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Fully-coupled aeroelastic simulation with fluid compressibility — For application to vocal fold vibration

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Abstract

In this study, a fully-coupled fluid–structure interaction model is developed for studying dynamic interactions between compressible fluid and aeroelastic structures. The technique is built based on the modified Immersed Finite Element Method (mIFEM), a robust numerical technique to simulate fluid–structure interactions that has capabilities to simulate high Reynolds number flows and handles large density disparities between the fluid and the solid. For accurate assessment of this intricate dynamic process between compressible fluid, such as air and aeroelastic structures, we included in the model the fluid compressibility in an isentropic process and a solid contact model. The accuracy of the compressible fluid solver is verified by examining acoustic wave propagations in a closed and an open duct, respectively. The fully-coupled fluid–structure interaction model is then used to simulate and analyze vocal folds vibrations using compressible air interacting with vocal folds that are represented as layered viscoelastic structures. Using physiological geometric and parametric setup, we are able to obtain a self-sustained vocal fold vibration with a constant inflow pressure. Parametric studies are also performed to study the effects of lung pressure and vocal fold tissue stiffness in vocal folds vibrations. All the case studies produce expected airflow behavior and a sustained vibration, which provide verification and confidence in our future studies of realistic acoustical studies of the phonation process.

Keywords

Fluid–structure interaction; Compressible fluid; Fully-coupled; Acoustic; Vocal fold vibration; Self-sustained vibration

1. Introduction

Fluid–structure interactions occur in many engineering applications and systems, as well as innate bio-mechanical functions in a human body. The self-sustained vocal folds vibration is such an application that intrinsically involves fluid (air) and structure (vocal folds) interactions. The vocal folds are a pair of flexible structures at the top of the trachea, where air travels through in the larynx and produces sounds due to its fluctuations.

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Over the past two decades, a number of numerical models are built to facilitate the understanding of the self-sustained vocal fold vibration, which span from multi-mass models where the vocal folds are modeled as mass–spring–damper system [1–4] to continuum models where the vocal folds are modeled using realistic physical representation. With the dramatic increase in computational capabilities and efficiencies in the recent years, more continuum vocal fold models have been developed using 2-D and 3-D finite elements, some of which are coupled with Bernoulli’s equation for fluid to study the mechanisms behind the self-sustained oscillation process [5–9].

These fluid–structure interaction continuum models can typically be categorized into two groups: one-way coupling where one field provides the solution information to another field and repeated over time cycles, and two-way coupling or fully-coupled fluid–structure interaction models where the fluid feeds the information to the solid and the solid feeds information back to the fluid, be it explicit or implicit in time. In general, one-way coupling between the fluid and the solid is where the motion and the deformation of the solid are only determined by the fluid pressure acting on the fluid–structure interface and the solid does not provide any feedback to the fluid domain. The one-way coupling approach includes controlled/manipulated solid motion by some pre-designed spatial–temporal function and superimposed solutions of the fluid and solid [10,11]. It is well known that one-way coupling often leads to divergence in the system solution or converging to an unphysical solution.

On the contrary, a fully-coupled fluid–structure system typically yields more stable and more realistic solution comparing to that of one-way coupling. The Immersed Boundary (IB) method is a numerical technique that involves two-way coupling between the fluid and the solid. A first attempt to use the IB method to simulate the airflow interacting with vocal folds using a 2-D model was done by Luo et al. [10]. In this work, the fluid was incompressible and viscous while the vocal folds were modeled with finite difference method with appropriate visco-elastic material properties. The coupling between the two domains was done via a “sharp interface” methodology, where the solid was driven by the traction boundary condition interpolated from the previous time step and solved as a quasi-steady state. The solid was then updated to a new position and a corresponding velocity (interior and boundary) field was obtained. The velocity at the solid boundary was then interpolated back to the fluid domain and applied as the “boundary” condition for the fluid. This “boundary” is not the boundary of the fluid domain, but rather, the fluid–structure interface inside the fluid domain. Using the IB method, Luo et al. [12] and Zheng et al. [13] further studied the effects of false vocal folds on glottal flow and vocal folds vibration. An improved solid model was also presented in [12], where the viscoelastic solid dynamics was solved using finite element method rather than finite difference as they did in [10], which provided a better and an easier way of applying solid boundary conditions, especially for complex geometries. A contact model was also included instead of their previous model where the vocal folds displacement was manipulated to avoid overlapping onto each other. More advanced work had also been done by Mittal’s group on representing the acoustic field by performing hydrodynamic/acoustic splitting method to model linearized perturbed compressible fluid equation [11]. However, this acoustic field was not coupled with a dynamic vocal fold model.

Despite the challenges, the fully-coupled approach to study vocal folds vibration has significantly advanced the capabilities in representing a feasible and realistic glottal system. As of date, there remains a major deficiency in all the sophisticated models to study the full interactions between aeroelasticity and aeroacoustics — they ignored the compressibility of air. The inclusion of compressibility in a fully-coupled fluid–structure glottal system is vitally important since the sound wave generation and propagation during vocal folds vibration are induced and facilitated by the compression of air. In fact, our initial attempt in using incompressible fluid for the coupled system resulted in reasonable magnitude of the vocal folds vibrations, but ultimately failed to reach a *sustained* vibration. Without a steady-state vibration, it is futile to perform any analysis on the results to understand the fundamental mechanism of sound production.

In this work, we design a new numerical tool that encompasses the compressibility of the fluid, its duly interactions with aeroelastic materials, as well as other features that are necessary to successfully simulate the laryngeal system. It is built based on a fully-coupled fluid–structure interaction algorithm, the modified immersed finite element method (mIFEM) [14]. The major distinction between the class of immersed finite element methods and that of the IB methods is that it utilizes the concept of principle of virtual work to remove the artificial effects from the overlapping background fluid, thus providing an independent and accurate solid material description. Both fluid and solid solvers use finite element approach where non-uniform and unstructured meshes can be used to handle complex geometries.

The versatility of the mIFEM provides us the perfect tool in modeling the vocal folds vibration. In this study, the dynamics of the vocal folds motion and deformation are governed by visco-elastic constitutive law and solved on a Lagrangian mesh using the finite element method, while the fluid is governed by Navier–Stokes equations describing compressible isentropic air that are represented using an Eulerian background mesh. We further utilize the numerical tool to perform a series of parametric studies using a range of lung pressure and vocal folds stiffness to evaluate the effects in the fundamental frequency and magnitude of the self-sustained vibration.

In this paper, we will first present the numerical method formulated for compressible fluid interacting with aeroelastic structures in Section 2. The solid model also includes a contact model. The 2-D vocal folds model is presented in Section 3. With a successful mesh convergence study, we first verify the vocal folds vibration in terms of vibration frequency and magnitude. A parametric study is conducted to further analyze the effect of lung pressure and vocal folds stiffness. Finally, the conclusions are drawn in Section 4.

2. Numerical method

2.1. Basic concept of the immersed finite element methods

The immersed finite element method (IFEM) is a volume-based finite element fluid–structure interaction numerical method. Finite elements are not only used as a way of discretization, but the entire IFEM formulation is derived from the weak form, or principle of virtual work, where the total work in the entire fluid–structure coupled system is

maintained. Even though the concept of “immersed” is similar to that of the immersed boundary (IB) method [15] where the solid is immersed in the fluid and the two domains are constructed independently, the major distinction of the two is that the solid is not just an interface as it is in the IB method, it actually occupies a volume in the computational domain. As such, the formulation for the IFEM must be volume-based, rather than point-based in the IB method. Similarly, the coupling scheme is derived through the balance of the total work in the entire computational space rather than a direct elastic force evaluation on each discrete point.

