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Self-calibrated correlation imaging with k-space variant correlation functions

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Abstract

Purpose—Correlation imaging is a previously-developed high-speed MRI framework that converts parallel imaging reconstruction into the estimate of correlation functions. The presented work aims to demonstrate this framework can provide a speed gain over parallel imaging by estimating k-space variant correlation functions.

Methods—Due to Fourier-encoding with gradients, outer k-space data contain higher spatial-frequency image components arising primarily from tissue boundaries. As a result of tissue-boundary sparsity in the human anatomy, neighboring k-space data correlation varies from the central to the outer k-space. By estimating k-space variant correlation functions with an iterative self-calibration method, correlation imaging can benefit from neighboring k-space data correlation associated with both coil sensitivity encoding and tissue-boundary sparsity, thereby providing a speed gain over parallel imaging that relies only on coil sensitivity encoding. This new approach is investigated in brain imaging, and free-breathing neonatal cardiac imaging.

Results—Correlation imaging performs better than existing parallel imaging techniques in simulated brain imaging acceleration experiments. The higher speed enables real-time data acquisition for neonatal cardiac imaging where physiological motion is fast and non-periodic.

Conclusion—With k-space variant correlation functions, correlation imaging gives a higher speed than parallel imaging and offers the potential to image physiological motion in real-time.

Keywords

Correlation imaging; Correlation function; High-speed MRI; Parallel imaging; Real-time imaging

Introduction

Magnetic resonance imaging (MRI) has a low data acquisition speed. This limits spatial/temporal resolution, introduces motion artifacts and reduces patient throughput in clinical practice (1–8). To meet clinical needs, high-speed MRI techniques are required. These techniques accelerate data acquisition with undersampling and reconstruct images from undersampled data using a special strategy. For example, partial Fourier imaging collects ~half k-space and generates the whole k-space by assuming data be conjugate symmetric about the k-space center (9,10). Parallel imaging reconstructs images using coil sensitivity

encoding provided by a multi-channel coil array (11–15). Compressed sensing generates images by enforcing data sparsity naturally existing in images (16–18). Data-sharing methods, including generalized series reconstruction (19–21), keyhole or data replacement strategy (22–24), constrained reconstruction (25,26), dynamic imaging by modeling (27,28), UNFOLD (29) and reduced field of view (FOV) imaging (30,31), utilize information sharing between dynamic images acquired from the same anatomy.

Although high-speed MRI has been an active research area for more than two decades, few techniques have been successfully translated into clinical applications. Currently, two parallel imaging techniques, SENSE (12) and GRAPPA (13), are clinical standards. They provide an acceleration factor of >2 for static imaging. In dynamic imaging, they have been implemented synergistically with data sharing methods. Two available techniques are k-t SENSE/BLAST (32,33) and k-t GRAPPA (34). These parallel imaging techniques are limited by inherent signal to noise ratio (SNR) and coil sensitivity encoding capability (12,35), which are both related to the configuration of a coil array. Specifically, they cannot use an acceleration factor higher than the number of coil elements (12). With this limitation, a MRI scan needs ~ 2 –3 minutes for sufficient volume coverage. Many patients such as children cannot complete a MRI session that often includes >10 scans. In dynamic imaging, k-t SENSE/BLAST and k-t GRAPPA are not fast enough to image physiological motion, e.g., respiration and heartbeats (4). To suppress motion, breath holding is required. As a result, uncooperative patients (e.g. neonates) cannot be scanned (36). In addition, since the whole k-space acquisition time is longer than a cardiac cycle, data segmentation is needed (37,38). This retrospective technique assumes cardiac motion be periodic and collects different k-space segments during the same phases of different cardiac cycles. In many cases where non-periodic cardiac motion exists (e.g. neonatal imaging), data segmentation may generate a k-space dataset with inconsistency across the segments, resulting in low contrast or image blurs (39). There is a clear need for higher imaging speed to enable real-time cardiac MRI.

The work presented here is a technical effort to seek a clinically-usable high-speed MRI technique that can overcome parallel imaging speed limitations. Specifically, we expect that the new technique provide a speed gain over existing parallel imaging techniques and offer the potential to image fast and non-periodic physiological motion in real-time. To that end, a high-speed MRI framework developed in our previous works (40–42), correlation imaging is introduced. In this framework, correlation functions are used to quantify data correlation at different k-space sampling positions and calculate a missing sample from the collected neighboring samples. It has been found that, with the estimate of k-space invariant correlation functions from calibration data, correlation imaging relies on data correlation associated only with coil sensitivity encoding and is equivalent to parallel imaging. To achieve better performance, correlation imaging is formulated into a constrained problem, providing an iterative approach to image reconstruction without calibration data. This allows for the estimate of k-space variant correlation functions that quantify data correlation associated with both coil sensitivity encoding and tissue-boundary sparsity in the human anatomy, thereby providing a speed gain over parallel imaging that relies only on coil sensitivity encoding. For feasibility demonstration, the new correlation imaging approach is investigated in brain imaging and neonatal cardiac imaging.

