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Ratio-adjustable power splitters for array-compressed parallel transmission

Xinqiang Yan^{1,2,†}, Zhipeng Cao^{1,3,†}, and William A. Grissom^{*,1,2,3,4}

¹Vanderbilt University Institute of Imaging Science, Nashville, TN, United States

²Department of Radiology, Vanderbilt University, Nashville, TN, United States

³Department of Biomedical Engineering, Vanderbilt University, Nashville, TN, United States

⁴Department of Electrical Engineering, Vanderbilt University, Nashville, TN, United States

Abstract

Purpose—To implement and validate low-loss ratio-adjustable power splitters for array-compressed parallel transmission (acpTx).

Methods—In acpTx, a small number of transmit channels drive a larger number of transmit coils, which are connected via an array compression network that implements optimized coil-to-channel combinations. Previous networks comprised a bank of power splitters, followed by attenuators to implement the amplitudes of the compression weights for each coil, but this resulted in high power dissipation in the network. Recognizing that an acpTx network need only implement relative attenuations between outputs, a ratio-adjustable power splitter (RAPS) circuit was developed which combines power splitting and relative attenuation, and has low insertion loss. RAPS circuits were experimentally validated and used to build an array compression network for a one-channel-to-four-coil spiral acpTx excitation experiment.

Results—Bench tests showed that the RAPS circuits came within 0.05 dB of the desired output ratios, and power dissipation was approximately 0.5 dB (10%). The spiral excitation experiment showed that the ability to optimally drive four coils with a single channel reduced excitation error by 46% compared to driving one coil, without using attenuators in the array compression network.

Conclusion—RAPS circuits enable the construction of low-loss array compression networks for parallel transmission.

Keywords

RF coils; MR engineering; ultra-high field MRI; RF pulse design; selective excitation; array compression; parallel transmission; optimization

*Corresponding author: Will Grissom, Department of Biomedical Engineering, Vanderbilt University, 5824 Stevenson Center, Nashville, TN 37235 USA, will.grissom@vanderbilt.edu, Twitter: @wgrissom.

†These authors contributed equally to this work.

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Introduction

Many-coil transmit arrays are desirable in parallel transmission (pTx) [1, 2], since with many coils multidimensional pulses can be shortened, more uniform radiofrequency (RF) shims can be produced, and specific absorption rate (SAR) can be more effectively controlled. However, the high cost and the large physical footprint and cabling requirements of the corresponding power amplifiers required to drive many-coil arrays has limited the number of transmit coils/channels that are used in practice, and most ultra-high field MR scanners in use today have only two or eight transmit channels. To overcome this limitation, array-compressed pTx (acpTx) was recently proposed [3, 4] in which a large number of coils is connected to a small number of channels via an array compression network that splits the input pulse waveforms to the coils and applies compression weights (attenuations and phase shifts) that are optimized jointly with the pulse waveforms. An array compression network is similar to a Butler matrix [5] in that it connects each input channel to multiple output coils, but instead of fixed phase shifts with equal power splits it implements connection weights that are optimized for a specific parallel excitation. We reported an initial hardware implementation of acpTx using equal Wilkinson power splitters followed by tunable attenuators which implemented the amplitudes of the compression weights [6], but this resulted in significant power dissipation in the network (59% in an accelerated spiral excitation experiment).

Recognizing that an acpTx network need only implement relative attenuations between outputs, here we describe ratio-adjustable power splitter (RAPS) circuits, which combine the duties of power splitting and applying relative attenuations, and should thereby enable array compression networks with minimal power dissipation. Although unequal Wilkinson splitters can be directly designed for arbitrary output ratios, the ratios are a function of branch-line impedances which are not readily adjustable in practice. In addition, Wilkinson splitters already require adjustable impedance transformation circuits and adjustable resistors to compensate port mismatching and isolation. In contrast, the output ratio of a RAPS circuit can be adjusted to any desired value by changing the length of a $50\ \Omega$ transmission line, without affecting matching or port isolation. RAPS circuits were validated in benchtop tests and in an acpTx experiment using an accelerated spiral excitation.