In IFEM, the fluid and solid domains are defined on two sets of meshes independently and governed by their own governing equations. The solid domain $\mathcal{Q}^s$ is completely immersed in its surrounding fluid $\mathcal{Q}^f$, depicted in Fig. 1. Since the solid domain is immersed in the fluid domain, the solid boundary Γ^s is also the common interface intersecting with the fluid domain, called the fluid–structure interface $\Gamma^{\text{FSI}} \equiv \Gamma^s$.

The fluid domain and the solid domain together form the entire computational domain, $\mathcal{Q} = \mathcal{Q}^f \cup \mathcal{Q}^s$. All variables associated with the solid are denoted with superscript ‘s’, while all of those associated with the fluid are denoted with superscript ‘f’. The key assumption of the IFEM algorithm is that the fluid exists everywhere in the entire computation domain, similar to the IB method. However, unlike the connected discrete points in representing the solid boundary in IB, the solid in IFEM occupies volume in the computational space. The part of the fluid that overlaps with the solid domain is called the ‘artificial’ fluid, $\tilde{\mathcal{Q}}$, since it does not physically exist. Note that by definition and implementation, the ‘artificial’ fluid domain $\tilde{\mathcal{Q}}$ and the solid domain $\mathcal{Q}^s$ do occupy the same space. The solid domain gets updated at every time step from its initial configuration $\mathcal{Q}_0^s$. As a result, the artificial domain also moves in the entire computational space. In fact, it is very important to understand that to incorporate the dynamics of the solid into the coupled system, the inertial force from the acceleration must be included. In the original IB method, the dynamics of the boundary is not considered, but rather, is carried by the dynamics of the fluid. The solid boundary has simply been solved as a quasi-static deformation.

The original IFEM [16–18] has been widely used to investigate various biomedical problems [19–23]. However, it is not the method of choice here due to its limitation in handling fluid and solid having large density differences and large Reynolds number flows as it is the case in the laryngeal flow simulation. In the modified IFEM (mIFEM), the algorithm is entirely re-formulated to accommodate for large Reynolds number flows or where the solid inertia is dominant in fluid–structure interactions, and when the solid and fluid have disparate density difference such as that between soft tissue and air. The mIFEM addresses these problems by re-formulating the coupling scheme and solving for the solid dynamics rather than imposing it. A more detailed derivation of the algorithm can be found in [14]. It has been rigorously derived and verified with 2-D and 3-D studies [24,14,25,26].

Using the concept of the mIFEM, we further incorporated the compressibility so that air can be appropriately presented as sound is produced by the compression of air, which is a necessary step to study the interactions between air and aeroelastic structures.

2.2. Compressible fluid solver for isentropic air

In this section, the compressible fluid with isentropic process is presented from its strong form to the weak form, along with a validation example of the implementation for the fluid solver. All the variables are pertinent to the fluid, unless specifically noted. Therefore, all the superscripts ‘f’ for the fluid variables are omitted temporarily in this section. In this laryngeal flow simulation, the fluid medium is fully presented for acoustic waves traveling through the larynx.

2.2.1. Isentropicity of air—An isentropic process is a reversible adiabatic process where the entropy S is constant. In normal acoustic situations air compression/expansion is much closer to isentropic than other processes such as isothermal process. When air is compressed by shrinking its volume V , for example, not only the pressure p increases, the temperature T increases as well, according to the ideal gas law $pV = nRT$ or $p = \rho RT$ where p is the absolute pressure. However, in a constant-entropy compression/expansion, temperature changes are not given time to diffuse away to thermal equilibrium. Heat diffusion occurs much slower than acoustic vibrations. Therefore, temperature field does not play a major role in the governing equation. The energy equation involving T does not need to be coupled with continuity and momentum equations. The pressure and density fluctuations from its equilibrium pressure and density, p_0 and ρ_0 are $p' = p - p_0$ and $\rho' = \rho - \rho_0$. For nearly incompressible flows with low Mach number, we can assume p' and ρ' are much smaller than the total pressure and the initial density, i.e. $p'/p \ll 1$ and $\rho'/\rho_0 \ll 1$.

Isentropic process further requires that $\left. \frac{\partial p}{\partial \rho} \right|_S = \frac{1}{c^2}$ where c is the speed of sound. With the nearly incompressibility assumption, it can be deduced to $p' = c^2 \rho'$. The isentropic relationship for air can then be written as:

$$p - p_0 = (\rho - \rho_0) c^2. \quad (1)$$

For dry air, the adiabatic index or isentropic expansion factor is $\gamma = 1.4$ and the specific gas constant is $R = 287.06 \text{ J/(kg K)}$. The temperature is chosen as a constant value of 273 K. The speed of sound c or the speed of the pressure wave in a fluid is dependent on the bulk modulus of the fluid, κ , where $c = \sqrt{\kappa/\rho}$. For an ideal gas, the isentropic bulk modulus $\kappa = \gamma p$. The speed of sound in air is then

$$c = \sqrt{\gamma p / \rho}. \quad (2)$$

2.2.2. Governing equations—The continuity equation of a compressible fluid is written as,

$$\rho_{,t} + \nabla \cdot (\rho \mathbf{v}) = 0. \quad (3)$$

Further utilization of the isentropic ideal gas law relationship, Eq. (1) and sound wave speed definition Eq. (2) can deduce the continuity equation (3) as:

$$p'_{,t} + \gamma(p_0 + p')(\nabla \cdot \mathbf{v}) + \mathbf{v} \cdot \nabla p' = 0. \quad (4)$$

To be clear, the fluctuation of pressure, p' , in the above equation is the typical pressure defined in the Navier–Stokes equations where pressure p is a relative measure. To be consistent with the remaining momentum equations that consists of the pressure p , from hereon p' in Eq. (5) is presented as p , and p_0 is the reference atmospherical pressure. The final continuity equation is then:

$$p_{,t} + \gamma(p_0 + p)(\nabla \cdot \mathbf{v}) + \mathbf{v} \cdot \nabla p = 0. \quad (5)$$

The momentum equation of the entire fluid domain that includes both real and artificial fluids can be written as follows,

$$\bar{\rho} \mathbf{v}_{,t} + \bar{\rho} \mathbf{v}(\nabla \cdot \mathbf{v}) = \nabla \cdot \boldsymbol{\sigma} + \mathbf{f}^{\text{FSI},f} \quad (6)$$

where $\bar{\rho}$ is the density and $\mathbf{f}^{\text{FSI},f}$ the fluid–structure interaction force, which are explained in the following. The constitutive relationship between the stress and state variables velocity $\mathbf{v}$ and pressure p for compressible fluid:

$$\boldsymbol{\sigma} = \left(-p + \frac{2}{3} \mu \nabla \cdot \mathbf{v} \right) \mathbf{I} + \mu (\nabla \mathbf{v} + (\nabla \mathbf{v})^T), \quad (7)$$

where μ is the dynamic viscosity of the fluid and $\boldsymbol{\sigma}$ is the fluid stress tensor.

