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More Unified Analysis of Medical Imaging System SNR Characteristics

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Abstract

The ideal observer signal-to-noise ratio has been derived from statistical decision theory for all of the major medical imaging modalities. This yields an absolute scale for image performance assessment and instrumentation design and optimization. Applications include: the functional dependence of detectable detail size on exposure or imaging time; a framework for comparing data acquisition techniques, e.g., Fourier methods vs reconstruction from projections in NMR imaging; calculations of realizable limits, e.g., the limiting gain of time-of-flight PET scanning. Measurements on human observers show that they can come close to ideal performance, except when the noise has negative correlations as in images reconstructed from projections. In this latter case they suffer a small but significant penalty.

Introduction

In the analysis of the sensitivity of medical imaging systems there are a number of structures that are as common to all of the systems as the harmonic oscillator is to so many problems in physics. The purpose of this paper is to sketch our general signal-to-noise ratio (SNR) analysis of the sensitivity of these imaging systems, emphasizing the common elements, and to give some applications of this general approach to particular systems.

The Ideal Observer SNR

The concept of the ideal observer is central to all of our analysis. The ideal observer is one who has access to the raw image data at the first stage of detection, and is able to extract all of the information in this data without its display as an image. The image evaluation problems which we know how to solve involve presenting an ideal observer with a well defined decision task and keeping score on his performance. For example, we might show noisy image data to the ideal observer and ask him to tell us whether it corresponds to a single lesion in the object or two lesions near coalescence. This is the generalization of the Rayleigh resolution task to a binary discrimination task for noisy images. It can also be thought of as the simple extension of the signal detection task where the observer is asked whether the noisy image data corresponds to a signal upon a background, or simply a uniform background. In the analysis of these tasks we shall speak of the difference signal S , which is the result of subtracting the two alternatives of the binary task.

The performance of the ideal observer as summarized, for example, by his per cent correct score in a two alternative forced choice (2AFC) experiment, is determined by the signal and noise characteristics of the image data at the first stage of detection. In the limit of low contrast images the following expression for the ideal observer signal-to-noise ratio uniquely determines his score (1), (2):

$$\text{SNR}_I^2 = \int \frac{[\Delta S]^2 \text{MTF}^2 G^2}{\text{NPS}} df \quad (1)$$

This requires knowledge of the difference signal, the slope of the large-area or macro-transfer characteristic G , the micro- or modulation transfer function (MTF), and the noise power spectrum (NPS) --all in absolute commensurate units. These functions of spatial frequency are integrated over a number of dimensions appropriate to the problem to give what is recognized as the matched filter SNR.

For photon noise limited images the quotient of the properly normalized $G^2 \text{MTF}^2$ and the NPS is identified as the equivalent number of input quanta, or noise equivalent quanta (NEQ), per unit area that for an ideal imaging system would give the same SNR as the real exposure quanta degraded in information content by the actual imaging system (1), (2), (3):

$$\text{SNR}_I^2 = \int [\Delta S]^2 \text{NEQ}(f) df \quad (2)$$

Most of the interesting radiation physics is contained in S ; however, we shall concentrate here on the function describing the detected radiation that is effective in carrying information, namely, NEQ. NEQ is a function of spatial frequency, indicating that the number of quanta available to image low frequencies is not available to the high frequencies. We refer to the NEQ bandwidth as the system aperture in frequency space. Some examples of the NEQ spectrum for conventional radiography (1) and for CT (2) at typical operating points are given in Fig. (1) and Fig. (2). We see that conventional radiographic systems operate with large area NEQ values of about $4 \times 10^4/\text{mm}^2$ and with an average bandwidth equivalent to an aperture of about 0.25 mm. The first and second generation CT head scanners operated with about 10^8 counts/cm along the periphery of the cut, with a bandwidth corresponding to an aperture of about 1 to 2 mm.

The system aperture is an essential concept in the unified view. It corresponds to a distance over which signal counts are irretrievably coupled to background noise counts by those contributors to system blur which do not also blur the noise. A list of system apertures for various modalities is given in Table I. We often refer to the system aperture as the terminal blur since it cannot be processed away by image processing, except in the non-physical limit of a noise free image source and detector.

For thermal noise limited images a concept of a spatial frequency dependent noise equivalent temperature can be defined by analogy with the noise equivalent quanta concept. We do not at present have measurements of NMR systems to demonstrate this concept.

Discrimination tasks such as the Rayleigh task mentioned above have been used by Wagner, Brown, and Hetz (4) to obtain new information on the coded aperture problem, where a more general SNR for high and low contrasts is required. They also have been found by Hanson (5) to have a sensitive dependence on the high frequency region of the NEQ spectrum. This emphasizes the need to make accurate image performance measurements such as HTP and NPS out to frequencies expected to be significant in the imaging task.

