Persistence length of collagen molecules based on nonlocal viscoelastic model

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Received: 5 April 2017 / Accepted: 16 August 2017 / Published online: 6 September 2017
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Abstract Persistence length is one of the most interesting properties of a molecular chain, which is used to describe the stiffness of a molecule. The experimentally measured values of the persistence length of the collagen molecule are widely scattered from 14 to 180 nm. Therefore, an alternative approach is highly desirable to predict the persistence length of a molecule and also to explain the experimental results. In this paper, a nonlocal viscoelastic model is developed to obtain the persistence length of the collagen molecules in solvent. A new explicit formula is proposed for the persistence length of the molecule with the consideration of the small-scale effect, viscoelastic properties of the molecule, loading frequency, and viscosity of the solvent. The presented model indicates that there exists a range of molecule lengths in which the persistence length strongly depends on the frequency and spatial mode of applied loads, small-scale effect, and viscoelastic properties of the collagen.

Keywords Collagen molecule · Persistence length, nonlocal elasticity · Viscoelastic property

1 Introduction

Collagen or tropocollagen molecules are crucial protein-based structural materials in the human body. These molecules provide mechanical stability, elasticity, and strength to many biological tissues including ligaments, tendons, and bone. They have the ability to adapt their properties in response to external stimuli, such as mechanical loads [1]. A collagen molecule consists of three helical polypeptide chains that are turned around each other to form a three-stranded rod-like molecule (Fig. 1) with a diameter of about 1.4 nm and a length of about 300 nm [2].

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Fig. 1 Molecular structure of collagen molecule

Finding the flexural rigidity of the collagen molecules is an essential prerequisite for understanding of mechanics of collagen fibers and tissues. The flexural rigidity of molecular chain is usually quantified by its persistence length, which describes a chain's resistance to thermal forces [3]. The persistence length of various biological molecules, including microtubules [4], actin filaments [5], intermediate filaments [6], DNA [7], and collagen molecules [8], were measured via several experimental and computational methodologies.

Over the past two decades, the persistence length of the collagen molecules has been estimated by different techniques such as electron microscopy analysis, optical tweezers, hydrodynamic method, etc. Unfortunately, considerable inconsistencies have developed in the literature, wherein the reported obtained values for the persistence length of the collagen molecules have varied from 14 to 180 nm [9]. The first contribution to the estimation of the persistence length of the collagen molecules was reported in 1973 by Utiyama et al. [2]. They calculated a value of 130 nm for the persistence length of tropocollagens by comparing the experimental data with the hydrodynamic theories of wormlike-coil. In another experimental study, Saito et al. [10] evaluated the persistence length of lugworm cuticle collagen from the intrinsic viscosity and the sedimentation constant. It was found that the persistence length ranges from 160 to 180 nm. Nestler et al. [11] evaluated the value of 170 nm for this parameter from viscoelastic measurements. Through an electron microscopy analysis, Hofmann et al. [12] found a value of 57 ± 5 nm for the persistence length of collagen molecule type I. In another study, the persistence length of type I collagen was measured using optical tweezers to pull a single collagen molecule [8]. The persistence length and contour length were found to be 14.5 and 309 nm, respectively. Based on the atomic and continuum modeling, Buehler [13] recommended the value 23.4 nm for the persistence length. Furthermore, the value 51.5 ± 6.7 was predicted on the basis of the coarse-grained model [14].

Based on the above discussion, it is clear that the values of the persistence length are scattered due to the effects of various factors such as the uncertainties of parameters, different experimental setup, definition of the parameters used to fit experimental data with theoretical models, etc. In all previous studies, the collagen molecules were treated as a one-dimensional elastic structure and their persistence lengths were estimated according to the elastic beam model and standard worm-like chain model. However, it has been shown that the individual collagen molecules present viscoelastic behavior [15, 16]. Furthermore, as the size of collagen molecules is sufficiently small, small-scale effects induced by collagen molecules and the discrete nature of the molecules become more important and cannot be ignored anymore. It is observed that the effects of small-scale, viscoelastic properties of the molecule, loading frequency, and viscosity of the solvent have been ignored in many previous studies. The question is how the small-scale effect and the viscoelastic properties affect the persistence length of the collagen molecules.

