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Reduction of Beam Hardening Artifacts in Cone-Beam CT Imaging via SMART-RECON Algorithm

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Abstract

When an automatic exposure control is introduced in C-arm cone beam CT data acquisition, the spectral inconsistencies between acquired projection data are exacerbated. As a result, conventional water/bone correction schemes are not as effective as in conventional diagnostic x-ray CT acquisitions with a fixed tube potential. In this paper, a new method was proposed to reconstruct several images with different degrees of spectral consistency and thus different levels of beam hardening artifacts. The new method relies neither on prior knowledge of the x-ray beam spectrum nor on prior compositional information of the imaging object. Numerical simulations were used to validate the algorithm.

Keywords

Beam hardening artifacts; C-arm Cone Beam Computer Tomography (CT); Spectral inconsistency; Consistency driven iterative image reconstruction

1. INTRODUCTION

When x-ray photons traverse through an image object, low energy photons are preferentially absorbed. This distorts the energy spectrum, increasing the mean energy. As a result, the x-ray beam is said to be hardened. Beam hardening effects are dependent on the composition of both the materials and the total length of the x-ray path. Due to inhomogeneities in tissues and variations of x-ray path length in x-ray CT acquisitions, the measured projection data are not spectrally consistent. This spectral inconsistency problem is exacerbated by the polychromatic nature of the entrance x-ray photons. When the acquired spectrally inconsistent projection data are used in a conventional filtered back projection reconstruction algorithm, Beam Hardening Artifacts (BHA) occur.

Using the empirical fact that the mass attenuation coefficient of almost all soft tissue types has the same energy dependence as the mass attenuation coefficient of water, a water correction scheme is often used to correct BHA. The residual BHA due to other highly attenuating objects such as bones may be further corrected by using some higher order correction methods. In a dual energy data acquisition where the projection data are measured from two different entrance x-ray spectra, the measured projection data can be decomposed into the equivalent path lengths of two material bases and thus, in principle, the spectrally independent projection data can be generated to reconstruct BHA free CT images. Other

empirical beam hardening correction schemes¹ have also been introduced to mitigate BHA. Some other iterative image reconstruction methods have also been introduced to reduce BHA in CT and these methods often require the knowledge of entrance x-ray spectra. For a fixed tube potential data acquisition such as that used in the majority of diagnostic CT scanners, the entrance x-ray spectrum may be measured, although this may be a cumbersome procedure. However, for some clinical cone beam CT systems, such as the C-arm cone-beam CT systems used in interventional procedures, automatic exposure control (AEC) during data acquisition leads to modulation of both tube potential and tube current during the data acquisition from one gantry angle to another. In this case, the known conventional beam hardening correction schemes are not as effective as in other cases without tube potential modulation during data acquisition. In this case, it is highly desirable to have an image reconstruction algorithm that can automatically group the spectrally inconsistent projection data into several relatively consistent subsets and each spectral subset of projection data can be used to reconstruct an individual CT image.

In this paper, the recently developed Synchronized Multi-Artifact Reduction with Tomographic RECONstruction (SMART-RECON) algorithm will be used to cope with the highly spectrally inconsistent data acquired from a C-arm cone beam CT system with tube potential modulation due to the use of AEC. As a result, several images can be reconstructed from a spectrally mixed data set. These images demonstrate different levels of BHA and thus any one of them with significantly reduced BHA can be selected for clinical use. Numerical simulations were used to validate the algorithm.

2. METHODS

2.1 Formulation of the reconstruction problem with spectrally inconsistent projection data

When a cone-beam CT data acquisition is performed to acquire data from one gantry angle to another, x-rays are attenuated differently from one view to another due to the change of intersection between x-rays and image object. For example, in one view angle, x-ray paths may pass through two highly attenuating objects. One example of this is the case when the x-rays pass laterally through the head, passing through the highly attenuating temporal bones from either side. In contrast, in the anterior/posterior direction x-rays only pass through one temporal bone. Therefore, from one view angle to another, acquired data are spectrally inconsistent with each other. Ideally, one would like to reconstruct a single image that corresponds to the spectral information at the given view angle. However, this is not possible for tomographic reconstruction. This reconstruction task can be gradually approachable if we notice that the spectral inconsistency degree is low for a relatively narrow view angle range. Therefore, one can hope to reconstruct several images with different x-ray spectral consistency levels. Each of these images corresponds to the average spectral information within a given angular range.