Methods

RAPS Circuit Design and Fabrication

Figure 1A shows the functional diagram of a one-input/two-output RAPS circuit, which comprises an equal Wilkinson splitter and a 3-dB 90° hybrid coupler connected by two $50\ \Omega$ transmission lines of different lengths. The RF signal (V_{in}) is fed to the Wilkinson splitter and equally divided into two parts (V_A and V_B), which are phase shifted by ϕ_1 and ϕ_2 radians by the transmission lines, resulting in V_C and V_D . The hybrid coupler sums V_C and V_D with different phase shifts to obtain the output signals V_1 and V_2 . Since V_1 and V_2 are weighted vector sums of V_C and V_D , their magnitudes depend on the relative phase between V_C and V_D . The circuit's scattering matrix and the amplitude relationships between its input and output ports are derived in the Appendix to be

$$\frac{|V_1|}{|V_2|} = \left| \cot \left(\frac{\phi_1 - \phi_2 - \pi/2}{2} \right) \right|, \quad [1]$$

$$V_{in}^2 = V_1^2 + V_2^2. \quad [2]$$

Equation 1 indicates that the output ratio has a cotangent dependence on the difference between the transmission line lengths, and can thus be tuned by adjusting those lengths. Equation 2 indicates that no input power is dissipated by the circuit under ideal conditions. In practice, it will dissipate some power due to imperfect components, coil matching, and coil coupling. Note that a RAPS circuit also applies different phase shifts to its outputs, which can be compensated by subsequent phase shifters in an acpTx network.

Figure 1B shows a fabricated RAPS circuit for 298 MHz (7 Tesla), made on a printed circuit board with dimensions $3 \times 7 \text{ cm}^2$. Two 50Ω coaxial cables (one fixed and one variable, RG-174, Belden Wire and Cable, Richmond, IN, USA) are used to change the relative phase shifts of the split signals and consequently the ratio between the output voltage amplitudes. Both the Wilkinson power splitter and the hybrid couplers were made with lumped-element designs. Figure 2 lists the specific values of all the lumped elements in the circuit, including non-magnetic ceramic capacitors (1000-V voltage rating, 1111C Series, Passive Plus, Huntington, NY, USA), non-magnetic resistors (American Technical Ceramics, Huntington Station, NY, USA) and inductors made with AWG-20 copper wire. As derived above, the only dissipative component in the circuit is the resistor, which will absorb a small amount of power due to imperfect components and coil matching. Therefore the circuit incurs negligible heating during use. Gerber files for the printed circuit board are available for download at <https://bitbucket.org/wgrissom/acptx>.

Bench Tests and B_1^+ Mapping Validation

A library of cable lengths was made for output ratios from 0 dB (equal splitting) to 7 dB in steps of 1 dB. Scattering (S-) parameters were measured for each ratio using a calibrated four-port network analyzer (E5071C, Keysight Technologies, Santa Rosa, CA, USA). Each ratio was further validated using B_1^+ mapping experiments. A circuit was inserted between one RF amplifier of a Philips Achieva 7T scanner (Philips Healthcare, Best, Netherlands) and one port of a Nova birdcage coil (Nova Medical, Wilmington, MA, USA). An axial B_1^+ map for each ratio was acquired in a 15 cm gel phantom using the DREAM method [7]. An additional scan was acquired with no RAPS circuit in-line as a power reference. The other port of the birdcage coil was not driven, and the input power and all parameters were the same for all scans. A Nova 32-channel receive-only coil was used for signal reception.

