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Active Localization and Tracking of Needle and Target in Robotic Image-Guided Intervention Systems

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Abstract

This paper describes a framework of algorithms for the active localization and tracking of flexible needles and targets during image-guided percutaneous interventions. The needle and target configurations are tracked by Bayesian filters employing models of the needle and target motions and measurements of the current system state obtained from an intra-operative imaging system which is controlled by an entropy-minimizing active localization algorithm. Versions of the system were built using particle and unscented Kalman filters and their performance was measured using both simulations and hardware experiments with real magnetic resonance imaging data of needle insertions into gel phantoms. Performance of the localization algorithms is given in terms of accuracy of the predictions and computational efficiency is discussed.

Index Terms

Medical Robotics; Image-Guided Interventions; Intra-Operative Image-Guidance; Needle Tracking; Bayesian Filtering; Active Sensing

I. Introduction

Needle insertion procedures are common for diagnostic or therapeutic purposes, for instance, for brachytherapy, tissue biopsy, and drug delivery [1]. In these procedures, a needle must be inserted in such a way that the needle tip hits a target of interest. For example, in brachytherapy, radioactive seeds are placed in a tumor. An important consideration in percutaneous intervention is the uncertainty of the needle, target, and obstacle positions. Needles are subject to bending during insertion, and the targets and obstacles may move, due to physiological motions such as breathing, or due to tissue deformations resulting from needle-tissue interaction. Intraoperative medical imaging systems, *e.g.*, X-ray fluoroscopy,

ultrasound, computerized tomography, and magnetic resonance imaging, can be used to provide real-time information about the needle and target locations, thus allowing the needle to be actively steered and enabling closed-loop robotic needle insertion.

In this paper, a probabilistic formulation of the problem of needle and target localization using intra-operative medical imaging is presented. The framework is intended to simultaneously localize a target which is embedded in tissue and a flexible needle as it travels toward the target. Probabilistic models account for noise and false detections in the intraoperative medical images, as well as, uncertainties in needle and target motions. Bayesian filters are used to track the motions of the needle and target, using the data obtained from the intra-operative imaging systems. An entropy minimization technique is then introduced to actively control the intraoperative imaging system.

The framework of models and algorithms is evaluated in simulations using artificial motion and imaging data, and in hardware experiments with real magnetic resonance imaging data of a needle being inserted toward a target in an artificial tissue phantom.

This study builds on the authors' previous work in [29]. In that study, particle filters were used in conjunction with an active localization framework to track a needle and a target during a simulated needle insertion. This paper extends that work by validating the framework using real imaging data collected during a needle insertion into a gel phantom in a hardware experiment and by comparing the particle filter performance with that of an unscented Kalman filter.

The rest of this paper is organized as follows. Related work and the problem formulation is presented in the remainder of this section. The proposed active localization and tracking algorithms are described in Section II. The image processing algorithms used in hardware experiments are presented in Section III. Experimental validation is done using both simulated data in Section IV and using magnetic resonance imaging data collected during needle insertions into tissue phantoms in Section V. Conclusions are made in Section VI.

A. Related Work

A great deal of work has been done on the subject of modeling and tracking needles during percutaneous interventions. DiMaio *et al.* [9] first proposed steering flexible needles to reach targets embedded in soft tissue without disturbing obstacles or critical areas within the tissue. DiMaio and Salcudean observed deformations that happen during needle insertion into a gel and simulated needle insertions using a quasistatic finite element method with measured tissue phantom parameters [10]. DiMaio and Salcudean [11] formulated a needle manipulation Jacobian using numerical needle deflection and soft tissue deformation models. In both simulation and experiments, they were able to steer needles to the target in the presence of obstacles in soft tissue by manipulating the needle base.

Okamura *et al.* [25] modeled insertion of deformable needles in elastic tissue. Their model includes a stiffness term that precedes puncture of the tissue capsule. After puncture, the force on the needle is composed of cutting and friction forces. Crouch *et al.* [8] developed a model of needle insertion force by using experimental optimization and finite element

modeling. They used an optimization approach in tandem with the FEM model that represented needle force models using piecewise cubic splines. This work determined that a static linear elastic tissue model combined with a dynamic force function accurately models needle insertion force.

Webster *et al.* [16], proposed a kinematic model that describe the trajectory of flexible bevel-tip needles in rigid tissue. Parameters were fit using experimental data. Their model did not consider the interaction of the needle with an elastic medium. Misra *et al.* [23] presented a two dimensional model for a bevel tip needle embedded in an elastic medium. Their mechanics based model is based on both microscopic and macroscopic observations of the needle interaction with soft gels.

Park *et al.* [28] demonstrated a nonholonomic kinematic model to describe how an ideal needle with bevel tip moves through firm tissue with a numerical example. The reachability criteria are proven for this model, and an inverse kinematics method based on the propagation of needle-tip pose probabilities is presented.

In more recent work, Khadem *et al.* [20] developed a dynamic model of needle steering in soft tissue, which relates needle deflection to insertion velocity and axial rotation of the needle, which can be regarded as control inputs for needle steering. Maghsoudi and Jahed [22] presented a dynamic model of needle-tissue interaction for needle movement in soft tissue, and investigated the role of tissue parameter estimation in needle insertions.

Most systems for tracking needles require specialized equipment, such as by placing electromagnetic beacons on the needle or in the patient's tissue. Cleary *et al.* [7] designed and implemented one such system using an electromagnetic tracking system with a sensor coil embedded in the needle and a patient model. Wong *et al.* [36] demonstrated the successful use of the SonicGPS ultrasound needle guidance system. In this system, a needle equipped with a radio transmitter is tracked and displayed on the ultrasound system's screen in real time.

Glozman and Shoham [14] developed a needle-steering system with fluoroscopic imaging-based guidance. The system can insert flexible needles with a custom RSPR 6-DOF robot designed to drive the needle tip in a predetermined, curved trajectory by maneuvering the needle base, based on a needle-tissue interaction model. The system can detect the needle tip position and considering natural obstacles, can updated the needle path in real-time. An inverse kinematics algorithm based on a virtual spring model is applied to calculate flexible needle model. More recently, Neubach and Shoham used the same model for flexible needle steering inside soft tissues under real-time ultrasound imaging [24].

Bayesian filtering has been used for numerous similar tracking applications. Dong *et al.* [12] proposed a framework for localizing needles in ultrasound image frames, in which they formulated the localization problem as a segmentation problem. Their proposed method can track needles in highly noisy ultrasound images during image-guided interventions using level set and partial differential equation based methods.

Koowal *et al.* [21] developed a system that uses ultrasound images to track a catheter during heart ablation procedures. The system uses a particle filter approach to track the expected pose of the catheter and thus the expected shape of the catheter as it appears on the ultrasound image. Tully [33] demonstrated BodySLAM, a system that uses an Extended Kalman filter estimator and snake shape estimation image processing to guide a HARP surgical robot. Asadian *et al.* [3] used multiple Kalman filters in sequence to track deformations of a flexible needle undergoing insertion and retraction forces.

