RESEARCH

Beamforming for hyperthermia treatment by training a weighted network of an ultrasonic array

Feng-cong Li^{1,2} , Yi-nan Zhao³, Peng-cheng Gong¹, Li Feng¹, Xiang-kui Wan^{2*} and Yan Li⁴

Abstract

We propose a beamforming algorithm based on waveform diversity for hyperthermia treatment of breast cancer using an ultrasonic array. The introduced array has a structure with a network connecting the feeding nodes and the array elements, and the objective of the algorithm is to train the weight matrix of the network to minimize the difference between the generated beam pattern and the ideal one. The training procedure of the algorithm, which is inspired by the idea of machine learning, comprises three parts: forward calculation, comparison, and backward calculation. The forward calculation maps the weight matrix to the beam pattern, and in the comparison step, the generated beam pattern is modified based on the error, and finally, the backward calculation maps the modified beam pattern to a refined weight matrix which performs better than the original one. An optimal weight matrix is obtained by iterative training. The effectiveness of the algorithm is demonstrated by using numerical simulations.

Keywords: Hyperthermia treatment, Breast cancer, MIMO Beamforming, Regression

Introduction

Hyperthermia is a treatment which kills cancer cells by exposing body tissue to high temperature [1, 2]. The local hyperthermia system for breast cancer treatment usually incorporates an ultrasonic array. The principle of this kind of system is heating tumors using ultrasonic energy emitted by the array, and the array has the capability to focus energy by modifying the waveform parameters of each array element [3]. Based on the similarity of the ultrasonic array to the array antenna used by radar systems [4], it is feasible to introduce the well-developed radar beamformer to the ultrasonic system. In fact, both of the clinical research community and the radar community can benefit from each other since they share a common goal: bringing energy to specific target areas, i.e., focused energy delivery [5, 6].

Phased array has been widely used by radar systems, especially in military applications since the array can focus the emitted electromagnetic energy to a specified

direction without rotating the antenna itself, which makes its scanning speed much higher than that of a mechanical scanning radar [7]. The principle of the phased array is not limited to electromagnetic wave. In theory, a phased array can focus any kind of wave energy in the space domain, including ultrasonic wave [8]; hence, phased array has been used in hyperthermia treatment for decades [9–11].

To improve the capability of an array, one can adopt specifically designed waveforms [12], and the radar community finds that by designing transmit waveforms for each individual element of an array, the limits of the phased array can be broken through [13, 14]. As a consequence, the array is no longer a phased array but a multiple-input multiple-output (MIMO) array, and a radar system utilizing this type of array is referred to as MIMO radar.

Nowadays the radar community is interested on boosting the performance of MIMO radar by designing transmit waveforms, and there are three categories of waveform design approaches [15]. The first one is maximizing the mutual information between the received signal and the response of the targets; the second one

*Correspondence: xiangkui_wan@126.com

² School of Electrical and Electronic Engineering, Hubei University of Technology, Wuhan 430068, People's Republic of China
Full list of author information is available at the end of the article

treats the transmit waveform as a filter to suppress signal dependent interference, and the third one is devised to synthesize beam pattern specifically for co-located MIMO radar. The term co-located contrasts with the term distributed, and unlike its counterpart, a distributed MIMO array can improve the performance of radar by utilizing spatial diversity [16–18], which is not considered in this study.

It has been demonstrated that the beam pattern of a co-located MIMO array depends on the correlation matrix of the waveforms [19]. Compared to its phased array counterpart, whose rank of the transmit signal's correlation matrix equals one, the rank of a waveform set used by a co-located MIMO array can be greater than one, and even full rank. This fact implies that the degree of freedom (DoF) of beamforming is much greater than that of a phased array [20–23]. However, a co-located MIMO array suffers signal to interference and noise ratio (SINR) loss inevitably because of its non-coherent property introduced by waveform diversity [24], and researchers provide refined methods to overcome this drawback [25].

As mentioned, the model of the hyperthermia system is very similar to that of a radar array; hence, it is possible to adopt the waveform diversity method to the ultrasonic array. Fortunately, unlike a radar system who needs to receive echoes bounced back from targets to accomplish SINR sensitive tasks such as detection, recognition, and tracking [26], the only thing the ultrasonic hyperthermia system needs to do is focusing energy to the tumor without spreading too much energy to healthy tissues, which means the non-coherent property of waveform diversity is no longer a flaw for hyperthermia treatment. In fact, a beamformer has already been incorporated and its effectiveness has been demonstrated by simulations [27, 28]. This method is based on the MIMO beamformer proposed in [19], which comprises a sequence of mathematical constraint optimization problems: (1) the optimization of the correlation matrix based on given tumor position, and (2) the optimization of the waveform set based on the optimal correlation matrix achieved by solving the previous problem.

In this study, we introduce a refined array structure which is originally designed for MIMO radar [29] to reduce the complexity of the beamformer. The structure has a network connecting the feeding nodes and the array elements. By utilizing this network and an orthogonal waveform set which can be obtained by using methods such as cyclic algorithm [30], the beam pattern becomes a function with respect to the weight matrix of the network rather than the correlation matrix of the waveform set, which means only the weight matrix needs to be optimized for a given scenario. Compared to the array using ordinary waveform diversity methods, the

proposed beamformer comprises only one optimization problem, rather than two. However, the optimization problem has a quartic objective function and a nonconvex feasible region, which makes it difficult to solve. We present an algorithm which trains the weight matrix iteratively to minimize the distance between the generated beam pattern and the ideal one.

