A hybrid phononic crystal for roof application

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Phononic crystal is a type of acoustic material, and the study of phononic crystals has attracted great attention from national research institutions. Meanwhile, noise reduction in the low-frequency range has always encountered difficulties and troubles in the engineering field. In order to obtain a unique and effective low-frequency noise reduction method, in this paper a low frequency noise attenuation system based on phononic crystal structure is proposed and demonstrated. The finite element simulation of the band gap is consistent with the final test results. The effects of structure parameters on the band gaps were studied by changing the structure parameters and the band gaps can be controlled by suitably tuning structure parameters. The structure and results provide a good support for phononic crystal structures engineering application.

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I. INTRODUCTION

The ability of interior noise suppression is an important aspect of improving the comfort performance of the vehicle. When the vehicle is at idle or driving, due to vibration noise transmitted to the roof by the engine, low-frequency vibration is one of the main noise sources in the cabin.¹ After research and development, for high and medium frequency noise has a relatively complete solution, such as the use of sound-absorbing cotton, micro-perforated plate, active noise reduction, and other related theories, but these methods are very difficult to completely solve the low-frequency noise, noise reduction has become an urgent problem to be solved. However, the use of phononic crystals provides a new way to solve these problems.

The roof of the car is composed of steel roof and plastic ceiling interior. The distance between the body steel roof and the interior ceiling is about 1–2 cm in most situations, which provides sufficient space to utilize the phononic crystal structure while reserving a safe distance.

The propagation of elastic wave in the phononic crystal structure is affected by its internal periodic structure.² The prohibition of propagation can be achieved in the frequency range of band gap. Based on the suppression mechanism of wave propagation using phononic crystals, multiple band gaps to control the wave selectively have been further demonstrated by Chen and Wang.^{3,4} the researchers designed various phononic crystals to achieve a lower, wider phononic crystal band gap.⁵ There are several physical mechanisms for the elastic band gap of phononic crystals: Besides Bragg scattering and local resonance, there should be an overlapping between them. Chen and Wang⁶ demonstrated the existence of complete band gaps attributed to the

coupling effects of local resonances and Bragg scattering, in which the intrinsic coupling effects are governed by the topological arrangements of the co-continuous structures and mechanical properties of constituent phases. To promote the engineering application of phononic crystals in vibration and noise reduction and mechanical engineering, the researchers conducted a lot of research work. In 2000, Liu *et al.* designed a phononic crystal structure that could obtain low-frequency band gaps in small size even when the local-resonant scatterers were not strictly periodic.⁷ Hassounani *et al.* studied the periodic phonon array of two-dimensional phononic crystals using silicon, in which the band gap ranges from about 16 to 18 kHz.⁸ Hsu *et al.* proposed a two-dimensional phononic crystal plate structure composed of rubber circular fillers embedded in the epoxy plate, and the band gap ranges from 880 to 938 Hz.⁹ Zhu *et al.* proposed a flat elastic material, which adds a stepped scatterer on the basis of the steel plate and the band gap ranges from 13 to 17 kHz.¹⁰ Li *et al.* investigated the local resonant full band gap of a two-dimensional phononic crystal plate using a composite scatterer phononic crystal plate. Compared with typical double-sided phononic crystal plates, the results show that the first complete band gap ranges from 300 to 500 Hz.¹¹

Most previous designs of the phononic crystals exhibit a band gap frequency range above 300 Hz, and there are few phononic crystals having a band gap range as low as 100 Hz. However, the reduction of low frequency noise is challenging in the improvement of NVH (Noise Vibration Harshness) performance, since the noise concentrated in 95–110 Hz cannot be eliminated by traditional vehicle noise attenuation materials. In this paper, due to the actual application requirement of vehicle NVH design, a new type of phononic crystal structure is proposed for low-frequency noise attenuation around 100 Hz, and its material composition and structure meet the requirements of vehicle and civil use.

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II. BAND STRUCTURES

A. Geometry and method of calculations

The unit cell of the proposed phononic crystals is shown in Fig. 1(a), which is fabricated by depositing the artificial resonant scatterer composed of rubber body neck and steel body head to a homogeneous rubber base plate. The lattice constant is $a = 10$ mm and the thickness of the substrate base plate is $e = 1$ mm. The diameter and height of the body neck (body head) are $d = 5$ mm and $h = 4$ mm ($D = 8$ mm and $H = 4.5$ mm), respectively. The total thickness is $T = e + h + H = 9.5$ mm, as shown in Fig. 1(b). This three-dimensional unit cell is periodic in x - and y -directions.