Image processing for the detection of needle tips in intraoperative medical images is an ongoing research topic. Green *et al.* [15] induced a vibration in a needle and used ultrasound to detect the Doppler effects of the vibration in order to localize a needle. Kaya and Bebek [18], [19] used the Gabor filter to detect needle orientations in ultrasound images and constructed a probability map to estimate the needle tip position. Okazawa *et al.* [26] used the Hough transform and coordinate mapping to localize curved needles in two-dimensional ultrasound images. Chatelain *et al.* [5] used a volume intensity difference technique to detect needles in three-dimensional ultrasound volume images and a Kalman filter to track the needle as it moved through the medium.

Vrooijink *et al.* [34] presented a needle steering system that can autonomously guide a steerable needle to a moving target. The system uses an online needle tracking method that can track flexible needles in 3-dimensions using 2-dimensional ultrasound images. The needle tracking employs an image processing algorithm that can locate needle centroid from cross sections 2-dimensional ultrasound images affected by the so called comet-tail-artifact.

This paper proposes a new approach for needle and target localization and tracking in needle-based percutaneous interventions using intra-operative medical imaging. To the best of the authors knowledge, there are no earlier studies where a robotic active sensing scheme was used for controlling the intra-operative imaging system for localization and tracking of the percutaneous instruments or anatomical targets. Furthermore, in this study, the localization and tracking of the needle and the target using intra-operative imaging is formulated in a probabilistic framework, which explicitly accounted for the uncertainties in the needle and target motions and the noise and detection errors in the intra-operative imaging system.

B. Problem Formulation

The general problem is formulated as follows. A target and a flexible needle are located inside some medium which is within the viewable workspace of some intra-operative imaging system. At each timestep, the intra-operative imaging system images one two-dimensional plane of the workspace. For simplicity, the image planes are kept orthogonal to the initial needle tip direction¹. Separate Bayesian filters are maintained to track the system's belief of the needle shape and target position. The core problem being addressed is the selection of the sensing action, which in this application is the image plane. In the approach used here, information maximization based on entropy minimization is used for sensing

¹This choice of image plane configurations is not fundamental to the proposed method. Arbitrary image plane configurations can be used without any modification to the algorithm.

action selection. The proposed system chooses the sensing action that maximizes the expected information gain. The goal of the system is to optimally localize and track the needle and the target. This paper explicitly focuses on the needle and target localization problems, as such, the needle steering control problem is outside its scope.

Algorithm 1

The basic algorithm of the entropy minimization-based active localization algorithm.

```

Initialize Bayes filter belief  $bel(x_0)$ 
Initialize initial control output  $u_0$ 
for each timestep  $t$  do
    Select  $u_t$  that maximizes information gain

     $z_t \leftarrow \text{Imaging}(u_t^I)$ 
     $bel(x_t) \leftarrow \text{BayesFilter}(bel(x_{t-1}), z_t)$ 
end for

```

At each timestep t , the active localization system calculates the control output u_t^I , which directs the imaging system to capture an image of the volume such that the selected image plane is most likely to maximize expected information gain about the system state x_t , which consists of the needle state x_t^N and target state x_t^T . It then uses the imaging system to obtain an image at u_t^I , processes the image to produce a measurement z_t that consists of the target and needle locations in the image plane (z_t^T and z_t^N , respectively), and uses the current measurement z_t to update the beliefs of the Bayesian filters which track the needle and target states $bel(x_t^N)$ and $bel(x_t^T)$ (Algorithm 1).

This formulation of the problem makes the implicit assumption that only one image plane can be imaged at each time step due to time and hardware constraints. If a function that explicitly models the cost of imaging an arbitrary plane is available, the cost function can be modeled in the entropy minimization calculations.

II. System Architecture

This section describes the overall architecture of the active localization system. The system consists of four major components: **1) An *Imaging System***, e.g., ultrasound, CT, MR, *etc.*, which is controlled by an active localization algorithm in order choose the image plane that will maximize the information it knows about the environment; **2) *Measurement Models***, which specify the likelihood of needle and target location measurements obtained from the imaging system for the given current states of the needle and the target; **3) *Motion Models***, which, when given a current state of the needle and target, specify the state transition probability density functions for their next states; and, **4) *Bayesian Filters***, which estimate the needle and target states.

A. Active Localization

In active localization, a system actively controls one or more of its control inputs in order to maximize information about the state of the system [4]. In this application there is a natural decomposition of the system's control inputs. The intra-operative medical imaging system can be controlled to actively localize the needle and the target, while the needle control inputs can be used to execute the task (such as to direct the needle towards the target). In this work, a greedy exploration algorithm based on entropy minimization is employed (Algorithm 2). In the algorithm, the system's beliefs about the needle and target states are represented using particle filters or unscented Kalman filters as described in Section II-B. In the particle filter implementation, the entropies of the beliefs are estimated using the differential entropy calculation approach proposed by Orguner [27], [30]. In the Kalman filter implementation, the entropy is directly calculated from the system covariance.

Algorithm 2

Greedy active localization algorithm based on entropy minimization. Here it is assumed that the needle control input u_t^N is separately determined by a needle motion planner and controller based on the current beliefs of the needle $bel(x_t^N)$ and target $bel(x_t^T)$ of needle and target states x_t^N and x_t^T . The dependence of the needle and target measurements on the configuration of the image plane u_t^I is explicitly included in the measurement model expressions for clarity. The algorithm calculates the expected entropy for every possible image plane configuration u_t^I and selects the one that is expected to increase the knowledge of the system the most.

```

function ActiveLocalization( $bel(x_t^N)$ ,  $bel(x_t^T)$ ,  $u_t^N$ )
  for  $i \leftarrow I_{min} \rightarrow I_{max}$  do
     $u_t^I$  = image plane  $i$ 
    sample  $\hat{x}_t^N \sim p(x_t^N | u_{t-1}^N, x_{t-1}^N)$ 
    sample  $\hat{z}_t^N \sim p(z_t^N | \hat{x}_t^N, u_t^I)$ 
     $bel'(x_{t+1}^N) \leftarrow \text{BayesFilter}(bel(x_t^N), u_t^I, \hat{z}_t^N)$ 
     $\rho_i^N \leftarrow \text{Entropy}(bel'(x_{t+1}^N))$ 
    sample  $\hat{x}_t^T \sim p(x_t^T | x_{t-1}^T)$ 
    sample  $\hat{z}_t^T \sim p(z_t^T | \hat{x}_t^T, u_t^I)$ 
     $bel'(x_{t+1}^T) \leftarrow \text{BayesFilter}(bel(x_t^T), u_t^I, \hat{z}_t^T)$ 
     $\rho_i^T \leftarrow \text{Entropy}(bel'(x_{t+1}^T))$ 

```

```

    end for

    return  $\text{argmax}_i(\rho_i^N + \rho_i^T)$ 

    end function

```

B. Bayesian Filters for Needle and Target Tracking

A Bayesian filter (Algorithm 3) is an estimator that relies on Bayes' Law to probabilistically estimate the current state of a time-varying system [31], [6]. They are commonly used for applications of this type, in which the estimate of the current state of a time-varying system is evolved as new measurements are made. In this work, two implementations of Bayesian filters are used: the unscented Kalman filter (UKF) [35] and the particle filter (PF) [2].