Assuming data are acquired within an angular range $\Theta = [\theta_i, \theta_f]$ where θ_i/f are the initial view angle position and the final view angle position respectively, we hope to reconstruct one image corresponding to an angular section θ . In term of vector notation used in iterative reconstruction literature, each of these images is an image vector and the union of these image vectors form a spatial-spectral image matrix $\mathbf{X}$. Therefore, the endpoint of the

image reconstruction task is to reconstruct this spatial-spectral image matrix $\mathbf{X}$ from the acquired data set. However, the update of each image column is determined by the discrepancy of the forward projection of the image column and the projection data acquired for the corresponding angular sector.

The major challenge in this reconstruction task is to eliminate the limited view artifacts for each of these image columns in the spatial-spectral image matrix $\mathbf{X}$. A similar situation has been handled in the recently published SMART-RECON method² where the spatial-temporal image matrix is reconstructed from data acquired in consecutive view angle sector.

2.2 SMART-RECON framework

The recently proposed SMART-RECON can be formulated as the following convex optimization problem:

$$\tilde{\mathbf{X}} = \arg \min_{\mathbf{X}} \left[\frac{1}{2} (\vec{\mathcal{Y}} - \mathcal{A} \vec{\mathbf{X}})^T \mathbf{D} (\vec{\mathcal{Y}} - \mathcal{A} \vec{\mathbf{X}}) + \lambda \|\mathbf{X}_A\|_* \right] \quad (1)$$

where $\vec{\mathcal{Y}}$ is the vectorized projection data and $\vec{\mathbf{X}}$ is the vectorized spatial-temporal image matrix. The prior image column vector, $\vec{\mathbf{X}}^p$, is used to augment the target spatial-temporal image matrix $\mathbf{X}$ to generate the augmented spatial-temporal matrix $\mathbf{X}_A$. The diagonal matrix $\mathbf{D}$ has the inverse of the noise variance of the log-transformed data as its diagonal elements. The notation $(\cdot)^T$ is the transpose operator. The parameter λ is introduced to control the balance between the data fidelity term and the regularizer strength. The nuclear norm of this latter matrix, i.e., $\|\mathbf{X}_A\|_*$, is used to regularize the reconstruction of the spatial-temporal matrix, $\mathbf{X}$.

$$\mathbf{X}_A = (\vec{\mathbf{X}}^p | \mathbf{X}) =: \begin{pmatrix} X_1^p & X_1^1 & X_1^2 & \cdots & X_1^T \\ X_2^p & X_2^1 & X_2^2 & \cdots & X_2^T \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ X_M^p & X_M^1 & X_M^2 & \cdots & X_M^T \end{pmatrix} \quad (2)$$

where $=:$ stands for a definition. The nuclear norm of this matrix, i.e., $\|\mathbf{X}_A\|_*$, is used to regularize the reconstruction of the spatial-temporal matrix, $\mathbf{X}$. Namely, the regularizer is given as below:

$$\|\mathbf{X}_A\|_* = \|\mathbf{U} \Sigma \mathbf{V}^T\|_* = \sum_r \sigma_r \quad (3)$$

where $\mathbf{X}_A = \mathbf{U} \Sigma \mathbf{V}^T$ is the singular value decomposition (SVD) of the matrix $\mathbf{X}_A$. In this decomposition $\mathbf{U}$ and $\mathbf{V}$ are two orthogonal matrices, and $\Sigma = \text{diag}\{\sigma_r\}$ is a diagonal matrix. The values σ_r ($r = 1, 2, \dots$) are also known as the singular values of the prior image augmented spatial-temporal image matrix $\mathbf{X}_A$. In this work, the FBP reconstruction from all available time-frames was used to generate the prior image. Projection data from the entire

short scan range was separated into 3 or 4 subsets corresponding to narrower angular spans. Each subset of the projection data forms the vector $\vec{\mathcal{Y}}$. Each reconstructed image volume corresponding to one subset forms the spatial-temporal image matrix $\mathbf{X}$.

