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Linear depolarization of lidar returns by aged smoke particles

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Abstract

We use the numerically exact (superposition) *T*-matrix method to analyze recent measurements of the backscattering linear depolarization ratio (LDR) for a plume of aged smoke at lidar wavelengths ranging from 355 to 1064 nm. We show that the unique spectral dependence of the measured LDRs can be modeled, but only by assuming expressly nonspherical morphologies of smoke particles containing substantial amounts of nonabsorbing (or weakly absorbing) refractory materials such as sulfates. Our results demonstrate that spectral backscattering LDR measurements can be indicative of the presence of morphologically complex smoke particles, but additional (e.g., passive polarimetric or bistatic lidar) measurements may be required for a definitive characterization of the particle morphology and composition.

OCIS codes

(290.5850) Scattering, particles; (290.1090) Aerosol and cloud effects; (290.1350) Backscattering; (290.5855) Scattering, polarization

1. INTRODUCTION

Soot particles represent an important category of tropospheric aerosols causing a direct radiative forcing of climate, affecting cloud formation, and reducing the albedo of ice and snow surfaces [1–8]. It is therefore essential to determine the global distribution of soot and soot-containing aerosols and their microphysical properties from satellite observations.

It is widely recognized that one of the most potent remotesensing tools for the optical characterization of morphologically complex particulates is the measurement of the linear depolarization ratio (LDR) with backscattering lidars [9–20]. Until quite recently, it had generally been believed that strong absorption of light by black carbon causes very small (and thereby hardly useful) LDR values. However, the observations of a smoke plume with the NASA Langley High Spectral Resolution Lidar-2 (HSRL-2) reported by Burton *et al.* [20] revealed highly unusual LDR values reaching 0.2 at the 355-nm lidar wavelength.

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Furthermore, the measured spectral dependence of the smoke LDR was distinctly different from that of dust aerosols.

The initial theoretical analysis of the observed LDR values in [20] (based on the results of [21]) was promising but somewhat inconclusive. Yet it appears to be important to demonstrate explicitly that specific complex morphologies of soot-containing aerosols can indeed reproduce the observed spectral dependence of the LDR and thereby confirm the potential of lidar depolarization measurements to identify and characterize smoke particles. This demonstration would be especially appropriate given the anticipated flight of a polarization lidar as part of the planned NASA Aerosol-Cloud-Ecosystem space mission (<http://acemission.gsfc.nasa.gov>).

Given the extreme complexity of the depolarization scattering phenomenon, it is imperative to analyze the LDR measurements reported in [20] on the basis of a first-principles scattering methodology involving a direct computer solver of the macroscopic Maxwell equations [22–24]. In this paper, we use for this purpose the highly efficient and numerically exact (superposition) T -matrix method.

2. LIDAR MEASUREMENTS

The real-valued so-called normalized Stokes scattering matrix typical of randomly oriented aerosol particles has the following block-diagonal structure [25,26]:

$$\tilde{\mathbf{F}}(\Theta) = \begin{bmatrix} a_1(\Theta) & b_1(\Theta) & 0 & 0 \\ b_1(\Theta) & a_2(\Theta) & 0 & 0 \\ 0 & 0 & a_3(\Theta) & b_2(\Theta) \\ 0 & 0 & -b_2(\Theta) & a_4(\Theta) \end{bmatrix}, \quad (1)$$

where Θ is the scattering angle and the (1, 1) element (conventionally referred to as the phase function) is normalized according to

$$\frac{1}{2} \int_0^\pi d\Theta a_1(\Theta) \sin\Theta = 1. \quad (2)$$

The LDR is then defined as

$$\delta = \frac{a_1(180^\circ) - a_2(180^\circ)}{a_1(180^\circ) + a_2(180^\circ)}. \quad (3)$$

