}^d$ using (15).
- vi. Segment the domain $D \approx D^\eta$ using the threshold $\eta > 0$, where D^η is defined as

$$D^\eta := \left\{ (x, y) \in \Omega_{z_0} : \sqrt{|\mathcal{M}_2(x, y)|^2 - F^2(x, y)} > \eta \right\}. \quad (22)$$

- vii. For $(x, y) \in \Omega_{z_0} \setminus D$, compute wrapped $B_{z,1}$ from (17).
- viii. Segment D^a, D^b, D^c and D^d and compute wrapped $B_{z,1}$ in D from (19).
- ix. Combine the (wrapped) values from D and $\Omega_{z_0} \setminus D$ to form $B_{z,1}$.
- x. Inject a current I_2 via the electrodes $\mathcal{E}_2^\pm$ and repeat Steps (iii)–(ix) to generate the wrapped data $B_{z,2}$.
- xi. If necessary, perform unwrapping (Ghiglia and Pritt 1998) to obtain $B_{z,1}$ and $B_{z,2}$.

3. Results

3.1. Numerical testing

We carried out a numerical test to validate the performance of the proposed method. We constructed a three dimensional realistic head model as shown in Figure 1, created from T1-weighted structural MRI data. The T1-weighted head data set (1 mm³ isotropic resolution) was segmented into tissue types including white matter, gray matter, blood, CSF, bone, muscle, skin and electrodes, using ScanIP v7.0 (Simpleware, Synopsys Inc., Exeter, UK). The segmented ScanIP model was then meshed using ScanFE (Simpleware, Synopsys Inc.,

Exeter, UK). Finally, the Simpleware mesh was imported into COMSOL 5.0 (COMSOL Inc., Burlington, MA, USA).

Figure 1(a) and (b) show the outer geometry and the internal structure of the model, respectively. In Figure 1(c), we show a schematic of the model with two pairs of electrodes $\mathcal{E}_1^\pm$ and $\mathcal{E}_2^\pm$ attached in order to inject two linearly independent currents $I_j(t)$ ($j = 1, 2$), each having amplitude $\tilde{I}(t) = 2\text{mA}$. Electrodes $\mathcal{E}_1^-$ and $\mathcal{E}_2^-$ are on the right and posterior head respectively. In this simulation, we set δ , the systematic phase artifact of the MRI machine, to be 0.

In terms of tissue relaxation parameters, MR signal strength for the spin echo sequence may be expressed

$$M = M_0(1 - e^{-\text{TR}/T_1})e^{-\text{TE}/T_2} \quad \text{in } \Omega, \quad (23)$$

as where M_0 is proton density, T_1 and T_2 are spin relaxation times and TE is the sequence echo time. Magnitude data for the realistic head phantom were generated using the MR tissue parameters shown in Table 1 (from (Kwon *et al.* 2016) and (MacIntosh and Graham 2013)) and (23) to produce MR magnitude images of the model, shown in the left image of Figure 2. We assumed a field of view (FOV) of $224 \times 224 \text{ mm}^2$, a matrix size of 128×128 , TE = 24 ms and TR = 1000 ms.

Conductivities for the different tissue compartments are shown in the right hand image of Figure 2, and are tabulated in Table 1. Conductivity values were the same as those used in (Meng *et al.* 2013). COMSOL 5.0 was used to compute current flow simulations within the phantom. A current density distribution $\mathbf{J}$ was generated corresponding to current flow from $\mathcal{E}_1^+$ to $\mathcal{E}_1^-$ and $\mathcal{E}_2^+$ to $\mathcal{E}_2^-$. These data were then converted to B_z distributions in a single model slice, at the same resolution as the magnitude image using a Fourier transform implementation of the Biot-Savart law (Seo and Woo 2013a). The resulting B_z images, $B_{z,1}$ and $B_{z,2}$, are shown in Figures 3 and 4(a) respectively.

We added Gaussian random noise with a standard deviation $s = 1/(2\gamma T_c \text{SNR})$ (T_c was assumed to be 24 ms) to the simulated $B_{z,1}$ data (Sadleir *et al.* 2005, Scott *et al.* 1992). Here, SNR is the signal to noise ratio observed in the magnitude image. We set SNR = 300. The resulting images in the focal slice are shown in Figure 3(b). From (5), we obtained the real and imaginary parts of the k-space data $\mathcal{S}(k_x, k_y)$ shown in Figure 3(c) and (d), respectively. We then skipped every second row of k-space data along the k_y direction, using zero-padding on alternate rows. An inverse Fourier transform was then applied to both sides of (5) to obtain the real and imaginary parts of $\mathcal{M}_2$, shown in Figure 3(e) and (f), respectively.