The above convex optimization problem 1 can be solved using the recently developed alternated updating method³ to decompose the original optimization problem into two sub-problems,

$$\vec{X}^{(k)} = \arg \min_{\vec{X}} \frac{1}{2} (\vec{\mathcal{Y}} - \mathcal{A} \vec{X})^T D (\vec{\mathcal{Y}} - \mathcal{A} \vec{X}) \quad (4)$$

$$\mathbf{X}_A^{(k+1)} = \arg \min_{\mathbf{X}_A} \frac{1}{2} \|\mathbf{X}_A - \mathbf{X}_A^{(k)}\|_F^2 + \lambda \|\mathbf{X}_A\|_* \quad (5)$$

where $\|\mathbf{X}\|_F^2 = \sum_{ij} X_{ij}^2$ is the Frobenius norm of the real matrix $\mathbf{X}$. The quadratic problem in Eq. (4) can be easily solved iteratively using the following update sequence:

$$\vec{X}^{(k)} = \vec{X}^{(k-1)} + \delta \mathcal{A}^T D [\vec{\mathcal{Y}} - \mathcal{A} \vec{X}^{(k-1)}] \quad (6)$$

After the k -th iteration of the image column vector $\vec{X}$, i.e., $\vec{X}^{(k)}$, is computed using the above formula, it is converted back to its spatial-temporal matrix form $\mathbf{X}^{(k)}$. The parameter δ is the updating step size. It was empirically fixed to 0.25 in this study. The modified denoising problem in Eq. (5) is then solved to obtain the denoised spatial-temporal matrix denoted by $\mathbf{X}^{(k+1)}$:

$$\mathbf{X}_A^{(k+1)} = SVT_\lambda(\mathbf{X}_A^{(k)}) \quad (7)$$

where the singular value thresholding (SVT) function $SVT_\lambda(\mathbf{X})$ is defined as follows,⁴

$$SVT_\lambda(\mathbf{X}) = U \left(\sum - \lambda \mathbf{I} \right)_+ V^T \quad (8)$$

$$\left(\sum - \lambda \mathbf{I} \right)_+ = \begin{cases} \sigma_i - \lambda, & \sigma_i \geq \lambda \\ 0, & \sigma_i < \lambda \end{cases} \quad (9)$$

where $\mathbf{I}$ is an identity matrix. This two-step optimization strategy has been proven to be linearly convergent.³ The implementation details can be referred to in Chen and Li.^{2,3,5}

2.3 Simulated phantom study

In our numerical phantom studies, the entire projection data set is separated into many angular sectors. Although it isn't necessary to avoid overlap, the 360° projection data was separated into 9 non-overlapped subset as shown in Figure 1 (b). All 9 subsets form the measurement matrix $\vec{y}$. Images reconstructed from each individual angular span form each individual image column in image matrix $\mathbf{X}$.

In numerical simulations, a water tank ($\varnothing=14\text{cm}$) with two bone inserts ($\varnothing=2\text{cm}$) were used to generate the fan-beam projection data. Poisson noise was added to the numerical simulation data by assuming of an entrance photon fluence of 1×10^6 photons/ray. The total view angle span in the numerical simulations was simulated as 360°. An image matrix size of 512×512 , corresponding to a 0.625mm^2 pixel size, was used in image reconstruction. 360 view angles were acquired over the short scan angular span with 888 detector elements crossing the entire scanning field of view. The polychromatic X-ray spectrum at 140 kVp with relatively soft filtration (0.5 mm Be + 0.35 mm Cu was simulated using the Spektr software package.)

For quantitative comparison, the the relative root mean square error (rRMSE) was calculated for the FBP reconstruction result, and for the SMART-RECON images from different angular span. The metric is defined as follows:

$$rRMSE(x) = \frac{\sqrt{\sum_i (x_i - x_i^{\text{mono}})^2}}{\sum_i |x_i^{\text{mono}}|} \times 100\% \quad (10)$$

where x is the image being assessed, x^{mono} is the ground truth reference image simulated from monochromatic X-ray projection data, and the subscript i denotes the pixel location in the image.