The formation mechanism of the band gap in the phononic crystal plate can be ascribed to the local resonance of the unit cell. In order to explore the underlying working mechanism, the acoustic equivalent circuit theory is adopted here to describe the unit cell. Figure 1(c) is the equivalent acoustic circuit of the resonator in the phononic crystal plate, which is a spring-mass system consisting of two subsystems with different vibration modes. Here the rubber body neck functions as a spring while the metal body head acts as a mass. Two vibration modes can be excited. The first one is the bending vibration represented by “ k_0 - m_0 ,” which is coupled with the anti-symmetric mode of the top steel plate. The other one is the longitudinal vibration represented by “ k_i - m_i ,” which is coupled with the symmetric mode of the top steel plate. Here, k_0 and k_i are equivalent stiffness, m_0 and m_i are equivalent mass. The two types of vibration modes can open the complete phononic band gaps and lead to complex coupled vibration modes between the two modes. Due to the complexity of vibration modes supported by the unit cell, numerical calculations are performed to characterize the band structure of the system. Additionally, the

irreducible Brillouin zone of the structure is shown in Fig. 1(d) and the band structure can be built by sweeping k_x and k_y through the entire first Brillouin zone $M - \Gamma - X - M$.

The band structure of the composite structure can be calculated from the elastic wave equation:

$$\rho(x, y, z) \ddot{u}_p = \partial_q [c_{pqmn}(x, y, z) \partial_n u_m] \quad (p, q, m, n = 1, 2, 3), \quad (1)$$

where the mass density $\rho(x, y, z)$ and the elastic constants $c_{pqmn}(x, y, z)$ are periodic functions of in-plane component. The calculations are performed by solving the eigenvalue problem of the unit cell with finite element method, as outlined below. First, the periodic boundary conditions (PBCs) are applied to the two sides of unit cell along the x -direction and y -direction based on the Bloch's theorem. The PBC in terms of displacement u_p can be defined as

$$u_p(x + mA, y + nA, z) = u_p(x, y, z) e^{-jk_x mA} e^{-jk_y nA}. \quad (2)$$

Here k_x and k_y are the wave-vector in the x -direction and y -direction, and m and n are arbitrary integer. On the other hand, the traction free boundary conditions are applied to the other surfaces, in which the stress component in the normal direction is zero. Second, the three-dimensional unit cell is meshed and divided into finite elements. The finite elements are tetrahedrons with four nodes, and each node has three translational degrees of freedoms u_x , u_y , and u_z represented by the Lagrange interpolation polynomials within the elements. Considering the harmonic time dependence of $e^{j\omega t}$, FEM (Finite Element Method) transforms the analysis into a generalized elastic eigenvalue problem expressed by the equations:¹²

$$[\mathbf{K} - \omega^2 \mathbf{M}] \mathbf{u} = \mathbf{0}, \quad (3)$$

FIG. 1. (Color online) (a) Schematic diagram and (b) cross-section view illustrating the unit cell of the phononic crystals for extremely low-frequency NVH. (c) Equivalent circuits of the unit cell. (d) The irreducible Brillouin zone.

where $\mathbf{K}$ and $\mathbf{M}$ are not the stiffness and mass matrix of the system. The matrices $\mathbf{K}$ and $\mathbf{M}$ are coupled with the items containing the Bloch vector. Inherent in these equations is a relationship between the angular frequency ω and the wave number k . For the existence of nontrivial solution $\mathbf{u}$, the determinant of the coefficient matrix should be zero, and the band structure can be built by sweeping k_x and k_y through the entire first Brillouin zone. An alternative searching arithmetic is employed to determine the eigenfrequency ω , which is similar to that used in the plane wave expansion method.

B. Low-frequency phononic band gap

In this study, the spatial discretization is fine enough for the convergence, which has been checked by the fact that the band structures do not change even if the number of discretization further increases. The choice of rubber as body neck material and steel as body head material is based on the strong contrast between their elastic properties. Figure 2(a) shows the band structures of the phononic crystal plate configuration. There is a complete phononic band gap in the low-frequency region below 300 Hz for this case, which is located between the fourth and the fifth transmission band and extends from 95 to 116 Hz. The gap width δf is 21 Hz and the corresponding relative bandwidth $\delta f/f_m$ is 20%, where f_m is the midgap frequency. For elastic wave excitation at the specific frequency within the band gap, each unit cell resonates and interacts with the elastic wave and the propagation of any acoustic modes within this phononic band gap is suppressed.

In order to explore the physical mechanism of the band structure formation, we further investigate the eigenmode of the unit cell, which reveals the specific form of the particular oscillatory patterns capable of maintaining their shape and propagating in a stable manner. Here the eigenmodes are characterized by the displacement field distribution in the unit cell. For a given set of k_x and k_y in the first Brillouin zone, a series of eigenfrequencies ω can be obtained. Each ω corresponds to an eigenvector $\mathbf{u}$, which determines a unique vibration mode of the composite structure. By assigning both the wave vector k_x and k_y as well as frequency ω , the

specific eigenmodes are excited at the lower and upper limits of the band gap.

The displacement fields of the eigenmodes A and B [marked in Fig. 2(a)] are plotted in Figs. 2(b) and 2(c), respectively. These two eigenmodes belong to the fifth transmission bands at Γ point (center of the Brillouin zone) and fourth transmission bands at X point (edge of the Brillouin zone). Three remarks should be noted.