At this point, it was possible to reconstruct $B_{z,1}$ from the data $\mathcal{M}_2$. Since $\delta = 0$ in this simulation, we had $B_z^c = B_z^a$ and $B_z^d = B_z^b$. We therefore computed the two possible B_z candidates B_z^a and B_z^b , shown in Figure 3(h) and (i), respectively.

It remained to recover B_z from B_z^a and B_z^b . We first segmented the subdomain D in (22) by setting $\eta = 10^{-6}$. D is shown in Figure 3(g). Using the proposed algorithm in this case, we segmented two regions, D^a and D^b shown in Figure 3(j) and (k), respectively. The resulting reconstructed image $B_{z,1}$ is shown in Figure 3(l).

We then repeated the same procedures to obtain the image of $B_{z,2}$, corresponding to the current injection from $\mathcal{E}_2^+$ to $\mathcal{E}_2^-$. The corresponding results are shown in Figure 4(b–l).

We confirmed the quality of reconstructed $B_{z,1}$ and $B_{z,2}$ images by also performing conductivity reconstruction using the harmonic B_z algorithm (Seo *et al.* 2003) via CoReHA 2.0 software (Jeon and Woo 2013). Figure 5 shows the reconstruction results and profile. To quantitatively evaluate reconstructed B_z images, we defined relative error (RE) measures for reconstructed B_z and σ as

$$RE_{B_z} = \frac{\|B_z^r - B_z^f\|_{L^2(\Omega_{z_0})}}{\|B_z^f\|_{L^2(\Omega_{z_0})}} \text{ and} \quad (24)$$

$$RE_{\sigma} = \frac{\|\sigma^r - \sigma^f\|_{L^2(\Omega_{z_0})}}{\|\sigma^f\|_{L^2(\Omega_{z_0})}}, \quad (25)$$

respectively. Here, B_z^f , B_z^r , σ^f and σ^r denote the image of B_z from fully sampled data (satisfying the Nyquist criterion) and 2-skipped undersampled k-space data, and conductivity images reconstructed using fully- and undersampled B_z images, respectively. In this experiment, we found $RE_{B_{z,1}} = 32\%$, $RE_{B_{z,2}} = 17\%$, and $RE_{\sigma} = 4\%$. Note that the low value for RE_{σ} was due to the fact that the error added to B_z data was Gaussian random noise, which produces little effect on conductivity images reconstructed using the harmonic B_z algorithm (Song *et al.* 2013).

3.2. Phantom experiment

To further evaluate the proposed method, we used a cylindrical tissue conductivity phantom. The phantom measured 45 mm in diameter and 42 mm in height. The phantom was filled with approximately 0.71 S/m conductivity agarose solution (3 g/L NaCl, 40g/L Agarose, and 1 g/L CuSO₄). To generate conductivity contrast, two objects (potato and chicken breast pieces) were placed inside the phantom. Four carbon-hydrogel electrodes (HUREV Co. Ltd, South Korea) were attached to the perimeter of the phantom, as shown in Figure 6(a).

Transverse images of the tissue phantom were acquired using a 7T MRI system (Bruker Biospin MRI, Billerica, MA), located at the Barrow Neurological Institute, Phoenix, AZ, USA. Spin echo sequences were used for MREIT imaging. The pulse sequence is shown in Figure 6(b). MREIT current injection pulses were synchronized with the MR pulse sequence via spectrometer-generated TTL control pulses. Currents were injected using a custom-

designed MREIT current source (Oh *et al.* 2006 2006) at 10mA amplitude and 11ms injection time (T_c). Separate images of currents $I_1^\pm$ and $I_2^\pm$ were acquired using two orthogonal pairs of electrodes. Scan parameters for spin echo imaging procedures were: FOV = 80×80 mm², matrix size = 128×128, TR/TE = 1000/24 ms, slice thickness = 3mm, number of averages = 2, T_c = 11ms. The total scan time for one current direction was 256 s.

From data acquired without current injection, we obtained the images of M and $e^{i\delta}$ from (3) and (4). Figure 7(a) shows the M image of a central phantom plane. Figure 7(b and c) show $\cos(\delta)$ and $\sin(\delta)$ respectively in the same plane.