The overall power loss of the RAPS circuit was calculated for each ratio using measured S-parameters as: $1 - |S_{12}|^2 - |S_{13}|^2$, where port 1 is the input port, and ports 2 and 3 are the output ports. This was compared to total power loss calculated using B_1^+ maps as:

$1 + (|B_{1,2}^+|/|B_{1,\text{ref}}^+|)^2 - (|B_{1,3}^+|/|B_{1,\text{ref}}^+|)^2$, where $B_{1,\text{ref}}^+$ is the measured map with no RAPS circuit in-line, and $B_{1,2}^+$ and $B_{1,3}^+$ are the measured maps when the outputs were used individually, with the other port terminated with a high-power 50 Ω load.

Array Compression Network

Figure 3A shows a previously reported compression network, which comprises a stage of equal splitters, followed by attenuators [6]. To minimize power dissipation, in Ref. [6] the coil weights implemented in this network were normalized by the largest weight so that one output had 0 dB attenuation, but 59% of the input power was still dissipated in the network in a spiral experiment. In contrast, since a RAPS circuit performs the combined duties of power splitting and applying relative attenuations, a one-channel-to- N -coil array compression network can be constructed solely from RAPS circuits and subsequent phase shifters, without attenuators. There are two basic network topologies: a $\log_2 N$ -stage “standard-tree” topology in which all outputs are in the last stage and are symmetric with respect to the input (Figure 3B,C), and an $N-1$ -stage “one-per-stage” topology in which each stage has one output port (Figure 3D,E). Both topologies comprise a total of $N-1$ RAPS circuits. Figures 3A and B show one-to-four network diagrams using standard-tree and one-per-stage topologies, respectively. Figures 3C and D show corresponding photos of connected RAPS circuits for each topology. Each has its potential advantages. Assuming the insertion loss of the RAPS circuits is approximately the same, each output of the standard-tree network sees the same insertion loss, which may simplify its setup. Conversely, if up to $\log_2 N-1$ coils have much larger weights than the others, then the one-per-stage network may be preferred since the coils with the largest weights can be output earlier in the network, where they will experience less insertion loss than coils with much smaller weights. The specific compression weights (amplitudes and phase shifts) implemented by a compression network are determined by acpTx pulse design, and a MATLAB (Mathworks, Natick, MA, USA) script to calculate all the RAPS circuit output ratios in a network (for either topology) from the amplitudes of the compression weights is provided as Supporting Information (acpTx_RAPS_network.m), along with a demo script (test_acpTx_RAPS_network.m). Phase shifts are applied by adjusting the coaxial cable lengths between the outputs of the network and the coils. The two topologies were compared in terms of simulated power dissipation in their resistors when configured in a one channel-to-four coil network and connected to a realistic four-element coil, using the measured S-parameter matrix and compression weights (relative amplitudes and phase shifts) from the spiral acpTx excitation experiment described below. Figure 4A depicts the simulation setup for each topology, and Figure 4B lists the coil array’s complex-valued S-parameter matrix. The return loss of each element was between -24.1 dB and -27.8 dB (average -26.3 dB), and mutual coupling between elements was between -14.8 dB and -33.9 dB (average -26.5 dB), with the strongest coupling occurring between next-adjacent neighbors [6]. The analysis was performed using the ANSYS Designer RF circuit simulation tool (ANSYS Designer, Canonsburg, PA, USA). Figure 4C lists the simulated power dissipation in each resistor of each network. In both networks, most of the power was dissipated in resistor R_3 , and the total power dissipation was approximately 2.5%. Which resistor dissipates the most power depends strongly on the specific compression weights. For example, equalizing the

output weights of R_3 's RAPS circuit caused R_2 to dissipate the most power in both networks.