Problem formulation

Notations

Array signal processing depends heavily on vectorized operations; hence we introduce following symbols for vectorized mathematical language.

Matrices and vectors are denoted by boldface upper and lower letters, respectively. The transpose and the Hermitian transpose are respectively represented by $(\cdot)^T$ and $(\cdot)^H$. The complex conjugate of a scalar, vector, and matrix is denoted by $(\cdot)^*$. The Euclidean norm of a vector is denoted by $\|\cdot\|$, while $\|\cdot\|_F$ denotes the Frobenius norm of a matrix. The Hadamard product is denoted by $\odot$. The l th element of a vector $\mathbf{x}$ is denoted by $(\mathbf{x})_l$. For a given matrix $\mathbf{X} \in \mathbb{C}^{N \times M}$ given by:

$$\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1M} \\ x_{21} & x_{22} & \cdots & x_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N1} & x_{N2} & \cdots & x_{NM} \end{bmatrix}, \quad (1)$$

$(\mathbf{X})_l$ denotes the transpose of the l th row vector of matrix $\mathbf{X}$, as follows:

$$(\mathbf{X})_l = [x_{l1} \ x_{l2} \ \cdots \ x_{lM}]^T, \quad (2)$$

and $(\mathbf{X})_{n,m}$ denotes the element of $\mathbf{X}$ located on the n th row and the m th column, i.e., x_{nm} . For a square matrix $\mathbf{X} \in \mathbb{C}^{N \times N}$, the vector-valued function $\text{diag}(\mathbf{X})$ is defined as follows:

$$\text{diag}(\mathbf{X}) = [(\mathbf{X})_{1,1} \ (\mathbf{X})_{2,2} \ \cdots \ (\mathbf{X})_{N,N}]^T. \quad (3)$$

Accordingly, for a given vector $\mathbf{x} \in \mathbb{C}^N$, the matrix-valued function $\text{Diag}(\mathbf{x})$ is given by:

$$\text{Diag}(\mathbf{x}) = \begin{bmatrix} (\mathbf{x})_1 & 0 & \cdots & 0 \\ 0 & (\mathbf{x})_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & (\mathbf{x})_N \end{bmatrix}. \quad (4)$$

Distance is an important concept for measuring similarities of vectors or matrices. In this study, the distance between two vectors $\mathbf{x}$ and $\mathbf{y}$ is defined by the Euclidean norm:

$$\text{dist}(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\|, \quad (5)$$

while the distance between two matrices $\mathbf{X}$ and $\mathbf{Y}$ is defined by Frobenius norm, as follows:

$$\text{dist}(\mathbf{X}, \mathbf{Y}) = \|\mathbf{X} - \mathbf{Y}\|_F. \quad (6)$$

For a mathematical object x belong to a metric space, the distance between x and a set S is given by:

$$\text{dist}(x, S) = \min_{y \in S} \text{dist}(x, y). \quad (7)$$

Accordingly, the projector onto a set can be defined as:

$$\text{proj}(x, S) = \arg \min_{y \in S} \text{dist}(x, y), \quad (8)$$

and projectors satisfy the following condition:

$$\text{dist}(x, \text{proj}(x, S)) = \text{dist}(x, S). \quad (9)$$

Pseudo inverse is a mathematical tool for calculating the best fit in the least square sense. For a given matrix $\mathbf{X}$, its pseudo inverse is denoted by $\mathbf{X}^\dagger$, which has the following analytical expression if the columns of $\mathbf{X}$ is linearly independent:

$$\mathbf{X}^\dagger = (\mathbf{X}^H \mathbf{X})^{-1} \mathbf{X}^H. \quad (10)$$

If $\mathbf{X}$ has linearly independent rows, the pseudo inverse is given by:

$$\mathbf{X}^\dagger = \mathbf{X}^H (\mathbf{X} \mathbf{X}^H)^{-1}. \quad (11)$$

For a given set S , the mask $\mathbf{m}(S)$ is a vector-valued function given by:

$$\mathbf{m}(S) = [1_S(1) \ 1_S(2) \ \cdots \ 1_S(\text{card}(S))]^T, \quad (12)$$

where $\text{card}(S)$ denotes the cardinality of S , and $1_S(\cdot)$ is the indicator function given by:

$$1_S(x) = \begin{cases} 1 & x \in S \\ 0 & \text{otherwise} \end{cases}. \quad (13)$$

Additionally, we introduce functional operators for building modularized algorithms, listed as follows. The composition operator is defined as:

$$(f \circ g)(x) = f(g(x)). \quad (14)$$

Based on the definition of composition, we have the functional power given by:

$$(f)^k(x) = \begin{cases} ((f)^{k-1} \circ f)(x), & k > 0 \\ x, & k = 0 \end{cases}. \quad (15)$$

The functional power can implement iterative process, and its efficient numerical implementation is provided in [31].

Array model

The ultrasonic hyperthermia system is somewhat similar to the phased array radar. All of the elements of the phased array transmit a single waveform $\mathbf{s} = [s_1, s_2, \dots, s_N]^T$ with different weights w_m , $m = 1 \dots M$ where M is the number of the elements. The structure of the array is shown in Fig. 1.

