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Abstract. Dynamic reference frames (DRFs) are a common component of modern surgical tracking systems; however, the limited number of commercially available DRFs poses a constraint in developing systems, especially for research and education. This work presents the design and validation of a large, open-source library of DRFs compatible with passive, single-face tracking systems, such as Polaris stereoscopic infrared trackers (NDI, Waterloo, Ontario). An algorithm was developed to create new DRF designs consistent with intra- and intertool design constraints and convert to computer-aided design (CAD) files suitable for three-dimensional printing. A library of 10 such groups, each with 6 to 10 DRFs, was produced and tracking performance was validated in comparison to a standard commercially available reference, including pivot calibration, fiducial registration error (FRE), and target registration error (TRE). Pivot tests showed calibration error (mean $\pm$ std) = 0.46 ± 0.1 mm, indistinguishable from the reference. FRE was 0.15 ± 0.03 mm, and TRE in a CT head phantom was 0.96 ± 0.5 mm, both equivalent to the reference. The library of DRFs offers a useful resource for surgical navigation research and could be extended to other tracking systems and alternative design constraints. © 2018 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: [10.1117/1.JMI.5.2.021215](https://doi.org/10.1117/1.JMI.5.2.021215)]

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1 Introduction

Biomedical researchers developing new surgical navigation systems often encounter a basic limitation with dynamic reference frames (DRFs): there are a limited number of sources for such tools and a limited number of designs available. While original equipment manufacturers such as NDI (Waterloo, Ontario) and their commercial partners often develop proprietary tool designs, the number of DRFs readily available for purchase is limited (e.g., only three with Polaris systems). Other companies producing commercially available DRFs include IZI Medical Products (Owings Springs, Maryland) and Traxtal (Toronto, Ontario); however, these sources also tend to offer a limited number of compatible DRFs (e.g., four with IZI Medical Products). Researchers developing new navigation systems^{1,2} commonly require a greater number and diversity of DRFs, for example, to track poses or the patient and multiple instruments simultaneously. Having a larger number of simultaneously tracked tools also benefits workflow in education/training centers for surgical navigation systems.^{3,4} In such environments, which often do not require FDA-approved or sterilizable devices, in-house resources such as three-dimensional (3-D) printers could offer a convenient means to produce such DRFs.

DRF design must follow specific criteria—both with respect to individually tracked tools and to a plurality of simultaneously tracked tools. These criteria ensure unique geometric characteristics for accurate tool identification and robust localization. Researchers sometimes resort to one-off, custom-built tools that may neglect such important design constraints. As

a consequence, the results obtained in one laboratory may be difficult to rigorously reproduce in another due to suboptimal DRF tool design.

Methods to improve the robustness and accuracy of various optical tracking systems have been studied extensively within the augmented reality and image guidance communities via design principles and/or enhancements of the reference frames or the tracker. With regard to the reference frames, the form and arrangement of fiducial markers can be chosen to provide DRFs with improved tracking accuracy, including guidelines for fiducial layout and spatial encoding.⁵ In the field of augmented reality, Bergamasco et al. developed a new fiducial marker system to improve localization and estimation accuracy as well as a framework of cyclic encoding patterns to improve robustness to noise and occlusion.⁶ Alternatively, Garrido-Jurado et al. focused on designs with larger intermarker distances and color to reduce occlusion and intermarker confusion.⁷ Others have developed methods to optimize the spatial arrangement of markers to generate unique DRFs. Pintaric and Kaufmann generated DRFs using a random local search to find optimal marker arrangements, yielding noncoplanar solutions with 500 to 1500 different DRF designs.⁸ Gierlach et al. generated “marker stars” similar to the arrangements of Pintaric and Kaufmann but much smaller using rapid prototyping techniques and decreased data processing by increasing the distinguishability of marker alignment.⁹ These and other ongoing research efforts present ways to improve the accuracy of DRF tracking. In this paper, we report an open-source library of DRFs that was validated (with an NDI tracker) and ready for download and production on a 3-D printer for research use. [Computer-aided

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design (CAD) files and tool definition files are available for download from Ref. 10]

DRF designs were generated adhering to intra- and intertool constraints using exhaustive search algorithms. Ten groups of mutually-compatible tools were derived and characterized via tool definition files. Tools were validated using NDI 6D Architect software, and experiments were conducted to evaluate accuracy in pivot calibration and point-based registration in comparison to a standard NDI reference tool.