Spiral acpTx Experiment

To validate a RAPS-based array compression network in an acpTx imaging experiment, RAPS circuits were used to build a one channel-to-four coil standard-tree array compression network optimized for an accelerated 2D spiral RF excitation pulse at 7T. Four consecutive elements of an eight-element gapped loop transmit array with induced current elimination decoupling [6] were used in the experiment (Figure 5A). The four elements' B_1^+ maps were measured in a transverse slice of a cylindrical gel phantom with diameter 17 cm using the DREAM method, with $24 \times 24 \text{ cm}^2$ FOV, $2 \times 2 \text{ mm}^2$ voxel size, 10 mm slice thickness, and 1000 ms TR (Figure 5B). A 32-channel Nova receive head array was used for all signal reception. An off-resonance map was also measured in the target slice using the dual-TE method [8] with the same resolution and FOV of the B_1^+ maps and a 0.5 ms TE difference. The maps were used to design a single-channel pulse and channel-to-coil weights that excited a 5.25 cm-diameter circle tangential to the edge of the phantom, using the iterative small-tip acpTx spiral pulse design method in Ref. [3], which repeatedly alternates between reducing a cost function comprising a squared-excitation error term [9] and a small waveform roughness penalty [10], and applying a singular value truncation to constraint the rank of the RF waveform matrix to one. Further details on this pulse design are provided in Ref. [3].

The output of the pulse design was a rank-one matrix $\hat{\mathbf{B}}$ whose columns contained the four RF waveforms for each of the two transmit channels. The singular value decomposition $\hat{\mathbf{B}} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$ of the matrix was calculated, and the array compression weights were taken to be elements of the first column of the right-singular vector ($\mathbf{V}^*$) matrix, where * denotes element-wise complex conjugation. The single-channel input RF pulse was taken to be the first column of the $\mathbf{U}\mathbf{\Sigma}$ matrix. The weights were implemented in the RAPS circuits and phase shifters of the array compression network. They were checked using a network analyzer, and a B_1^+ map was measured for the compressed single channel array.

A 5.3 ms slew rate-limited spiral-in trajectory was used for pulse design with excitation-FOV 9 cm ($1.9\times$ acceleration in the 17-cm phantom) and maximum slew rate 120 mT/m/s. For comparison to the acpTx pulse, single-channel pulses using the same trajectory were designed for a CP mode combination of the coil elements, and using the single coil element whose B_1^+ map most overlapped the targeted circle. The acpTx and single coil element pulses' excitation patterns were imaged using a 3D gradient echo sequence with TE/TR = 1.2/100 ms, flip angle = 1° , FOV = $240 \times 240 \times 186 \text{ mm}^3$, and voxel size = $2 \times 2 \times 6 \text{ mm}^3$. To remove receive sensitivity variations from the imaged excitation patterns, a receive sensitivity map was estimated as the ratio of a GRE image acquired with a small-tip-angle hard pulse, and the compressed B_1^+ map, and was then divided out of the spiral excitation images.

Results

Bench Tests and B_1^+ Mapping Validation

Figure 6 shows S-parameter measurements of a RAPS circuit with target output ratios between 0 dB (equal splitting) and 7 dB. Figure 6A shows the input-output S-parameters and the measured ratios; the errors between desired and measured output ratios were all less than 0.05 dB. Figure 6B plots total power loss versus output ratio, which was approximately 10% (≈ 0.5 dB) across ratios. Figure 6C shows that all ports were well-matched, with return loss better than -16 dB across ratios ($< 2\%$ power reflection). Figure 6D shows that the isolation between the two output ports was better than -22 dB across ratios ($< 1\%$ power crosstalk). Good matching and isolation is critical in a multi-stage network to prevent build-up of mismatching and cross-coupling between RAPS circuits. Figure 7A shows B_1^+ maps measured across output ratios. Figure 7B plots predicted (using S-parameter measurements) and measured average B_1^+ at each ratio. The measured and predicted ratios matched well, with errors less than 4%. Figure 7C shows calculated power loss based on the measured average B_1^+ . As expected from the bench measurements, the power loss was low ($< 12\%$).