Before defining $\bar{\rho}$, it is first worth emphasizing that the Navier–Stokes equations are solved in both real fluid and artificial fluid domains simultaneously. Effectively, we can think of it as a one-fluid approach where the artificial fluid behaves like the solid. Distinguishing the artificial fluid from the real fluid is accomplished by an indicator function. With this indicator function, material properties such as density ρ in Eq. (6) can be properly assigned such that:

$$\bar{\rho} = \rho^f + (\rho^s - \rho^f) I(\mathbf{x}), \quad (8)$$

in which ρ^s and ρ^f are the solid and fluid densities, respectively, and I is the indicator function used to identify the artificial fluid ($I = 1$) and the real fluid ($I = 0$). Since the solid position gets updated at every time step, the indicator field has to be updated as well at every time step based on relative position of the solid structure. The indicators of the interior points are easy to define. However, with the interface cutting through fluid elements, the

nodal indicator values in a fluid element need to be re-considered by solving a Poisson's equation [27,28],

$$\nabla^2 I = \nabla \cdot \int_{\Gamma^s} \mathbf{n} \phi(\mathbf{x} - \mathbf{x}^s) d\Gamma^s \quad (9)$$

where $\mathbf{n}$ again is the outward normal of the interface Γ^s , $\mathbf{x}$ and $\mathbf{x}^s$ are the coordinates of the fluid grid and the solid nodes, respectively. Solving this Poisson's equation yields a relatively smooth transition of the indicator values between the two domains across the interface. Even though the material properties at the interface may not be exact, the overall exactness of the system solution is *not* compromised. We must emphasize the major difference between the mIFEM and the typical immersed boundary (IB) method is the fact that in the IB method, the entire solid *is* the interface while our mIFEM represents the solid as a volume where every node inside the volume has the exact material property properly represented. Another interpretation is that the total energy in the system is still conserved despite the slightly smeared interface between the real and the artificial fluid.

The fluid–structure interaction force $\mathbf{f}^{\text{FSI},f}$ in Eq. (6) is distributed from the solid volume to the fluid volume as an external force so that the fluid feels the existence of the solid. Note again this force is not just a boundary nodal force, but rather it is evaluated at every solid point including the solid interior. The derivation of this force is shown later in Section 2.4.

2.2.3. Finite element weak form—With the governing equations (3) and (6), let us assume there exist defined finite-dimensional trial solution and test function spaces for velocity and pressure: $\mathcal{V}_v, \mathcal{V}_p, \mathcal{J}_v$ and $\mathcal{J}_p$. The goal is to find $\mathbf{v} \in \mathcal{V}_v$ and $p \in \mathcal{V}_p$ such that $\forall \delta \mathbf{v} \in \mathcal{J}_v$ and $\delta p \in \mathcal{J}_p$.

This work follows the stabilized equal-order finite element formulation developed in Refs. [29,30]. This stabilization technique was designed for incompressible flows as detailed in [29]. Although compressibility is included in this governing equations, the formulation derived for isentropic air is still regarded as nearly incompressible fluid where the energy equation is not solved simultaneously with continuity and momentum equations. Stabilized test functions $\delta \tilde{v}_i$ and $\delta \tilde{p}$ along with the stabilization parameters τ^m and τ^c defined in [29] are used:

$$\delta \tilde{v}_i = \delta v_i + \tau^m v_k \delta v_{i,k} + \tau^c \delta p_{,i}, \quad (10a)$$

$$\delta \tilde{p} = \delta p + \tau^c \delta v_{i,i}, \quad (10b)$$

where τ^m and τ^c are the scalar stabilization parameters dependent of the computational grid, time step size, and flow variables.

The weak form of the continuity equation (3) is obtained by multiplying the pressure test function $\delta\tilde{p}$ and integrating over Ω ,

$$\int_{\Omega} (\delta p + \tau^c \delta v_{i,i}) [p_{,t} + \gamma(p_0 + p)(\nabla \cdot \mathbf{v}) + \nabla p \cdot \mathbf{v}] d\Omega = 0. \quad (11)$$

Similarly, the semi-discrete weak form of the momentum equation (6) is achieved by multiplying the velocity test function $\delta\tilde{v}_i$,

$$\int_{\Omega} (\delta v_i + \tau^m v_k \delta v_{i,k} + \tau^c \delta p_{,i}) \left[\bar{\rho}(v_{i,t} + v_j v_{i,j}) - f_i^{\text{FSI},f} \right] d\Omega - \int_{\Omega} \delta v_i \sigma_{ij,j}^f d\Omega + \sum_e \int_{\Omega_e} (\tau^m v_k \delta v_{i,k} + \tau^c \delta p_{,i}) \sigma_{ij,j} d\Omega = 0. \quad (12)$$

Applying integration by parts and the divergence theorem, we can rewrite Eq. (12) as

$$\begin{aligned} & \int_{\Omega} (\delta v_i + \tau^m v_k \delta v_{i,k} + \tau^c \delta p_{,i}) \left[\bar{\rho}(v_{i,t} + v_j v_{i,j}) - f_i^{\text{FSI},f} \right] d\Omega - \int_{\Gamma_{hi}} \delta v_i h_i d\Gamma \\ & + \int_{\Omega} \delta v_{i,j} \sigma_{ij} d\Omega - \sum_e \int_{\Omega_e} (\tau^m v_k \delta v_{i,k} + \tau^c \delta p_{,i}) \sigma_{ij,j} d\Omega = 0, \end{aligned} \quad (13)$$

where individual finite elements are denoted with the subscript e and the stabilization supplementary terms are evaluated at the element interiors.

Let us assume that there is no traction applied on the fluid boundary, *i.e.* $\int_{\Gamma_{hi}} \delta v_i h_i d\Gamma = 0$, the final weak form yields

$$\begin{aligned} & \int_{\Omega} (\delta v_i + \tau^m v_k \delta v_{i,k} + \tau^c \delta p_{,i}) \left[\bar{\rho}(v_{i,t} + v_j v_{i,j}) - f_i^{\text{FSI},f} \right] d\Omega + \int_{\Omega} \delta v_{i,j} \sigma_{ij} d\Omega \\ & - \sum_e \int_{\Omega_e} (\tau^m v_k \delta v_{i,k} + \tau^c \delta p_{,i}) \sigma_{ij,j} d\Omega + \int_{\Omega} (\delta p + \tau^c \delta v_{i,i}) [p_{,t} + \gamma(p_0 + p)v_{j,j} + v_j p_{,j}] d\Omega = 0. \end{aligned} \quad (14)$$

The full finite element discretization is then performed by discretizing the test and trial functional spaces into finite elements. The nonlinear systems are solved with the Newton–Raphson method. GMRES iterative algorithm is used to minimize the equation residuals [31,32]. With this stabilization technique, we did not observe any deterioration in the solution convergence.

2.2.4. Numerical validation of air wave propagation—To verify the compressible Navier–Stokes implementation with an isentropic flow, we examine wave propagation in a duct with both of its ends closed after an initial Gaussian pulse applied at the inlet. Wave propagation speed and reflective wave speed are examined and compared to analytical

solutions. The units used in these studies are the cm–gram–s (CGS) units. The rectangular duct has a size of 5.0 cm in length and 0.6 cm in width. The model setup is shown in Fig. 2.

At the beginning of the simulation a Gaussian planar wave is introduced at the left end, and hereafter both ends are closed. The top and bottom boundaries of the duct have free-slip boundary condition. The Gaussian pulse applied on the left inlet boundary is

$U_0 = 6 \cdot \exp\left(-\frac{(t-5 \times 10^{-5})^2}{2 \cdot (1.5 \times 10^{-5})^2}\right)$, where the inlet velocity U_0 has a unit of cm/s and time t in s. A uniform mesh with 1875 quadrilateral elements is used in this case; the time step size is set to be 2.5×10^{-8} s, sufficiently small to capture the wave propagation. The fluid is isentropic air with an equilibrium density of $\rho_0 = 1.293 \text{ kg/m}^3$ at 273 K, a dynamic viscosity of 1.8×10^{-5} Pa s, and an initial pressure set as the atmospherical pressure of $p_0 = 101.3 \text{ kPa}$. The theoretical speed of acoustic wave propagation can be estimated as

$$c = \sqrt{\gamma p_0 / \rho_0} \approx 331.33 \text{ m/s}.$$

The snapshots of the traveling wave are shown in Fig. 3. The wave travels in the duct and reflects off the closed ends. The reflective wave form is identical to the initial wave.