Methods

Framework of correlation imaging

Correlation imaging (40,41) relies on the estimation of correlation functions. A correlation function between two data acquisition channels is defined as:

$$c_{ij}(\mathbf{k}) = \sum_{\text{all } \mathbf{k}'} d_i(\mathbf{k}' + \mathbf{k}) d_j^*(\mathbf{k}') \quad (1)$$

where i and j are channel indices ranging from 1 to N , $\mathbf{k}$ and $\mathbf{k}'$ are multi-dimensional k-space or k-t space position vectors, $*$ is the conjugate operation, and $\{d_i(\mathbf{k}), i=1, 2, \dots, N\}$ represents N -channel imaging data with full-Fourier encoding. Our previous works (40,41) demonstrated that correlation functions are related to image reconstruction from undersampled data by the following linear equations:

$$\sum_{i=1}^N \sum_{\mathbf{k}' \leq -\mathbf{K}_u} c_{ij}(\mathbf{k} - \mathbf{k}') c_t(\mathbf{k} - \mathbf{k}') u_{mi}(\mathbf{k}') = c_t(0) c_{mj}(\mathbf{k}), \text{ for all } j, \mathbf{k} \text{ such that } u_j(\mathbf{k}) \neq 0 \quad (2)$$

where $\{u_{mi}(\mathbf{k}), i=1, 2, \dots, N\}$ represents a set of linear reconstruction operators for a channel m ($m=1, 2, \dots, N$) with a dimension of $2\mathbf{K}_u+1$ in k-space ($\mathbf{K}_u$ may be empirically determined based on anatomy structure), and $c_i(\mathbf{k})$ can be calculated from a priorly known undersampling trajectory $t_s(\mathbf{k})$ by

$$c_t(\mathbf{k}) = \sum_{\text{all } \mathbf{k}'} t_s(\mathbf{k}' + \mathbf{k}) t_s(\mathbf{k}').$$

With the resolved reconstruction operators from Eq. 2, an estimate of fully-sampled data for channel m , $\hat{d}_m(\mathbf{k})$ can be calculated by

$$\hat{d}_m(\mathbf{k}) = \sum_{i=1}^N \sum_{\mathbf{k}' \leq -\mathbf{K}_u} a_i(\mathbf{k} - \mathbf{k}') u_{mi}(\mathbf{k}') \quad (3)$$

where $\{a_i(\mathbf{k}) = d_i(\mathbf{k}) t_s(\mathbf{k}), i=1, 2, \dots, N\}$ represents undersampled N -channel data.

Effects of coil sensitivity and anatomy sparsity on correlation functions

A correlation function (Eq. 1) measures the linear relationship between two data samples in a distance of $|\mathbf{k}|$ from the same channel or two different channels. Typically, data samples are less correlated if they are located further away from each other. This manifests as the magnitude decrease of a correlation function with the k-space distance $|\mathbf{k}|$, as illustrated in Figure 1(a). For convenience of discussion, a parameter "correlation distance", $\mathbf{K}_u$ is used to represent the distance beyond which the correlation function is close to zero, i.e., data samples are uncorrelated. It follows that, if k-space is undersampled, a missing sample is

linearly correlated to those collected samples only in a distance smaller than $\mathbf{K}_u$. For this reason, the correlation distance determines the dimension of reconstruction operators, i.e., $\{u_m(\mathbf{k}), i=1, 2, \dots, N\}$ is non-zero only if $|\mathbf{k}| \leq \mathbf{K}_u$ in Eqs. 2 and 3. Then if correlation functions decrease more slowly with $|\mathbf{k}|$ (a larger $\mathbf{K}_u$), reconstruction operators will have more non-zero values (a larger dimension), implying a missing sample may be estimated from more neighboring samples with Eq. 3. This is beneficial because image reconstruction takes advantage of more data relationship. Therefore, correlation imaging performs better if the magnitude of correlation functions decreases at a lower rate with the increase of $|\mathbf{k}|$. Practically, it is desirable that a 2D correlation function has a spread-out pattern, i.e., the magnitude of a correlation function is well above zero over a large area around the k-space center (Figure 1b). This spread-out pattern indicates that data correlation exists in those k-space samples even far away from each other.

In parallel imaging, multi-channel images are equal to the multiplication of an image with multi-channel coil sensitivities. As a result of coil sensitivity encoding (image-space multiplication), the correlation functions of multi-channel images are related to those of an original image and multi-channel coil sensitivities by k-space convolution (40,41). This convolution may generate a spread-out pattern around the k-space center in multi-channel correlation functions, introducing neighboring k-space data correlation. For example, in an extreme case where the image is random noise, i.e., the neighboring k-space data are uncorrelated ($\mathbf{K}_u=0$), the correlation function becomes a Dirac function. After coil sensitivity encoding, the k-space convolution may generate a new correlation function with non-zero values at $|\mathbf{k}|>0$ and a correlation distance of $\mathbf{K}_u>0$.

A medical image always has neighboring data correlation associated with anatomy structure. In most cases, however, the correlation functions arising from image contents are dominated by those associated with coil sensitivity encoding. As shown in Figure 1(c), if imaging data are collected using a locally sensitive coil, the sensitivity-encoded image has much stronger k-space data correlation (a much more spread-out correlation function) than the original image. Due to its dominance, the correlation functions associated with coil sensitivity encoding may be directly estimated from multi-channel calibration data and used in correlation imaging (40,41). Here attention should be paid to a special case where the image has anatomy sparsity (Figure 1c, right column). The correlation function of a sparse image has a strong spread-out pattern in reference to that of a non-sparse image (Figure 1c, left column). Mathematically, a sparse image may be modeled as the multiplication of a non-sparse image and a window function with values close to zero in some regions and equal to one elsewhere. This image-space multiplication can enhance neighboring k-space data correlation as coil sensitivity encoding does. The presented work is an investigation on how to improve correlation imaging with correlation functions arising from both coil sensitivity encoding and anatomy sparsity.

Tissue-boundary sparsity variation of k-space data

MRI relies on gradient encoding that decomposes an image into different components in a spatial-frequency domain (k-space). The low-frequency components (central k-space data) primarily contain image contrast information that covers most of the FOV. The high-

frequency components (outer k-space data) arise mostly from tissue boundaries that are naturally sparse in the human anatomy. For this reason, k-space data have tissue-boundary sparsity varying with the distance to the k-space center (the increase of spatial frequencies). As illustrated in Figure 2, the image generated from an outer k-space region (inverse Fourier transform of the data in an outer k-space region) contains more information about sparse tissue boundaries than that from a central k-space region. As a result of higher anatomy sparsity, the correlation function of an outer k-space region has a more significant spread-out pattern than that of a central k-space region. This indicates that the neighboring data samples in outer k-space, due to both coil sensitivity encoding and tissue-boundary sparsity, are more correlated than that in central k-space where coil sensitivity encoding is dominant.