Spiral acpTx Experiment

Figure 8A plots the intended versus measured compression weights. The average/maximum error between the two was 0.3/0.7% in amplitude and 0.7/2.0 degrees in phase. Figure 8B shows the B_1^+ map that was predicted based on the designed compression weights, and the map that was measured after implementing those weights in the network. Most of the B_1^+ errors were in areas of low MR signal in the DREAM images, where image division in the B_1^+ calculation amplified noise in the maps. The predicted and measured excitation patterns produced by the single-coil pulse (Coil 3 in Figure 5A) and the acpTx pulse are shown in Figure 9. The predicted root-mean-square excitation errors were 28% (single-coil) and 15% (acpTx). The higher error of the single-coil pattern was due to blurring of the circle and larger sidelobes, which are indicated with arrows in the patterns. The close match between the predicted and measured B_1^+ maps and excitation patterns indicates that the compression network's function was not influenced by the static or gradient magnetic fields even when placed close to the RF coils, due to its compact dimensions and non-magnetic components. The CP pulse's excitation pattern was not imaged since its predicted root-mean-square excitation error was higher than the single-coil pulse's error, and implementing it would have required an additional network to be constructed.

Discussion and Conclusion

Summary of Results

We described and validated ratio-adjustable power splitter (RAPS) circuits, which allow a desired power-split ratio to be implemented by adjusting transmission line lengths. Their performance was validated in bench tests and by B_1^+ mapping. Array compression networks can be built from them in standard-tree or one-per-stage configurations; simulations with a realistic coil's scattering parameters showed that similar power losses are incurred in each

configuration, so the choice between the two should be based on other practical considerations as discussed in the Methods. A two-stage, one-channel-to-four-coil standard-tree acpTx network was built from RAPS circuits and validated in a spiral acpTx experiment. Compared to a previously-described array compression network using attenuators to adjust the magnitude weights [6], the RAPS-based network had much lower power dissipation and a simpler structure. Array compression networks based on RAPS circuits could be built into transmit coil arrays for specific applications or anatomies, or application-specific boxes that could be used with general-purpose arrays such as multi-element body coils. The networks could be fixed or tunable; if fixed, the weights they implement could be determined from offline RF pulse designs for specific pulse sequences in multiple human body models, so that they apply to the general population. Tunable networks that could be remotely tuned would better adapt an array to a specific sequence and subject. This would require remotely-controlled phase shifters in the RAPS circuits and between the network and coil array, which could be implemented using piezoelectric actuators that adjust reflection-type hybrid couplers [11], or several other ways [12–18]. One example application would be RF shimming with a single amplifier, where the RAPS circuits would provide the ability to tune the shims by adjusting transmission line lengths/phase shifts.

Circuit Losses

As shown in Equation 2, no input power is dissipated in a RAPS circuit under ideal conditions, independent of output ratio. However, the Wilkinson splitter, the hybrid coupler, and the connecting cables and connectors still induce insertion losses. In this work, the Wilkinson splitter and hybrid coupler were made using lumped-element designs, and had an insertion loss between 0.22 dB and 0.27 dB. We found that this loss dominated the total power loss of the RAPS circuits, as evidenced by the fact that the total loss of each RAPS circuit (between 0.47–0.61 dB/10–13%) was approximately equal to the sum of these losses. These losses were also significantly larger than power dissipation due to coil mismatch and coupling, which in this acpTx experiment was simulated to be approximately 2.5% for the entire network, using either topology. Circuit losses could be reduced by using distributed-element designs such as microstrip lines, though the circuits would be physically larger. Overall though, the power dissipated in a RAPS-based network will be much lower than in an attenuator-based network such as was described in Ref. [6]: for the spiral pulse compression weights designed here, an ideal attenuator-based network would have incurred at least 3.7 dB/58% power dissipation, before accounting for insertion losses which would be similar to a RAPS-based network.