Next, we injected current horizontally, to obtain half of the k-space data, as shown in Figure 8.

Note that in real experiments there may be a large error in reconstructed B_z values near the boundary, a portion $\mathcal{C} \subset D$, due to the approximation of M from (3). Such an error can be monopole in type (Song *et al.* 2013) and this would propagate through images more than random noise. Hence, it may be necessary to apply an algorithm to block the propagation of monopole type errors. Therefore, to block monopole error propagation in phantom data, we first detected the boundary of the phantom, the curve $\mathcal{C} \subset D$, and set

$$B_z = \frac{1}{|\mathcal{C}_h|} \int_{\mathcal{C}_h} B_z(x, y) dx dy, \quad \text{on } \mathcal{C}, \quad (26)$$

where $\mathcal{C}_h$ is the tubular neighborhood of $\mathcal{C}$ with width $2h$. Further details of the error blocking algorithm may be found in (Song *et al.* 2013).

Using the 2-skipped undersampled k-space data and the method described in (Song *et al.* 2013), we obtained the reconstructed $B_{z,1}$ shown in Figure 8. Finally we injected current vertically and repeated the process. The reconstructed $B_{z,2}$ is shown in Figure 9. The relative errors of the reconstructed $B_{z,1}$ and $B_{z,2}$ compared to fully sampled data were 19% and 24% respectively, (Table 2).

Figure 10 illustrates the results of the reconstructed conductivity with the data sets $B_{z,1}$ and $B_{z,2}$ using fully sampled (a) and undersampled data (b). Figure 10(c) compares reconstructed conductivity profiles along the red line overlaid on Figure 10(a). The relative error of the reconstructed conductivity compared to fully sampled data was around 30%. In this case, while monopole type errors were controlled with method of (Song *et al.* 2013), there was still some propagation of errors to the reconstructed conductivity distribution. Artifacts near anomaly edges were observed in the reconstructed conductivity from undersampled B_z data, and are discussed below.

4. Discussion and Conclusion

The B_z and conductivity reconstructions demonstrated here showed a fairly faithful correspondence to fully sampled data, with errors in conductivities reconstructed from 2-skipped undersampled B_z data of less than 5% in simulations, and around 30% in tissue

phantom data. We believe that to some extent, artifacts in phantom reconstructions were caused by the composition of the phantom changing after no current magnitude data were collected. Inspection of magnitude data gathered before and after current administration showed that there had been changes in the neighborhood of the electrodes, most likely because of bubbles formed during passage of 10 mA currents. The changes occurred about both pairs of electrodes, but the artifact was accentuated in the vertical direction because this was the phase encode direction. Acquisition of the M and δ data after current administration may therefore result in improved reconstruction quality. This factor will probably not be as important where smaller currents are applied. These residual wrapping artifacts may be avoided by increasing the field of view such that foldover artifacts do not overlap with the main image, but this will reduce overall image resolution (or increase noise levels, if resolution is maintained). We note that some residual wrapping artifacts were also present in phantom reconstructions with added noise (Figure 3(l), 5(b)). We found that relative errors in reconstructed B_z data did not depend monotonically on noise alone. We determined that errors in reconstruction were also caused by errors in the discontinuity-based segmentation process. If the data reconstructed in Section 3.1 were segmented exactly (using *a priori* information), relative errors in $B_{z,1}$ and $B_{z,2}$ reduced to $4.98 \times 10^{-5}\%$ and $9.83 \times 10^{-5}\%$ respectively. Future implementations of this algorithm should use a more complex, automatic segmentation method. Overall, this technique assumes that recovered B_z data are continuous, a property that may be violated in low MR signal regions or where noise is non-uniform. Therefore, future work should investigate if the algorithm performance is improved by applying MREIT denoising, smoothing, and in-painting algorithms (Lee *et al.* 2005, Lee *et al.* 2006, Lee *et al.* 2011).