To further quantitatively verify our results, we examine the acoustic wave speed by using the concept of acoustic impedance. The characteristic specific acoustic impedance of a medium, say the air, is $\rho_0 c$, the acoustic pressure applied to the system is the pressure variation p (not the absolute pressure), the acoustic ‘current’ is the fluid velocity u . Therefore, the relationship $\rho_0 c = p/u$ is obtained [33–35]. Here, we plot the pressure variation normalized by density p/ρ_0 against velocity u at various axial locations across the duct ($x = 0, 1, 2, 3 \text{ cm}$). All the data collected at each of these axial locations collapsed onto a single line. The acoustic wave speed is successfully captured as shown in the linear regression of these data points yielding a slope of 33138.7 cm/s in Fig. 4, recovering the theoretical speed of sound.

2.3. Solid dynamics with contact model

The solid dynamics governing equation is:

$$\rho^s \ddot{\mathbf{u}}^s = \nabla \cdot \boldsymbol{\sigma}^s, \quad (15)$$

where ρ^s is the solid density, $\mathbf{u}^s$ is the solid displacement, $\ddot{\mathbf{u}}^s$ is the solid acceleration, and $\boldsymbol{\sigma}^s$ is the solid stress tensor. A visco-elastic material is used here to describe the vocal folds:

$$\boldsymbol{\sigma}^s = \mathbf{C} : \boldsymbol{\varepsilon}^s + \eta \frac{\partial \boldsymbol{\varepsilon}^s}{\partial t} \quad \text{in } \Omega^s, \quad (16)$$

where the Cauchy strain tensor $\boldsymbol{\varepsilon}^s = \frac{1}{2} [\nabla \mathbf{u}^s + (\nabla \mathbf{u}^s)^T]$, assuming small strain, and the parameter $\mathbf{C}$ is the fourth-order tensors and η is the viscosity of the material. The constitutive model has to be easily replaced in the solid equation if other types of materials are required.

Noting that in the IB and the original IFEM, the solid displacement $\mathbf{u}^s$ is not solved, but rather, it is imposed based on the background fluid solution. Such an imposed solid solution often leads to unrealistic or extremely large solid deformation, particularly for large Reynolds number flows, since the solid dynamics is not being accounted for. The mIFEM approach corrects this problem by solving the solid dynamics and then imposing it back to the artificial part of the fluid.

Either Dirichlet or Neumann boundary conditions are applied at the solid boundary Γ^s which is equivalent to the fluid–solid interface Γ^{FSI} , where

$$\mathbf{u}^s = \mathbf{q} \quad \text{on } \Gamma^{sq}, \quad (17)$$

$$\boldsymbol{\sigma}^s \cdot \mathbf{n} = \mathbf{h} \quad \text{on } \Gamma^{sh}. \quad (18)$$

The vector $\mathbf{n}$ is the outward normal of the solid surface, therefore also of the fluid–structure interface, towards the fluid. To ensure the existence and the uniqueness of the solid solution, it is required that $\Gamma^s = \Gamma^{sq} \cup \Gamma^{sh}$, $\Gamma^{sq} \cap \Gamma^{sh} = \emptyset$, and $\Gamma^{sq} \neq \emptyset$.

When the vocal folds vibrate, the deformation of the vocal folds may become large enough for the vocal folds to overlap, especially in the case with high pressure input. It is unrealistic and unphysical for the vocal folds to overlap, and without proper contact treatment, the solution would not produce the correct fluid–structure interactions.

Here, we implement a contact model with the augmented Lagrangian method [36], to treat vocal folds that are in contact. The augmented Lagrangian method takes the advantages of both the penalty method and the Lagrangian multiplier method. The Lagrangian multiplier method takes the contact pressure as unknowns in the formulation, and essentially makes the contact problem a constrained minimization problem; whereas the penalty method removes the constraints explicitly from the variational formulation by assuming the contact pressure is proportional to the degree of the penetration, and thus makes the contact problem an unconstrained optimization, which is easier to implement but often sacrifices accuracy for numerous reasons, ill-conditioning being a major concern [37].

The augmented Lagrangian method [36], often considered as a compromise between the two methods mentioned above, retains the accuracy of the Lagrangian multiplier method while uses the penalty method to facilitate an iterative procedure. Ultimately, it minimizes between penetration depth and the contact force. The procedure is described as follows: after an initial contact is made (at iteration number $k = 0$), we first evaluate the penetration magnitude in the normal direction to the contacting surface $g(\mathbf{x}^{(0)}(t))$ where no contact pressure, λ , or penalty force, $\varepsilon g(\mathbf{x})$, is yet applied, with ε being the penalty coefficient. We then iterate and solve the following minimization problem with the variational form until the penetration depth g at all locations is below a tolerance threshold $g_{\max}$:

$$G(\mathbf{x}^{(k)}, \delta \mathbf{x}) + \int_{\Gamma_C} \left[\lambda^{(k)}(\mathbf{x}) + \varepsilon g(\mathbf{x}^{(k)}) \right] \mathbf{n}(\mathbf{x}) \cdot \delta \mathbf{x} d\Gamma = 0. \quad (19)$$

The contact pressure is updated in each iteration incrementally based on the penalty force with $\lambda^{(k+1)}(\mathbf{x}) = \lambda^{(k)}(\mathbf{x}) + \varepsilon g(\mathbf{x}^{(k)})$. This optimization process is iterated over until the penetration is reduced to be under a threshold such that $g(\mathbf{x}) < g_{\max}$ for all $\mathbf{x}$ where $\mathbf{x}$ is the admissible deformation. $G(\mathbf{x}, \delta \mathbf{x})$ is the virtual work done without contact forces for an arbitrary variation of the deformation $\delta \mathbf{x}$. Γ_C is the boundary in contact, which is a subset of the boundary where natural boundary condition is applied, while $\mathbf{n}(\mathbf{x})$ is the surface normal vector on the opposing boundary indicating the direction of contact forces to be exerted. The second term in Eq. (19) can be considered as the virtual work by the additional “force” that is imposed due to contact. In the case where the opposing surface remains rigid, $g(\mathbf{x})$ and $\mathbf{n}(\mathbf{x})$ can be determined with ease: e.g., if the opposing rigid surface is the centerline $\mathbf{x}_C$ s.t. $y = y_C$, then $\mathbf{n}(\mathbf{x}) = (0, 1)$ everywhere and $g(\mathbf{x}) = (\mathbf{x} - \mathbf{x}_C) \cdot \mathbf{n}(\mathbf{x})$.

2.4. Coupling of the fluid and solid domains

Two-way coupling is done in completion for the fluid to interact with the embedded solid, similar to the IB method. However, since the solid occupies a volume, the interaction force is evaluated quite differently. Rather than just the forces acting on a boundary, the force exists everywhere in the solid domain including the interior to balance the work done in the overlapping domain.

2.4.1. Force distribution from solid to fluid—Here, we first provide a bit of background in the derivation of the fluid–structure interaction force. In the original IFEM [16,17], this force was simply:

$$\mathbf{f}^{\text{FSI},s} = \nabla \cdot \boldsymbol{\sigma}^s - \nabla \cdot \boldsymbol{\sigma}^f - (\rho^s - \rho^f) \left(\frac{D\mathbf{v}^f}{Dt} \right) \quad \text{in } \Omega^s, \quad (20)$$

where the terms are derived from the balance of the forces in the overlapping domain after the solid velocity has been imposed by the fluid, which indicates that the fluid dynamics drives the solid movement and deformation.