2 Methods

2.1 Dynamic Reference Frame Design

A passive infrared (IR) optical tracking system emits and detects the reflected IR light typically from retroreflective-coated spheres attached to a rigid tool frame, although retroreflective planar stickers may also be used. Once the 3-D locations of these spheres are determined with respect to the epipolar geometry of the stereoscopic camera, their geometric configuration is matched against profiles stored in tool definition files. While this process uniquely identifies a given marker configuration, certain geometric constraints need to be satisfied to robustly identify multiple DRFs simultaneously.

2.1.1 Intratool design constraints

Basic DRF design criteria pertain to the number and spatial arrangement of retroreflective spherical markers (denoted by m_i , where $i = 1 \dots M$ and M is the total number of spherical markers on a tool). As shown in Fig. 1(a), each pair of spherical markers on the tool forms a segment with a total number of segments equal to $M(M-1)/2$. A single-face DRF has an arrangement of markers that is coplanar, and a large distance between spherical markers helps to mitigate occlusion. Collinear DRFs may also be generated similar to the NDI stylus (#8700340), but they have one fewer degree of freedom than a noncollinear DRF, allowing 3-D localization but DRF orientation in only two directions. Additional specifications would have to be met for the tracker to provide all three orientations of a collinear tool. Noncoplanar DRFs are also possible though this paper focused on coplanar to reduce bulkiness.^{8,9} Single-face tools commonly have three to six spherical markers, with $M = 3$ as the minimum required for rigid registration. While a larger number of spherical markers improves robustness to partial occlusion by monotonically decreasing the TRE, it also complicates the design and

tends to increase the overall size; hence, since the most significant decrease in TRE occurs from three to four markers, we limited designs to the commonly used $M = 4$. To present a uniquely identifiable tool geometry, each segment length (defined as the distance between spherical markers, denoted d_i) must be unique and satisfy certain constraints enforced by the tracker system. The minimum distance ($d_{\min}$) allows the tracker to distinguish two DRFs that are in proximity to one another, and the minimum difference ($|\nabla d_{\min}|$) between lengths helps to determine the location and orientation of such tools in 3-D space. To yield tools of reasonable size for practical use, a maximum segment length ($d_{\max}$) is also enforced. Thus, each tool satisfies the following intrabody design constraints: (1) $d_{\min} \leq d_i \leq d_{\max}$ and (2) $|d_i - d_j| \geq |\nabla d_{\min}|, \forall (i \neq j)$.

In the experiments below, a four-marker tool constitutes six length segments. Different trackers may enforce other parameter values for geometric constraints. For example, Polaris Vicra requires $d_{\min} \geq 30$ mm and $|\nabla d_{\min}| \geq 3.5$ mm, and Polaris Spectra requires $d_{\min} \geq 40$ to 50 mm and $|\nabla d_{\min}| \geq 3.5$ mm. Other Polaris systems require $d_{\min} \geq 50$ mm and $|\nabla d_{\min}| \geq 5$ mm. To be consistent across all NDI Polaris trackers, we selected $d_{\min} = 52$ mm, $d_{\max} = 100$ mm, and $|\nabla d_{\min}| = 5$ mm in the current work, as shown in Figs. 1(a) and 2.^{11,12}

2.1.2 Intertool design constraints

Additional interbody constraints are required to robustly distinguish the pose of multiple tools, such that they can be tracked simultaneously. Given a pair of tools (A, B), two segments are said to be similar if $|d_i^A - d_j^B| < |\nabla d_{\min}|$, recognizing that such similar segment pairs may still be valid tools, provided that $|a_{ij}^A - a_{ij}^B| \geq \theta_{\min}$, where a is the angle associated with the segments, as shown in Fig. 1(b). These constraints prevent geometrically similar or mirrored tools to be included in the same set. In the experiments below, $\theta_{\min}$ was selected to be 2 deg to satisfy the NDI Polaris guidelines.¹¹