The LDR values measured by Burton *et al.* [20] at the three NASA HSRL-2 wavelengths (355, 532, and 1064 nm) are summarized in Table 1. The measured values are reported as mean $\pm$ one standard deviation for the sample, while systematic measurement uncertainties from the NASA HSRL-2 are given in parentheses. Also shown are the resulting approximate ranges of the measured LDR values, each calculated as the mean $\pm$ the sum of the systematic

measurement uncertainty and one standard deviation. The observations were performed during the Colorado deployment of the DISCOVER-AQ (Deriving Information on Surface Conditions from Column and Vertically Resolved Observations Relevant to Air Quality) field mission on 17 July 2014 and pertain to a plume of wildfire smoke situated at a ~8-km altitude. Figure 1 reveals indeed that the spectral dependence of the smoke LDR is distinctly different from that observed typically for dust-dominated aerosol.

3. MODEL PARTICLE MORPHOLOGIES

It has been well documented that the morphology of smoke particulates can change dramatically during their aging [27–33]. The LDR values computed for fluffy and compact aggregates consisting of pure black-carbon monomers are very small (typically smaller than a few percent [17,34,35]) and cannot explain the observed values indicated in Table 1. Therefore, we need to consider alternative particulate morphologies lacking spherical symmetry and involving significant amounts of a nonabsorbing (or weakly absorbing) refractory material.

One such morphology considered in [21] is a spherical sulfate host with a partially imbedded soot aggregate. Four other model morphologies of aged soot-containing particulates are shown in Fig. 2. The so-called closed-cell morphology depicted in Fig. 2(a) (hereinafter model 1) represents the process of accumulation of a refractory material around individual soot monomers constituting a compact cluster formed after the collapse of the initial fluffy soot aggregate. Figure 2(b) shows a spherical sulfate aerosol hosting a completely imbedded compact soot cluster (hereinafter model 2). Figure 2(c) shows two spherical sulfate particles in contact, each encapsulating a compact soot aggregate (hereinafter model 3). Finally, Fig. 2(d) shows a concentric core-mantle spheroid intended to model a high-density aspherical soot core enveloped by a layer of sulfate material (hereinafter model 4). The scattering properties of models 1–3 were computed using the random-orientation superposition *T*-matrix method developed by Mackowski [36], while those of model 4 were quantified using the random-orientation *T*-matrix program by Quirantes [37].

4. NUMERICAL RESULTS

Extensive *T*-matrix computations for various realizations of model 2 have shown that this morphology causes LDR values too small to reproduce the observed depolarization of lidar returns by smoke particulates. Specifically, linear depolarization is typically ~1% or smaller at all wavelengths. The most likely explanation of this finding is that the outer boundary of such particles is spherical and dominates (i.e., suppresses) the resulting LDRs. Therefore, we will exclude model 2 from the following discussion.

A. Model 1

It has been demonstrated in [34,38] that the overall morphology of a random soot aggregate is well represented by a fractal cluster parameterized by the following statistical scaling law:

$$N = k_0 \left(\frac{R_g}{a} \right)^{D_f}, \quad (4)$$

where a is the monomer mean radius, k_0 is the fractal prefactor, D_f is the fractal dimension, N is the number of monomers in the cluster, and R_g is the radius of gyration. The latter is a measure of the overall radius of the aggregate and is defined by

$$R_g^2 = \frac{1}{N} \sum_{i=1}^N r_i^2, \quad (5)$$

where r_i is the distance of the i th monomer to the cluster's center of mass. The fractal dimension serves as a quantitative measure of the aggregate morphology. D_f values close to 3 represent densely packed aggregates, whereas chain-like branched clusters can have significantly smaller values. The fractal prefactor is also related to the state of compactness of a fractal particle in that for a fixed fractal dimension the packing density tends to be smaller for smaller k_0 .