After years of experimenting with the method including a full-implicit attempt [38,39], some limiting cases surfaced such as it cannot handle large density disparities between the fluid and the solid, e.g. 1000, and it cannot handle large Reynolds number cases where the solid is distorted unrealistically as it follows the fast moving fluid. A significant restructure of the algorithm was warranted, which then ultimately leads to the development of the modified IFEM (mIFEM) [24,14] where the density variation between the fluid and the solid is handled implicitly, as shown previously in Section 2.2.2. It further solves for the solid dynamics rather than imposing it. This concept also allows modular implementation of the fluid and the solid solvers as both dynamics are considered simultaneously. As a result, the derived the fluid–structure interaction force in the mIFEM becomes:

$$\mathbf{f}^{\text{FSI},s} = \nabla \cdot \boldsymbol{\sigma}^s - \nabla \cdot \boldsymbol{\sigma}^f - \rho^s \left(\frac{D\mathbf{v}^s}{Dt} - \frac{D\mathbf{v}^f}{Dt} \right) \quad \text{in } \Omega^s, \quad (21)$$

where $\boldsymbol{\sigma}^s$ and $\boldsymbol{\sigma}^f$ are the solid and the fluid stresses, respectively. The operator D represents the total derivative. This interaction force is evaluated on solid nodes as solid nodal forces $\mathbf{f}^{\text{FSI},s}$, which are then distributed onto the background fluid nodes using the interpolation function, ϕ , as follows,

$$\mathbf{f}^{\text{FSI},f} = \int_{\Omega^s} \mathbf{f}^{\text{FSI},s} \phi(\mathbf{x} - \mathbf{x}^s) d\Omega^s. \quad (22)$$

The interpolation function, ϕ , is a function of the distance from a fluid node $\mathbf{x}$ to a solid node $\mathbf{x}^s$, which essentially smooths out the forces at the interface. Since the interface intersects the background fluid elements, the distributed forces on the fluid domain cover the entire solid domain and some number of element layers beyond the interface. This number of element layers dictates the ‘sharpness’ of the interface. If a sharp layer needs to be realized to capture the distinct discontinuities, in material property or solutions, between the solid and fluid domains, then fewer number of element layers and a refined background fluid grid are desired. However, too few interface layers may often result in singularity forces which can lead to sensitive solution convergence. Detailed discussion on the choices of the interpolation functions can be found in [18]. In capturing the respective vocal fold’s open and closed positions, extremely sharp interface is represented by setting the background fluid grid to be of high resolution where the motion of the vocal folds takes place. Also a small dilation parameter that controls the thickness of the interpolation is used so that ‘spreading’ of the forces or velocity is minimal, while maintaining fast numerical convergence.

2.4.2. Velocity interpolation—Once the Navier–Stokes equations are solved, which yields the real fluid velocity and pressure fields, and artificial fluid velocity field that ideally is the same as the solid velocity field. To couple the fluid solution back to the solid as part of the fully-coupled process, the fluid velocity and stress at the interface are being interpolated back to the solid as the boundary condition, such that the no-slip and/or traction boundary conditions are satisfied at the fluid–solid interface:

$$\mathbf{q} = \left[\int_{\Omega} \mathbf{v}^f \phi(\mathbf{x} - \mathbf{x}^s) d\Omega \right] \Delta t \quad \text{on } \Gamma^{\text{sq}}, \quad (23)$$

$$\mathbf{h} = \left[\int_{\Omega} \boldsymbol{\sigma}^f \phi(\mathbf{x} - \mathbf{x}^s) d\Omega \right] \cdot \mathbf{n} \quad \text{on } \Gamma^{\text{sh}}. \quad (24)$$

Here, ϕ is again the interpolation function; Δt is the time step size and $\mathbf{n}$ is the outward normal of the fluid–structure interface. These two boundary conditions simply correspond to the kinematic and dynamic boundary conditions at the interface. A kinematic condition is to relate the solid velocity to the fluid velocity at the interface, which is no-slip, *i.e.* $\mathbf{v}^s = \mathbf{v}^f$. A dynamic condition concerns with the force balance at the interface, which stipulates stress continuity at the interface, $\boldsymbol{\sigma}^s \cdot \mathbf{n} = \boldsymbol{\sigma}^f \cdot \mathbf{n}$. Applying only kinematic boundary condition or no-slip condition on the entire interface can often make the fluid solver very ‘stiff’. The traction boundary condition on the other hand is more easily handled and yields better numerical convergence. The no-slip condition is automatically satisfied in the last term on the right hand side of Eq. (21), where $\mathbf{v}^f = \mathbf{v}^s$ is enforced through the interaction force. Allowing both types of boundary conditions (but not overlapping) is important in cases where the solid has displacement constraints on Γ^s while the others can be applied using either Eq. (23) or Eq. (24). Regardless of the boundary condition type, the kinematic condition and dynamic conditions must be achieved at the end of each coupling process within the time step.

3. 2-D vocal folds model

3.1. Vocal folds physiology

The human vocal folds are a pair of pliable structures located within the larynx at the top of the trachea. In order to produce sound in voice, the lungs blow air against the pair of closed vocal folds causing the immediate increase of the air pressure beneath the vocal folds. This pressure pushes the two vocal folds apart from each other and a jet of air, sometimes called glottal jet, escapes from the gap between the vocal folds, as illustrated in Fig. 5(a). As the air rapidly flows through the larynx, it induces a decrease in pressure and the vocal folds are brought together collaborated with the elastic recoil of the deformed tissue, shown in Fig. 5(b). Once the vocal folds close, the pressure beneath the vocal folds rises again, where the compressed air forces its way back through the open passageway to the downstream, repeating the process and driving sustained vibration of the vocal folds. It is important to note that here the air is compressible that undergoes an isentropic process. The frequency of vocal folds vibration is also known as the fundamental frequency of human voice, which is in the range of 100–150 Hz for males and 150–200 Hz for females. The vocal fold vibration can be strongly disturbed by various diseases, such as laryngeal cancer, vocal fold paralysis and edema, etc., which are the causes of voice disorders. A clear understanding of the mechanism of vocal fold vibration during phonation is important for providing effective diagnosis and treatment of voice disorders.

Researchers have been studying the vocal fold vibration for decades experimentally, theoretically, and numerically. Van den Berg [40] started the experimental study of the glottal flow using Bernoulli’s equation to relate the transglottal pressure to the glottal volume flow. Scherer, et al. [41] measured the pressure distribution along the walls of a static vocal fold model. Cranen [42] performed measurements of transglottal pressure in live subjects, with associated difficulties in experimental control and characterizing glottal shape. Deverge et al. [43] used scaled-up models and measured pressure in trachea and glottis, and downstream of the vocal folds while one of the model vocal folds was moved. More

recently, Krane et al. [44] used Digital Particle Image Velocimetry (DPIV) to obtain time-resolved measurements of the whole velocity field through a cycle of vocal fold wall motion. The purpose of these experimental findings is to have a full understanding of the physical mechanism of the phonation process and navigate ways to characterize the entire energy flow (from airstream to glottal jet dissipation to sound production) in a glottal system. Using computational means as we demonstrate here, a more thorough analysis can be obtained. Particularly in this study, fundamental and ‘measurable’ quantities such as pressure, volume flow rate, glottis opening width, are examined to show the validity of the proposed numerical approach and to be used as a baseline for more complex analysis used in the future.

3.2. Model setup

3.2.1. Model geometry and setup—A 2-D vocal folds in a vocal tract is modeled in this study, shown in Fig. 6. The numerical model setup follows a vocal fold acoustic experimental setup designed by Campo and Krane [45], which is a simplified real-size model. The vocal tract is represented by a long straight rectangular air duct that is 31.514 cm in length and 2.794 cm in width, while the two symmetric vocal folds are placed in the middle of the vocal tract giving sufficient duct length in both upstream and downstream regions for the compressive air to travel through the glottis without significant boundary effects, physically or numerically. The vocal fold model has a base length of 1.505 cm that is attached to the vocal tract wall and a height of 1.397 cm. The convergent angle, which is defined as the angle formed at the lips of the vocal fold is 10° at the resting condition, based on the M5 model in [46]. The vocal fold in this model is composed of two parts, the cover and the body layers, both of which are considered as viscoelastic materials, where the density is $\rho^s = 1.0 \text{ g/cm}^3$, the Poisson’s ratio is $\nu = 0.3$ and the damping factor is $\eta = 10$ poise. The elastic moduli for the two parts, however, are different. The cover layer is typically softer than the body. Studies have measured the Young’s moduli for the tissues to be in the range between 2.7 and 110 kPa [47–49]. Our initial study has the body elastic modulus to be 4 times higher than that of the cover, where $E_{\text{cover}} = 10 \text{ kPa}$ and $E_{\text{body}} = 40 \text{ kPa}$.