2.2 Design Methodology

2.2.1 Algorithm for generating individual tools

The brute-force search algorithm illustrated in Fig. 2 was implemented to generate potential designs for single-face, four-marker DRFs. Five segment lengths (d_{1-5} , of a total of six lengths) were systematically selected within the range of

Fig. 1 Illustration of tool design parameters and constraints. (a) Intrabody tool constraint: lengths must differ by at least $|\nabla d_{\min}| = 5$ mm. (b) Intertool tool constraint: for similar length pairs between tools (< 5 mm difference in length), the angles must differ by $\theta_{\min} = 2$ deg. (c) Example DRF (NDI #8700339) used as a reference in experiments throughout this work, annotated to highlight example lengths and angles that satisfy inter- and intrabody design requirements.

Fig. 2 Flowchart of an exhaustive search algorithm for generating a single, valid DRF. An example run within $d_{\min} = 52$ mm and $d_{\max} = 100$ mm yielded ~750,000 individual valid designs.

valid lengths. Each length d was individually incremented by 1 mm in exhaustive iteration within the range $[d_{\min}, d_{\max}]$. After each increment, all segments were checked against intrabody tool constraints, with d_6 computed using the law of cosines as shown in Fig. 2. If the design failed any intrabody constraint, then the search continued by incrementing segment lengths until exhaustion. Finally, the resulting designs that passed all criteria were then compared against the current library of designs (denoted as *toolList*) and redundancies (i.e., tools for which $[d_1, d_6]$ were equivalent under rotation or reflection) were eliminated.

2.2.2 Algorithm for generating mutually compatible groups of tools

Mutually compatible groups were generated from the large *toolList* resulting from that algorithm of Fig. 2 using a subsequent selection algorithm shown in Fig. 3. First, a subset of all tools compatible with a known initial tool was created (e.g., compatible with the NDI reference tool #8700339). A tool was randomly selected from this subset and added to the group if it passed interbody design constraints (versus tools

already added to the group). This process was repeated until the search was exhausted, resulting in a “group” of ~6 to 10 compatible tools. Since compatibility within a group depends on the existing members, multiple runs with randomly selected tools resulted in different mutually compatible groups.

2.2.3 3-D printing of dynamic reference frames

Resulting groups of tool designs (each represented simply by two cross-lengths and the interposing angle) were rendered as DRFs using OpenSCAD,¹³ as shown in Fig. 4. Mounting posts compatible with NDI retroreflective spheres were generated in SolidWorks (Dassault Systèmes, Concord, Massachusetts). A variety of 5-mm-diameter clearance holes were created along and through each cross length (with set-screw holes) for attaching extensions, such as a pointer tool. Each tool was thus rendered as CAD file ready for 3-D printing. For the experiments described below, we printed tools using a Lulzbot TAZ 6 (Lulzbot, Loveland, Colorado).

While the Lulzbot TAZ 6 can print with a variety of filament materials, the most common are ABS/ABS+ and PLA/PLA+. ABS has high strength and flexibility, is scratch and heat

Generate a group of rigid-body tools

Fig. 3 Flowchart of an algorithm for forming groups of mutually compatible tools designs from the exhaustive search *toolList* resulting from Fig. 2. In the current work, 10 such groups were formed (denoted A, B, ..., K), each containing 6 to 10 mutually compatible DRFs.

Fig. 4 Example DRF generated using the algorithm described in Fig. 2. CAD models were exported as STL files ready for 3-D printing. The models included posts for attachment of retroreflective markers at locations corresponding to $d_1, \dots, d_6$ as well as features such as posts, an X-shaped support frame; pass-through holes for attaching a pointer with set screws; and a name designator (e.g., APPLE-2 for group A, tool 2).

resistant, nonhygroscopic, and can be polished with acetone. Disadvantages of ABS include strong fumes during production and temperature-dependent shrinkage. ABS+ shares the same qualities as ABS but is better suited for printing large objects due to improved thermal stability. The DRFs used in this work were printed using PLA, which prints at a lower temperature thereby minimizing shrinkage and warping. PLA+ has the qualities of PLA but is stronger, smoother, and crack resistant. Possible disadvantages of PLA are that it can become brittle when exposed to sunlight/ultraviolet light and is more reactive with water than ABS. Such effects should be negligible, if the tools are kept in a dry environment and not exposed to extended periods of direct sunlight.