In previous works on correlation imaging (40,41), the whole k-space is reconstructed with correlation functions estimated from fully-sampled calibration data, e.g. auto-calibration signals (ACS). This approach utilizes k-space invariant data correlation associated with coil sensitivity encoding, but cannot benefit from k-space variant data correlation associated with tissue-boundary sparsity. In the presented work, k-space variant correlation functions are introduced for correlation imaging. By reconstructing different k-space regions with different correlation functions, correlation imaging may utilize data correlation associated with both coil sensitivity and tissue-boundary sparsity, providing a gain over existing parallel imaging techniques.

Region-by-region constrained k-space reconstruction

Correlation imaging is self-consistent, i.e., if real data are used to estimate correlation functions with Eq. 1, correlation imaging reconstruction (Eqs. 2–3) will generate the same data, i.e., $\hat{d}_m(\mathbf{k}) = d_m(\mathbf{k})$. With self-consistency, correlation imaging reconstruction may be formulated as a constrained problem:

$$\text{Minimize } \|\hat{d}_i(\mathbf{k})t_s(\mathbf{k}) - a_i(\mathbf{k})\|, \quad i=1, 2, \dots, N \quad (4a)$$

$$\text{Subject to } \{\hat{d}_i(\mathbf{k}), i=1, 2, \dots, N\} \text{ satisfy Eqs. 1 – 3.} \quad (4b)$$

where $\{\hat{d}_i(\mathbf{k}), i=1, 2, \dots, N\}$ are the reconstructed N -channel k-space data, and $\{a_i(\mathbf{k}), i=1, 2, \dots, N\}$ are the N -channel data collected with an undersampling trajectory $t_s(\mathbf{k})$. Here Eq. 4a is a data fidelity constraint requiring that the reconstructed and the collected data are the same at the sampling positions. Equation 4b represents a self-consistent constraint imposed by correlation imaging. This constrained problem can be resolved with a projection onto convex set (POCS) algorithm that has been used previously in many image reconstruction techniques, e.g. JSENSE, SPIRiT, SAKE (14,43,44). In POCS iterative reconstruction, the constraints 4a and 4b may be alternatively applied until satisfactory images are generated. It should be noted that Eq. 4 does not require the calibration of reconstruction operators from a separate set of fully-sampled data, making it possible to implement region-by-region reconstruction with k-space variant correlation functions as follows.

It has been experimentally found that POCS convergence with Eq. 4 may benefit from the availability of some fully-sampled data at the beginning of iterations. This agrees with what has been found in SAKE (44). For this purpose, the presented work uses minor fully-sampled data (~ 4 phase encoding lines for 2D imaging and $\sim 4 \times 4$ phase encoding lines for 3D imaging). To introduce k-space variant correlation functions, iterative correlation imaging is run in a region-by-region fashion. This approach divides the k-space into a region in the central k-space and several bands in the outer k-space (e.g., 2 outer bands in Figure 3). Every outer k-space band overlaps its neighboring band or the central k-space region. Image reconstruction is performed sequentially from the central region to the outer k-space bands: The central k-space region is first reconstructed with available fully-sampled data (from data collection). Every outer k-space band is then divided into multiple local regions (e.g. 8 regions in Figure 3), each of which is reconstructed iteratively using Eq. 4. Here, due to the overlaps between outer and inner k-space regions, some fully-sampled data (from previous reconstruction) are always available before the reconstruction of an outer k-space region. This ensures POCS convergence in outer k-space reconstruction.

In vivo MRI data collection

Human imaging data were acquired in compliance with the regulations of the institution's human ethics committee. The following studies were conducted:

1. **2D brain imaging:** A set of high-resolution brain images was collected on a 1.5T clinical MRI scanner using an 8-channel head coil and a 2D inversion recovery (IR) turbo spin echo (TSE) sequence (FOV 220×220 mm, matrix size 512×512 , TR/TE/TI 3060/126/500 ms, flip angle 90° , TSE factor 32, phase encoding along the left–right direction).
2. **3D brain imaging:** A brain imaging experiment was conducted on a 3T clinical MRI scanner using an 8-channel head coil array and a 3D T1 enhanced ultrafast gradient echo sequence (FOV $240 \times 240 \times 240$ mm, matrix $240 \times 240 \times 154$, TR/TE 9.3/3.7 ms, flip angle 8° , frequency encoding along the superior-inferior direction, scan time ~ 6 minutes).
3. **Neonatal cardiac imaging:** A real-time free-breathing neonatal cardiac imaging experiment was conducted on a 1.5T clinical MRI scanner using a 10-channel body coil array and a balanced steady-state free precession sequence (short-axis single slice, FOV 160×160 mm, matrix 128×128 , TR/TE 3.5/1.1 ms, flip angle 45° , scan time ~ 25 seconds). To achieve a temporal resolution of ~ 50 ms, the 2D data were undersampled by a factor of 4 in the central k-space (inner 24 phase encoding lines) and 16 in the outer k-space. A total of 4 phase encoding lines were fully sampled around the k-space center. Non-real time dynamic images were also collected using a data segmentation method with 16 segments per k-space.

Experimental implementation and performance characterization

In all the presented experiments, the k-space is divided into 17 regions as illustrated in Figure 3 for region-by-region reconstruction with iterative correlation imaging. This k-space

partition is not optimum, but has been found to be sufficient for feasibility demonstration, which is the primary purpose of this work. For convenience, the k-space partition is specified by a central k-space region A with $L_A \times W_A$ samples, an outer k-space band B with $L_B \times W_B$ samples and the overlap region widths of M_{AB} and M_{BC} samples, as illustrated in Figure 3.