Since sound generation greatly involves the compression of air, the working fluid in the vocal tract is represented as compressible air governed by the ideal gas law at 273 K under an isentropic process. The air density $\rho^f = 1.3 \times 10^{-3} \text{ g/cm}^3$ and viscosity $\mu = 1.8 \times 10^{-4} \text{ g/cm s}$ are used. Constant total pressure boundary condition is applied at the duct inlet on the left, and the stress-free outflow boundary condition is applied at the exit of the duct on the right, as shown in Fig. 6. No-slip and no-penetration boundary conditions are applied on the duct interior walls and the vocal fold surface. For the contact model the penetration tolerance is set to $g_{\text{max}} = 1 \times 10^{-4} \text{ cm}$.

It is worth mentioning that the material parameters and computational boundary conditions must be properly set. In the case of simulating vocal fold vibrations, if the material properties or input parameters are set beyond a reasonable range or improper boundary condition types are applied, the vocal folds may either cannot fully close or cannot produce

sustained steady vibration, which would not be a surprising outcome if the parameters and conditions do not realistically represent the physical system.

In healthy larynx physiology, the top and bottom vocal folds are symmetric about the center line and have identical material properties, although the resulting jet flow is often observed to be asymmetric in experiments and numerical simulations. One such example is our result with a full-space model as shown in Fig. 7, where the glottal jet is seen diverting towards one side rather than following the center line.

For the purpose of illustrating the capability of our numerical model, obtaining meaningful measurements, and capturing the key features of the vocal folds vibration, all the numerical results shown here are obtained by a so-called “half-space” model. In this study we create a half-space model by assuming the fluid field is symmetric about the center line with a symmetric boundary condition. The causes of glottal jet asymmetry and its effects on the vocal folds vibration will be examined in our subsequent studies.

3.2.2. Mesh convergence test—Before the numerical analysis, a mesh convergence test must be performed for this model to examine the effects of mesh resolution in the coupled solution accuracy. For non-conforming meshing techniques in modeling fluid–structure interactions such as the IFEM used in this study, mesh convergence test can be complicated. Since there are two sets of meshes involved, incompatible meshing may result in inaccurate coupled solutions. A very careful mesh convergence study of the IFEM was previously shown in [18]. Moreover, in particular for this vocal folds study, the meshing near the glottis opening is especially critical in capturing the opening and closing processes and a sustained vibration. This is another reason why conforming meshing techniques are not suitable for this study as the closing of the vocal folds may involve frequent and intensive mesh reconstruction. Here, we design 4 sets of background fluid meshes, with each one varying in either stream-wise (x -) or gap-width (y -) direction. Since the glottis gap width is measured in the y -direction, it is more important to study the resolution in the y -direction. Ideally, the element size is sufficiently smaller than the gap size. Non-uniform meshing is used in all the meshes to accommodate the complex nature of the geometries. Therefore, the total number of nodes and elements is not proportionally varied. The mesh sizes listed here are based on the local resolution near the center line. The detailed information for the 4 meshes is listed in Table 1.

To determine the fidelity of a mesh, we examine two quantities that are directly measured from the simulation using that mesh, the distance between vocal fold and centerline (Fig. 8(a)) and the vocal fold vibration frequency (Fig. 8(b)). Despite differences in element numbers and element sizes, Meshes 1, 2, and 3 produce the same glottis gap width during the steady-state vibration and the vocal fold vibration reaches the same frequency, while Mesh 4 fails to produce a sustained vibration due to the low resolution in the vicinity of the glottis gap. This outcome indicates that there is a lower bound in the mesh size, particularly near the tip of the vocal folds where instantaneous variations of the flow need to be captured.

Based on the outcome of the mesh convergence test, Mesh 2 is chosen as the optimum mesh which has 24308 quadrilateral elements and 24836 nodes in the fluid domain, and 6476

triangle elements and 4306 nodes in the solid domain. Comparing to Mesh 3, finer mesh resolution is imposed near the center line. The time step size is chosen to be 2×10^{-5} s to ensure the stability of the numerical algorithm.

3.2.3. Computational resources—The computing resource to perform these computations is the IBM Blue Gene Q system located in the Center of Computation Innovations at Rensselaer Polytechnic Institute, with each computing node consisting of a 16-core 1.6 GHz A2 processor and 16 GB of DDR3 memory. The source code is efficiently parallelized with MPI, please refer to [24] for the computing efficiency studies. For each run, 1024 cores are utilized, which takes approximately 40 h to complete 10000 time steps of simulation. With the time step we choose, it is 50–60 cycles of the vocal folds vibration, which is more than sufficient number of cycles to reach steady-state vibrations.

3.3. Analysis

To perform the analysis on the glottal flow and the vocal fold vibration using simulations, a few physical quantities are first defined. Then, a benchmark case that uses all physiological input parameters from a healthy vocal tract is set up. This benchmark study is to demonstrate that the solution technique provided here has the capability in yielding a sustained vocal folds vibration without any additional enforcement or manipulation of the physical interactions. It also provides us a basic understanding of the fundamental mechanism between the air flow and the vocal folds interactions through detailed analysis. Further parametric studies of lung pressure and vocal fold stiffness are then presented. The goal is to understand how the inlet pressure and the stiffness of the vocal folds can affect the vocal folds vibration, which enhances our understanding of the dynamics of the coupled system. A total of nine cases are studied, with the corresponding quantities such as the peak velocity, maximum glottis width, maximum volume flow rate, fundamental and reduced frequencies, and Reynolds number at the glottis, yielded from the simulations listed in Table 2. The exact definitions of these quantities pertaining to the vocal folds study are defined as follows, in Section 3.3.1.

3.3.1. Definitions of physical quantities

Glottis width: The glottis width is defined as the size of the narrowest glottal opening which varies with time as the vocal fold vibrates. Specifically for the half-space model, half of the glottis width, h , is defined as the minimum distance from the vocal fold surface to the center line (or the symmetric boundary), as labeled in Fig. 9. The full glottis width is assumed to be $2h$. $h_{\max}$ is denoted as the maximum glottis opening during a vibrational cycle in a half-space model.

Volume flow rate: The volume flow rate is also measured at different cross sections. Since it is to be evaluated in a 2-D model, the “volume” flow rate is effectively the rate of flow at a particular x as: $Q_C = \int_{x=C} v_x dy$. In this study, the flow rate is evaluated at three different locations (-2.2 cm, 0 cm, 0.8 cm), which respectively fall into the subglottal, glottal and supraglottal regions (labeled in Fig. 9). The origin of the coordinates is set at the trailing edge of the vocal fold.

Reynolds number: The Reynolds number in phonation is typically defined using the peak velocity at the glottis as, $Re = \rho U_{\max}(2h_{\max})/\mu$. Here, $U_{\max}$ is the maximum glottal jet velocity and $2h_{\max}$ is the maximum glottis width during the opening phase of each vibrational cycle. ρ and μ are the air density and dynamic viscosity, respectively. Since the air density only varies slightly in compressible flows, the initial constant ρ_0 is used when evaluating the Reynolds number.