Alternative Designs

Alternative RAPS circuit designs are possible. Figure 10 shows two possibilities: the Wilkinson splitter can be replaced with a 3 dB hybrid coupler in which the isolation port is terminated with 50 Ω , or both the splitter and coupler could be replaced with 180-degree (rat-race) hybrid couplers.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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Appendix

RAPS circuit scattering parameters and input-output relationships

The RAPS circuit's transmission and reflection coefficients are given by its scattering (S)-matrix. The general form of an S -matrix for a three-port network such as a RAPS circuit is:

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, \quad [3]$$

which relates the waves incident on each port (a_1, a_2, a_3) to the waves reflected from each port (b_1, b_2, b_3). However, as shown in Figure 1A, the RAPS circuit comprises three individual circuits (a two-way Wilkinson power splitter, phase shifters and a hybrid coupler) connected in series, and its S -matrix cannot be obtained by directly multiplying the individual circuits' S -matrices. Instead, in the following we use the approach of Ref. [19] and convert each individual circuit's S -matrix to a transmission (T)-matrix, which is arranged with the intent of cascading multiple circuits and has the following form for a three-port network with one input and two outputs:

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \\ b_2 \\ b_3 \end{bmatrix}, \quad [4]$$

where port 1 is the input and ports 2 and 3 are the outputs. The overall RAPS circuit's T -matrix is then equal to the product of the individual circuits' T -matrices, and is converted back to an S -matrix for interpretation.

Using the voltage definitions in Figure 1A, the S -matrix of the Wilkinson power splitter is:

$$\begin{bmatrix} V_{\text{in}}^- \\ V_A^- \\ V_B^- \end{bmatrix} = \mathbf{S}_{\text{wps}} \begin{bmatrix} V_{\text{in}}^+ \\ V_A^+ \\ V_B^+ \end{bmatrix} = \frac{-j}{\sqrt{2}} \begin{bmatrix} 011 \\ 100 \\ 100 \end{bmatrix} \begin{bmatrix} V_{\text{in}}^+ \\ V_A^+ \\ V_B^+ \end{bmatrix}, \quad [5]$$

where '−' denotes reflected waves and '+' denotes incident waves. This matrix can be converted to T -matrix form, obtaining:

$$\begin{bmatrix} V_{\text{in}}^- \\ V_{\text{in}}^+ \end{bmatrix} = \mathbf{T}_{\text{wps}} \begin{bmatrix} V_A^+ \\ V_B^+ \\ V_A^- \\ V_B^- \end{bmatrix} = \frac{-j}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} V_A^+ \\ V_B^+ \\ V_A^- \\ V_B^- \end{bmatrix}. \quad [6]$$

Similarly, the $\mathbf{S}$ and $\mathbf{T}$ -matrices of the transmission lines/phase shifters connecting the splitter and hybrid coupler are given by:

$$\begin{bmatrix} V_A^- \\ V_B^- \\ V_C^- \\ V_D^- \end{bmatrix} = \mathbf{S}_{\text{ps}} \begin{bmatrix} V_A^+ \\ V_B^+ \\ V_C^- \\ V_D^- \end{bmatrix} = \begin{bmatrix} 0 & 0 & e^{-j\phi_1} & 0 \\ 0 & 0 & 0 & e^{-j\phi_2} \\ e^{-j\phi_1} & 0 & 0 & 0 \\ 0 & e^{-j\phi_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} V_A^+ \\ V_B^+ \\ V_C^- \\ V_D^- \end{bmatrix}, \quad [7]$$

and

$$\begin{bmatrix} V_A^- \\ V_B^- \\ V_C^+ \\ V_D^+ \end{bmatrix} = \mathbf{T}_{\text{ps}} \begin{bmatrix} V_C^+ \\ V_D^+ \\ V_C^- \\ V_D^- \end{bmatrix} = \begin{bmatrix} e^{-j\phi_1} & 0 & 0 & 0 \\ 0 & e^{-j\phi_2} & 0 & 0 \\ 0 & 0 & e^{-j\phi_1} & 0 \\ 0 & 0 & 0 & e^{-j\phi_2} \end{bmatrix} \begin{bmatrix} V_C^+ \\ V_D^+ \\ V_C^- \\ V_D^- \end{bmatrix}, \quad [8]$$

and the matrices for the hybrid coupler are given by:

$$\begin{bmatrix} V_C^- \\ V_D^- \\ V_1^- \\ V_2^- \end{bmatrix} = \mathbf{S}_{\text{hc}} \begin{bmatrix} V_C^+ \\ V_D^+ \\ V_1^+ \\ V_2^+ \end{bmatrix} = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & j & 1 \\ 0 & 0 & 1 & j \\ j & 1 & 0 & 0 \\ 1 & j & 0 & 0 \end{bmatrix} \begin{bmatrix} V_C^+ \\ V_D^+ \\ V_1^+ \\ V_2^+ \end{bmatrix}, \quad [9]$$

and

$$\begin{bmatrix} V_C^- \\ V_D^- \\ V_1^+ \\ V_2^+ \end{bmatrix} = \mathbf{T}_{\text{hc}} \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_1^- \\ V_2^- \end{bmatrix} = \frac{-1}{\sqrt{2}} \begin{bmatrix} j & 1 & 0 & 0 \\ 1 & j & 0 & 0 \\ 0 & 0 & -j & 1 \\ 0 & 0 & 1 & -j \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_1^- \\ V_2^- \end{bmatrix}. \quad [10]$$

Multiplying the $\mathbf{T}$ matrices together, we obtain the $\mathbf{T}$ matrix for the overall circuit:

$$\mathbf{T}_{\text{RAPS}} = \mathbf{T}_{\text{wps}} \mathbf{T}_{\text{ps}} \mathbf{T}_{\text{hc}} = \frac{j}{2} \begin{bmatrix} je^{-j\phi_1} + e^{-j\phi_2} & e^{-j\phi_1} + je^{-j\phi_2} & 0 & 0 \\ 0 & 0 & je^{j\phi_1} - e^{j\phi_2} & -e^{j\phi_1} + je^{j\phi_2} \end{bmatrix},$$

[11]

which can be converted back to an S -matrix for the overall circuit, yielding:

$$S_{\text{RAPS}} = \frac{j}{2} \begin{bmatrix} 0 & je^{-j\phi_1} + e^{-j\phi_2} & e^{-j\phi_1} + je^{-j\phi_2} \\ je^{-j\phi_1} + e^{-j\phi_2} & 0 & 0 \\ e^{-j\phi_1} + je^{-j\phi_2} & 0 & 0 \end{bmatrix} = e^{-j\frac{\phi_1 + \phi_2 - 3\pi/2}{2}} \begin{bmatrix} 0 & \cos\left(\frac{\phi_1 - \phi_2 - \pi/2}{2}\right) & -\sin\left(\frac{\phi_1 - \phi_2 - \pi/2}{2}\right) \\ \cos\left(\frac{\phi_1 - \phi_2 - \pi/2}{2}\right) & 0 & 0 \\ -\sin\left(\frac{\phi_1 - \phi_2 - \pi/2}{2}\right) & 0 & 0 \end{bmatrix}. \quad [12]$$

where the first row/column corresponds to the input, and the second and third correspond to the outputs. We can make the following inferences from Equation 12:

1. $S_{11} = S_{22} = S_{33} = 0$, regardless of ϕ_1 and ϕ_2 , which indicates that impedance matching (typically 50Ω) is independent of the output ratio.
2. $S_{23} = S_{32} = 0$, regardless of ϕ_1 and ϕ_2 , which indicates that the isolation between ports is independent of output ratio.
3. The output voltage amplitude ratio has a cotangent dependence on the difference between the transmission line lengths:

$$\frac{|S_{12}|}{|S_{13}|} = \frac{|V_1|}{|V_2|} = \left| \cot\left(\frac{\phi_1 - \phi_2 - \pi/2}{2}\right) \right|. \quad [13]$$

4. $|S_{12}|^2 + |S_{13}|^2 = 1$, regardless of ϕ_1 and ϕ_2 , which indicates that under ideal conditions, no input power (to port 1) is dissipated in the circuit.
5. $|S_{21}|^2 + |S_{23}|^2 = |S_{21}|^2 - 1$ and $|S_{31}|^2 + |S_{32}|^2 = |S_{31}|^2 - 1$, which indicates that power reflected at the output ports due to imperfect coil matching, or incident on the ports due to coil coupling (which may be the more significant effect in practice), will be dissipated in the Wilkinson splitter's resistor or passed out of the input port and dissipated in another network stage or the RF power amplifier output. The amount of dissipated power will depend on the voltage ratio and the relative phases and amplitudes of the reflected/incident power.