Iterative correlation imaging is validated using fully-sampled brain imaging data. Imaging acceleration is simulated by manual undersampling. SENSE (12), GRAPPA (13), SPIRiT (14) and SAKE+ESPIRiT (44) are used as reference methods. Here, SENSE, GRAPPA and SPIRiT are calibrated with fully-sampled central k-space data and only outer k-space is undersampled. SAKE+ESPIRiT and correlation imaging use 4 fully-sampled central k-space lines for 2D reconstruction and 4×4 fully-sampled central k-space lines for 3D reconstruction. Both central and outer k-space regions are undersampled and thus they are run with a higher acceleration factor than the other methods. To characterize the reconstruction performance, an error image is defined as the difference image between the reconstructed and real (from fully sampled data) images. Quantitative evaluation is given by the reconstruction error defined as the ratio of the square Root of the Sum of Squares (RSS) of the error image to that of the real image in percentage.

The real-time imaging capability of correlation imaging is experimentally investigated in neonatal cardiac imaging where respiration and heartbeats may generate fast and non-periodic physiological motion. Data segmentation (37, 38), k-t SENSE (32, 33) and k-t GRAPPA (34) are used as reference methods. All the data are collected with free-breathing. Here k-t SENSE, k-t GRAPPA and correlation imaging are implemented with off-line reconstruction from the same raw data collected in real-time. In data segmentation, images are reconstructed with online tools provided by the MRI manufacturer.

Results

Method validation with publically available data

This experiment uses a set of brain imaging data provided by the downloaded ESPIRiT software (14,44). Figure 4(a) shows a few examples of correlation functions calculated from the data in different k-space regions. It can be seen that correlation functions have considerable variation across the k-space. This provides rationales for k-space variant correlation functions in correlation imaging.

To validate iterative correlation imaging, SAKE+ESPIRiT (14,44) is used as a reference. The central (48×48 samples) and the outer k-space regions are undersampled differently. The k-space partition for region-by-region reconstruction uses a central k-space region A with 48×48 samples, an outer k-space band B with 84×84 samples and the overlap region widths of 12 and 16 samples. In every experiment, two methods are compared with the same fully-sampled data and undersampling factors. Figure 4(b) shows a few results. It can be seen that iterative correlation imaging gives lower reconstruction errors and less artifacts than SAKE+ESPIRiT when the outer k-space is undersampled by a high factor (2×3). These differences may be understood by examining the reconstruction of every local k-space region. Figure 5 shows the plots of local reconstruction errors against iteration steps in three regions (A, B

and C in Figure 4a). It can be seen that both iterative correlation imaging and SAKE converge well in central k-space reconstruction except that the former converges faster. In outer k-space reconstruction, iterative correlation imaging and ESPIRiT give comparable performance with a low undersampling factor (2×1). However, they perform differently with a high undersampling factor (2×3): Iterative correlation imaging always converges well. In comparison, although the whole outer k-space reconstruction error converges well in ESPIRiT, the reconstruction error in a local outer k-space region (e.g. region C) may increase with iterations. This manifests as higher reconstruction errors and more artifacts in the final images.

Image reconstruction with 2D imaging data

Figure 6 shows the reconstruction results from the 2D brain imaging scan. Correlation imaging is compared to SENSE, GRAPPA, SPIRiT, and SAKE+ESPIRiT. In this experiment, the central k-space contains 48 phase encoding lines. SENSE, GRAPPA and SPIRiT are performed with the central k-space fully sampled for calibration. SAKE+ESPIRiT and correlation imaging are performed with the central k-space undersampled by a factor of 2. A total of 4 fully-sampled central k-space phase encoding lines are used to improve iterative reconstruction convergence. The k-space partition for region-by-region reconstruction uses a central k-space region A with 48×48 samples, an outer k-space band B with 180×180 samples and the overlap region widths of 12 and 16 samples. All the methods are compared in a series of experiments with the outer k-space undersampling factors ranged from 2 to 8.

In reference to the real image (Figure 6a), it is found that correlation imaging gives lower reconstruction errors than the other methods (Figure 6b) with the same outer k-space undersampling factors. This gain is more significant when the undersampling factor is high (>4). Figure 6(c) shows a few examples of reconstructed images. It can be seen that these methods give comparable image quality at low undersampling factors (top row) while correlation imaging gives fewer artifacts or lower noise than the other methods at high undersampling factors (bottom row).

Image reconstruction with 3D imaging data

The 3D brain imaging data are undersampled along the left-right (240 samples) and the anterior-posterior (154 samples) directions in which the coil array have multiple elements, i.e., 2D undersampling is applied on the axial plane (perpendicular to frequency-encoding direction) for imaging acceleration. Three experiments are simulated with the scan times of ~ 2 , ~ 1 and ~ 0.75 minutes respectively. These correspond to: 1) full-sampling in the central k-space (64×56 samples) and an undersampling factor of 2×2 in the outer k-space, 2) an undersampling factor of 2×1 in the central k-space (48×32 samples) and an undersampling factor of 2×3 in the outer k-space, and 3) an undersampling factor of 2×2 in the central k-space (48×32 samples) + an undersampling factor of 2×4 in the outer k-space. In SENSE, GRAPPA and SPIRiT, the fully-sampled central k-space data are used for calibration. In SAKE+ESPIRiT and correlation imaging, 4×4 fully-sampled central k-space phase encoding lines are used to improve the convergence of iterative reconstruction. The k-space partition for region-by-region reconstruction uses a central k-space region A with 48×32 samples, an

outer k-space band B with 128×96 samples and the overlap region widths of 12 and 16 samples.

As shown in Figure 7, all the methods generate good image quality with a scan time of ~2 minutes. With a scan time of ~1 minute, correlation imaging, SPIRiT and SAKE+ESPIRiT give slightly better performance than SENSE and GRAPPA. With a scan time of ~0.75 minutes, SENSE and GRAPPA cannot provide interpretable images. Both SPIRiT and SAKE+SPIRiT give noticeable artifacts. In comparison, correlation imaging gives good image quality.