From Fig. 1 we find that the phased array has a network with one input node connected to M output nodes with weighted links, and the superposition of all the waveforms is a function with respect to the direction θ , shown as follows:

$$y(\theta) = \mathbf{a}^H(\theta) \mathbf{w}, \quad (16)$$

where $\mathbf{w} = [w_1, w_2, \dots, w_M]^T$ is the weight vector, $\mathbf{a}(\theta)$ is a vector-valued function widely known as steering vector in radar community. For uniform linear array, $\mathbf{a}(\theta)$ has the following form:

$$\mathbf{a}(\theta) = [e^{j\omega_c d \sin \theta / c} \ e^{j2\omega_c d \sin \theta / c} \ \cdots \ e^{jM\omega_c d \sin \theta / c}]^T \quad (17)$$

where d is the distance between adjacent elements, ω_c denotes the angular frequency of the carrier, and c is the speed of light. The following nonnegative function based on (16) is referred to as beam pattern which is used to describe the energy distribution in the space domain of a given weight vector $\mathbf{w}$:

$$P(\theta) = |y(\theta)|^2 = \mathbf{a}^H(\theta) \mathbf{R} \mathbf{a}(\theta), \quad (18)$$

where $\mathbf{R} = \mathbf{w} \mathbf{w}^H$. Obviously, $\text{rank}(\mathbf{R}) = 1$. For a given scenario, the objective function of beamforming is defined as follows:

$$J(\mathbf{w}) = \int_{-\pi/2}^{\pi/2} |P(\theta) - \tilde{P}(\theta)|^2 d\theta, \quad (19)$$

where $\tilde{P}(\theta)$ is the ideal beam pattern in the current scene. By solving the following minimization problem, the optimal weight vector can be obtained:

$$\begin{aligned} \min_{\mathbf{w}} \int_{-\pi/2}^{\pi/2} |P(\theta) - \tilde{P}(\theta)|^2 d\theta \\ \text{s.t. } \mathbf{w} \in S_{\text{con}}, \end{aligned} \quad (20)$$

where S_{con} denotes the feasible region of the weight vector. However, the analytical expression of the ideal pattern $\tilde{P}(\theta)$ is usually not achievable in practice, and the integration also complicates the beamformer; hence the objective function is converted to the following form by sampling the ideal pattern:

$$J(\mathbf{w}) = \sum_{\theta \in S_\theta} |P(\theta) - \tilde{P}(\theta)|^2, \quad (21)$$

where S_θ is a set of angular samples. By substituting (21) into (20) we find that the pattern design problem is in fact a regression problem, which is an active research domain in the artificial intelligence. We may adopt any available regression method to solve the problem. However, the approximation capability of a phased array is quite limited since the rank of the matrix $\mathbf{R} = \mathbf{w}\mathbf{w}^H$ equals one, which means the DoF is quite limited.

In the radar community, the researchers believe that the limit of phased array can be broken through by introducing co-located MIMO array whose structure is shown in Fig. 2. This structure does not need a network since each element of the array has its own transmit waveform.

The baseband form of the collection of the coded waveforms used by the MIMO radar can be expressed by a matrix $\mathbf{S} \in \mathbb{C}^{N \times M}$ shown as follows:

$$\mathbf{S} = [\mathbf{s}_1 \ \mathbf{s}_2 \ \cdots \ \mathbf{s}_M] = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1M} \\ s_{21} & s_{22} & \cdots & s_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ s_{N1} & s_{N2} & \cdots & s_{NM} \end{bmatrix}, \quad (22)$$

where M is the number of the elements, and N is the length of the waveforms. The mechanism of the beamformer of the co-located MIMO radar is explained as follows.

For an arbitrary instant n of the transmitting stage, the coded waveform elements emitted by the array is the n th row of $\mathbf{S}$, and can be denoted as $(\mathbf{S})_n$ according to (2). Considering that the carriers of the array elements are identical, the MIMO array can be seen as a phased array within the period of a coded waveform element, and $(\mathbf{S})_n$ acts as the weight vector. Hence, the instantaneous beam pattern has the following form according to (18):

$$P_n(\theta) = \mathbf{a}^H(\theta)(\mathbf{S})_n(\mathbf{S})_n^H \mathbf{a}(\theta), \quad (23)$$

and the average beam pattern of the MIMO array is given by:

$$\begin{aligned} \bar{P}(\theta) &= \frac{1}{N} \sum_{n=1}^N P_n(\theta) \\ &= \mathbf{a}^H(\theta) \left(\frac{1}{N} \sum_{n=1}^N (\mathbf{S})_n(\mathbf{S})_n^H \right) \mathbf{a}(\theta). \end{aligned} \quad (24)$$

Fig. 2 The structure of a MIMO array

Obviously, the average beam pattern has the same form shown in (18), yet the matrix $\mathbf{R}$ is different, shown as follows:

$$\begin{aligned} \mathbf{R} &= \frac{1}{N} \sum_{n=1}^N (\mathbf{S})_n(\mathbf{S})_n^H \\ &= E\{(\mathbf{S})_n(\mathbf{S})_n^H\}. \end{aligned} \quad (25)$$

The total beam pattern is obtained by $P(\theta) = N\bar{P}(\theta)$. Considering that N is a constant, we adopt the average beam pattern as $P(\theta)$ without loss of generality. It is obvious that $\text{rank}(\mathbf{R})$ is not limited to one. By substituting (25) and (18) into (21), the regression problem can be described as follows:

$$\begin{aligned} \min_{\mathbf{S}} \quad & \sum_{\theta \in S_\theta} \left| \mathbf{a}^H(\theta) \mathbf{R} \mathbf{a}(\theta) - \tilde{P}(\theta) \right|^2 \\ \text{s.t.} \quad & \mathbf{S} \in S_{\text{con}}, \end{aligned} \quad (26)$$

where S_{con} denotes the feasible region of the waveform set. The problem shown in (26) is quite difficult to solve using ordinary regression method since (1) the objective function is quartic and (2) the constraint set S_{con} is usually nonconvex in practical applications. As mentioned, currently the main stream beamformer is based on the one provided in [19], which comprises two constraint optimization problems.