In this paper we proposed and demonstrated a novel data acquisition method for MREIT that potentially reduces acquisition time, independent of pulse sequence, by a factor just under four. The method may possibly be useful in MREIT applications to follow fast conductivity changes, such as those occurring during irreversible electroporation (IRE) (Chauhan *et al.* 2016, Kranjc *et al.* 2015), or neural activity (Sadleir *et al.* 2010). Further, because it involves measurement of full magnitude and systematic phase distortion distributions before current is applied, the method provides a means to reduce imaging current dose. It may also be possible to combine this method with parallel imaging acquisition and reconstruction by dividing k-space acquisition between multiple coils. Undersampling and reconstruction of B_z data using the methods described here may potentially mitigate some sensitivities causing EPI artifact. For example, sampling fewer lines of k-space per TR may enable use of lower bandwidth (leading to improved SNR), or, if bandwidth is maintained, shorter echo trains, and may lead to less susceptibility artifact or sensitivity to poor main magnetic field shimming (Bernstein *et al.* 2004). Undersampling may also allow time to employ alternative gradient pulse patterns (Duerk and Simonetti 1991) that create less burden for EPI correction techniques. Future studies should be conducted to establish if this approach is useful in vivo compared in the presence of other EPI artifact sources.

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Appendix A

Proof of (10)

Proof

From the first and the second equations of (9) we have

$$\mathcal{T}_{1/2} \cos(\nu B_z + \delta) = \frac{2\Re(\mathcal{M}_2) - M \cos(\nu B_z + \delta)}{\mathcal{T}_{1/2} M}, \quad \text{and} \quad (\text{A.1})$$

$$\mathcal{T}_{1/2} \sin(\nu B_z + \delta) = \frac{2\Im(\mathcal{M}_2) - M \sin(\nu B_z + \delta)}{\mathcal{T}_{1/2} M} \quad \text{in } \Omega_{z_0}.$$

Since

$$\mathcal{T}_{1/2} \cos^2(\nu B_z + \delta) + \mathcal{T}_{1/2} \sin^2(\nu B_z + \delta) = 1,$$

from the third equation of (9) we obtain

$$\Re(\mathcal{M}_2) \cos(\nu B_z + \delta) + \Im(\mathcal{M}_2) \sin(\nu B_z + \delta) = F. \quad (\text{A.2})$$

Substitute (A.2) into the third equation of (9), we obtain

$$|\mathcal{M}_2|^2 \cos^2(\nu B_z + \delta) - 2F\Re(\mathcal{M}_2) \cos(\nu B_z + \delta) + F^2 - [\Im(\mathcal{M}_2)]^2 = 0 \quad (\text{A.3})$$

and

$$|\mathcal{M}_2|^2 \sin^2(\nu B_z + \delta) - 2F\Im(\mathcal{M}_2) \sin(\nu B_z + \delta) + F^2 - [\Re(\mathcal{M}_2)]^2 = 0. \quad (\text{A.4})$$

Solving the equations (A.3) and (A.4) in terms of $\cos(\nu B_z + \delta)$ and $\sin(\nu B_z + \delta)$, respectively, we obtain (10).

Appendix B

Proof of Observation 2.1

Proof

(i) We assume that $B_z = B_z^{j^*}$, where j^* is a map from Ω_{z_0} to $\{a, b, c, d\}$. Then for $j \in \{a, b, c, d\} \setminus \{j^*\}$, from (15), we have

$$\sin(\nu B_z^{j^*}) - \sin(\nu B_z^j) = \frac{\alpha \Re(\mathcal{M}_2) \cos \delta - \beta \Im(\mathcal{M}_2) \sin \delta}{|\mathcal{M}_2|^2} \sqrt{|\mathcal{M}_2|^2 - F^2}.$$

For the proof of (i), it is sufficient to show that

$$|\mathcal{M}_2|^2 - F^2 = \mathcal{T}_{1/2} M^2 - \frac{|\mathcal{M}_2|^2 - (M^2 + \mathcal{T}_{1/2} M^2)}{4M^2}. \quad (\text{B.1})$$

in Ω_{z_0} . Indeed, from (9), we get

$$\begin{aligned} 4|\mathcal{M}_2|^2 &= \{M \cos(\nu B_z + \delta) + \mathcal{T}_{1/2} [M \cos(\nu B_z + \delta)]\}^2 \\ &\quad + \{M \sin(\nu B_z + \delta) + \mathcal{T}_{1/2} [M \sin(\nu B_z + \delta)]\}^2 \\ &= M^2 + \mathcal{T}_{1/2} M^2 + 2M \mathcal{T}_{1/2} M \cos[\nu B_z + \delta - \mathcal{T}_{1/2}(\nu B_z + \delta)]. \end{aligned}$$

From the above identity and (11), we obtain

$$4MF = |\mathcal{M}_2|^2 + [M^2 - \mathcal{T}_{1/2} M^2] \quad \text{in } \Omega_{z_0}.$$

The above two identities yield that

$$2F = M + \mathcal{T}_{1/2} M \cos[\nu B_z + \delta - \mathcal{T}_{1/2}(\nu B_z + \delta)].$$

Hence,

$$4[|\mathcal{M}_2|^2 - F^2] = \mathcal{T}_{1/2} M^2 \sin^2[\nu B_z + \delta - \mathcal{T}_{1/2}(\nu B_z + \delta)].$$

This completes the proof of (i).