Free-breathing real-time neonatal cardiac imaging

Figure 8(a) shows an ECG record in the neonatal cardiac imaging experiment. It can be seen that R-R interval may reach as short as ~100 ms or as long as ~800 ms, indicating cardiac motion is fast and non-periodic. With free-breathing, respiration may further enhance non-periodic motion. Consequently if data segmentation is used to fill k-space retrospectively with the data collected in different cardiac cycles, non-periodic cardiac motion may introduce considerable data inconsistency, manifesting as temporal contrast loss or image blurs. In this case, real-time data acquisition is required.

Figure 8(b) shows the experimental results from data segmentation and real-time imaging with free-breathing. It is found that data segmentation cannot generate good contrast between systolic and diastolic images. In real-time imaging, k-t SENSE and k-t GRAPPA suffer from considerable noise/artifacts and give poor image quality. In comparison, correlation imaging can generate systolic and diastolic images with good quality and contrast. Here the k-space partition for region-by-region reconstruction uses a central k-space region A with 24×24 samples, an outer k-space band B with 64×64 samples and the overlap region widths of 8 and 10 samples.

Discussion

A mathematical view of correlation imaging

The mathematical formulation of correlation imaging is similar to that of parallel imaging in many aspects (40,41). For example, correlation imaging models k-space reconstruction as the linear prediction from undersampled data. This is equivalent to GRAPPA and SPIRiT, which model a missing k-space sample as the linear combination of its neighbors. In image-space, correlation imaging performs reconstruction by the pixel-wise multiplication of reconstruction operators and multi-channel aliased images. The image-space reconstruction operators play the same role as the unfolding matrix in SENSE. The primary difference of correlation imaging from parallel imaging is that correlation functions are explicitly used to generate the final reconstruction (Eqs. 1–3). This links imaging acceleration to the estimate of correlation functions. The presented work demonstrates an approach to accelerating data acquisition by estimating k-space variant correlation functions.

Physical mechanisms underlying k-space variant correlation functions

Due to Fourier-encoding with gradients, an image is decomposed into different spatial-frequency components in k-space. At tissue boundaries, MRI signals vary fast, generating high spatial-frequency components that make the primary contribution to outer k-space data. The natural tissue-boundary sparsity in the human anatomy may introduce considerable outer k-space data correlation. In parallel imaging, reconstruction operators (e.g. SENSE unfolding matrix, GRAPPA linear weights, or SPIRiT self-consistent weights) are calculated from central k-space calibration data and used to reconstruct the whole k-space. Due to the dominance of data correlation associated with coil sensitivity encoding in central k-space, parallel imaging performance relies on and is limited by a multi-channel coil array. The presented work gives a gain over parallel imaging because image reconstruction utilizes both coil sensitivity encoding and outer k-space tissue-boundary sparsity through the estimate of k-space variant correlation functions from the data in every local region. For the same reason, this gain is not expected in central k-space reconstruction that relies only on coil sensitivity encoding, but primarily in outer k-space reconstruction that can benefit from tissue-boundary sparsity.

Figure 9 shows the k-space reconstruction errors from three brain imaging examples presented in Figures 4–7. It can be seen that correlation imaging and SAKE give comparable reconstruction errors in most of the central k-space because they both rely on data correlation associated with multi-channel coil sensitivities. In contrast, correlation imaging gives considerably lower outer k-space reconstruction errors than ESPIRiT because outer k-space region-by-region reconstruction benefits from data correlation associated with tissue-boundary sparsity through the estimate of k-space variant correlation functions. It should be mentioned that the gain of correlation imaging over SENSE, GRAPPA and SPIRiT also arises from outer k-space reconstruction although their k-space reconstruction errors are not presented. This is straightforward because these conventional techniques require fully-sampled central k-space data for calibration and have zero or minor central k-space reconstruction errors.

From a different perspective, correlation imaging with k-space variant correlation functions is a shift-variant system that offers the ability to optimize k-space reconstruction in a region-by-region fashion. Most conventional techniques use a single set of operators to reconstruct all the data and may be considered as a linear shift-invariant (LSI) system. As a single set of reconstruction operators cannot take into account the k-space variation of neighboring data correlation associated tissue-boundary sparsity, the local k-space reconstruction is not optimized. This explains Figure 5, which shows that ESPIRiT reconstruction, although providing good error convergence for the whole outer k-space reconstruction, does not converge well in several local outer k-space regions. With k-space variation, a shift-variant system gives a better solution to image reconstruction than a LSI system.

K-space partition and computation cost

Computation is a practical concern in iterative region-by-region reconstruction. From Eq. 4, it can be seen that every iteration needs a correlation imaging operation with the computation cost comparable to that of SENSE or GRAPPA. The total computation cost is

thus roughly (iteration number)×(region number) times higher than SENSE or GRAPPA. Ideally, correlation functions should be point-wise different, i.e., every missing k-space data sample should be calculated with different correlation functions. With a region number equal to the number of missing samples, however, the computation cost is high. To reduce the computation cost, k-space may be divided into a number of local regions, e.g., 17 regions in Figure 3. In the presented experiments, this k-space partition is found to be sufficient for generating a gain over parallel imaging in outer k-space reconstruction. The computation cost is also affordable: The reconstruction of a set of 8-channel 200×200 2D images needs ~500 ms using MATLAB (MathWorks Inc., Natick, MA) on a workstation with 2.80 GHz Intel Xeon CPU. This computation burden is ~5 times higher than that of linear methods (GRAPPA or SENSE), comparable to that of nonlinear GRAPPA (45), and ~20% of that of many non-linear methods such as SPIRiT or SAKE+ESPIRiT. It is not as high as expected because the data size of a local region is much smaller than that of the whole k-space, and with less data in region-by-region reconstruction, every correlation imaging iteration runs faster than a standard SENSE or GRAPPA. And similar to parallel imaging, it is apparent that this computation cost may change greatly if the number of channels is high (e.g. 32 or 64).