Consistent with the previous discussion of model 1, we will assume that each monomer in the aggregate shown in Fig. 2(a) consists of a spherical soot core with a radius a_s located in the center of a spherical sulfate host with a radius a . The refractive indices of soot at the three lidar wavelengths were estimated according to Eqs. (19a) and (19b) of [39] and are as follows: $1.66284 + 0.715235i$ at 355 nm, $1.73156 + 0.600028i$ at 532 nm, and $1.81895 + 0.590511i$ at 1064 nm. The corresponding sulfate refractive indices were interpolated from the tabulated values at the 50% relative humidity in [40] and are as follows: 1.3813 at 355 nm, 1.3684 at 532 nm, and 1.3595 at 1064 nm.

Our T -matrix results show that model 1 is capable of reproducing the observed spectral dependence of the LDR. As an example, in Table 2 we list the LDR values computed for 10 random realizations of the model 1 morphology with the following fixed fractal parameters: $k_0 = 1.2$, $D_f = 2.6$, $N = 125$, $a_s = 20$ nm, and $a = 6a_s$. It can be seen indeed that the LDR values in Table 2 follow the observed spectral trend and are approximately consistent with the ranges of the measured LDRs in Table 1.

Similarly, in Table 3 we show the results of T -matrix computations for the same fixed values of the parameters k_0 , D_f , N , and a_s , but for seven values of a ranging from $5.7a_s$ to $6.3a_s$. Again the resulting LDRs, at least those for $a \geq 6a_s$, are largely consistent with Table 1.

B. Model 3

Table 4 summarizes the results of T -matrix computations for 10 random realizations of the model-3 morphology assuming that each spherical sulfate host encapsulates a soot fractal aggregate with $k_0 = 1.2$, $D_f = 2.6$, $N = 125$, and $a_s = 20$ nm. The host radius is 232 nm, and the corresponding soot volume fraction is 0.08. The soot and sulfate refractive indices are the same as in the preceding subsection. Table 5 is similar, but illustrates the sensitivity of

the modeled LDRs to the host radius when the soot inclusions remain the same. Again, most of these model-3 T-matrix results are consistent with the LDR ranges in Table 1.

C. Model 4

Finally, Tables 6 and 7 summarize select T -matrix results for the model-4 morphology. The overall shape of the core-mantle spheroidal particle is characterized by the axis ratio $e = a/b$, where b is the rotational (vertical) axis of the corresponding ellipse and a is the horizontal axis. The axis ratio is assumed to be the same for both the soot core and the sulfate shell. To suppress the effect of scattering resonances on the modeled LDRs, each result is averaged over a narrow power law distribution of shell sizes while assuming that the core remains the same. Accordingly, R_{eff} in Tables 6 and 7 is the effective equal-volume-sphere radius of the entire core-mantle particle and R_{core} is the monodisperse equal-volume-sphere radius of the soot core. The effective variance of the power law distribution [41] is fixed at 0.01. The sulfate and soot refractive indices are assumed to be wavelength-independent. The former is fixed at 1.44, while the latter is $m_{\text{soot}} = 1.75 + 0.435i$ in Table 6 and $m_{\text{soot}} = 1.67 + 0.27i$ in Table 7. Again, the reader can verify that the majority of modeled LDR values in these tables are consistent with the LDR ranges given in Table 1.

5. DISCUSSION

The main objective of this paper is rather limited: to demonstrate that complex morphologies of aged soot-containing aerosols can reproduce the unique spectral dependence of linear depolarization observed for an aged smoke plume by Burton *et al.* [20] (see Fig. 3). Perhaps the most important outcome of our study is that achieving this objective requires the use of expressly nonspherical overall morphologies containing substantial amounts of nonabsorbing (or weakly absorbing) refractory materials (referred to generically as “sulfates”). We have shown that the measured spectral LDR values can be reproduced by a range of model morphologies and a range of model soot and sulfate refractive indices. We leave it up to the experts in aerosol physics and chemistry to discuss which morphological models and/or refractive indices are more or less realistic. It is obvious, however, that spectral LDR measurements can indeed be used to identify the presence of morphologically complex smoke particles, even though additional observations (e.g., with a passive polarimeter [42] or a bistatic lidar [43,44]) may be required to narrow down the plausible ranges of particle morphology (including size) and composition.