Algorithm 3

The basic Bayesian filter algorithm [32]. $bel(x_t)$ is the algorithm's belief about the system state x at time t , u_t is the control input at time t , and z_t is the measurement.

```

function BayesFilter( $bel(x_{t-1}), u_t, z_t$ )

     $\overline{bel}(x_t) \leftarrow \int p(x_t|u_t, x_{t-1}) bel(x_{t-1}) dx_{t-1}$ 

     $bel(x_t) \leftarrow \eta p(z_t|x_t) \overline{bel}(x_t)$ 

    return  $bel(x_t)$ 

end function

```

Algorithm 4

The particle filter algorithm. The posterior is represented by the particle set χ_t which is made up of M particles, and is altered according to the measurement z_t and the control input u_t . The resampling step has the effect of discarding unlikely hypotheses in favor of more likely ones, as determined by the weight vector W_t .

```

function ParticleFilter( $\chi_{t-1}, u_t, z_t$ )

     $\overline{\chi}_t \leftarrow \emptyset$ 
     $W_t \leftarrow \emptyset$ 
    for  $i \leftarrow 1 \rightarrow M$  do

        sample  $x_t^i \sim p(x_t|u_t, x_{t-1}^i)$ 

         $w_t^i \leftarrow p(z_t|x_t^i)$ 

        add  $x_t^i$  to  $\overline{\chi}_t$  and  $w_t^i$  to  $W_t$ 

    end for

    for  $i \leftarrow 1 : M$  do

```

```

 $\chi_t \leftarrow \text{LowVarianceResample}(\overline{\chi}_t, W_t)$ 
end for
return  $\chi_t$ 
end function

```

1) Particle Filter—The particle filter (Algorithm 4) is a sequential Monte Carlo sampling approach that is a nonparametric implementation of the Bayesian filter [2]. In the particle filter, the posterior belief is represented by set of random samples drawn from the posterior. These samples are known as *particles*. Given a large enough set χ of particles, the distribution of the set approximates the true posterior belief $bel(x_t)$.

The algorithm constructs its current particle set χ_t from its previous estimate χ_{t-1} , a current control input u_t and a measurement z_t . Like the general Bayesian filter, it has two basic steps of operation. First, for each particle x_{t-1}^i in χ_{t-1} , it generates a hypothetical current state x_t^i based on the control u_t and the state transition distribution. Then the newly-generated particles are assigned weights w_t^i based on the measurement evaluation and resampled according to their weight, which tends to eliminate particles which are unlikely to be the true state of the system.

2) Unscented Kalman Filter—Like the particle filter, the Kalman filter [17] is an implementation of the general Bayesian filter algorithm. The belief at time t is represented by a mean μ_t and a covariance matrix Σ_t , and like the particle filter is updated using a current control input u_t and a measurement z_t . The basic Kalman filter algorithm assumes linear system dynamics, *i.e.*, that the current state is a linear function of the last state and that observations are linear functions of the state. Because these properties cannot be guaranteed in the system being studied here, it is necessary to linearize the system. This is done using the unscented Kalman Filter (UKF) [35].

Algorithm 5

The unscented Kalman filter algorithm. R_t and Q_t are the process and prediction noise covariance matrices, respectively. χ is the vector of sigma points, and μ_t and Σ_t are the mean and covariance of the current state estimate. In the calculations of sigma points below $\beta = \sqrt{n+\lambda}$, where n is the dimensionality and $\lambda = \alpha^2(n+\kappa)-n$, with the scaling parameters α and κ determining the sigma points' spread from the mean.

function UnscentedKalmanFilter($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$)

```

 $\chi_{t-1} \leftarrow [\mu_{t-1}, \mu_{t-1} + \beta \sqrt{\Sigma_{t-1}}, \mu_{t-1} - \beta \sqrt{\Sigma_{t-1}}]$ 

 $\chi_t^* \leftarrow g(\mu_t, \chi_{t-1})$ 

```

$$\bar{\mu}_t \leftarrow \sum_{i=0}^{2n} w_m^i \chi_t^{*i}$$

$$\bar{\Sigma}_t \leftarrow \sum_{i=0}^{2n} w_c^i (\chi_t^{*i} - \bar{\mu}_t)(\chi_t^{*i} - \bar{\mu}_t)^T + R_t$$

$$\bar{\chi}_t \leftarrow [\bar{\mu}_t, \bar{\mu}_t + \beta \sqrt{\bar{\Sigma}_t}, \bar{\mu}_t - \beta \sqrt{\bar{\Sigma}_t}]$$

$$\bar{Z}_t \leftarrow h(\bar{\chi}_t)$$

$$\hat{z}_t = \sum_{i=0}^{2n} w_m^i \bar{Z}_t^i$$

$$S_t = \sum_{i=0}^{2n} w_c^i (\bar{Z}_t^i - \hat{z}_t)(\bar{Z}_t^i - \hat{z}_t)^T + Q_t$$

$$K_t \leftarrow \sum_{i=0}^{2n} w_c^i (\bar{\chi}_t^i - \bar{\mu}_t)(\bar{Z}_t^i - \hat{z}_t)^T$$

$$\mu_t \leftarrow \bar{\mu}_t + K_t(z_t - \hat{z}_t)$$

$$\Sigma_t \leftarrow \bar{\Sigma}_t - K_t S_t K_t^T$$

return μ_t, Σ_t

end function

The UKF (Algorithm 5) uses the *unscented transform* to create an approximation of the underlying function by the use of *sigma points*. These are denoted by χ and are located at the mean μ_t and along the principal axes of the covariance matrix Σ_t at a symmetric distance from μ_t . Each sigma point χ_i has two weights associated with it, w_m^i , which is used to calculate $\bar{\mu}_t$, and w_c^i which is used to calculate $\bar{\Sigma}_t$. These sigma points are used as input to the state transition probability function g . This is used to generate $\bar{\mu}_t$ and $\bar{\Sigma}_t$, the mean and covariance of the transformed Gaussian. The same procedure is then used to produce $\bar{Z}_t$, the vector of likelihoods of observations given the calculated sigma points.