To simplify the process, we introduce a refined array structure which is originally designed for MIMO radar [29], as shown in Fig. 3. This figure implies that the input and output nodes form a network, of which the links connecting the input and output nodes are weighted, and

the weight of the link connecting the p th input node with the q th output node is denoted as w_{qp} . All of the weights form a matrix shown as follows:

$$\mathbf{W} = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1M} \\ w_{21} & w_{22} & \cdots & w_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ w_{M1} & w_{M2} & \cdots & w_{MM} \end{bmatrix}. \quad (27)$$

As mentioned, this structure is originally proposed for MIMO radar to focus electromagnetic energy to given directions. Unlike radar, the hypothermia system should focus the ultrasonic energy on a specific area where tumor exists rather than a direction; hence, the beam pattern with respect to angle should be replaced with a new one with respect to coordinates to describe energy distribution in the space domain. According to (18), we need a new type of steering vector whose argument is coordinate rather than direction, and the new steering vector should take the tissue attenuation into consideration. Moreover, the array for breast is not a linear array, which means that unlike the one shown in (17), the new steering vector depends not only on the distance between adjacent elements, but also on the position of all the elements. In other words, this steering vector is the combination of array and channel model if we regard the tissue as a channel. Here we introduce the expression of the steering vector used in [27], which has the following form:

$$\mathbf{a}(\mathbf{r}) = \begin{bmatrix} \frac{\exp(j2\pi f_c \|\mathbf{r}_1 - \mathbf{r}\| v^{-1})}{\sqrt{\|\mathbf{r}_1 - \mathbf{r}\|}} \\ \frac{\exp(j2\pi f_c \|\mathbf{r}_2 - \mathbf{r}\| v^{-1})}{\sqrt{\|\mathbf{r}_2 - \mathbf{r}\|}} \\ \vdots \\ \frac{\exp(j2\pi f_c \|\mathbf{r}_M - \mathbf{r}\| v^{-1})}{\sqrt{\|\mathbf{r}_M - \mathbf{r}\|}} \end{bmatrix}, \quad (28)$$

where $\mathbf{r}$ is the coordinate vector corresponding to a point in the tissue; v denotes the speed of the sound, and $\mathbf{r}_m$, $m = 1 \dots M$ is the position vector of the m th array element. According to (25), (27), (28), and Fig. 3, the beam pattern is given by:

$$P(\mathbf{r}) = \mathbf{E} \left\{ \mathbf{a}^H(\mathbf{r}) \mathbf{W} (\mathbf{S}_n) (\mathbf{S}_n^H \mathbf{W}^H \mathbf{a}(\mathbf{r})) \right\}. \quad (29)$$

To simplify the system, we adopt orthogonal waveform sets which can be synthesized using cyclic algorithms [30], majorization minimization algorithms [32], spectral approximation algorithms [31, 33], and derivative based algorithms [34]. An orthogonal waveform set satisfies the following condition:

$$\mathbf{E} \left\{ (\mathbf{S}_n) (\mathbf{S}_n^H) \right\} = \mathbf{I}, \quad (30)$$

and the beam pattern is simplified as follows:

$$P(\mathbf{r}) = \mathbf{a}^H(\mathbf{r}) \mathbf{W} \mathbf{W}^H \mathbf{a}(\mathbf{r}). \quad (31)$$

The beamforming problem can now be described by the following mathematical optimization problem:

$$\begin{aligned} \min_{\mathbf{W}} \sum_{\mathbf{r} \in S_r} & \left| \mathbf{a}^H(\mathbf{r}) \mathbf{R} \mathbf{a}(\mathbf{r}) - \tilde{P}(\mathbf{r}) \right|^2 \\ \text{s.t. } & \mathbf{W} \in S_{\text{con}}, \end{aligned} \quad (32)$$

where $\mathbf{R} = \mathbf{W} \mathbf{W}^H$, and S_{con} has the following form:

$$S_{\text{con}} = \{ \mathbf{W} \in \mathbb{C}^{M \times M} : \mathbf{W} \odot \mathbf{W}^* \mathbf{1}_{M \times 1} = \mathbf{1}_{M \times 1} \}, \quad (33)$$

which guarantees that all of the array elements have the same average transmit power.

Here we consider the array and the breast tissue comprise a system whose output is the beam pattern, as shown in Fig. 4.

For a given scenario in which the tumor is located, we can build an ideal beam pattern. To make the system work as expected, we need to train the system to optimize the weight matrix $\mathbf{W}$, and consequently, minimize the difference between the output beam pattern and the ideal one.

Weights optimization

From the discussion above, we know that the problem has a quartic objective function with a nonconvex feasible region S_{con} , which makes it difficult to solve using ordinary method. In this section we propose an iterative algorithm inspired by the idea of machine learning.

From Fig. 4 we know that the problem shown in (32) is in fact a function approximation problem, i.e., regression. The problem can be solved by designing a learning mechanism, and the one we designed is shown in Fig. 5.

The forward calculation in Fig. 5 is based on (31). To complete the training procedure, we still need the following terms:

1. an ideal beam pattern;
2. the inverse system including the reverse propagation whose input is the beam pattern and output is the correlation matrix, and the inverse array network whose input is the correlation matrix and output is the weight matrix;
3. a mechanism to make the weight matrix satisfy the constraint.