2.3 Experimental Validation of Tool Designs

2.3.1 Intra- and intertool compatibility for simultaneous tracking

The NDI 6D Architect software was used to create a tool definition file for each tool, describing its geometry as a ROM file identifiable by the tracker system.¹² The 6D Architect software was also used (via the built-in “Geometry Test”) to validate individual tools and their mutual compatibility. Tools failing the geometry test for either intra- or intertool constraints were rejected. In principle, no tools resulting from the design process described above should fail the geometry test, and while this was the case for all individual tools (i.e., intratool validation), a small number of anomalous failures were observed based on intertool incompatibility (subject to future investigation). All tools described below (10 groups, each with 6 to 10 DRFs) passed both intra- and intertool geometry tests.

2.3.2 Validation with a basic tracked pointer: pivot calibration

Pivot calibration was tested for each DRF using a ball-point stylus (adapted from the MicronTracker stylus; Claron, Toronto, Ontario) as shown in Fig. 5(a). Each DRF was rigidly attached to the stylus through its center clearance hole, and the tool definition file was defined such that the center hole coincided with the tool origin. The distance between the tool origin and the stylus tip was 53 mm for all DRFs. Pivot calibration of the tool-tip was performed using 10 repeated trials per tool. Each trial was conducted on the optical bench setup shown in Fig. 5(b) with an NDI Polaris Vicra tracker. Since the geometric accuracy of pivot calibration depends on robust estimation of the pose, errors in this process reflect the accuracy and robustness of the tracker as well as the DRF itself. Tool orientation and position measurements ($\mathbf{R}_i, p_i$) were formulated as an $Ax = b$ problem to solve for the tool-tip pose using¹⁴

$$[\mathbf{R}_i | -\mathbf{I}][p_{\text{rel}} | p_{\text{abs}}]^T = [-p_i], \quad (1)$$

where the analytical solution yields the tool-tip offset (p_{rel}) from the DRF origin, and p_{abs} is the position of the pivot (conical “divot” in a steel plate on the optical bench) in the tracker coordinate frame. The 3-D error distribution for each sample was found by subtracting the position of the divot transformations from the measured using PLA, which prints at a low error ε in the least-squares sense¹⁵

$$[\varepsilon_i] = [\mathbf{R}_i | -\mathbf{I}][p_{\text{rel}} | p_{\text{abs}}]^T - [-p_i]. \quad (2)$$

The root mean square error (RMSE) of the resulting error distribution was calculated according to

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^n \varepsilon_i^2}{n}}, \quad (3)$$

where n was the total number of samples collected for each pivot calibration.

2.3.3 Measurement of fiducial registration error

The validity of the tools was also measured in application to point-based registration.¹⁶ For each trial, a DRF was secured to a motorized stage by an adjustable, passive static arm

Fig. 5 Experimental setup. (a) Pivot calibration tool. The distance from the DRF origin to the pointer tip was fixed at 53 mm for all tools, with each DRF attached to the pointer via the central clearance hole. (b) Illustration of the experimental setup for validation of DRF designs. A computer-controlled motion system on an optical bench was used in combination with NDI trackers and a selection of test objects for pivot calibration (steel divot), FRE measurements (grid points via computer-controlled motion stages), and TRE measurements (a CT head phantom mimicking anatomy of the skull).

(Noga, Shlomi, Israel). The motorized stage consisted of three BiSlide computer-controlled stages (Velmex, Bloomington, New York) and a VXM stepping motor controller (Velmex, Bloomington, New York) with a specified accuracy of 0.01 mm. Each DRF was moved through a grid of 36 points covering $25 \times 25 \text{ cm}^2$ [Fig. 5(b)] well within the field of measurement of the Vicra tracker. Specifically, the grid was centered at location $d = [-1068.6 \text{ mm}, -33.3 \text{ mm}, 57.6 \text{ mm}]$ from the tracker origin, compared to a depth of field of 1336 mm. The specified accuracy of tracking a single marker within this volumetric region was 0.25 mm.¹⁷ The location of each grid point was measured using both a tracked DRF ($\mathbf{x}_{\text{tracked}}$) and grid location according to computer-controlled motorized stage translations ($\mathbf{x}_{\text{true}}$). Point-based rigid registration was performed between these “tracked” and “true” point sets, yielding fiducial registration error (FRE) according to