(ii) is a direct conclusion of (i).

References

Bernstein, MA., King, KF., Zhou, XJ. Handbook of MRI Pulse Sequences. London, UK: Academic Press; 2004.

- Chauhan M, Vidya Shankar R, Ashok Kumar N, Kodibagkar V, Sadleir RJ. Multi-shot echo-planar MREIT for fast imaging of conductivity, current density and electric field distributions. *Magnetic Resonance in Medicine*. 2018; 79:71–82. [PubMed: 28205251]
- Duerk JL, Simonetti OE. Theoretical aspects of motion sensitivity and compensation in echo-planar imaging. *J. Magn. Reson. Imaging*. 1991; 1:643–50. [PubMed: 1823169]
- Ghiglia, DC., Pritt, MD. Two dimensional phase unwrapping theory, algorithms, and software. New York: John Wiley & Sons; 1998.
- Hamamura MJ, Muftuler LT. Fast imaging for magnetic resonance electrical impedance tomography. *Magn. Reson. Imaging*. 2008; 26:739–45. [PubMed: 18499381]
- Jeon K, Woo EJ. CoReHA 2.0: A software package for in vivo MREIT experiments. *Comput. Math. Method. Med*. 2013; 2013:8.
- Joy M, Scott G, Henkelman M. In-vivo detection of applied electric currents by magnetic resonance imaging. *Mag. Res. Imag*. 1989; 7:89–94.
- Kim HJ, Lee BI, Cho Y, Kim YT, Kang BT, Park HM, Lee SY, Seo JK, Woo EJ. Conductivity imaging of canine brain using a 3T MREIT system: postmortem experiments. *Physiol. Meas*. 2007; 28:1341–53. [PubMed: 17978419]
- Kranjc M, Markelc B, Bajd F, Caemazar M, Sersa I, Blagus T, Miklavcic D. In situ monitoring of electric field distribution in mouse tumor during electroporation. *Radiology*. 2015; 274:115–23. [PubMed: 25144647]
- Kwon OI, Chauhan M, Kim HJ, Jeong WC, Wi H, Oh TI, Woo EJ. Fast conductivity imaging in magnetic resonance electrical impedance tomography (MREIT) for RF ablation monitoring. *Int. J. Hyperther*. 2014; 30:447–55.
- Kwon OI, Sajib SZK, Sersa I, Oh TI, Jeong WC, Kim HJ, Woo EJ. Current density imaging during transcranial direct current stimulation using DT-MRI and MREIT: algorithm development and numerical simulations. *IEEE Trans. Biomed. Eng*. 2016; 64:168–75.
- Larkman DJ, Nunes RG. Parallel magnetic resonance imaging. *Phys. Med. Biol*. 2007; 52:R15–R55. [PubMed: 17374908]
- Lee BI, Lee SH, Kim TS, Kwon O, Woo EJ, Seo JK. Harmonic decomposition in PDE-based denoising technique for magnetic resonance electrical impedance tomography. *IEEE Trans. Biomed. Eng*. 2005; 52:1912–20. [PubMed: 16285395]
- Lee SH, Seo JK, Park C, Lee BI, Woo EJ, Lee SY, Kwon O, Hahn J. Conductivity image reconstruction from defective data in MREIT: Numerical simulation and animal experiment. *IEEE Trans. Med. Imag*. 2006; 25:168–76.
- Lee CO, Jeon K, Ahn S, Kim HJ, Woo EJ. Ramp-preserving denoising for conductivity image reconstruction in magnetic resonance electrical impedance tomography. *IEEE Trans. Biomed. Eng*. 2011; 58:2038–50. [PubMed: 21693387]
- MacIntosh BJ, Graham SJ. Magnetic resonance imaging to visualize stroke and characterize stroke recovery: a review. *Front Neurol*. 2013; 4:60. [PubMed: 23750149]
- Meng ZJ, Sajib SZK, Chauhan M, Sadleir RJ, Kim HJ, Kwon OI, Woo EJ. Numerical simulations of MREIT conductivity for brain tumor detection. *Comput. Math. Method. Med*. 2013; 2013:10.
- Minhas, A., Woo, EJ., Lee, SY. Magnetic flux density measurement with balanced steady state free precession pulse sequence for MREIT: a simulation study; Proceedings 31st Annual International Conference of the IEEE EMBS; 2009. p. 2276-8.
- Nam HS, Kwon OI. Non-iterative conductivity reconstruction algorithm using projected current density in MREIT. *Phys. Med. Biol*. 2008; 53:6947–61. [PubMed: 19001695]
- Oh TI, Cho Y, Hwang YK, Oh SH, Woo EJ, Lee SY. Improved current source design to measure induced magnetic flux density distributions in MREIT 2006. *Journal of Biomedical Engineering Research*. 2006; 27:30–7.
- Park HM, Nam HS, Kwon OI. Magnetic flux density reconstruction using interleaved partial Fourier acquisitions in MREIT. *Phys. Med. Biol*. 2011; 56:2059–73. [PubMed: 21386139]
- Sadleir R, Grant S, Zhang SU, Lee BI, Pyo HC, Oh SH, Park C, Woo EJ, Lee SY, Kwon O, Seo JK. Noise analysis in MREIT at 3 and 11 Tesla field strength. *Physiol. Meas*. 2005; 26:875–84. [PubMed: 16088075]