Motivated by these ideas, in the present study, a nonlocal viscoelastic model is developed to obtain a deeper understanding of the persistence length of the collagen molecules for the first time in the literature. In this connection, Eringen's nonlocal elasticity theory [17] is used to incorporate small-scale effects. This theory has been successfully used to solve problems involving the mechanics of the biological as well as non-biological nanoscopic structures [18–20]. The nonlocal theory provides an efficient mechanism for capturing small-scale effects. Furthermore, mean field Langevin equation of the chain [21] is utilized and the viscoelastic material properties, calculated by Gautieri et al. [15], are considered. Here, the collagen molecule is considered as a linear viscoelastic beam and the fluid around the molecule is modeled as Stokes flow with small Reynolds number. The effects of the viscoelastic property of the collagen molecule, viscosity of the external fluid, nonlocal parameter, and loading frequency are examined in detail.

2 Mathematical formulation

The collagen molecule is a thermally deformable rod-like molecule and it can be proposed as a nonlocal viscoelastic Euler-Bernoulli beam model. On the other hand, the statistical behavior of the collagen can be described by the worm-like chain theory. In order to obtain the effective bending stiffness of the collagen, the worm-like chain is assumed to have the same length, boundary, and loading conditions. The effective persistence length can thus be determined by equating the deformations of the chain and beam. Recent investigations have revealed that the electrostatic stiffening effect may be important topics for some semiflexible polymers [22]. Thus, the persistence length may be written as the sum of an intrinsic persistence length and an electrostatic persistence length. The electrostatic persistence length is highly sensitive to salt concentration [23]. However, the electrostatic stiffening effect has not been explored for the collagen molecule. Therefore, in this study, the intrinsic persistence length arising from nonelectrostatic interactions is only predicted. The interested reader can find additional details of the concept of electrostatic persistence length in Ref. [24]. Furthermore, we do not consider any kinds of structural transition of the molecules, which is the reason of the length-dependent persistence length of short DNA.

The collagen molecule in solvent can be modeled as a linear viscoelastic beam surrounded by viscous fluid. Based on the nonlocal Euler-Bernoulli beam theory [20], the equation of motion for the collagen molecule can be expressed as

$$\frac{\partial^2}{\partial x^2} \left(-\tilde{E}I \frac{\partial^2 w}{\partial x^2} \right) + f + \mu^2 \frac{\partial^2}{\partial x^2} \left[\rho A \frac{\partial^2 w}{\partial t^2} - f \right] = \rho A \frac{\partial^2 w}{\partial t^2}, \quad (1)$$

where w is the transverse deflection, t is the time, ρ is the mass density of the collagen molecule and A is effective area of the molecule. Moreover, $\tilde{E}$ denotes Young's modulus, I is the cross-sectional moment of inertia, μ is the nonlocal scale coefficient and f is the external transverse excitation. It should be noted that the value of the nonlocal scale coefficient depends on the crystal structure and the nature of the physics under investigation. Unfortunately, no experiments have been conducted to predict the magnitude of

the nonlocal scale coefficient for the collagen molecule. Therefore, the nonlocal scale coefficient $0 \leq \mu \leq 0.15$ L is adopted in our investigation. It was shown that the viscoelastic behavior of an individual collagen molecule could be successfully simulated by the Kelvin–Voigt model [15]. On the basis of the Kelvin–Voigt model, for a material with viscoelastic properties, one can write

$$\tilde{E} = E + E^* \frac{\partial}{\partial t}, \quad (2)$$

in which E is the elastic modulus and E^* is the viscoelastic parameter which are assumed to be time independent. The transverse excitation can be expressed as:

$$f = q - \alpha \frac{\partial w}{\partial t}, \quad (3)$$

where q is the distributed transverse load. The second term in Eq. (3) represents the hydrodynamic drag per unit length, which is defined by [25]

$$\alpha = \frac{2\nu_f \pi (\eta^2 - 1)}{1 - \eta^2 + (1 + \eta^2) \ln \eta}, \quad \eta = \frac{R}{R_1}. \quad (4)$$