Advantages in preserving image details

The presented work is based on the mathematical framework (Eqs. 1–3) of correlation imaging (40–41) and the finding of k-space variation of correlation functions associated with tissue-boundary sparsity (42). Its major contribution is the development of a self-calibrated iterative algorithm that allows for the estimation of k-space variant correlation functions without calibration data and enables region-by-region reconstruction for improved outer k-space reconstruction. It should be noted that many advanced image reconstruction techniques also takes into account tissue-boundary sparsity using total-variation penalty (46). Compared with total-variation penalty and previous correlation imaging techniques, the presented approach is advantageous for the preservation of image details. Figure 10 shows an example of image reconstruction with the 2D brain imaging data. Here the presented approach is compared to the wavelet-space correlation imaging (42) and the basic correlation imaging with total variation penalty (40,41,46). It can be seen that all three methods generate comparable errors and artifact/noise level. However, the presented approach gives better performance in image-detail preservation. This benefit should be attributed to the iterative reconstruction with regionally-different correlation functions in outer k-space reconstruction. It should be pointed out that iterative algorithm plays a critical role as the converged reconstruction is dependent primarily on the constraints of POCS and this avoids the propagation of errors from the central to the outer k-space in region-by-region reconstruction.

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References

1. Carpenter A, Leemis L, Rapir A, Phillips D, Phillips G. Managing Magnetic Resonance Imaging Machines: Support Tools for Scheduling and Planning. *Health Care Management Science*. 2011; 14(2):158–173. [PubMed: 21533751]
2. Hussain S, Wielopolski P, Martin D. Abdominal magnetic resonance imaging at 3.0 T: problem or a promise for the future. *Top Magn Reson Imaging*. 2005; 15(4):325–335.
3. Jackson E, Ginsberg L, Schomer D, Leeds N. A review of MRI pulse sequences and techniques in neuroimaging. *Surgical Neurology*. 1997; 47(2):185–199. [PubMed: 9040824]
4. Rehwald W, Wagner A, Sievers B, Kim R, Judd R. Cardiovascular MRI: Its Current and Future Use in Clinical Practice. *Expet Rev Cardiovasc Ther*. 2007; 5(2):307–321.
5. Vargas M, Delavelle J, Kohler R, Becker C, Lovblad K. Brain and Spine MRI Artifacts at 3 Tesla. *Journal of Neuroradiology*. 2009; 36(2):74–81.
6. Edelman RR. The history of MR imaging as seen through the pages of radiology. *Radiology*. 2014; 273(2 Suppl):S181–S200. [PubMed: 25340436]
7. Francois CJ. Advances in CT and MR technology. *Perspect Vasc Surg Endovasc Ther*. 2012; 24(3): 128–136. [PubMed: 23334534]
8. Feinberg DA, Yacoub E. The rapid development of high speed, resolution and precision in fMRI. *NeuroImage*. 2012; 65(2):720–725.
9. Noll DC, Nishimura DG, Macovski A. Homodyne detection in magnetic resonance imaging. *IEEE Trans Med Imag*. 1991; 10(2):154–163.
10. Margosian P, Schmitt F, Purdy D. Faster MR imaging: imaging with half the data. *Health Care Instrumentation*. 1986; 1:195–197.
11. Sodickson DK, Manning WJ. Simultaneous acquisition of spatial harmonic (SMASH): ultra-fast imaging with radiofrequency coil arrays. *Magn Reson Med*. 1997; 38(4):591–603. [PubMed: 9324327]
12. Pruessmann KP, Weiger M, Scheidegger MB, Boesiger P. SENSE: Sensitivity encoding for fast MRI. *Magn Reson Med*. 1999; 42(5):952–962. [PubMed: 10542355]
13. Griswold MA, Jakob PM, Heidemann RM, Nittka M, Jellus V, Wang J, Kiefer B, Haase A. Generalized autocalibrating partially parallel acquisitions (GRAPPA). *Magn Reson Med*. 2002; 47(6):1202–1210. [PubMed: 12111967]
14. Lustig M, Pauly J. SPIRiT: Iterative self-consistent parallel imaging reconstruction from arbitrary k-space. *Magn Reson Med*. 2010; 64:457–471. [PubMed: 20665790]
15. Roemer PB, Edelstein WA, Hayes CE, Souza SP, Mueller OM. The NMR phase array. *Magn Reson Med*. 1990; 16(2):192–225. [PubMed: 2266841]
16. Jung H, Sung K, Nayak KS, Kim EY, Ye JC. k-t FOCUSS: A General Compressed Sensing Framework for High Resolution Dynamic MRI. *Magn Reson Med*. 2009; 61:103–116. [PubMed: 19097216]
17. Gamper U, Boesiger P, Kozerke S. Compressed sensing in dynamic MRI. *Magn Reson Med*. 2008; 59(2):365–373. [PubMed: 18228595]
18. Lustig M, Donoho D, Pauly JM. Sparse MRI: The application of compressed sensing for rapid MR imaging. *Magn Reson Med*. 2007; 58:1182–1195. [PubMed: 17969013]
19. Hess CP, Liang Z-P, Lauterbur PC. Maximum Cross-Entropy Generalized Series Reconstruction. *Int J Imag Syst Tech*. 1999; 10:258–265.
20. Liang Z-P, Lauterbur PC. A generalized series approach to MR spectroscopic imaging. *IEEE Trans Med Imag*. 1991; 10:132–137.
21. Webb AG, Liang Z-P, Magin RL, Lauterbur PC. Application of reduced encoding MR imaging with generalized series reconstruction (RIGR). *J Magn Reson Imaging*. 1993; 3:925–928. [PubMed: 8280985]
22. van Vaals JJ, Brummer ME, Dixon WT, Tuithof HH, Engels H, Nelson RC, Gerety BM, Chezmar JL, den Boer JA. "Keyhole" method for accelerating imaging of contrast agent uptake. *J Magn Reson Imaging*. 1993; 3(4):671–675. [PubMed: 8347963]