$$\text{FRE} = \min_{\theta} [\mathbf{x}_{\text{true}} - T_{\theta}(\mathbf{x}_{\text{tracked}})]^2, \quad (4)$$

where θ is the rigid-body transformation relating the tracker and grid-point coordinate systems. Five trials were conducted for each DRF, and the distribution of the RMSE was obtained in comparison to that of the NDI reference tool (#8700339).

2.3.4 Measurement of TRE

A rigid anthropomorphic head phantom [denoted simply as “Phantom” in Fig. 5(b)] featuring 12 divots on its surface was used for measurement of target registration error (TRE). For each registration, a leave-one-out approach was used in which 11 divots were used as registration fiducials and the 12th designated as the target point. The true coordinates for all divots were defined by manual localization in CT (estimated

accuracy of localization $\pm 0.15 \text{ mm}$, approximately half the voxel size). The pivot calibration function in the NDI Track software was used to apply the tool-tip offset to the DRF. The distance between the DRF origin was the same as in the pivot calibration. The pointer tool was manually positioned at each divot location on the phantom, and the mean of ~ 200 samples recorded by the tracker was calculated. Measurements for 12 divots were repeated 10 times for each tool, giving a total of 120 measurements for each DRF. Point-based registration was performed between tracked locations and the corresponding CT locations. The TRE was given by the distance between the transformed location of each tracked target point and the true location defined in CT.

3 Results

3.1 Open-Source Library of Dynamic Reference Frames

Figure 6 shows the resulting open-source library containing 10 groups of mutually compatible DRFs. Each group is designated by a letter (A, B, ..., K), with 6 to 10 tools in each group. The first tool in each group is geometrically equivalent to the NDI reference tool (#8700339). All 10 sets are available for research use via download from Ref. 10.

To assess the diversity (geometric dispersion) of tool configurations within a mutually compatible group, each tool was represented as a point in the 6-D space according to its six segment lengths (d_i), and the Euclidean distance between each pair of tools was calculated. Figure 7 shows the distribution of distances within groups, demonstrating an overall mean $\pm$ standard deviation of $25.14 \pm 7.24 \text{ mm}$. One-way analysis of variance test failed to find any significant differences with $p = 0.41$. Thus, different groups exhibited similar

Fig. 6 Open-source library of DRFs. (left) Photograph of the A (“Apple”) group comprising 10 mutually compatible tools produced on a 3-D printer and outfitted with NDI retroreflective markers. The hole pattern in the background marks 1 in. (2.54 cm) scale. (right) Illustration of CAD renderings for the 10 groups, denoted A–K.

Fig. 7 Distribution of intertool distances within a group, where each DRF is represented as a point in 6-D space.

geometric dispersion in a way that satisfied the specified design configurations.

3.2 Experimental Validation

3.2.1 Intertool compatibility: multiple tool tracking

Geometric tests performed using 6D Architect passed all the intra- and intertool constraints, and the individual tool definition ROM files were loaded into the NDI Track software for simultaneous tracking. Figure 8 shows robust identification and simultaneous tracking of six example tools.

3.2.2 Pivot calibration

Figure 9(a) shows the pivot calibration error (RMSE of the spread in tool-tip pivot calibration) for each of the 10 tools in group A. The calibration errors were within 0.27 to 0.67 mm. In group A, the mean error was 0.46 ± 0.12 mm compared to 0.44 ± 0.06 mm for the NDI reference tool. A Student *t*-test for group A compared to the reference tool failed to detect statistically significant difference ($p = 0.49$). Pivot calibration error may result from the design of the DRF, the distance between the DRF origin and the pointer tip, the range of angles, the face orientation to the camera, and the divot interface in which the tool is pivoted. These potential sources

Fig. 8 Illustration of simultaneous multiple tool tracking (six tools from the A group) using an NDI Polaris Vicra. A colored circle is superimposed on the photograph to identify the six tools evident in NDI Track.