- Sadleir R, Grant S, Zhang SU, Oh SH, Lee BI, Woo EJ. High field MREIT: setup and tissue phantom imaging at 11 T. *Physiol. Meas.* 2006; 27:S261–S70. [PubMed: 16636417]
- Sadleir RJ, Grant SC, Woo EJ. Can high-field MREIT be used to directly detect neural activity? Theoretical considerations. *NeuroImage*. 2010; 52:205–16. [PubMed: 20382240]
- Scott GC, Joy MLG, Armstrong RL, Henkelman RM. Measurement of nonuniform current density by magnetic resonance. *IEEE Trans. Med. Imag.* 1991; 10:362–374.
- Scott GC, Joy MLG, Armstrong RL, Henkelman RM. Sensitivity of magnetic-resonance current-density imaging. *J. Mag. Res.* 1992; 97:235–254.
- Seo JK, Yoon JR, Woo EJ, Kwon O. Reconstruction of conductivity and current density images using only one component of magnetic field measurements. *IEEE Trans. Biomed. Eng.* 2003; 50:1121–4. [PubMed: 12943280]
- Seo JK, Woo EJ. Magnetic resonance electrical impedance tomography (MREIT). *SIAM Rev.* 2011; 53:40–68.
- Seo, JK., Woo, EJ. *Nonlinear Inverse Problems in Imaging*. Chichester: John Wiley & Sons; 2013a.
- Seo, JK., Woo, EJ., Katcher, U., Wang, Y. *Electro-Magnetic Tissue Properties MRI*. London: Imperial College Press; 2013b.
- Seo JK, Woo EJ. Electrical tissue property imaging at low frequency using MREIT. *IEEE Trans. Biomed. Eng.* 2014; 61:1390–9. [PubMed: 24759274]
- Song Y, Kwon H, Jeon K, Jung YM, Seo JK, Woo EJ. Analysis and blocking of error propagation by region-dependent noisy data in MREIT. *SIAM J. Sci. Comput.* 2013; 35:B912–B924.
- Song Y, Jeong WC, Woo EJ, Seo JK. A method for MREIT-based source imaging: simulation studies. *Phys. Med. Biol.* 2016; 61:5706–23. [PubMed: 27401235]
- Song Y, Ammari H, Seo JK. Fast magnetic resonance electrical impedance tomography with highly undersampled data. *SIAM J. Imag. Sci.* 2017; 10:558–77.
- Woo EJ, Seo JK. Magnetic resonance electrical impedance tomography (MREIT) for high-resolution conductivity imaging. *Physiol. Meas.* 2008; 29:R1–26. [PubMed: 18799834]

Figure 1. Human brain model constructed using COMSOL 5.0, showing (a) the outer geometry of the model; (b) cutaway diagram showing internal model structures; (c) pairs of electrodes $\mathcal{E}_1^\pm$ and $\mathcal{E}_2^\pm$, attached to the object to allow injection of two linearly independent currents.