23. Jones RA, Haraldseth O, Muller TB, Trinck PA, Oksendal AN. K-space substitution: a novel dynamic imaging technique. *Magn Reson Med*. 1993; 29(6):830–834. [PubMed: 8350729]
24. Hu X. On the "keyhole" technique. *J Magn Reson Imaging*. 1994; 4:231. [PubMed: 8180466]
25. Liang Z-P, Boada F, Haacke EM, Lauterbur PC, Smith MR. Constrained reconstruction methods in MR imaging. *Rev Magn Reson Med*. 1992; 4:67–185.
26. Haacke EM, Liang Z-P, Izen SH. Superresolution reconstruction through object modeling and estimation. *IEEE Trans Acoust, Speech, Signal Processing*. 1989; 37:592–595.
27. Liang Z-P, Jiang H, Hess CP, Lauterbur PC. Dynamic imaging by model estimation. *Int J Imag Sys Tech*. 1997; 8:551–557.
28. Jacob M, Zhu X, Ebel A, Schuff N, Liang Z-P. Improved model-based magnetic resonance spectroscopic imaging. *IEEE Trans Med Imag*. 2007; 26:3201–3226.
29. Madore B, Glover GH, Pelc NJ. Unaliasing by Fourier-encoding the overlaps using temporal dimension (UNFOLD) applied to cardiac imaging and fMRI. *Magn Reson Med*. 1999; 42:813–828. [PubMed: 10542340]
30. Hu X, Parrish T. Reduction of field of view for dynamic imaging. *Magn Reson Med*. 1994; 31:691–694. [PubMed: 8057824]
31. Brummer ME, David MP, Hong CY, Pettigrew RI, Millet-Roig J, Dixon WT. Noquist: Reduced field-of-view imaging by direct Fourier inversion. *Magn Reson Med*. 2004; 51:331–342. [PubMed: 14755659]
32. Tsao J, Boesiger P, Pruessmann KP. k-t BLAST and k-t SENSE: Dynamic MRI with high frame rate exploiting spatiotemporal correlations. *Magn Reson Med*. 2003; 50:1031–1042. [PubMed: 14587014]
33. Jung H, Ye JC, Kim EY. Improved k-t BLAST and k-t SENSE using FOCUSS. *Phys Med Biol*. 2007; 52:3201–3206. [PubMed: 17505098]
34. Huang F, Akao J, Vijayakumar S, Duensing GR, Limkeman M. k-t GRAPPA: A k-space implementation for dynamic MRI with high reduction factor. *Magn Reson Med*. 2005; 54:1172–1184. [PubMed: 16193468]
35. Ohliger MA, Grant AK, Sodickson DK. Ultimate Intrinsic Signal-to-Noise Ratio for Parallel MRI: Electromagnetic Field Considerations. *Magn Reson Med*. 2003; 50:1018–1030. [PubMed: 14587013]
36. Wang Y, Grimm RC, Rossman PJ, Debbins JP, Riederer SJ, Ehman RL. 3D Coronary MR Angiography in Multiple Breath-Holds Using a Respiratory Feedback Monitor. *Magnetic Resonance in Medicine*. 1995; 34:11–16. [PubMed: 7674888]
37. Bluemke DA, Boxerman JL, Atalar E, McVeigh ER. Segmented K-space cine breath-hold cardiovascular MR imaging: Part 1. Principles and technique. *Am J Roentgenol*. 1997; 169:395–400. [PubMed: 9242742]
38. Bluemke DA, Boxerman JL, Mosher T, Lima JA. Segmented K-space breath-hold cardiovascular MR imaging: Part 2. Evaluation of aortic vasculopathy. *Am J Roentgenol*. 1997; 169:401–407. [PubMed: 9242743]
39. Kellman P, Ched'hotel C, Lorenz CH, Mancini C, Arai AE, McVeigh ER. High spatial and temporal resolution cardiac cine MRI from retrospective reconstruction of data acquired in real time using motion correction and resampling. *Magnetic Resonance in Medicine*. 2009; 62(6):1557–1564. [PubMed: 19780155]
40. Li Y, Dumoulin C. Correlation imaging for multiscan MRI with parallel data acquisition. *Magn Reson Med*. 2012; 68(6):2005–2017. [PubMed: 22374782]
41. Li Y. Correlation Imaging with Arbitrary Sampling Trajectories. *Magnetic Resonance Imaging*. 2014; 32(5):551–562. [PubMed: 24629517]
42. Li Y, Wang H, Tkach J, Roach D, Woods J, Dumoulin C. Wavelet-Space Correlation Imaging for High-Speed MRI without Motion Monitoring or Data Segmentation. *Magn Reson Med*. 2015; 74:1574–1586. [PubMed: 25470230]
43. Ying L, Sheng J. Joint image reconstruction and sensitivity estimation in SENSE (JSSENSE). *Magn Reson Med*. 2007; 57:1196–1202. [PubMed: 17534910]

44. Shin PJ, Larson PEZ, Ohliger MA, Elad M, Pauly JM, Vigneron DB, Lustig M. Calibrationless Parallel Imaging Reconstruction Based on Structured Low-rank Matrix Completion. *Magn Reson Med*. 2014; 72:959–970. [PubMed: 24248734]
45. Change Y, Linag D, Ying L. Nonlinear GRAPPA: a kernel approach to parallel MRI reconstrucion. *Magn Reson Med*. 2012; 68(3):730–740. [PubMed: 22161975]
46. Rudin LI, Osher S, Fatemi E. Nonlinear total variation based noise removal algorithms. *Physica D: Nonlinear Phenomena*. 1992; 60:259–268.