Here, ν_f is the viscosity of the external fluid, R is the effective collagen molecule, and R_1 is the distance from the center line to the position where the induced viscous flow vanishes. The details of mathematical derivation for developing Eq. (4) have been reported in Ref. [25]. Substituting Eqs. (2) and (3) into Eq. (1), one obtains

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} + E^* I \frac{\partial^5 w}{\partial x^4 \partial t} + \alpha \frac{\partial w}{\partial t} - \mu^2 \left[\rho A \frac{\partial^4 w}{\partial x^2 \partial t^2} + \alpha \frac{\partial^3 w}{\partial x^2 \partial t} \right] = q - \mu^2 \frac{\partial^2 q}{\partial x^2}. \quad (5)$$

Alternatively, the statistical behaviors of flexible molecules can be described by the theory of a worm-like chain. Considering a collagen molecule as a worm-like chain, the mean field approximation of the Langevin motion of the molecule can be expressed as [21]

$$\rho A \frac{\partial^2 w_c}{\partial t^2} + L_p k_B T \frac{\partial^4 w_c}{\partial x^4} + \alpha \frac{\partial w_c}{\partial t} = q_c, \quad (6)$$

wherein w_c is the transverse deflection of chain, L_p is the persistence length, k_B denotes the Boltzmann constant and T is temperature. In original Langevin equation of motion, there is a random force term, which is ignored in Eq. (6). Therefore, q_c is the deterministic part of the transverse external excitation.

The deformation of the collagen molecule can be estimated by using Eq. (5) and Eq. (6) separately. In this way, the similar boundary supports and loading condition are considered. For reasons of simplicity, only the simply supported boundary condition is considered at both ends of the molecule. The displacements that satisfy the boundary condition can be written as:

$$w(x, t) = W_\perp \sin\left(\frac{n\pi x}{L}\right) \exp(i\Omega t), \quad (7)$$

$$w_c(x, t) = W_c \sin\left(\frac{n\pi x}{L}\right) \exp(i\Omega t), \quad (8)$$

where L is the contour length, Ω is the angular frequency, and n is the mode number. Furthermore, the transverse load is presented as a sinusoidal wave form:

$$q(x, t) = q_c(x, t) = q_0 \sin\left(\frac{n\pi x}{L}\right) \exp(i(\Omega t + \phi)), \quad (9)$$

where q_0 is a constant and φ is the relative phase between the input force and the vibration displacement. Substitution of the expansions for w and q from Eqs. (7) and (9) into the equation of motion (5) leads to

$$W_{\perp} = \frac{q_0 \left(1 + \frac{\mu^2 n^2 \pi^2}{L^2}\right) \exp(i\phi)}{EI \frac{n^4 \pi^4}{L^4} - \rho A \Omega^2 \left(1 + \frac{\mu^2 n^2 \pi^2}{L^2}\right) + i\Omega \left[E^* I \frac{n^4 \pi^4}{L^4} + \alpha \left(1 + \frac{\mu^2 n^2 \pi^2}{L^2}\right)\right]}. \quad (10)$$

Similarly, based on the worm-like chain model, the deformation of the collagen molecule can be written as follows

$$W_c = \frac{q_0 \exp(i\phi)}{L_p k_B T \frac{n^4 \pi^4}{L^4} - \rho A \Omega^2 + i\Omega \alpha}. \quad (11)$$

Comparing Eq. (10) and Eq. (11) through the following relation

$$|W_{\perp}|^2 = |W_c|^2, \quad (12)$$

an effective persistence length of the collagen molecule is obtained as

$$L_p = \frac{L^4}{k_B T} \left[\frac{\rho A \Omega^2}{n^4 \pi^4} + \sqrt{\left(\frac{EI}{(L^4 + \mu^2 n^2 \pi^2 L^2)} - \frac{\rho A \Omega^2}{n^4 \pi^4} \right)^2 + \frac{E^{*2} I^2 \Omega^2}{(L^4 + \mu^2 n^2 \pi^2 L^2)^2} + \frac{2\Omega^2 E^* I \alpha}{n^4 \pi^4 (L^4 + \mu^2 n^2 \pi^2 L^2)}} \right] \quad (13)$$