When the integral for SNR^2 of Equation (1) is carried out for the elementary signal detection task, the result has the form:

$$SNR_t^2 = C^2 Q d^n$$

where C is the signal contrast, Q refers either to exposure quanta, noise equivalent quanta, or image exposure time, and d refers to the diameter of the lesion to be detected.

The existence of such power laws for imaging systems is well-known but the power cited for n is frequently incorrect, or at least over-simplified. We give our rigorously derived results for the exponent n as a function of imaging modality and lesion size in Table II; "large diameter" and "small diameter" regimes are defined with respect to the size of the system aperture, e.g. large means much greater than the system aperture, etc.

We see first of all the general result that for imaging in two dimensions the exposure or imaging time t depends on the inverse 4th power of lesion diameter, and in three dimensions on the inverse 6th power of lesion diameter (6)-- for small diameter lesions. There is always a large diameter regime in which the dependence is not so severe. One application of this distinction is to the problem of "resolution" in conventional and tomographic radiography.

At any given state of technology it is always possible to move from the 4th or 6th power dependence to, say, a 3rd or 5th power dependence merely (1) for the cost of smaller sources and detector apertures--with noticeable improvement in detail resolution without an exposure cost. This has been taking place over the last five years in x-ray CT. It is also possible to achieve this in positron emission tomography. The situation is somewhat more complicated, however/ for radionuclide imaging and single photon emission CT which are intrinsically dependent on hardware collimation. Analysis of the dose/ or exposure/ resolution function for these systems requires the detailed analysis of ref. 4.

When increased field gradients are used in NMR imaging to obtain improved resolution, the detection bandwidth must be increased to cover the same object format; there is then a commensurate increase in noise. To improve resolution in the orthogonal direction requires an increase in the imaging time. Improved resolution in HMR then, unlike the x-ray case, cannot be obtained without increasing the exposure or imaging time.

Our analysis generates the exact form of the SNR as a function of lesion and aperture size for Gaussian lesions and apertures. We find the form of the SNR not only in the limiting cases but over the transition region. One interesting result we find is that CT methods for NMR (referred to as projection reconstruction, or PR methods) using m views have an m -fold advantage over non-PR techniques for estimating large area or low frequency information due to their use of m times as many estimates of the information in the low frequency regime. This advantage gradually diminishes to unity in the limit of pixel size lesions. It must be balanced, however, against the considerations of the next section.

Finally, the analysis can be used to determine the gain realizable from time-of-flight (TOF) measurements in positron emission tomography (PET) scanning for the limit of perfect TOF resolution. We obtain for the case where the lesion diameter, $d_0 = a_0^{1/2}$, approaches the size of the region scanned, D , a gain of unity, which is obvious. In the other limit, $a_0^{1/2} \rightarrow 0$, the limiting gain is D/d_{AP} , where d_{AP} is the aperture diameter. This gain can be quite appreciable, currently on the order of 5 – 10.

Performance of Real Observers

The performance of real observers can be measured by the true positive and false positive scores in yes/no type experiments or by their percent correct scores in 2AFC experiments. These scores can be transformed into SNRs by the use of appropriate inverse error functions (7). The performance of the ideal observer in the same experiment can either be measured with a computer simulation or by calculating the SNR from the imaging parameters. The ratio

$$OBS \text{ EFF} = \frac{SNR_{Real}^2}{SNR_{Ideal}^2} \quad (3)$$

then expresses a statistical efficiency for the real observer and represents the fraction of the information in the image that he extracts in performing it.

Before proceeding with this discussion, it is necessary to introduce the SNR of an almost- or quasi-ideal observer. This observer ignores the true character of the noise correlations in the image, treating the noise as if it were white. For images whose noise is colored in the frequency domain, or correlated in the spatial domain, the performance of this quasi-ideal observer is not optimal since he effectively makes an incorrect assumption about the independence of the noise samples. This quasi-ideal observer SNR is given by

$$SNR_{ii}^2 = \frac{G^2 [\int [\Delta S]^2 MTF^2 df]^2}{\int NPS [\Delta S]^2 MTF^2 df} \quad (4)$$

This SNR has been used by Otto Schade (8), Wagner, Brown, and Pastel (2), and P.F. Judy (9). Its denominator may be identified as the unnormalized noise granularity function (10).