of errors were minimized by keeping the face orientation, the divot location, the range of angles, and the distance between the pointer tip and DRF origin the same for each test. As a result, the distribution of errors shown in Fig. 9(a) primarily reflects the design of the DRF. A leave-one-out analysis (Satterthwaite's approximate t test) of the error distribution for each DRF showed each tool performed similarly—except DRFs 3 and 4 (higher error) and DRF 6 (lower error). The variations in pivot calibration error for these DRFs were small (though statistically significant), and while they could be attributable to the DRF configuration itself, we attribute the observation primarily to slight variations in the quality and lack of reproducibility of manual pivoting.

3.2.3 Measurement of FRE

Figure 9(b) shows the FRE measured on the computer-controlled tracking bench for group A. FRE was in the range 0.10 to 0.25 mm, with mean error of 0.15 ± 0.03 mm compared to 0.19 ± 0.11 mm of the reference tool. A statistically significant reduction in FRE was suggested by Student t -test for group A ($p = 0.0047$), but the small difference is likely attributed to the larger physical size of the custom designs. The accuracy of the Polaris Vicra System is calculated as the RMS variation of the mean number of readings about the true 3-D location calculated across different locations throughout the measurement volume.¹⁷ Satterthwaite's approximate t test did not demonstrate a statistically significant difference ($p = 0.16$) between the error distribution for each DRF.

3.2.4 Measurement of TRE

Finally, Fig. 9(c) shows the TRE measured in the anthropomorphic head phantom registered to CT. The error distributions from repeat trials are shown for group A compared to the reference tool. TRE was measured to be 0.96 ± 0.47 mm compared to 0.90 ± 0.44 mm for the reference tool. There was no statistically significant difference ($p = 0.16$) in TRE measured for the custom tools versus the reference tool or for any individual DRF versus the rest of the group.

4 Discussion and Conclusions

A large library of DRFs was designed by exhaustive search consistent with geometric constraints for single-DRF (intratool) tracking as well as multiple-DRF (intertool) tracking for NDI Polaris surgical trackers. The library generated in the current work features 10 groups of mutually compatible DRFs, each with 6 to 10 tools. All tools passed compatibility checks in NDI 6D Architect. Measurements of tool-tip pivot calibration precision as well as FRE on a computer-controlled navigation bench and TRE in a CT head phantom validated basic tracking performance of each tool compared to a standard NDI reference tool. The library was converted to 3-D printable CAD files

Fig. 9 Experimental validation of DRF designs. Measurements are shown for the 10 DRFs in group A. (a) Pivot calibration error. (b) FRE. (c) TRE. The dashed horizontal line marks the average median value of the custom DRFs.

available in the public domain (Open-source library available for download from Ref. 10) for use in research and education, providing a potentially useful resource for researchers in image-guided surgery and other applications involving the development of novel, multiple tracked DRFs.

While different systems have various limits on the number of simultaneously tracked DRFs (e.g., up to 6 tools with Polaris Vicra and 15 tools with Polaris Spectra), tools can be selected from a particular group generated in this work to meet the needs of a particular system. While the presented algorithms addressed DRF designs relevant to single-face, passive tracking systems, the methods could be adapted to other multiface and/or active tracking systems. Additional geometric constraints for other multiface and/or active tracking systems could be incorporated to the algorithms to further extend the library. The DRFs are also not limited to NDI trackers specifically, and they could likely be used with other trackers (e.g., the Atracsys system, Puidoux, Switzerland) if the design constraints imposed by such tracking systems are consistent with those of NDI. A passive IR optical tracking system emits and detects the reflected IR light typically from retroreflective-coated spheres attached to a rigid tool frame, though retroreflective planar stickers may also be used.

Potential sources of geometric error in the DRF designs produced and tested in this work include 3-D printing errors (e.g., thermal stability) and tolerance in attaching the spheres to mounting posts as well as the usual factors affecting tracker localization error, accuracy of the computer-controlled motion system (for measurement of FRE), and truth definition (for measurement of TRE). Geometric errors in 3-D printing and mounting the sphere could cause errors if the actual dimensions significantly deviate from the tool definition. In the experiments reported above, the segment lengths (after locating the markers in NDI 6D Architect) were found to be within 0.3 ± 0.3 mm of the specified design. Certain plastics (e.g., ABS) may be prone to thermal effects causing shrinkage or warping during the print process. Therefore, care in material selection is important to ensure that geometric specifications for each tool are met during 3-D printing.