Equation (13) provides a closed-form formula for the persistence length of a collagen molecule of finite length. From Eq. (13), it is easily seen that the persistence length of the collagen molecules depends on various parameters including the contour length, the bending rigidity, the viscoelastic parameter, the spatial mode, and frequency of applied loading, the small-scale effect, and the viscosity of the external fluid. In the next section, the effects of the mentioned parameters are numerically examined. It should be noted that, in the special case of $E^* = 0$ and $n = 1$, we have

$$L_p = \frac{EI}{k_B T} \frac{L^4}{(L^4 + \mu^2 \pi^2 L^2)}, \quad (14)$$

which is similar to that presented by Gao and Lei [26] for the protein microtubule. Specifically, when the small-scale effect of the collagen molecules is neglected ($\mu = 0$), Eq. (14) can be reduced to the classical expression, which is usually used to estimate the persistence length of the flexible molecules [27], i.e.,

$$L_p = \frac{EI}{k_B T}. \quad (15)$$

Fig. 2 Persistence length of a collagen molecule as a function of the contour length for different values of the loading frequency

3 Results and discussion

In order to confirm the validity of the present formulation, the persistence length of the collagen monomer is calculated and compared with experimental data that was reported by Sun et al. [8]. The results of these experiments demonstrated that, by fitting to the worm-like chain model, the collagen monomer has a persistence length of 14.5 nm. Properties of the collagen in the validation analysis are considered to be the same as mentioned in Ref. [8]. Young's modulus of the collagen is considered as $E = 12.2$ GPa, radius $r = 0.28$ nm, and the contour length is 309 nm. Using the present formulation and the mentioned parameters, the predicted persistence length is about 14.3 nm, which is in good agreement with that reported by Sun et al. [8].

Accurate numerical simulations require an appropriate choice of the basic quantities. This work was possible owing to recently obtained experimental and numerical data from several groups describing physical and structural properties of the collagen molecules. In the numerical simulations, the temperature is taken to be $T = 300$ K, the mass density to be 1340 kg/m^3 [9], the effective radius is 0.55 nm, and the viscoelastic parameter, E^* , is 3.84 Pa s [15].

Generally, there are contradictory opinions on whether the persistence length of the semiflexible polymer depends on its contour length. The experimental results indicate a dependence of the persistence length upon the contour length. The dependence of persistence length on its contour length may contribute to explain the broad range of measured persistence lengths found in the literature for the collagen molecule. To elucidate the relationship between the persistence length and contour length of the collagen molecule, the effects of the counter length and the loading frequency on the persistence length of the collagen molecule are

Fig. 3 Persistence length of a collagen molecule as a function of the contour length for both elastic and viscoelastic models

Fig. 4 Persistence length of a collagen molecule as a function of the contour length for different mode numbers

depicted in Fig. 2. The numerical results are computed for five different values of the loading frequency. In this case, we take $n = 1$, $\mu = 0.1 L$, $\nu_f = 0.001$ Pa s, and $E = 6$ GPa. It can be seen from Fig. 2 that for $L < 150$ nm, the persistence length is not sensitive to the loading frequency. However, for a specific contour length, the persistence length increases as the loading frequency increases if L is greater than 150 nm. It is also observed that the persistence length of the collagen molecule is independent of the contour length for low-frequency case. Furthermore, results show that the persistence length of the collagen depends on its contour length. It can be seen that the persistence length increases with an increase in the counter contour for high frequency case. This is the same as the experimental results given by Pampaloni et al. [28] for microtubules. They found an increase of microtubule persistence length with the total contour length.