C. Probabilistic Motion Models

This section describes the probabilistic motion models used with the needle and target tracking algorithms. These models are used to represent the motion of the needle and target in the Kalman and particle filters. The motion models specify the state transition probability density functions for the needle and the target and are used in the prediction step of the Bayesian filters used in the algorithms.

1) Target Motion Model—The target is assumed to be a spherical object, with radius R , undergoing Brownian random motion to account for the displacements of the target as a result of unmodeled tissue deformations. The state of the target at time t consists of the x, y ,

z coordinates of the center of the target, and will be denoted as $x_{g,t}$. Then each coordinate of the state of the target at time $t + \Delta_t$ is drawn from a Gaussian probability density function

$$p(x_{g,t+\Delta_t}|x_{g,t}) = \mathcal{N}(x_{g,t+\Delta_t}; x_{g,t}, \phi \Delta_t) \quad (1)$$

where ϕ is the variance of the Brownian motion for unit time, and $\mathcal{N}(\cdot; \mu, \sigma^2)$ is the normal distribution function with mean μ and variance σ^2 .

2) Needle Motion Model—The needle is modeled kinematically as a piecewise array of K cubic splines, γ (Fig. 2). At time t , the shape of the needle is given by the parametric equation

$$\gamma_{j,t}(\gamma) = \mathbf{a}_{j,t} + \mathbf{b}_{j,t}\lambda + \mathbf{c}_{j,t}\lambda^2 + \mathbf{d}_{j,t}\lambda^3, \\ j=0..K-1, 0 \leq \lambda \leq 1, \quad (2)$$

where $\mathbf{a}_{j,t}$, $\mathbf{b}_{j,t}$, $\mathbf{c}_{j,t}$, $\mathbf{d}_{j,t} \in \mathbf{R}^3$ are the coefficients of the spline segment j at time t , K is the number of segments, and λ is the parameter of the spline. The continuity of the needle is enforced by the boundary conditions

$$\gamma_{j,t}(1) = \gamma_{j+1,t}(0) \quad (3)$$

$$\frac{\gamma'_{j,t}(1)}{\|\gamma'_{j,t}(1)\|} = \frac{\gamma'_{j+1,t}(0)}{\|\gamma'_{j+1,t}(0)\|}, \\ j=0..K-2, 0 \leq \lambda \leq 1 \quad (4)$$

i.e., the last point of a spline is the first point of the next spline, and the tangents of the splines are equal at the shared points. The state of the spline at time t , denoted by γ_t , can then be defined uniquely by the Cartesian coordinates of the endpoints of all its control points and their tangents, subject to the continuity conditions above.

The insertion of the needle is modeled as three distinct motion phases (Algorithm 6). The first phase is the change in the needle configuration as a result of the motion of the needle base, *i.e.*, the displacement of the entry point of the needle into the tissue and the change of its tangent direction. The second phase is the axial insertion of the needle into the tissue. The third phase is the addition of random perturbations to the shape of the needle due to various unmodeled tissue-needle interaction forces (*e.g.*, friction, tissue deformation, etc.), which modify the state of the needle by perturbing the positions and tangents of the control points. Figure 3 shows examples of probabilistic needle motion resulting from Algorithm 6. Phases 1 and 2 of this procedure are presented in the Appendix.

Algorithm 6

Algorithm for calculating the current needle state γ_{t+1} from the last needle state γ_t and the needle control input u_t^N , which consists of the displacement of the needle entry point $\mathbf{d}_b$, the rotation of the needle base tangent direction R_b , insertion length l_{ins} , and rotation R_{tip} of the needle tip insertion direction. The model includes random perturbations to the needle motion. The calculation of the perturbation terms $\mathbf{d}_p$, R_{p1} , l_p , and R_{p2} has not been included due to space constraints.

```

function NeedleMotion( $\gamma_t, \mathbf{d}_b, R_b, l_{ins}, R_{tip}$ )
     $\hat{\mathbf{d}}_b \leftarrow \mathbf{d}_b + \mathbf{d}_p$ 
     $\hat{R}_b \leftarrow R_{p1} R_b$ 
     $\hat{l}_{ins} \leftarrow l_{ins} + l_p$ 
     $\hat{R}_{tip} \leftarrow R_{p2} R_{tip}$ 
     $\gamma_1 \leftarrow \text{NeedleBaseMotion}(\gamma_t, \hat{\mathbf{d}}_b, \hat{R}_b)$ 
     $\gamma_2 \leftarrow \text{NeedleInsertion}(\gamma_1, \hat{l}_{ins}, \hat{R}_{tip})$ 
     $\gamma_{t+1} \leftarrow \text{RandomPerturbation}(\gamma_2)$ 
return  $\gamma_{t+1}$ 
end function

```

D. Probabilistic Measurement Models

In many medical imaging technologies such as ultrasound and computed tomography, the images are acquired serially and each image represents a single slice of the patient. These images are typically processed using an image processing algorithm of some type in order to detect features of interest. Here, the features of interest are the presence or absence of the target or the needle in the image, and if either is present, to determine its location in the image. As with any sensor system, measurement of target and needle locations using the intra-operative medical imaging system is prone to measurement errors due to noise. The measurement models described here explicitly model the inherent uncertainty in the simulated sensor measurements, characterized in the form of a conditional probability distribution function $p(z_t|x_t)$. Here, x_t is the system state and includes the needle and target locations as well as the current image plane, and z_t is the measurement vector.

In the following measurement models, image processing is treated as a black box. The specific image processing algorithms employed are presented in Section III.²

1) Needle Measurement Model—The output of the needle measurement model is two logical variables, $z_{nd,t}$ and $z_{td,t}$ which indicate, respectively, whether the needle and target have been detected in the current image. Its output is two image coordinate variables, $z_{nc,t}$ and $z_{tc,t}$ which indicate the locations of the detected needle and target on the image plane. Furthermore, the measurements of the needle and the target are assumed to be independent,

²The same measurement models are employed in both the simulation studies (Section IV) and the experiments with real magnetic resonance imaging data (Section V).

i.e., $p(z_t|x_t) = p(z_{n,t}|x_t)p(z_{g,t}|x_t)$. For notational simplicity, the measurement outputs for detection and location the needle and target are denoted by $z_{n,t}$ and $z_{g,t}$ respectively.

The needle measurement model using the intra-operative imaging system models both types of needle detection errors, i.e., false positives and misses, as well as error in image locations of correctly detected needle. In a noise-free sensor, the needle would be detectable in an image if the needle intersects the current image plane, and the intersection point on the image plane would be the corresponding image coordinates of the detected needle. The visibility of the needle in the ideal noise-free image will be denoted by the logical variable I_n , where a true value corresponds to an actual intersection between the needle and the image plane, and the corresponding image coordinates will be denoted by q_n .