Fundamental and reduced frequencies: The fundamental frequency, f , is evaluated by taking the Fast Fourier Transform (FFT) of all the directly measurable quantities, namely, glottis width, volumetric flow rate, fluid velocity and pressure. The fluid velocity and pressure at the glottis region are used for frequency analysis so that they are consistent with the glottis width and the flow rate, which are more conservative quantities. The reduced frequency is computed based on the fundamental frequency as: $f^* = fL/U_{\max}$, where L is the length of the base of the vocal fold. It is $L = 1.505$ cm in the current model, as indicated in Fig. 9.

The definitions of both Re and f^* are suggested by [44]. For typical life-scale problems, the Re number can reach as high as 8000 and reduced frequency f^* in the range of 0.1–0.01.

3.3.2. Benchmark case—The benchmark case of this study is listed as Case 4 in Table 2 where the inlet pressure (the lung pressure) is set to be 1 kPa, and the Young's modulus for the cover layer is $E_{\text{cover}} = 10$ kPa and $E_{\text{body}} = 40$ kPa for the body. Similar input parameters and material properties are also used in other numerical studies in Refs. [5,50,8,13,12]. A few snapshots of the vocal folds from a closed position to an opened position as the air travels and pushes through the glottis are shown in Fig. 10.

At the beginning of the simulation, the vocal fold starts at a resting position. The time histories of the glottis width and the volume flow rate at $x = -0.1$ cm (the tip of the vocal fold) are shown in Figs. 11(a) and 11(b), respectively. As shown in the two plots, the vocal fold starts its consistent vibration pattern after 0.05 s and reaches steady vibration after around 0.2 s. During the steady vibration, the maximum glottis width reaches 0.1066 cm and the minimum glottis value is zero, which indicates the vocal folds are able to shut off the airflow completely. Using FFT for both h and Q , the fundamental frequencies of the vocal folds vibration, are found to be the same at $f = 234$ Hz. The Fourier transform is shown in Fig. 11(c).

A more detailed view of each cycle during the steady vibration (physical time $t = 0.2210$ – 0.2254 s) is shown in Fig. 12. Volume flow rate, velocity, and pressure are closely examined at glottal, subglottal, and supraglottal regions. Here, the time axis is normalized, where $T^* = 0$ is the instance when the glottis width reaches its minimum, which is also when glottis closes. The time interval when the glottis width increases from its minimum to maximum is called 'open quotient' and decreases from maximum back to minimum is called 'close quotient'. In this case, the glottis reaches fully opened position right after $T^* = 0.5$.

The volume flow rate reaches its maximum when the glottis fully opens, see Fig. 12(a). At fully closed position, the volume flow rate is zero at the glottis ($x = 0$ cm) indicating that the

vocal fold shuts off the airflow completely. Meanwhile, both subglottal ($x = -2.2$ cm) and supraglottal ($x = 0.8$ cm) regions are seen to have negative flow rates when the glottis is closed. This is due to the fact that even as the cover shuts off the flow, the body is still being carried backward which causes a reversed air flow that generates a negative volume rate. Such phenomena is also observed in Krane's experimental study with a scaled-up model with prescribed vocal fold motion [51]. However, it has not been captured in other numerical simulations such as finite element model [7,8] and Immersed Boundary method [13,12], where these numerical models numerically block the fluid flow by enforcing the flow rate to be zero when the glottis width is less than a critical value to avoid the contact between two vocal fold walls.

The velocity over one cycle at subglottal, glottal, supraglottal regions is shown in Fig. 12(b). The glottal velocity is also considered as the glottal jet velocity and its maximum value during the vibration reaches 6658.1 cm/s. Therefore, the Reynolds number of the glottal flow is $Re = 4731.7$ and the reduce frequency is $f^* = 0.052893$; both fall in the range of the life-scale problems. The glottal velocity has a quick rise as the glottis starts to open, and then increases slowly before it fully opens. Just before the glottis closes, the glottal velocity has a second quick rise, and then drops to zero when the vocal folds shut off the air flow. The same waveform of the glottal velocity is also found by other numerical and experimental studies [12,51].

The pressure over one cycle at subglottal, glottal, supraglottal regions is shown in Fig. 12(c). The subglottal pressure reaches its maximum right after the vocal fold shut offs the air flow, when the air in the subglottal region is compressed by the air flow from the upstream and the vocal fold surface. The high subglottal pressure then pushes the vocal folds apart and makes the glottis open again. After the glottis opens, the glottal pressure decreases due to the rapid increasing of the glottal velocity, from the Bernoulli's effect. These observations are also consistent with those found in other numerical studies [5,8].

3.3.3. Effect of inlet pressure—The effect of the inlet pressure (or lung pressure) is examined in this study. Seven different lung pressures ranging from 0.7 kPa to 1.3 kPa (Cases 1–7 in Table 2) are investigated while all the other physical parameters are set to be exactly the same. The simulations for the 7 cases yield similar results, where the directly measured quantities, such as the gap width and volume flow rate, showed the vocal fold vibration reaches a steady state vibration around 0.2 s, although higher inlet pressure may reach to steady vibration slightly earlier. The power spectra of the volume flow rates for all seven cases are found to be almost the same at 234 Hz, shown in Fig. 13. It indicates that the fundamental frequency is independent of the inlet pressure. The same conclusion was also drawn in Zhang's experimental study [52]. In their single layer vocal fold acoustic model, the fundamental frequency does not change with the inlet pressure either. However, there are debates on the effects of the inlet pressure on the fundamental frequency. Titze predicts that the fundamental frequency should increase with the growing inlet pressure, using small-amplitude approximation [53]. Different numerical models tend to have different conclusions on how the inlet pressure affects the fundamental frequency. Luo [12] and Tao [54] report that there does not exist any clear relationship between the vibration frequency and lung pressure. Rosa [50] concludes that the vibration frequency decreases with the lung

pressure grows. Therefore, further experimental and numerical studies are required to reveal the relationship between the inlet pressure and the fundamental frequency.

Fig. 14(a) shows the glottis width for each cycle with normalized time scale T^* using their own fundamental frequency. As expected, the higher the pressure, the larger the maximum glottis width, which means the vibration magnitude also increases. It also correlates to higher volume flow rates that may lead to a stronger signal, or a louder voice during phonation. In fact, as shown in Fig. 14(b), we also find an approximate linear relationship between the maximum glottis width and the square root of the inlet pressure. Such rough linear relationship is also predicted in analytical and numerical studies. Titze [53] and Titze and Durham [55] conducted experiments with warm humidified air through the glottis and found a linear relationship between the vocal fold's oscillation amplitude and the square root of the subglottal pressure. Tao et al. [8] and Jiang and Titze [56] later used their finite element model simulation to confirm this relationship between the impact pressure and oscillation amplitude. Our glottal pressure study is consistent with these findings.

3.3.4. Effect of vocal fold stiffness—To study the effect of stiffness on the vocal fold vibration, three sets of different vocal folds stiffnesses are examined; they are listed in Table 2 as Cases 4, 8, and 9. The benchmark, Case 4, has $E_{\text{cover}} = 10$ kPa and $E_{\text{body}} = 40$ kPa, denoted as 'stiff' case. Case 8 is a 'soft' case that has $E_{\text{cover}} = 5$ kPa and $E_{\text{body}} = 20$ kPa. Cases 4 and 8 have the same stiffness ratio of 4. Case 9 has a lower stiffness ratio between the cover and the body where $E_{\text{cover}} = 20$ kPa and $E_{\text{body}} = 40$ kPa for the body. All other numerical parameters are kept exactly the same including the same pressure input of 1 kPa, Poisson's ratio μ is 0.3 and the damping factor η is 10 poise for both cover and body layers. Based on the results, there are no fundamental differences between Case 4 and Case 9. For direct comparisons, we will closely examine Cases 4 and 8 since the stiffness ratios for both cases are the same. The stiff case starts steady vibration after 0.2 s as it takes a little longer for the soft case to get to the steady vibration. The wave speed in the vocal fold is proportional to the square root of the Young's modulus. Therefore, with the same damping factor and density, the stiffer tissue reaches the steady vibration faster than the softer one.