Figure 1.

Coil sensitivity encoding and anatomy sparsity both affects correlation functions. (a) A correlation function measures data correlation between two samples located in a distance of $|\mathbf{k}|$. The magnitude of a correlation function decreases with $|\mathbf{k}|$ because two data samples, if located further away, tend to be less correlated. A correlation distance, K_u is defined as the distance within which data samples are correlated. (b) Correlation imaging estimates a missing sample from neighboring collected samples based on correlation functions. A larger correlation distance can benefit correlation imaging because image reconstruction may use more neighboring data correlation. In 2D imaging, it is desirable to have correlation functions with a spread-out pattern (right) instead of those with a concentrated pattern (left). A spread-out pattern indicates more neighboring data are correlated. (c) Both coil sensitivity encoding and anatomy sparsity may affect the correlation function of an image (left column) by introducing a spread-out pattern around the k-space center (center and right columns).

Figure 2.

Illustration of tissue-boundary sparsity variation with a brain image: A central and an outer k-space region may each generate an image by inverse Fourier transform. The image generated from outer k-space data have higher tissue-boundary sparsity than that from central k-space data. Correspondingly, the correlation function shows a stronger spread-out pattern around the k-space center, indicating more neighboring data are correlated in the outer than in the central k-space. This k-space variant data correlation associated with tissue-boundary sparsity is used to improve correlation imaging in the presented work.

Figure 3.

Region-by-region reconstruction with iterative correlation imaging: The k-space is divided into a central k-space region (labelled as "A"), and two outer k-space bands (labelled as "B" and "C"). These 3 regions (A, B and C) have overlaps. Every outer k-space band (B and C) is circumferentially divided into 8 roughly-equal rectangular regions. Image reconstruction is run sequentially from the central to the outer k-space regions. In every region, iterative correlation imaging (POCS) is run with some available fully-sampled data (collected or reconstructed). For convenience, several dimension parameters (L_A , W_A , L_B , W_B , M_{AB} , M_{BC}) are defined as illustrated in the figure. This k-space partition can be specified by the size of the central k-space region A ($L_A \times W_A$ samples), the size of the outer k-space band B ($L_B \times W_B$ samples), and the overlap region widths (M_{AB} and M_{BC} samples).

Figure 4.

Validation of iterative correlation imaging in reference to SAKE+ESPIRiT using a set of publicly available brain imaging data. (a) Correlation functions from different k-space regions show different patterns. The outer k-space correlation functions show stronger spread-out patterns than the central k-space correlation function due to higher tissue-boundary sparsity. (b) A few final reconstructed images are presented. Region-by-region iterative correlation imaging gives lower reconstruction errors and less artifacts (marked by arrows) than SAKE+ESPIRiT when a high undersampling factor (2×3) is used in outer k-space regions. At the bottom-right corner of each image is the reconstruction error.

Figure 5.

The validation experiments (Figure 4) indicate that iterative correlation imaging and SAKE +ESPIRiT perform differently in local reconstruction. Iterative correlation imaging converges faster than SAKE in central k-space reconstruction. In outer k-space regions, iterative correlation imaging always converges well. In comparison, although ESPIRiT reconstruction error over the whole outer k-space decreases with iterations (right column), the error of a local outer k-space region may increase (center column).

Figure 6.

Reconstruction results from 2D brain imaging data. a) Real image for reference. (b) Plots of reconstruction errors against outer k-space acceleration factors for SENSE, GRAPPA, SPIRiT, SAKE+ESPIRiT and correlation imaging. (c) Examples of reconstructed images from different methods with a low outer k-space undersampling factor (top row) and a high outer k-space undersampling factor (bottom row).

Figure 7.

Reconstruction results from 3D brain imaging data. (a) Real image for reference. (b) Three different imaging acceleration experiments are simulated with the scan times of ~2, ~1 and ~0.75 minutes respectively. In reference to the real image, SENSE, GRAPPA, SPIRiT, SAKE+ESPIRiT and correlation imaging give comparable performance at a low speed (scan time 1 minute) while correlation imaging gives lower noise and less artifacts (marked by arrows) at a high speed (scan time < 1 minute) than the other methods.

Figure 8.

Experimental results from free-breathing neonatal cardiac imaging. (a) An ECG record shows that the cardiac cycles vary widely within a range of 100–800 ms, indicating this neonate has fast and non-periodic cardiac motion. (b) Data segmentation cannot provide good contrast between diastolic and systolic images (left column). Two images show little differences due to the loss of temporal contrast. The conventional parallel imaging techniques (k-t SENSE and k-t GRAPPA) suffer from considerable noise and artifacts (center two columns). Correlation imaging (right column) provides better image quality and contrast than the other methods.

Figure 9.

The k-space reconstruction error maps of SAKE+ESPIRiT and correlation imaging in the reconstruction experiments on publicly available brain imaging data (left column), 2D brain imaging data (center column), and 3D brain imaging data (right column). The maps for two different reconstruction methods are presented in the same scale. It can be seen that two methods give comparable errors in central k-space while correlation imaging gives considerably lower errors in outer k-space, indicating the gain of correlation imaging over parallel imaging arises primarily from improved outer k-space reconstruction due to the use of data correlation associated with tissue-boundary sparsity.

Figure 10.

Using the 2D brain imaging data with an outer k-space undersampling factor of 6, correlation imaging with k-space variant correlation functions (presented work) is compared to wavelet-space correlation imaging (reference 42) and basic correlation imaging in combination with total variation penalty (references 40,41,46). Here the reconstruction experiments are performed with the same parameters as those in Figure 6. The top row shows the reference image (most-left column) and the reconstructed images with reconstruction errors at the right-bottom corners. The bottom row shows the zoomed-in images (image details) located in the rectangular boxes on the top row. With respect to the reference (most-left column), the presented approach preserves image details better than the other methods (marked by the arrow) due to improved outer k-space reconstruction.