The ideal beam pattern

The objective of the training procedure is to focus the energy of the ultrasound to the area where tumor exists. According to the law of conservation of energy, the ideal beam pattern should equal zero on healthy tissue to squeeze the energy to the tumor. The ideal beam pattern is a continuous function with respect to spatial coordinates; however, as explained in Sect. 2.2, we sample the ideal beam pattern to obtain its discrete form, as shown in Fig. 6.

In Fig. 6 the dot and plus symbols denote the positions of the samples on healthy and tumor tissues, respectively. We denote the set of the sample positions as:

$$\mathbf{S}_r = \{\mathbf{r}_1 \ \mathbf{r}_2 \ \cdots \ \mathbf{r}_L\} = S_{r_T} \cup S_{r_H}, \quad (34)$$

where L is the number of samples, and S_{r_T} and S_{r_H} denote the sets of sample positions located in tumor and healthy tissues, respectively. To simplify the notations, we also define two sets: S_{l_T} and S_{l_H} , which comprise the subscripts l of $\mathbf{r}_l \in S_{r_T}$ and $\mathbf{r}_l \in S_{r_H}$, respectively, shown as follows:

$$\begin{cases} S_{l_T} = \{l : \mathbf{r}_l \in S_{r_T}\}, \\ S_{l_H} = \{l : \mathbf{r}_l \in S_{r_H}\}. \end{cases} \quad (35)$$

The ideal beam pattern should satisfy the following condition to squeeze the ultrasonic energy to the tumor tissue:

$$\tilde{P}(\mathbf{r}) = 0, \quad \forall \mathbf{r} \in S_{r_H}. \quad (36)$$

To make the expression tractable using vectorized language, we define a matrix-valued function which is given by:

$$\mathbf{P}(\mathbf{S}_r, \mathbf{W}) = \mathbf{A}^H(\mathbf{S}_r) \mathbf{W} \mathbf{W}^H \mathbf{A}(\mathbf{S}_r) \in \mathbb{C}^{L \times L}, \quad (37)$$

where

$$\mathbf{A}(\mathbf{S}_r) = [\mathbf{a}(\mathbf{r}_1) \ \mathbf{a}(\mathbf{r}_2) \ \cdots \ \mathbf{a}(\mathbf{r}_L)] \in \mathbb{C}^{M \times L} \quad (38)$$

is a matrix comprises steering vectors. The relationship between $\mathbf{P}(\mathbf{S}_r, \mathbf{W})$ and $P(\mathbf{r})$ can be written as follows according to (37):

$$\text{diag}(\mathbf{P}(\mathbf{S}_r, \mathbf{W})) = [P(\mathbf{r}_1) \ P(\mathbf{r}_2) \ \cdots \ P(\mathbf{r}_L)]^T. \quad (39)$$

Consequently, the equation shown in (36) can be rewritten as follows:

$$\tilde{\mathbf{P}} \in S_{\text{obj}} = \{\mathbf{P} \in \mathbb{C}^{L \times L} : (\mathbf{P})_{l,l} = 0, \quad l \in S_{l_H}\}. \quad (40)$$

For a given $\mathbf{P}$, the distance between $\mathbf{P}$ and S_{obj} is given by:

$$\text{dist}(\mathbf{P}, S_{\text{obj}}) = \min_{\tilde{\mathbf{P}} \in S_{\text{obj}}} \text{dist}(\mathbf{P}, \tilde{\mathbf{P}}) = \sqrt{\sum_{l \in S_{l_H}} |(\mathbf{P})_{l,l}|^2} \quad (41)$$

Hence, the projector onto S_{obj} has the following form:

$$\begin{aligned} \text{proj}(\mathbf{P}, S_{\text{obj}}) &= \arg \min_{\tilde{\mathbf{P}} \in S_{\text{obj}}} \text{dist}(\mathbf{P}, \tilde{\mathbf{P}}) \\ &= (\mathbf{1}_{L \times L} - \text{Diag}(\mathbf{m}(S_{l_H}))) \odot \mathbf{P}, \end{aligned} \quad (42)$$

where $\mathbf{m}(\cdot)$ is the masking vector defined in (12). This projector applies the minimal modification on the generated beam pattern according to (36) in order to implement the learning process.

The inverse system

The inverse system comprises the reverse propagation and inverse array network blocks shown in Fig. 5. Physically speaking, the reverse propagation can be seen as the propagation on a reversed time axis, and mathematically speaking, the reverse propagation is an inversion of the propagation equation. The vectorized expression of the propagation process is given by (37), and the reverse propagation can be implemented based on the least-square method shown as follows:

$$\begin{aligned}\hat{\mathbf{R}} &= \mathbf{A}^{\dagger H}(\mathbf{S}_r) \mathbf{P} \mathbf{A}^{\dagger}(\mathbf{S}_r) \\ &= \mathbf{A}^{\dagger H}(\mathbf{S}_r) \mathbf{A}^H(\mathbf{S}_r) \mathbf{W} \mathbf{W}^H \mathbf{A}(\mathbf{S}_r) \mathbf{A}^{\dagger}(\mathbf{S}_r) \\ &= \mathbf{W} \mathbf{W}^H.\end{aligned}\quad (43)$$