Higher vibration frequency is expected when large stiffness is given for both the cover and the body, as in Case 8. The power spectra of the volume flow rates for both stiff and soft cases are shown in Fig. 15. The fundamental frequencies are found to be $f_{\text{stiff}} = 234$ Hz and $f_{\text{soft}} = 169$ Hz for the stiff and soft cases, respectively.

The glottis width and volume flow rate for both the soft and stiff cases during one cycle are plotted in Fig. 16. To compare the two cases, the time scale is again normalized by their respective fundamental frequency. The maximum glottis width over one cycle for the soft case is larger than that for the stiff case. Moreover, the wave forms of the flow rate and glottis width are also very different. As shown in Fig. 16(a), the glottis width for the soft case decreases more drastically in the middle of the open quotient and then increases again. After the glottis width reaches its maximum, it drops to its minimum much faster in the soft case than in the stiff case. Such difference is caused by the different vibration pattern between the soft and stiff cases. For the soft vocal folds, up-down motion of the inferior edge is much more pronounced, which causes the glottis width to decrease in the middle of

the open quotient. The volume flow rate changes similarly to the glottis width, because during the glottis opening the flow rate directly depends on the glottis width. In Fig. 16(b), the flow rate also indicates that the time interval of air shutting off is longer for the stiff case than for the soft case, which means that it takes less time for the pressure to build up and push the vocal folds open when the vocal folds stiffness is smaller.

Overall, we observed that vocal fold stiffness has a significant effect on the vocal fold vibration. This result shows that higher vocal fold stiffness leads to higher fundamental frequency because the natural frequency of a material is proportional to the square root of the stiffness. Softer vocal fold material also provides larger vocal folds vibration magnitude because softer vocal folds are easier to be pushed and tend to have larger vibration magnitude, which results in larger maximum glottis width and volume flow rate during each cycle.

4. Conclusions and outlook

In this work, we successfully developed a numerical tool to simulate a *self-sustained* vocal fold vibration. This simulation involves a complex fluid–structure interaction process and requires a robust numerical technique. Based on our modified immersed finite element method (mIFEM) that we have been developing over the past decade to study fluid–structure interactions, two additional features are implemented: fluid compressibility and solid contact model. The governing equations for this coupled system are compressible Navier–Stokes equations for the airflow and linear viscoelastic material constitutive law with layered structure for the vocal folds. The dynamic solutions for the airflow and vocal folds are strongly coupled with each other at every time step. Our approach has two unique advantages over the existing methods: (1) the vocal folds behavior is accurately predicted by solving the solid *dynamic* governing equation using the fully coupled algorithm, which is essential in these high-Reynolds-number laryngeal flows, and (2) the compressibility of the air flow and vocal folds are taken into consideration, which more accurately captures the nature of the air flow.

Based on the numerical results, we observed robust self-sustained vibration of the vocal folds. The flow field (velocity and pressure fields) at the steady vibration agrees well with measurements obtained from published experimental data [52,51] and the results obtained from other numerical studies [5,8,12]. Our parametric studies further showed that the inlet pressure, or the lung pressure, does not affect the fundamental frequency of the vibration but instead, higher inlet pressure leads to larger vibration magnitude. An approximate linear relationship between the vibration magnitude and the square root of the inlet pressure is found, which agrees with previous experimental and analytical findings. On the other hand, the stiffness of the vocal fold seems to have a direct effect on the fundamental frequency, as expected. Low fundamental frequency and larger magnitude vibration are observed for softer vocal folds.

We recognize that the simulation was not performed in 3-D, therefore the vocal fold vibration pattern would be somewhat deviated from reality. However, with the concern of the

tremendous computational cost and additional complexities in the analysis, it is not done in this initial study, even though the tool presented here is implemented for 3-D.

We intend to further show and compare the differences in symmetric and asymmetric laryngeal airflows, and study and discuss the dynamic and energetic impact of glottal jet asymmetry on the vocal folds vibration as well as the resultant phonation. We also expect to further this study with more detailed control volume analysis to fully understand and identify the effects of fluid compressibility, friction on the vocal folds surface, as well as air viscosity. More work is to be done on carefully designing boundary treatment in the aeroacoustic model which will help in identifying acoustic sources in the phonation process.

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Fig. 1.
Computational domain decomposition.

Fig. 2.
Geometry and boundary conditions of acoustic wave traveling in a closed rectangular duct.

Fig. 3.
Pressure profile showing propagation of an acoustic wave in a rectangular duct.

Fig. 4.

Pressure variation over density versus velocity at several locations along the duct to calculate acoustic wave speed.

(a) From closed position to fully open position.

(b) From opened position to fully closed position.

Fig. 5.

A schematic drawing of the glottis opening and closing process.

Fig. 6.
Model setup for a 2-D two-layer vocal folds in a long duct.

Fig. 7.

Vorticity contours of the glottal jet over in one vibration cycle in a full-space model.

(a) Distance between vocal fold and centerline produced by the 4 meshes.

(b) Vibration frequency produced by the 4 meshes.

Fig. 8.
Mesh convergence tests specific to vocal fold simulation studies.

Fig. 9.
Labels and definitions for variables of interest.

Fig. 10.

Vorticity contours of the glottal jet over one vibration cycle in a half-space model.

(a) Half glottis width.

(b) Flow rate at $x = -0.1$ cm.

(c) FFT.

Fig. 11.

Time history and Fourier transform of flow rate Q at $x = -0.1$ cm and glottis width h .

Fig. 12.

Volume flow rate, velocity, and pressure during one cycle measured at $x = -2.2$ cm (subglottal region; '♦'), $x = 0$ cm (glottal region; '□'), and $x = 0.8$ cm (supraglottal region; '○').

Fig. 13.
Fourier transform of volume flow rates obtained from different pressure inputs.

Fig. 14.
Relationship between maximum glottis width and inlet pressure.

Fig. 15.

Fourier transform of volume flow rate for stiff (green ‘–’) and soft (blue ‘–’) cases. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 16.
Half glottis width and volume flow rate during one cycle for stiff (‘♦’) and soft (‘○’) cases.

Table 1

Meshing information for mesh convergence test.

	Mesh size — x	Mesh size — y	# of elements	# of nodes
Mesh 1	0.002 cm	0.014 cm	26992	27480
Mesh 2	0.005 cm	0.014 cm	24308	24836
Mesh 3	0.005 cm	0.028 cm	8750	9060
Mesh 4	0.010 cm	0.028 cm	6905	7215

Table 2

List of cases with input parameters (pressure inlet P_{in} and Young's modulus E of the vocal folds) and their corresponding output quantities.

Case	P_{in} (kPa)	E (kPa) [cover, body]	f (Hz)	U_{max} (cm/s)	h_{max} (cm)	Q_{max} (cm ³ /s)	f^*	Re
1	0.7	10, 40	234	5318.2	0.0390	84.99	0.066220	2995.9
2	0.8	10, 40	234	5852.5	0.0430	105.2	0.060174	3635.1
3	0.9	10, 40	234	6208.7	0.0461	123.5	0.056722	4134.3
4	1.0	10, 40	234	6658.1	0.0492	145.8	0.052893	4731.7
5	1.1	10, 40	234	6813.4	0.0506	156.9	0.051688	4979.8
6	1.2	10, 40	234	6597.5	0.0526	160.6	0.053379	5012.6
7	1.3	10, 40	234	7361.7	0.0534	187.2	0.047838	5678.3
8	1.0	5, 20	169	6543.0	0.0574	185.7	0.038873	5424.9
9	1.0	20, 40	258	6291.0	0.0392	113.5	0.061722	3562.1