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Fig. 1.
Spectral dependence of the LDRs observed for dust-dominated aerosol and smoke [20].

Fig. 2.
Model morphologies of soot-containing particulates.

Fig. 3. Graphical summaries of Tables 3 and 5 (upper panel) and Tables 6 and 7 (lower panel). The shaded area shows the range of measurement uncertainty according to Table 1.

Table 1

Measured Spectral Values of the Linear Depolarization Ratio and Their Ranges [20]

	$\delta(355\text{nm})$	$\delta(532\text{ nm})$	$\delta(1064\text{ nm})$
Measured Range	0.203 ± 0.036 (0.017) [0.150, 0.256]	0.093 ± 0.015 (0.011) [0.067, 0.119]	0.018 ± 0.002 (0.007) [0.009, 0.027]

Table 2*T*-matrix Results for Ten Fractal-Parameter- Equivalent Realizations of Model 1

Realization	δ (355 nm)	δ (532 nm)	δ (1064 nm)
1	0.222	0.056	0.015
2	0.238	0.060	0.021
3	0.237	0.066	0.012
4	0.229	0.060	0.015
5	0.252	0.062	0.013
6	0.224	0.068	0.015
7	0.243	0.054	0.012
8	0.237	0.055	0.010
9	0.239	0.072	0.013
10	0.226	0.056	0.014

Table 3*T*-matrix Results for Seven Versions of Model 1

a/a_s	δ (355 nm)	δ (532 nm)	δ (1064 nm)
5.7	0.289	0.054	0.015
5.8	0.274	0.057	0.015
5.9	0.259	0.060	0.015
6	0.244	0.063	0.016
6.1	0.228	0.067	0.016
6.2	0.212	0.072	0.016
6.3	0.199	0.078	0.016

Table 4*T*-matrix Results for Ten Fractal-Parameter- Equivalent Realizations of Model 3

Realization	δ (355 nm)	δ (532 nm)	δ (1064 nm)
1	0.225	0.093	0.016
2	0.242	0.105	0.018
3	0.220	0.097	0.017
4	0.264	0.110	0.019
5	0.267	0.107	0.019
6	0.232	0.101	0.017
7	0.252	0.102	0.018
8	0.235	0.095	0.016
9	0.264	0.108	0.019
10	0.262	0.109	0.019

Table 5*T*-matrix Results for Six Versions of Model 3

Host radius (nm)	δ (355 nm)	δ (532 nm)	δ (1064 nm)
230	0.276	0.091	0.017
232	0.248	0.103	0.017
234	0.224	0.116	0.018
236	0.202	0.130	0.018
368	0.196	0.113	0.024

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Table 6

T-matrix Model-4 Results for $m_{\text{oot}} = 1.75 + 0.435i$

R_{eff} (nm)	R_{core} (nm)	e	δ (355 nm)	δ (532 nm)	δ (1064 nm)
450	150	1.2	0.241	0.092	0.011
670	200	1.1	0.197	0.090	0.008
640	250	1.1	0.147	0.076	0.015
600	250	1.15	0.228	0.127	0.027
550	150	0.9	0.256	0.089	0.009
550	250	0.9	0.208	0.110	0.022

Table 7

T-matrix Model-4 Results for $m_{\text{soot}} = 1.67 + 0.27i$

R_{eff} (nm)	R_{core} (nm)	e	δ (355 nm)	δ (532 nm)	δ (1064 nm)
470	150	1.2	0.244	0.097	0.008
550	200	1.15	0.207	0.099	0.013
510	250	1.15	0.164	0.110	0.015
600	250	1.15	0.225	0.118	0.018
550	250	0.9	0.204	0.094	0.014
600	300	0.9	0.226	0.132	0.027