The final part of the inverse system is the inverse array network. From Fig. 3 and according to (31) we know that the function of the forward network is to generate the correlation matrix $\mathbf{R}$ by multiplying the weight matrix $\mathbf{W}$ and its Hermitian transpose; hence, the inverse network should apply the following decomposition:

$$\mathbf{R} = \hat{\mathbf{W}} \hat{\mathbf{W}}^H, \quad (44)$$

and this operation can be implemented by eigenvalue decomposition, as:

$$\hat{\mathbf{W}} = \mathbf{V} \mathbf{D}^{1/2}, \quad (45)$$

where $\mathbf{V}$ and $\mathbf{D}$ are comprised of the eigenvectors and eigenvalues of $\mathbf{R}$, respectively, as follows:

$$\mathbf{R} = \mathbf{V} \mathbf{D} \mathbf{V}^H. \quad (46)$$

Consequently, the inverse system whose input is the beam pattern and output is the weight matrix has the following form:

$$\mathbf{Q}(\mathbf{S}_r, \mathbf{P}) = \hat{\mathbf{W}}, \quad (47)$$

where $\mathbf{P}$ is a given beam pattern, and $\hat{\mathbf{W}}$ is obtained by decomposing $\hat{\mathbf{R}}$ according to (44), while $\hat{\mathbf{R}}$ is obtained according to (43).

The constraint

The generated weight matrix $\hat{\mathbf{W}}$ by passing a given beam pattern $\mathbf{P}$ through the inverse system $\mathbf{Q}(\mathbf{S}_r, \cdot)$ may not satisfy the constraint shown in (33), which means the backward calculation needs an extra step to modify the weight matrix to make it satisfy the constraint. The minimum modification expression, i.e., the projector onto the constraint set can be obtained by solving the following problem:

$$\begin{aligned}\min_{\mathbf{W}} \text{dist}^2(\mathbf{W}, \hat{\mathbf{W}}) \\ \text{s.t. } \mathbf{W} \in S_{\text{con}}.\end{aligned}\quad (48)$$

Let $\mathbf{W}$ and $\hat{\mathbf{W}}$ have the following block forms:

$$\begin{cases} \mathbf{W} = [\boldsymbol{\alpha}_1 & \boldsymbol{\alpha}_2 & \cdots & \boldsymbol{\alpha}_M]^T \\ \hat{\mathbf{W}} = [\boldsymbol{\beta}_1 & \boldsymbol{\beta}_2 & \cdots & \boldsymbol{\beta}_M]^T, \end{cases} \quad (49)$$

and the problem shown in (48) has the following equivalent form:

$$\begin{aligned}\min_{\boldsymbol{\alpha}_m, m=1 \dots M} \sum_{m=1}^M \|\boldsymbol{\alpha}_m - \boldsymbol{\beta}_m\|^2 \\ \text{s.t. } \|\boldsymbol{\alpha}_m\| = 1, \quad m = 1 \dots M.\end{aligned}\quad (50)$$

The Lagrangian is shown as follows:

$$\mathcal{L} = \sum_{m=1}^M \left(\|\boldsymbol{\alpha}_m - \boldsymbol{\beta}_m\|^2 + \lambda_m (\|\boldsymbol{\alpha}_m\|^2 - 1) \right), \quad (51)$$

and its derivatives are given by:

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial \boldsymbol{\alpha}_m^*} = (1 + \lambda_m) \boldsymbol{\alpha}_m - \boldsymbol{\beta}_m \\ \frac{\partial \mathcal{L}}{\partial \lambda_m} = \|\boldsymbol{\alpha}_m\|^2 - 1. \end{cases} \quad (52)$$

By solving this equation, we have:

$$\boldsymbol{\alpha}_m = \boldsymbol{\beta}_m / \|\boldsymbol{\beta}_m\|, \quad m = 1 \dots M, \quad (53)$$

which means the row vectors of $\mathbf{W}$ are the corresponding normalized row vectors of $\hat{\mathbf{W}}$, and consequently the projector has the following analytical expression:

$$\begin{aligned}\text{proj}(\hat{\mathbf{W}}, S_{\text{con}}) &= \left[\frac{\boldsymbol{\beta}_1}{\|\boldsymbol{\beta}_1\|} \quad \frac{\boldsymbol{\beta}_2}{\|\boldsymbol{\beta}_2\|} \quad \cdots \quad \frac{\boldsymbol{\beta}_M}{\|\boldsymbol{\beta}_M\|} \right]^T \\ &= \text{Diag}^{-\frac{1}{2}} \left((\hat{\mathbf{W}} \odot \hat{\mathbf{W}}^*) \mathbf{1}_{M \times 1} \right) \hat{\mathbf{W}}.\end{aligned}\quad (54)$$

The training algorithm

The proposed method is modularized; each of the blocks in Fig. 5 corresponds to a function or operator: the array network and propagation block correspond to (37), the modification based on the difference between the generated and the ideal beam pattern corresponds to (42), the reverse propagation and inverse network correspond to (43) and (45), and the projector onto S_{con} corresponds to (54). By assembling the corresponding functions/operators using operator composition and functional power, we obtain an iterative algorithm shown as follows, and we name it the iterative array network training algorithm (IANTA):

$$\begin{aligned}\mathbf{W}^{(K)} &= (\text{proj}(\cdot, S_{\text{con}}) \circ \mathbf{Q}(\mathbf{S}_r, \cdot) \circ \text{proj} \\ &\quad (\cdot, S_{\text{obj}}) \circ \mathbf{P}(\mathbf{S}_r, \cdot))^K (\mathbf{W}^{(0)}).\end{aligned}\quad (55)$$

The step-by-step list of IANTA is as follows:

Step 1: Let the iteration counter $k \leftarrow 0$, and initialize the weight matrix, e.g., $\mathbf{W}^{(0)} = \mathbf{I}_{M \times M}$;

Repeat the following steps:

Step 2: Calculate the beam pattern $\mathbf{P}(S_r, \mathbf{W}^{(k)})$ according to (37);

Step 3: Modify the beam pattern by utilizing $\text{proj}(\cdot, S_{\text{obj}})$ according to (42);

Step 4: Calculate the weight matrix $\hat{\mathbf{W}} = \mathbf{Q}(S_r, \mathbf{P})$ according to (47);

Step 5: Project the weight matrix onto the constraint set $\mathbf{W}^{(k+1)} = \text{proj}(\hat{\mathbf{W}}, S_{\text{con}})$ according to (54), and let $k \leftarrow k + 1$;

Until $k \geq K$ or $\|\mathbf{W}^{(k)} - \mathbf{W}^{(k-1)}\|_F < \eta$, where η is the tolerance.

Numerical examples

We provide a numerical example to demonstrate the effectiveness of IANTA. The breast model we used here is a semicircle of which the radius is 50mm, and its center is located at $[0 \text{ mm}, 0 \text{ mm}]^T$. A tumor located at $[5 \text{ mm}, 35 \text{ mm}]^T$ is a circle with its radius equals 8 mm. The distribution of the 51 array elements is shown in Fig. 6, and the distance between adjacent elements is 1.5 mm. The samples are uniformly distributed in tissue with the distance between neighboring samples equals 0.125 mm.

We also introduce the delay-and-sum (DAS) beamformer for comparison. The beam pattern generated by the weighted array network trained by IANTA is shown in Fig. 7, while the one generated by DAS is shown in Fig. 8.

From these two figures we find that by utilizing the network array structure with a weight matrix trained by IANTA, the energy is evenly distributed in the tumor, while the beam pattern of the DAS beamformer shows that only part of the tumor tissue is heated, and the ultrasonic energy is highly unevenly distributed.

The convergent property is shown in Fig. 9 by the normalized error defined as follows:

$$\epsilon_k = \frac{\text{dist}(\mathbf{P}^{(k)}, S_{\text{obj}})}{\text{dist}(\mathbf{P}^{(0)}, S_{\text{obj}})}, \quad (56)$$

where $\mathbf{P}^{(k)}$ denotes the beam pattern generated by the k th iteration, and $\text{dist}(\mathbf{P}^{(k)}, S_{\text{obj}})$ is defined in (41).

From Fig. 9 we find that the training procedure iteratively reduces the value of the objective function as expected. Consequently, The ultrasonic energy is suppressed in the healthy tissue step by step. We also notice that the convergent rate, i.e., the negative derivative of the convergent curve, is reduced while the training continues. This phenomenon has the following explanation. The value of the objective function shown in (32) is reduced as the iterations proceed, indicating that the distance between the generated beam pattern and the ideal one becomes small, and consequently, the scale of the modification in step 3 according to (42) of IANTA is decreased; hence, the step size of the consecutive iterations is also decreased, and this fact slows down the convergence speed. We believe that this drawback can be overcome by incorporating second order information of the objective function.

Conclusion

An array structure originally developed for MIMO radar is introduced to hyperthermia treatment of breast cancer, and we provide it with a beamformer name IANTA for targeted energy delivery. By training the network iteratively, the beamformer can focus ultrasonic energy to a specified region. Numerical simulation demonstrates that the array with an optimized weight matrix can deliver ultrasonic energy to the tumor, and the targeted energy is evenly distributed compared to its phased array counterpart.

Author details

¹ Hubei Collaborative Innovation Center for High-efficiency Utilization of Solar Energy, Hubei University of Technology, Wuhan 430068, People's Republic of China. ² School of Electrical and Electronic Engineering, Hubei University of Technology, Wuhan 430068, People's Republic of China. ³ Harbin Institute of Technology, Harbin, People's Republic of China. ⁴ University of Southern Queensland, Toowoomba, Australia.

Acknowledgements

This work is supported by the Shenzhen Science and Technology Program (No. JCYJ20150630145302222), the Doctoral Scientific Research Foundation of Hubei University of Technology (4301/00037), the National Natural Science Foundation of China (No. 61601178), the National Natural Science Foundation of China (No. 61571182), and the Science and Technology Key Project of Nansha District (No. 2016CX002).

Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 24 July 2017 Accepted: 23 November 2017