In the actual (noisy) imaging system, the needle imaging is assumed to have true positive and false positive rates of $\alpha_{n,tp}$ and $\alpha_{n,fp}$, respectively. Then, the true and false positive probabilities can be written, respectively, as:

$$\begin{aligned} p(\text{TP}|x_t) &= \begin{cases} \text{if } I_n=\text{true}: & \alpha_{n,tp} \\ \text{if } I_n=\text{false}: & 0 \end{cases}, \\ p(\text{FP}|x_t) &= \begin{cases} \text{if } I_n=\text{true}: & (1-\alpha_{n,tp})\alpha_{n,fp} \\ \text{if } I_n=\text{false}: & \alpha_{n,fp} \end{cases}. \end{aligned} \quad (5)$$

Adding the true and false positive rates yield the needle detection probability

$$\begin{aligned} p(z_{nd,t}=\text{true}|x_t) &= p(\text{TP}|x_t) + p(\text{FP}|x_t) \\ &= \begin{cases} \text{if } I_n=\text{true}: & \alpha_{n,tp} + (1-\alpha_{n,tp})\alpha_{n,fp} \\ \text{if } I_n=\text{false}: & \alpha_{n,fp} \end{cases} \end{aligned} \quad (6)$$

and

$$p(z_{nd,t}=\text{false}|x_t) = 1 - p(z_{nd,t}=\text{true}|x_t). \quad (7)$$

For a true positive (TP), the measured needle location in the actual (noisy) imaging system is assumed to have zero-mean Gaussian noise. For a false positive (FP), the measured locations is assumed to be uniformly distributed over the image plane. Then, for each of the two coordinates $i = 1, 2$ on the image,

$$p(z_{nc,t}^i|\text{TP}, x_t) = \mathcal{N}(z_{nc,t}^i; q_n^i, \sigma_{n,i}^2), \quad (8)$$

$$p(z_{nc,t}^i | FP, x_t) = \mathcal{U}(z_{nc,t}^i; \min_i, \max_i), \quad (9)$$

where $\min_i, \max_i$ is the minimum and maximum coordinates of the image plane in the corresponding dimension, and $\mathcal{U}(\cdot; \min, \max)$ is the uniform distribution function in the domain $[\min, \max]$.

Combining the probability density functions for the needle detection and coordinates, using the conditional probability and total probability equations, yields the probability density function of the needle measurement model as:

$$\begin{aligned} p(z_{nd,t}, z_{nc,t}^1, z_{nc,t}^2 | x_t) &= p(z_{nc,t}^1, z_{nc,t}^2 | z_{nd,t}, x_t) p(z_{nd,t} | x_t) \\ &= p(z_{nc,t}^1, z_{nc,t}^2 | TP, x_t) p(TP | x_t) + p(z_{nc,t}^1, z_{nc,t}^2 | FP, x_t) p(FP | x_t) \\ &= p(z_{nc,t}^1 | TP, x_t) p(TP | x_t) p(z_{nc,t}^2 | TP, x_t) p(TP | x_t) + p(z_{nc,t}^1 | FP, x_t) p(FP | x_t) p(z_{nc,t}^2 | FP, x_t) p(FP | x_t), \end{aligned} \quad (10)$$

where, in the last step, the independence of the measured needle location's noise in the two dimensions is used.

It is important to note that, in the derivations above, the dependence of the needle measurement to the configuration of the imaging plane has not been explicitly included in the equations to simplify the notation.

III. Image Processing Algorithms

This section presents the image processing algorithms used in experiments using real intra-operative imaging data. The needle is detected by looking for dark pixels near the expected intersection of the needle with the image plane. Because the problem formulation assumes that the target is spherical, the image processing method used to detect is to search for objects of the expected shape.

A. Image Processing for Needle Detection

The speed of an intra-operative imaging system is inversely proportional to the resolution of its images. As such, the needle detection model described here is intended to be capable of detecting a needle in low-resolution intra-operative imaging data, in which the cross-sectional area of the needle is small relative to the imaging resolution. Additionally, the images contain noise, the magnitude of which may be large relative to the difference between the needle and the background data. Because of this, it is very difficult to definitively identify the needle in any given image if the search is done in a global manner.

The image processing algorithm makes the assumption that the needle's approximate insertion location is known and that a model exists that can reliably detect the needle when its approximate location is known. Then, as the needle is inserted into the medium, the model biases its search so that locations near the expected needle location are given priority.

This measurement bias is done using the current needle estimate provided by the Bayesian filter. This is done as follows: 1) The image I which was acquired by the intra-operative imaging system is filtered using a point-detection mask H to obtain the intermediate image I' . 2) The intersection point p_{N,P_I} of the current needle estimate and the image I is calculated. 3) I' is multiplied with a 2-dimensional Gaussian function G centered at p_{N,P_I} to produce I'' . 4) If the highest-valued pixel q_{max} in I'' exceeds a preset threshold T , the location q_{max} is returned as the detected needle.

For the particle filter, this is done by first finding the intersection of each needle particle with the image plane. The intersection of the i th needle particle with the image plane is considered the mean μ_i of a Gaussian function on the plane. The standard deviation σ of each Gaussian is set to a small constant, and the procedure above is followed. The final needle detection is done by a majority vote procedure of the candidate detections produced by each needle particle, as shown in Figure 4.

For the Kalman filter, the mean μ of the Gaussian is centered at the intersection of the image plane and the Kalman filter's needle belief. The standard deviation σ is calculated by using the unscented transform to find sigma points on the plane, which define the spread of the needle at one standard deviation.

B. Image Processing for Target Detection

Because this algorithm is designed to detect targets which are roughly spherical, the algorithm for detecting a target in a given image is straightforward. The original image's gradient is convolved with a Sobel edge detecting filter and the result cropped to remove the edge of the gel [13]. Next, an elliptical Hough transform[37] is applied to search for ellipses that approximately match the expected radius of the target in the slice. This is done a number of times, and the best-fitting ellipses are averaged to give a final location of the detected target.

The image processing algorithm for target detection is purposely kept simple because of the simple nature of the phantoms under consideration here. In more complicated phantoms and in real tissue, more complicated shape detection algorithms would be used to detect the target.

IV. Simulation Results

The proposed algorithms were initially validated in simulation, by evaluating the needle and target tracking performances in a simulated needle insertion task.³ In these simulation experiments, a simulated magnetic resonance imaging system is used as the intra-operative

³The simulation studies reported in this section, and the experiments using real magnetic-resonance imaging data reported in Section V employed the same algorithms and models, except for image processing. Specifically, no image processing was performed in

imaging system. The slices of the magnetic resonance images are orthogonal to the initial needle insertion direction for simplicity. The scanner has needle and target measurement error variances of $\sigma_{n,i}^2 = \sigma_{g,i}^2 = (0.2 \text{ mm})^2$, with $a_{n,fp} = 0.02$, $a_{n,tp} = 0.98$, $a_{g,fp} = 0.01$, and $A_o = 75 \text{ mm}^2$. In order to reduce the computational complexity, at the expense of reduced resolution, MR scanner image planes are assumed to be positioned in 1 mm increments. The needle is aimed at a target with a radius of approximately 5mm that is located approximately 60 mm from the needle entry point. During insertion, the needle bevel angle and control point locations are perturbed slightly, and the target undergoes Brownian motion with $\phi \ t = (0.1 \text{ mm})^2$. The needle was inserted straight towards the target in 10 steps, 10 mm per step.