DRF designs were created in this work via algorithms of exhaustive search (resulting in ~750,000 individually valid marker designs), and the resulting groups of mutually compatible groups were dependent on the initialization and the randomization in selecting designs that satisfied intratool compatibility. Each group was initialized with a known NDI reference tool (i.e., the first tool in each group is geometrically identical to the NDI reference tool #8700339), ensuring that the tools in the resulting library can be used to augment existing sets of NDI tools. When candidate DRFs are randomly selected from the pool of valid individual tools to test for intertool compatibility, groups with difference geometric configurations were found during multiple trials. Since the validity of a new candidate tool is dependent on the geometry of all tools previously in the group, a variable number of mutually compatible tools were found for different groups, ranging from 6 to 10 in the groups reported here. Using different initializations and randomizations, it is certainly possible to find sets with more than 10 members each. It also merits consideration in future work to

consider “optimal” search of the 6-D (d_i) space to maximize geometric dispersion of individual tools, rather than incremental, exhaustive identification as in the current work.

Disclosures

No conflicts of interest, financial or otherwise, are declared by the authors.

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References

1. A. Uneri et al., “TREK: an integrated system architecture for intraoperative cone-beam CT-guided surgery,” *Int. J. Comput. Assist. Radiol. Surg.* **7**(1), 159–173 (2012).
2. D. J. Costa and R. Sindwani, “Advances in surgical navigation,” *Otolaryngol. Clin. North Am.* **42**(5), 799–811 (2009).
3. R. M. Satava, “Surgical education and surgical simulation,” *World J. Surg.* **25**(11), 1484–1489 (2001).
4. F. Tendick et al., “A virtual environment testbed for training laparoscopic surgical skills,” *Presence Teleoperators Virtual Environ.* **9**(3), 236–255 (2000).
5. J. B. West and C. R. Maurer Jr., “Designing optically tracked instruments for image-guided surgery,” *IEEE Trans. Med. Imaging* **23**(5), 533–545 (2004).
6. F. Bergamasco et al., “An accurate and robust artificial marker based on cyclic codes,” *IEEE Trans. Pattern Anal. Mach. Intell.* **38**(12), 2359–2373 (2016).
7. S. Garrido-Jurado et al., “Automatic generation and detection of highly reliable fiducial markers under occlusion,” *Pattern Recognit.* **47**(6), 2280–2292 (2014).
8. T. Pinteric and H. Kaufmann, “A rigid-body target design methodology for optical pose-tracking,” in *ACM Symp. on Virtual Reality Software and Technology (VRST)*, pp. 73–76 (2008).
9. D. Gierlach, A. Gustus, and P. van der Smagt, “Generating marker stars for 6D optical tracking,” in *IEEE Int. Conf. on Biomedical Robotics and Biomechatronics (RAS/EMBS)* (2012).
10. <http://istar.jhu.edu/downloads>
11. “NDI 6D Architect User Guide,” Northern Digital Inc., Waterloo Ontario, Canada (2004).
12. “Polaris Tool User Guide,” Northern Digital Inc., Waterloo Ontario, Canada (2007).
13. M. Kintel, “OpenSCAD - The programmers solid 3D CAD modeller,” (05 May 2017).
14. Z. Yaniv, “Which pivot calibration?” *Proc. SPIE* **9415**, 941527 (2015).
15. B. Ma et al., “Is pose-based pivot calibration superior to sphere fitting?” *Proc. SPIE* **10135**, 101351U (2017).
16. J. M. Fitzpatrick, J. B. West, and C. R. Maurer, “Predicting error in rigid-body point-based registration,” *IEEE Trans. Med. Imaging* **17**(5), 694–702 (1998).
17. “Polaris Vicra User Guide,” Northern Digital Inc., Waterloo Ontario, Canada (2007).

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