Published online: 12 December 2017

References

- Nguyen PT, Abbosh AM, Crozier S. 3-D focused microwave hyperthermia for breast cancer treatment with experimental validation. *IEEE Trans Antennas Propag.* 2017;65(7):3489–500.
- Nguyen PT, Abbosh A, Crozier S. Three-dimensional microwave hyperthermia for breast cancer treatment in a realistic environment using particle swarm optimization. *IEEE Trans Biomed Eng.* 2017;64(6):1335–44.
- Von Aulock WH. Properties of phased arrays. *Proc IRE.* 1960;48(10):1715–27.
- Liu J, Foiret J, Stephens DN, Le Baron O, Ferrara KW. Development of a spherically focused phased array transducer for ultrasonic image-guided hyperthermia. *Phys Med Biol.* 2016;61(14):5275.
- Song D, Wang W, Xu Z, Xiong Z, Kirubarajan T. Focused energy delivery with protection for precision electronic warfare. *IEEE Trans Aerosp Electron Syst.* 2016;52(6):3053–64.
- Curto S, Prakash P. Design of a compact antenna with flared ground-plane for a wearable breast hyperthermia system. *Int J Hyperth.* 2015;31(7):726–36.
- Kikuchi H, Yoshikawa E, Ushio T, Mizutani F, Wada M. Application of adaptive digital beamforming to Osaka University phased array weather radar. *IEEE Trans Geosci Remote Sens.* 2017;55(7):3875–84.
- Shi K, Que K, Guo D. Flexible ultrasonic phased-array probe. *Tsinghua Sci Technol.* 2004;9(5):574–7.
- Benkeser PJ, Frizzell LA, Ocheltree KB, Cain CA. A tapered phased array ultrasound transducer for hyperthermia treatment. *IEEE Trans Ultrason Ferroelectr Freq Control.* 1987;34(4):446–53.
- Knudsen M, Hartmann U. Optimal temperature control with phased array hyperthermia system. *IEEE Trans Microw Theory Tech.* 1986;34(5):597–603.
- Sathiaselalan V, Iskander MF, Howard GCW, Bleehe NM. Theoretical analysis and clinical demonstration of the effect of power pattern control using the annular phased-array hyperthermia system. *IEEE Trans Microw Theory Tech.* 1986;34(5):514–9.
- Pérez MC, García R, Hernández A, Jimnez A, Diego C, Urea J. SoC-based architecture for an ultrasonic phased array with encoded transmissions. *IEEE Trans Circuits Syst I.* 2015;62(3):873–80.
- Ahmed S, Alouini M. A survey of correlated waveform design for multi-function software radar. *IEEE Aerosp Electron Syst Mag.* 2016;31(3):19–31.
- Gong PC, Wang WQ, Li FC, So HC. Sparsity-aware transmit beamspace design for FDA-MIMO radar. *Signal Process.* 2018;144:99–103.
- Cheng ZY, He ZS, Zhang SM, Li J. Constant modulus waveform design for MIMO radar transmit beampattern. *IEEE Trans Signal Process.* 2017;65(18):4912–23.
- Akçakaya M, Nehorai A. MIMO radar sensitivity analysis for target detection. *IEEE Trans Signal Process.* 2011;59(7):3241–50.
- Chernyak V. Multisite radar systems composed of MIMO radars. *IEEE Aerosp Electron Syst Mag.* 2014;29(12):28–37.
- Chen P, Qi C, Wu L. Antenna placement optimisation for compressed sensing-based distributed MIMO radar. *IET Radar Sonar Navig.* 2017;11(2):285–93.
- Stoica P, Li J, Xie Y. On probing signal design for MIMO radar. *IEEE Trans Signal Process.* 2007;55(8):4151–61.
- Li H, Zhao Y, Cheng Z, Feng D. Correlated LFM waveform set design for MIMO radar transmit beampattern. *IEEE Geosci Remote Sens.* 2017;14(3):329–33.
- Shi J, Jiu B, Liu Y, Yan J, Liu H, Hu L. Fast transmit waveform design method for interference mitigation in simultaneous multibeam MIMO scheme. *Electron Lett.* 2016;52(13):1166–8.
- Aldayel O, Monga V, Rangaswamy M. Tractable transmit MIMO beampattern design under a constant modulus constraint. *IEEE Trans Signal Process.* 2017;65(10):2588–99.
- Hassanien A, Vorobyov SA. Transmit energy focusing for DOA estimation in MIMO radar with colocated antennas. *IEEE Trans Signal Process.* 2011;59(6):2669–82.
- Daum F, Huang J. MIMO radar: snake oil or good idea? In: 2008 42nd Asilomar Conference on Signals, Systems and Computers, pp. 183–187.
- Ahmed S, Alouini MS. MIMO-radar waveform covariance matrix for high SINR and low side-lobe levels. *IEEE Trans Signal Process.* 2014;62(8):2056–65.
- Song X, Willett P, Zhou S, Luh PB. The MIMO radar and jammer games. *IEEE Trans Signal Process.* 2012;60(2):687–99.
- Guo B, Li J. Waveform diversity based ultrasound system for hyperthermia treatment of breast cancer. *IEEE Trans Biomed Eng.* 2008;55(2):822–6.
- Zeng X, Li J, McGough RJ. A waveform diversity method for optimizing 3-D power depositions generated by ultrasound phased arrays. *IEEE Trans Biomed Eng.* 2010;57(1):41–7.
- Ahmed S, Alouini MS. MIMO radar transmit beampattern design without synthesising the covariance matrix. *IEEE Trans Signal Process.* 2014;62(9):2278–89.
- He H, Stoica P, Li J. Designing unimodular sequence sets with good correlations—including an application to MIMO radar. *IEEE Trans Signal Process.* 2009;57(11):4391–405.
- Zhou ZQ, Li FC, Zhao YN. Alternating projections for designing locally orthogonal waveform pairs. *Inf Sci.* 2016;369:267–86.
- Song J, Babu P, Palomar DP. Sequence set design with good correlation properties via majorization-minimization. *IEEE Trans Signal Process.* 2016;64(11):2866–79.
- Zhao YN, Li FC, Zhang T, Zhou ZQ. Computational design of optimal waveforms for MIMO radar via multi-dimensional iterative spectral approximation. *Multidimens Syst Signal Process.* 2016;27(1):43–60.
- Wang YC, Dong L, Xue X, Yi KC. On the design of constant modulus sequences with low correlation sidelobe levels. *IEEE Commun Lett.* 2012;16(4):462–5.