To illustrate the effect of viscoelastic properties of the collagen molecule, the persistence length of the molecule is predicted and plotted in Fig. 3, with a comparison to the results determined by the elastic model from the present model by setting $E^* = 0$. For numerical calculations in this case, we take $n = 1$, $\mu = 0.1 L$, $\nu_f = 0.001$ Pa s, $E = 6$ GPa and $\Omega = 120$ kHz. It is clearly seen from Fig. 3 that the elastic model predicts smaller persistence lengths as compared to the viscoelastic results.

To investigate the effect of mode number on the persistence length, we take $\mu = 0.05 L$, $\nu_f = 0.001$ Pa s, $E = 6$ GPa and $\Omega = 150$ kHz. The results are shown in Fig. 4. It can be seen from Fig. 4 that the mode number has significant effects on the persistence length. It can be observed that the persistence length decreases with an increase in the mode number. In higher modes ($n > 1$), the persistence length is independent of the contour length.

Fig. 5 Persistence length of a collagen molecule as a function of the contour length for different nonlocal scale coefficients

Fig. 6 Persistence length of a collagen molecule as a function of the contour length for different Young's moduli

The persistence lengths of the collagen molecule predicted for various contour lengths with different nonlocal scale coefficients are plotted in Fig. 5. To determine the results, the other parameters are $n = 1$, $\nu_f = 0.001$ Pa s, $E = 6$ GPa and $\Omega = 200$ kHz. It can be observed that the persistence length decreases with an increase in the nonlocal scale coefficient. The reduction in the persistence length is due to the nonlocal effect, which reduces the stiffness of the material. This is the same as the result given by Gao and Lei [26] for the microtubule.

According to results from molecular dynamics simulations, the Young modulus of the collagen molecule ranges from 6 to 16 GPa in solvated conditions [15, 16]. To investigate the effect of Young's modulus on the persistence length, the results are obtained with Young's modulus ranging from 6 to 10 GPa, all with fixed parameters including $n = 1$, $\mu = 0.1$ L, $\nu_f = 0.001$ Pa s and $\Omega = 100$ kHz. These results are plotted in Fig. 6. As expected, the persistence length increases with the increase in the Young's modulus.

Finally, the variations of the persistence length with increasing of the contour length for three different values of viscosity of the external fluid are plotted in Fig. 7. In this case, the following parameters are used in computing the numerical results: $n = 1$, $\mu = 0.1$ L, $E = 6$ GPa and $\Omega = 120$ kHz. It is found that the persistence length increases when the viscosity of the external fluid increases. Moreover, it is observed that the persistence length without considering the effect of external fluid is independent of the contour length.

Fig. 7 Persistence length of the collagen molecule as a function of contour length for different values of viscosity of external fluid

4 Conclusions

A nonlocal viscoelastic beam model in conjunction with the mean field Langevin equation of the chain was utilized to predict the persistence length of the collagen molecule. In spite of some experimental and numerical achievements in prediction of the persistence length of the collagen molecule, to the author's knowledge, there has been no attempt to tackle the problem described in the present investigation. Considering the viscoelastic property of the collagen molecule and including the small-scale effect, effect of the external viscous fluid and loading conditions are the main contributions of the present paper. Based on the numerical results, the following main conclusions can be drawn. Results clearly showed how the persistence length of the collagen molecule depends on its contour length. It was found that the persistence length of the molecule is smaller than its contour length. This means that physically the molecule is relatively flexible. This is the same conclusion reported by An et al. [29]. Furthermore, it was shown that the persistence length, without considering the effect of the viscoelastic properties, is smaller than that including the effect of the viscoelastic behavior. In addition, other factors including the Young modulus, loading mode and frequency, viscosity of external fluid and small-scale effect, can also significantly influence the persistence length of the collagen molecule. It should be noted that the proposed persistence length in this study is known as a bending persistence length. Another type of persistence length is also defined for biological helices. This type of persistence length was called *torsional persistence length* [30]. The prediction of the torsional persistence length of the collagen would be an interesting issue for future work. Finally, I believe that the present model can provide feasible explanations for contradictory experimental observations in the literature.

Compliance with ethical standards

Conflict of interest The author declares that there are no conflicts of interest.

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