Although the model used in the active localization algorithm assumes a nominally straight needle, the actual needle used in the simulations were given a bevel tip that varied from 0° to 6° tip deflection per insertion step. The total angular deviation of the needletip therefore varies up to 60° and the total needletip deviation from nominal was approximately 40mm for the 100mm insertion. The needle is inserted in open loop mode, without any explicit feedback control, in order to investigate if the needle is successfully localized or not.

A. Particle Filters

The particle filter algorithm is suitable for both local and global localization problems since particle filters can accommodate arbitrary (including, multi-modal and non-Gaussian) belief probability distributions. In order to validate both local and global localization performance of the particle filter-based algorithm, the initial target position was assumed to be unknown to the localization algorithm. As a result, the needle insertion task used in the simulation was set up as two phases, namely, initial target localization and needle insertion. Prior to the needle insertion, the target region was scanned linearly in 15 steps to determine the initial location of the target. In this initial target localization phase, only the target localization algorithm was executed since the needle insertion has not started yet, and the imaging was performed for each image plane in a linear fashion. Once the target was localized, the needle insertion proceeded as usual, during which both the needle and target localization algorithms were executed. During the initial target localization phase, 500 target particles were used to localize the target. During the needle insertion phase, 25 target particles and 100 needle particles were used for tracking, respectively, the target and the needle.

The performance of the particle filter algorithm was measured by performing 10 simulated needle insertions for this experiment and measuring the error between the actual and estimated (using expected value of the belief functions) locations of the needle tip and target. Fig. 5 shows the active localization algorithm's beliefs of the needle and target at four different steps in a sample execution of the task. As it can be seen from the figure, the active localization algorithm was successfully able to accurately capture the deviated shape of the needle.

simulation studies, as the simulation directly generated the output of the image processing algorithm; whereas, the experiments with real imaging data employed the image processing algorithms presented in Section III.

The resulting absolute needle tip and target localization errors were 1.07 ± 1.19 mm and 0.83 ± 0.34 mm (mean $\pm$ standard deviation), respectively. The needle tip tracking errors per needle bevel are shown in Table I.

B. Unscented Kalman Filters

While the unscented Kalman Filter is suitable for tracking an object, it is not suitable to solving a global localization problem since the unscented Kalman filter assumes that the beliefs are Gaussian. Therefore, the initial target localization phase is not performed with the unscented Kalman filter. Instead, an initial estimate of the target location is assumed to be available to the UKF, and the algorithm directly proceeds with needle insertion, following a similar procedure described above for the particle filter case. As with the particle filter algorithm, the performance of the unscented Kalman filter implementation was measured by performing 30 repetitions of an open-loop insertion experiment with a simulated needle with a bevel tip that varied from 0 to 6 degrees.

Fig. 6 shows the active localization algorithm's beliefs of the needle and target at the end of one localization trial. As with the particle filter, it can be seen that the active localization algorithm was successful in tracking the needle as it deformed during insertion.

The resulting absolute needle tip and target localization errors were 0.58 ± 0.53 mm and 1.85 ± 0.45 mm (mean $\pm$ standard deviation), respectively. The needle tip deviations are given in Table II. These results demonstrate that the Kalman filter, with no knowledge of the needle's curvature, is capable of accurately tracking a needle with reasonable deformation.

These results indicate that, given imaging algorithms that can detect the objects with some reliability, both the particle filter and the unscented Kalman filter are capable of successfully tracking a needle and a target as the needle is inserted into a medium.

V. Experimental Validation

This section describes the experiments done using real imaging data collected from experiments in which a needle was inserted into a tissue phantom. This was done to demonstrate the viability of the algorithms on non-simulated data. As with the simulation experiments, tracking was performed by particle filters and unscented Kalman Filters and the results compared. The models used in the experiments and the experimental results are as follows.

A. Data Collection

Data were collected using magnetic resonance imaging during trials in which a needle was inserted into a 1% agarose gel medium in which a target was suspended roughly 80mm from the needle entry point. The needle was inserted approximately 10mm in open-loop fashion at each step, and after each step the entire volume was imaged. This was repeated until the needle reached the edge of the gel. There was no external motion induced on the target, i.e., target embedded in the tissue phantom only moved in response to the needle-tissue interaction forces.

Magnetic resonance imaging was chosen as the intraoperative imaging methodology because it provides the ability to image an entire volume, allowing the data to be sliced in arbitrary ways in post-processing, and providing a known reference frame. In order to collect the data in a manageable time, data were collected at a relatively low resolution - the volume of data collected at each timestep was $64 \times 128 \times 256$ voxels, each voxel having dimensions of $2 \times 1 \times 1$ mm. At this resolution, the MR system took approximately two minutes to image the volume at each timestep. During data collection, the entire volume of the gel phantom was imaged at each timestep. This was done so that the active localization algorithms would be able to arbitrarily select any one image plane during each processing step during offline analysis, thus allowing the processing to be performed exactly as it would be during an actual needle-insertion procedure in which full-volume imaging at each timestep takes an infeasible amount of time.

Because the data were collected using a 1.5 Tesla MR scanner, a standard steel surgical needle could not be safely used in this study. Instead, a nylon ersatz needle was used and fed into the gel using a plastic feeding device built for the purpose. A bevel tip was cut in the needletip, enabling it to be bent in order to provide a greater or lesser degree of needle flexion during the trial, as desired.

Data were collected during three open-loop needle insertion trials - one in which the needle hit the target, one in which the needle closely missed the target, and one in which the needle widely missed the target. The lateral deviation of the needletips from straight for these trials was approximately 21mm, 19mm, and 50mm, respectively.

Unlike the simulated experiments, the target position is fixed during the trials. Additionally, while the needle shape may deform due to the interaction with the tissue phantom, the needle does not fluctuate stochastically in the same way that the needle in the simulation experiments did.

Figure 7 shows example images slices from each of the three trials, showing the needle curvature.

In order to simulate the operation of an intraoperative imaging system, the imaging data were sliced such that the slices were orthogonal to the initial needle insertion direction. The needle location in each slice was marked manually for later evaluation purposes and the marked files stored separately, and the unmarked data were used by the imaging algorithm as input data. Figures 8 shows sample images containing a target and a needle, respectively.