Figure 2.

Magnitude and conductivity image of computational phantom. (a) MR magnitude distribution M ; (b) conductivity distribution σ in the slice Ω_{20} . The image M was estimated for a 3T MR system from the formula (23), using T1 and T2 values from Table I.

Figure 3.

Results of the reconstructions for current injected through $\mathcal{E}_1^\pm$. (a) is the image of the $B_{z,1}$ image from Biot-Savart law for current flow from $\mathcal{E}_1^+$ to $\mathcal{E}_1^-$; (b) is the $B_{z,1}$ image with noise; (c) and (d) are, respectively, $\Re(S^f)$, $\Im(S^f)$; (e) and (f) are, respectively, $\Re(\mathcal{M}_2)$ and $\Im(\mathcal{M}_2)$; (g) is D^η by choosing $\eta = 10^{-6}$; (h) and (i) are, respectively, the $\sin(\nu B_{z,1}^a)$ and $\sin(\nu B_{z,1}^b)$; (j) and (k) are, respectively, D^a and D^b ; (l) is the reconstructed $B_{z,1}$ using the proposed method.

Figure 4.

Results of reconstruction when the current was injected through $\mathcal{E}_2^{\pm}$. The meaning of each image in this figure is the same as in Figure 3.

Figure 5.

Reconstructed conductivity images and profiles. Data were reconstructed as relative conductivity images, with the background conductivity assumed to be 1 S/m. (a) conductivity distribution reconstructed from fully sampled $B_{z,1}$ and $B_{z,2}$ data; (b) conductivity reconstructed from 2-skipped undersampled $B_{z,1}$ and $B_{z,2}$ using the proposed algorithm; (c) profiles of the conductivity images (shown along the dotted red line shown in the left image) corresponding to fully sampled (red) and undersampled (blue) data.

(a)

(b)

Figure 6. Photograph of phantom and pulse sequence used in the phantom experiment. (a) Phantom object containing agar background with included potato and chicken anomalies. (b) Spin-echo pulse sequence.

Figure 7. Distribution of M and δ obtained from (3) and (4) without current injection. (a) M , (b) $\cos(\delta)$ and (c) $\sin(\delta)$.

Figure 8.

Results of the phantom experiment with horizontally injected current. Parts (a) and (b) show, respectively, images of real and imaginary parts of the 2-skipped undersampled k-space data; (c) and (d) are, respectively, the real and imaginary parts of the $\mathcal{M}_2$; (e) is the segmented D for a given η ; (f), (g), (h) and (i) are, respectively, the segmented D^a , D^b , D^c and D^d ; (j) is the image of $\mathcal{C}$; (k), (l), (m) and (n) are, respectively, the images of $\sin(\nu B_z^a)$, $\sin(\nu B_z^b)$, $\sin(\nu B_z^c)$ and $\sin(\nu B_z^d)$; (o) is the reconstructed B_z using the proposed method.

Figure 9. Results of the phantom experiment when the current is injected vertically. The meaning of each image in this figure is the same as those in Figure 8.

Figure 10.

Reconstructed conductivity images and profiles for phantom experiment. As in Figure 5, (a) shows the conductivity reconstructed from fully sampled data, (b) shows reconstructed conductivity from 2-skipped undersampled data and (c) compares conductivity profiles for each method along the red dotted line overlaid on (a).

Table 1

Values of M_0 , $T1$, $T2$ and σ values for model tissues at 3 T, showing sources

Tissue	M_0	$T1(ms)$	$T2(ms)$	$\sigma(S/m)$	Source
Skin	0.1	371	133	0.24	(Kwon <i>et al.</i> 2016), (Meng <i>et al.</i> 2013)
White matter	0.72	1060	70	0.06	(MacIntosh and Graham 2013), (Meng <i>et al.</i> 2013)
Gray matter	0.69	1820	100	0.09	(MacIntosh and Graham 2013), (Meng <i>et al.</i> 2013)
CSF	1	3817	1442	1.8	(MacIntosh and Graham 2013), (Meng <i>et al.</i> 2013)
Skull	0.01	1240	37	0.041	(Kwon <i>et al.</i> 2016)

Table 2

Comparison of Errors in B_z and reconstructed conductivities for undersampled data compared relative to fully sampled data, for both model and phantom objects.

Object	RE		
	$B_{z,1}$	$B_{z,2}$	σ
Model	32	17	4
Phantom	19	24	30