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Figure 1.

A) Functional diagram of a ratio-adjustable power splitter (RAPS) circuit. B) A fabricated RAPS circuit on a printed circuit board, with dimensions 3 cm × 7 cm. All components are non-magnetic (inductors, capacitors, connectors and cables). The ratio is adjusted by swapping coaxial cables, which are connectorized.

Figure 2. RAPS circuit diagram with all element values. The Wilkinson splitter is a $\lambda/4$ design, and the hybrid coupler is a $\lambda/8$ design.

Figure 3.

(A,B,D) Diagrams of one-channel-to-four-coil acpTx networks using a conventional splitter plus attenuator topology (A) [6], and RAPS-based topologies: “standard-tree” (B) and “one-per-stage” (D). (C,D) Photographs of connected RAPS circuits based on the standard-tree (C) and one-per-stage (D) topologies. The spiral acpTx experiment used the standard-tree topology. Both topologies include phase shifters between the outputs and the coils (not shown), to compensate phase shifts between the outputs and implement the compression weight phase shifts.

Figure 4.
A) Illustration of the simulation setup to compare power dissipated in RAPS circuit resistors for each network topology, using the coil and compression weights (relative amplitudes and phase shifts) from the spiral acpTx excitation experiment. The resistor in each RAPS circuit is labeled, and the coil is represented by its measured S-parameter matrix, which is given in (B). C) Power dissipated in each resistor of each topology.

Figure 5.

A) The arrangement of the 8-element transmit array, phantom, and circular target excitation pattern. B) Normalized B_1^+ maps for each of the 4 coil elements used in this study. Note that due to electrodynamics the fields are rotated clockwise compared to the locations of the coils themselves.

Figure 6. RAPS circuit bench test results. A) Transmission coefficients between the inputs and outputs. Port 1 is the input port, and ports 2 and 3 are the output ports. S_{12} - S_{13} is the ratio (in dB) between the two output ports. B) Calculated power loss based on the measured S_{12} and S_{13} in (A). C) Impedance match of each port (S_{11} , S_{22} and S_{33}). D) Isolation between the two output ports (S_{23}).

Figure 7.

B_1^+ mapping results with RAPS circuits placed between an amplifier and one port of a birdcage coil. A) Axial B_1^+ maps versus voltage ratio in a 15 cm ball phantom. The two rows correspond to the two outputs, and each map was acquired with the other output port terminated with 50 Ω . B) Plots of predicted and measured B_1^+ ratios. C) Power loss calculated from the B_1^+ maps in (A).

Figure 8.

A) Intended and measured relative amplitudes and phase shifts applied by the acpTx one channel-to-four coil network. B) Predicted and measured B_1^+ amplitude maps in the acpTx configuration, and the differences between the predicted and measured maps.

Figure 9. Spiral acpTx excitation experiment results. Top row: Measured single-coil (coil 3) and compressed single-channel B_1^+ amplitude maps. Middle row: Predicted excitation patterns and root-mean-squared excitation errors for the single-coil and acpTx pulses. Bottom row: Measured excitation patterns for the single coil and acpTx pulses. Arrows indicate higher blurring and side-lobes that contribute to the higher excitation error in the single-coil excitations.

Figure 10.

Alternative RAPS circuits. A) A circuit in which the Wilkinson splitter is replaced with a 3-dB 90-degree hybrid coupler whose isolation port is terminated with $50\ \Omega$. B) A circuit in which both the Wilkinson splitter and the 90-degree hybrid coupler are replaced with a 180-degree (rat-race) hybrid coupler.