B. Experimental Results

The above measurement models were implemented using MATLAB and the algorithms were tested using the three magnetic resonance imaging data trials. The performance of the needle filter was evaluated by measuring the error between the real needletip, which was determined by marking the image slices manually, and the filter's belief of the needletip location. The performance of the target filter was similarly measured by finding the difference between the target filter's belief and the real target location. Because the data is

comprised of asymmetric voxels, errors are reported both in units of millimeters and voxel units.

Because the particle filter is a stochastic algorithm, the results vary by trial and the mean and standard deviation are reported. The only stochastic elements in the UKF algorithm are in the sampling which is used in the entropy minimization step. As the number of samples used in this step increases, the algorithm converges on the ideal solution. In the trials used in this study, the algorithm chose an identical sequence of image planes for each trial, and consequently the error for each trial was identical.

Table III shows the results for the particle filter tracking the three needle insertion trials. Table IV shows the results in voxels taking into account asymmetric voxels. These results indicate that the algorithms are able to track the needle to within 1.5mm (1.5 voxels) during the trials where the needle deviation is approximately 20mm from straight, and to within 4.5mm (4.5 voxels) during the trial where the needle deviation is approximately 50mm. Target localization errors are within 4mm (2.5 voxels). Table V shows the results for the Kalman filter. The results are comparable, with needle tracking within 1.5mm (1.5 voxels) for small deflection error case and 5.3mm (5 voxels) for large deflection error case, and target tracking within 2mm (2 voxels).

Figure 9 shows the needle and target being tracked by the particle filter during one trial, and demonstrates the entropy minimization algorithm calculating the image plane that will produce the largest expected information gain in the next timestep.

VI. Conclusions

This work serves as a proof of concept for an active sensing approach to needle and target localization during needle insertion procedures that are guided by intra-operative imaging system. In trials using real data, the algorithms are able to localize the needle to a mean precision of approximately 5mm (5 voxels) in all cases and a mean precision of approximately 1.5mm (1.5 voxels) in cases where the needle bevel is not severe. In all experiments, the needle model assumed a straight needle, and had no knowledge of the bevel of the real needle. The experiments with a severe needle bevel were performed in order to establish the algorithm's performance when its needle model is highly erroneous. The needle was found to perform well until a per-step angular deviation error of approximately 6° was encountered. This serves to establish a reasonable upper bound for acceptable performance of the system with the parameters used in these experiments.

At first sight, the absolute values of the needle tracking errors reported in Tables III and V may appear large, especially in comparison with typical clinical (manual) needle insertion accuracy levels. However, in the hardware experiments, the needle tip was intentionally modified to create an exaggerated beveling of the tip that would lead to deflection of the needle from a straight needle path. The lateral deviation of the needle from straight insertion path for each of the three experimental trials were approximately 21mm, 19mm, and 50mm, respectively (which would lead to such needle insertion errors in a manual insertion). In the experiments, the needle localization/tracking algorithm assumed a straight nominal needle

tip (leading to a straight needle insertion path), and, as such, the proposed localization/tracking algorithm was able to eliminate approximately 90% of the lateral deviation error in each of the trials. It is also important to note that, the proposed algorithm would also be able to account for a known bevel angle of the needle through the R_{tip} parameter (Algorithm 8).

The image processing algorithms demonstrated here have shown the ability to extract target and needle feature information from noisy images. In particular, the needle detection algorithm shows the ability to detect a needle which has a cross-sectional area less than a pixel, and is able to accurately detect the needle in image slices where the needle is obscured by noise. Intra-operative imaging data of greater precision would lead to more precise localization.

The presented study focused on proof-of-concept of the proposed active localization and tracking methods. As such, this work was not done with computation time as a chief consideration. Specifically, most of the code base is written in MATLAB r2014a, which is an interpreted language, for ease of implementation, debugging, and validation. For a reasonable number of particles or samples, the particle filter and unscented Kalman filter implementations both compute each timestep in approximately 2–4 minutes using an Intel Core Duo CPU at 3.16 GHz. For a production system, considerable speedup would be achieved by rewriting the implementations in a faster language and/or parallelizing the algorithms. Furthermore, computation time can be reduced by such methods as using the geometric knowledge of the system state to reduce the search space for the entropy minimization algorithm. Additionally, on-line real-time execution of the active sensing algorithm requires substantial engineering work for low-level interfacing to the hardware of the magnetic resonance imaging scanner, to be able to control the imaging parameters on-line. This system integration work was outside the scope of the present study, and has been left for future work.

In the present study, the proposed algorithm was validated in simulation, and on benchtop using tissue phantoms. Further validation of the developed algorithms and methods in *ex vivo* and *in vivo* studies has been left for future work. Images of heterogenous tissues (as would be encountered in *ex vivo* and *in vivo* studies) are noisier and susceptible to image artifacts, and, as such, may require specialized image processing algorithms to address. *In vivo* studies would introduce additional challenges resulting from complex physiological motions of the tissue (e.g., due to breathing).

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Appendix

This appendix presents the phases of needle motion in more detail. Needle motion consists of three phases: perturbation of the insertion point and propagation of the perturbation (Algorithm 7), axial insertion of the needle (Algorithm 8), and addition of random perturbations. Figure 10 shows an example of the axial insertion procedure.

Algorithm 7

The first needle motion phase is the calculation of the change in the spline array caused by translational and rotational motions $\mathbf{d}_b$ and R_b of its insertion point. This is accomplished by perturbing the first spline's base and then propagating the perturbation through all K splines in the spline array under the assumption that the spline is rigid. The perturbed spline parameters are found and mixed with the previous spline parameters according to a weight parameter w . The mixing function is omitted here for brevity.

```

function NeedleBaseMotion( $\gamma$ ,  $\mathbf{d}_b$ ,  $R_b$ )
  for  $i \leftarrow 0 \rightarrow K-1$  do
     $\mathbf{p}_0 \leftarrow \gamma(0) + \mathbf{d}_b$ 

     $\mathbf{t}_0 \leftarrow \frac{R_b \gamma'_i(0)}{\|R_b \gamma'_i(0)\|}$ 

     $\mathbf{p}_1 \leftarrow (\gamma(0) + \mathbf{d}_b) + R_b(\gamma(1) - \gamma(0))$ 

     $\mathbf{t}_1 \leftarrow \frac{R_b \gamma'_i(1)}{\|R_b \gamma'_i(1)\|}$ 

     $\hat{\gamma}_i \leftarrow [\mathbf{p}_0, \mathbf{t}_0, -3\mathbf{p}_0 + 3\mathbf{p}_1 - 2\mathbf{t}_0 - \mathbf{t}_1, 2\mathbf{p}_0 - 2\mathbf{p}_1 + \mathbf{t}_0 + \mathbf{t}_1]$ 

     $\mathbf{d}_b \leftarrow \mathbf{p}_1 - \gamma(1)$ 
  end for

  return WeightedCubicSplineMixture( $\hat{\gamma}, \gamma, w$ )
end function