3. RESULTS

Numerical simulation results are presented in Figure 2. The polychromatic FBP image was generated from the projection simulated with the spectrum shown in Figure 1 (c). The monochromatic FBP image was generated from the projection simulated with the δ spectrum with the energy as the mean energy of the spectrum used in Figure 1 (c). Images in Figure 2 are the SMART-RECON reconstructed images using the data corresponding to the angular span shown in Figure 1 (b). This images demonstrate that the beam hardening effect presents differently in each angular span. Corresponding to the given angular span separation mode in Figure 1 (b), SMART-RECON image corresponding to angular sector 2 shows less BHA then images corresponding to sector 1 and 3 because the X-ray photons corresponding to the angular sector 2 pass more high concentrated material than that corresponding to sector 1 and 3. One may notice that the residual limited-view artifacts still present in each images. This is easy to understand since in the static imaging case, SMART-RECON can fully eliminate limited-view artifacts if the selected parameter λ equals to the largest singular value of the image matrix. However, if we really do this, the BHA will

present equally in each image. Hence, the parameter λ has to be fine tuned according the the attenuation distribution and the angular span separation mode.

To study the performance dependence on the phantom anisotropy, the elliptical shape phantom was designed and studied. In the phantom 2, the length ratio between major axle and minor axle is 5:1. The fan beam geometry was used. The total 240 views were equally distributed over 240 degree short scan angular range with the assumed 60 degree fan angle. The polychromatic spectrum was used the same as the circle phantom. Due to the anisotropic shape, the mean energy of the exiting spectrum along horizontal direction is higher than that along vertical direction. In a word, the spectral inconsistency level over the entire projection angular range is higher. Based on this, in Figure 3 (a), more severe BHA can be observed along horizontal direction using FBP as reconstruction algorithm. In SMART-RECON, the entire shore scan data were separated into 3 non-overlapped subsets. Intuitively, the exiting spectra within the second sector have relatively lower mean energy than that in other sectors because rays within this sector pass through at most one high attenuating objects while rays within the first and third sectors may pass through two high attenuating objects. Hence, data within the second sector have higher spectral consistency and should appear negligible BHA using SMART-RECON. We can observe this in the Figure 3 (b)–(d).

The rRMSEs were calculated for all images reconstructed with FBP and SMART-RECON to quantify the ability of BHA reduction. The rRMSE were calculated locally as the region-of-interest (ROI) is selected as the center region between two high contrast inserts. The rRMSE result was shown in Figure 4. The rRMSE between polychromatic FBP and monochromatic FBP was shown as the purple straight line. The quantification result in Figure 4.

4. DISCUSSIONS AND CONCLUSIONS

Over the past decades, many algorithms have been proposed to mitigate BHA. In this work, a new technique was proposed to achieve BHA reduced reconstruction. For a short scan CT acquisition, smaller angular span often corresponds to better spectral consistency compared to that of the entire short scan angular range and thus potentially less severe BHA in the reconstructed CT image. SMART-RECON method can be applied to mitigate spectral inconsistency induced BHA without explicit knowledge of x-ray spectrum, without additional calibration procedures, or other additional post-processing methods. Numerical simulations show the potential of SMART-RECON to help mitigate BHA.

To reconstruct a image volume using the data angular range narrower than short scan range using FBP, severe limited view artifacts make images useless. A recently proposed algorithm SMART-RECON can be used to mitigate spectral inconsistency artifacts and eliminate limited view artifacts simultaneously. To eliminate limited-view artifacts, two things are special in SMART-RECON: (1) the nuclear norm of the spatial-spectral matrix is used to promote similarity between different image columns and thus to mitigate limited view artifacts; (2) a limited-view artifacts free prior image is used to augment the spatial-spectral matrix to further sparsify images and to reduce limited view artifacts.

Algorithm is operated in a local workstation equipped with an nVidia GTX Titan Z GPU (nVidia, Santa Clara, CA). It takes 30s to complete one iteration. Normally, 15 iterations are sufficient to achieve the empirical convergence.

Instead of eliminating BHA, different levels of BHA present in images corresponding to different angular sectors. Performance of the algorithm in mitigating BHA depends on the distribution of high attenuating materials in the imaging object and data grouping manner. Robustness will be validated in phantom with more complicated distribution of high attenuating materials and different data grouping manner.

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Figure 1.
Experiment setup and spectrum used in this study.

Figure 2.

Image results for circle phantom. All images are shown with a W/L: 1000/800.

Figure 3.

Image results for elliptical phantom. All images are shown with a W/L: 1000/800.

Figure 4. Quantitative comparison in elliptical phantom. The ROI used to calculate rRMSE is indicated as the center region between two high contrast inserts.