Now we may rewrite the definition of observer efficiency

$$\text{OBS EFF} = \frac{\text{SNR}_{\text{Real}}^2}{\text{SNR}_{\text{I(deal)}}^2} \left(\times \frac{\text{SNR}_{\text{II}}^2}{\text{SNR}_{\text{II}}^2} \right) = \frac{\text{SNR}_{\text{Real}}^2}{\text{SNR}_{\text{II}}^2} \times \frac{\text{SNR}_{\text{II}}^2}{\text{SNR}_{\text{I}}^2} = 0.5 \pm 0.2 \times \left(\begin{array}{c} \text{Observer} \\ \text{Reconstruction} \\ \text{Efficiency} \end{array} \right)$$

(5)

The first factor in this expression has been found by Burgess and collaborators to cluster about 50% for a wide range of observer detection and discrimination tasks as long as there is enough contrast in the image (7), (11), (12). We shall define the second factor as the “reconstruction/observer efficiency” since, in practice, it is generally only appreciably different from unity for images that have been reconstructed from projections. In these images the negative correlations are not used optimally by the observer, leading to this reconstruction/observer efficiency loss. (The inverse of the reconstruction/observer efficiency is referred to as the reconstruction/observer penalty (13).) A list of these efficiencies is given in Table III. The values follow from using the appropriate noise power spectra in Eqs. (1) and (4).

The ideal observer beats this penalty by either using a complicated averaging scheme in testing the image for the presence of a lesion/ or equivalently by (p)re-whitening or uncorrelating the image noise before testing for the lesion. A poor (but practical) man's version of a pre-whitening filter--with respect to the CT negative noise correlations--can be achieved by means of gross image smoothing. Joseph has found that in CT this requires first expanding the data scale to maintain numerical precision and then using a filter function with a very poor high frequency response (14).

The smallest value in Table III occurs for 3D reconstruction from planar integrals. We therefore expect to see a re-opening of the question of image smoothing for data acquired by that technique. Our analysis shows that optimal smoothing can raise this factor from 1/3 to about 2/3; the corresponding optimal smoothing for 2 dimensional CT raised the factor from $2/\pi = 0.6$ to about 0.8 (2). That is, the relative effect is much greater in the 3 dimensional case.

Non-Gaussian Statistics

In principle, the above analysis is only rigorous for additive Gaussian noise. However, for photon images the multiplicative Poisson noise becomes additive Gaussian noise in the limit of low contrast imaging, and so the above analysis applies in that limit. In ref. (4) the high contrast case was considered, including the Poisson statistics of $\times$ and gamma rays, in a study of the multiplex advantage of coded aperture imaging.

In ref. (15) and (16) the fundamentals of statistical decision theory from which the above analysis derives were applied to the Rayleigh statistics that result from envelope detection of signals returned from ultrasound scattering phantoms. In ref. (17) this work was extended to the more general case of Rician statistics. The results fall within the general framework given here, but additional considerations are required. These will not be treated here.

Exposure Efficiency

Finally, some considerations of system efficiency and exposure optimization are given in refs. (1), (2), (18) including effects of scatter and beam energy (19), (20). There, the generalization of some of the above concepts to detective quantum efficiency, and the generalization of this to exposure and dose efficiency are given. These concepts involve normalization of the above SNR quantities (squared) by exposure quanta or dose. At that point the distance between the performance of the actual system being evaluated and the physically ideal system can be determined.

Conclusions

The ideal observer SNR derived from statistical decision theory leads to an absolute scale of image system performance evaluation: it requires measurement of the large area and micro-area transfer characteristics and the noise power spectrum at a given operating point as a function of spatial frequency. (The commonly measured “pixel variance” does not appear in this analysis.) This information can then be used to see how far a given system falls short of the optimal design, to project possible improvements, and to chart the optimization procedure.

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Fig. 1.
(left): NEQ spectrum for three calcium tungstate screen/film systems (ref. 1) at O.D. = 1.0.

Fig. 2.
(below): NEQ spectrum for first & second generation CT systems at typical operating conditions (ref. 2).

Table I
The Aperture or the Terminal Blur

RADIOLOGY: Focal spot & portion of screen (NEQ bandwidth)	
CT, DR:	$\left\{ \begin{array}{l} \text{Focal spot \& detector aperture} \\ \text{(sampling greatly complicates)} \end{array} \right.$
RN: Collimator, detector	
NMR:	$\left\{ \begin{array}{l} 1/T_2 \\ \text{Field inhomogeneity} \\ \text{Chemical shift} \end{array} \right.$

Table II

Ideal power Law:	C²Q	dⁿ=const
MODALITY	Large d	Small d
Photog/Proj Imaging	2	n 4
2D CT - line f s	3	4
2D FT	2	4
3D CT -planar f s	5	6
3D FT	3	6

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Table III
Reconstruction/Observer Efficiency

2D PR (CT)	---	$2/\pi$
3D PR		
LINE INTEGRALS	-	$\pi/4$
PLANAR INTEGRALS	-	$1/3$

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