```

Algorithm 8

In the second needle insertion phase, the needle is inserted axially by some distance l_{ins} . A new quadratic spline extension γ_{ext} is found which has starting point $\mathbf{p}_s$, starting tangent $\mathbf{t}_s$, ending tangent $\mathbf{t}_e$, and length l_{ins} . Finally, convex optimization is used to find the minimum-curvature cubic spline of length $\text{length}(\gamma) + l_{ext}$ that most closely fits the composite spline found by concatenating the spline extension to the end of the previous spline (Figure 10).

```

function NeedleInsertion( $\gamma$ ,  $l_{ins}$ ,  $R_{tip}$ )

```

$$\mathbf{t}_1 \leftarrow \frac{\gamma'_{K-1}}{\|\gamma'_{K-1}\|}$$

```


$$\mathbf{t}_2 \leftarrow R_{tip} \frac{\gamma'_{k-1}}{\|\gamma'_{k-1}\|}$$



$$\mathbf{n} \leftarrow \frac{\mathbf{t}_2 \times \mathbf{t}_1}{\|\mathbf{t}_2 \times \mathbf{t}_1\|}$$



$$\mathbf{u} \leftarrow \frac{\mathbf{t}_1 + \mathbf{t}_2}{\|\mathbf{t}_1 + \mathbf{t}_2\|}$$



$$\mathbf{v} \leftarrow \mathbf{n} \times \mathbf{u}$$



$$R_{WL} \leftarrow [\mathbf{u} \ \mathbf{v} \ \mathbf{n}]$$



$$\theta \leftarrow \text{atan2}(\|\mathbf{t}_2 \times \mathbf{t}_1\|, \mathbf{t}_1 \cdot \mathbf{t}_2)$$



$$d \leftarrow \frac{2l_{ins}}{\text{asinh}(\tan \frac{\theta}{2} + \cot \frac{\theta}{2} + \sec \frac{\theta}{2})}$$



$$\gamma_{ext} \leftarrow R_{WL} \begin{bmatrix} 0 & d & 0 \\ 0 & d \tan(\frac{\theta}{2}) & -d \tan(\frac{\theta}{2}) \\ 0 & 0 & 0 \end{bmatrix} + [\gamma(1) \ \mathbf{0} \ \mathbf{0}]$$



$$\gamma' \leftarrow \text{AppendSplines}(\gamma \ \gamma_{ext})$$



$$\gamma_{new} \leftarrow \text{CubicSplineFit}(\gamma', \text{length}(\gamma) + l_{ins})$$


return  $\gamma_{new}$ 

end function

```

Fig. 10.

One step of axial needle insertion. The original spline (blue) is augmented with a spline extension (green) and a minimum-curvature final spline (red) is fitted.

Fig. 1.

The Markov model of the system. The needle state x_t^N depends on the previous state x_{t-1}^N and the needle control input u_t^N , while the target state x_t^T depends only on the previous target state. The measurement z_t depends on the target state, needle state, and the measurement control input u_t^i which is determined by the active localization algorithm.

Fig. 2.

Schematic representation of the spline array used to model the needle. The two splines γ_0 and γ_1 are uniquely defined by their endpoints and the tangents of the endpoints.

Fig. 3.

(a) An example 4 control point needle, showing flexion of the needle as it is partially inserted into the simulated tissue. (b) 20 samples from the posterior distribution of the needle shape after execution of the needle command shown in (a).

Fig. 4.

An image plane showing needle particle intersections with the image plane (red), the standard deviation of the Gaussians (green) and the detected needle (blue).

Fig. 5.

The particle filter tracking the needle and target. The blue line (invisible) is the actual needle shape, the green lines are the shapes of the particles in the needle belief function, the large semi-transparent sphere is the actual target, the red circles are the centers of the particle in the target belief function, and the semi-transparent plane is the current imaging plane. (a) Initial particle distribution when the location of the target is unknown. (b) The target is localized at the end of the linear scanning phase. (c) Particle distribution at the mid point of the needle insertion. (d) Particle distribution at the end of the task. Figures show that the algorithm successfully localizes the needle and target configurations.

Fig. 6.

The unscented Kalman filter tracking a simulated needle and target. The blue line is the actual needle shape, the red line is the UKF filter's mean needle belief, and the red sphere represents the target belief.

Fig. 7. MR images of the three needle insertion trials. The needle is not completely visible in any single image slice.

(a)

(b)

Fig. 8.

(a) A sample orthogonally-sliced MR image showing the target. (b) A sample orthogonally-sliced MR image showing the needle intersecting with the image plane. The needle is very faint relative to the background noise in this image and is indicated by the arrow.

Fig. 9.

(a) The particle filter tracking the needle. Green lines represent the particles comprising the filter's belief and red circles represent the target filter's particles. The true target position is given by the dark ellipsoid and the black dots represent the real needle position. (b) The active localization algorithm calculating the image plane that it will use in the next timestep according to the system state shown in (a). The thick blue line shows the total expected information gain of each imaging plane, while the red line shows the contribution expected from the needle, and the dashed green line shows the expected information gain due to the target. (c) The particle filter needle detection image processing algorithm. (d) The result of the image processing algorithms detecting the needle and target locations.

TABLE I

Needle localization mean errors and standard deviation (in mm) for the particle filter in simulation.

deflection angle per step	mean error (std. dev.)
0°	0.41 (0.13)
1°	0.64 (0.10)
2°	0.75 (0.51)
3°	0.58 (0.36)
4°	0.41 (0.14)
5°	0.73 (0.43)
6°	3.81 (1.19)

TABLE II

Needle localization mean errors and standard deviation (in mm) for the UKF filter in simulation.

deflection angle per step	mean error (std. dev.)
0°	0.05 (0.00)
1°	0.13 (0.00)
2°	0.21 (0.03)
3°	0.28 (0.06)
4°	1.28 (0.20)
5°	1.40 (0.04)
6°	0.70 (0.13)

TABLE III

Needle and target localization errors in mm for the particle filter tracking real data.

trial	needle error (std. dev.)	target error (std. dev.)
1	1.26 (0.04)	3.84 (2.03)
2	1.44 (0.25)	3.87 (1.70)
3	4.34 (0.43)	2.01 (1.04)

TABLE IV

Needle and target localization errors in voxel units for the particle filter tracking real data.

trial	needle error (std. dev.)	target error (std. dev.)
1	1.23 (0.05)	3.24 (2.23)
2	1.43 (0.25)	3.58 (1.87)
3	4.33 (0.44)	1.95 (1.23)

TABLE V

Needle and target localization errors for the unscented Kalman filter tracking real data in mm and voxels.

trial	needle error		target error	
	mm	voxels	mm	voxels
1	1.27	1.13	1.52	1.16
2	1.42	1.41	1.81	1.61
3	5.23	4.76	1.45	1.42