- (1) Obviously, the body neck and body head exhibit torsional vibration in both cases. The complex displacement fields in multi-directions do not have rigorous longitudinal or bending variations, which should be ascribed to the coupling between the two basic vibration modes.
- (2) In Fig. 2(c), the dominant particle motion occurs inside the body neck and body head, while a little motion occurs inside the top steel plate and the base rubber plate. However, In Fig. 2(b), the displacement component u_2 that lies in the plane of the phononic crystal plate surface dominates the oscillatory patterns in the top steel plate and rubber base plate.
- (3) These modes with special k values located at the lower and upper limits of the band gap. We have also investigated the other modes with k values located at general positions of the transmission bands, and the results show that the torsional vibration character maintains.

III. RECONFIGURE THE BAND GAP

As illustrated in the preceding section, since the local resonant band gap is dependent on the interaction between the unit cell's resonant characteristics and the long-wavelength traveling wave in the top steel plate, the material and structural parameters of the unit cell are crucial in the configuration of the band gap frequency. Therefore, we investigate the band structure with different parameters of the unit cell in this section.

A. Effect of different materials on band gap

In order to characterize the influence of materials, we first tested the effect of the material on the band gap. Figures

FIG. 2. (Color online) (a) The band structure. (b), (c) Displacement fields of the eigenmodes corresponding to points A and B in panel (a). The black lines denote the initial boundaries of the unit cell.

FIG. 3. (Color online) (a) Variation of the band structures with the body head of (a) aluminum, (b) steel, (c) lead, and (d) gold. The corresponding displacement fields of the eigenmodes are plotted in the insets of (a)–(d), respectively. Complete phononic band gap extent as a function of the body head material.

3(a)–3(d) depict the band structures with the body head changing from aluminum, steel, lead, to gold, respectively. We fix the materials of the base plate and body neck, as well as the structural parameters. It is found that with the increase of the material density, the upper (lower) band-edge frequency decreases from 172 Hz (151 Hz) with aluminum body head to 92 Hz (60 Hz) with gold body head, as illustrated by the band gap extent in Fig. 3(e) which is extracted from the band structures. This red-shift phenomenon should be attributed to the increased effective mass in the equivalent mass-spring system. In addition, the first band gap of the phononic crystal plate extends from $\delta f = 21$ Hz with aluminum body head to $\delta f = 32$ Hz with gold body head. We also investigated the band gap evolution with the variation of the contrast of mechanical properties, and confirm that the working frequency of the band gap gradually decreases when the scatter head density increases, which is consistent with the trend shown in Fig. 3

B. Effect of body head diameter on the band gap

In practical application, since the metal materials such as gold is expensive while lead material is harmful to the human body, we prefer to configure the working frequency

of the band gap by adjusting the geometry parameters while employing specific cheap and harmless materials. Therefore, we investigate the band structure with the diameter of the body head ranging from $D = 4$ to 10 mm, as shown in Figs. 4(a)–4(d). It is found that the band gaps are sensitive to the diameter D . Its upper (lower) band-edge frequency red shifts from 218 Hz (202 Hz) with $D = 4$ mm to 97 Hz (70 Hz) with $D = 10$ mm. In addition, the first band gap of the phononic crystal plate extends from $\delta f = 16$ Hz with $D = 4$ mm to $\delta f = 27$ Hz with $D = 10$ mm. This phenomenon should also originates from the significantly increased effective mass in the equivalent mass-spring system, due to the increased volume of the body head.

C. Effect of body neck diameter on the band gap

We further investigate the band structure with the diameter of the body neck ranging from $d = 4$ to 10 mm, as shown in Figs. 5(a)–5(d). It is found that the band gaps exhibit reverse evolution trend with respect to the diameter of the body head shown in Fig. 4. Its upper (lower) band-edge frequency blue shifts from 89 Hz (71 Hz) with $d = 4$ mm to 211 Hz (162 Hz) with $d = 10$ mm. In addition, the band gap width of the phononic crystal plate extends from $\delta f = 18$ Hz

FIG. 4. (Color online) (a) Variation of the band structures with the diameter of the body head $D =$ (a) 4 mm, (b) 6 mm, (c) 8 mm, and (d) 10 mm. The corresponding displacement fields of the eigenmodes are plotted in the insets of (a) to (d), respectively. (e) Complete phononic band gap extent as a function of D .

FIG. 5. (Color online) (a) Variation of the band structures with the diameter of the body neck $d =$ (a) 4 mm, (b) 6 mm, (c) 8 mm, and (d) 10 mm. The corresponding displacement fields of the eigenmodes are plotted in the insets of (a)–(d), respectively. (e) Complete phononic band gap extent as a function of d .

with $d = 4$ mm to $\delta f = 49$ Hz with $d = 10$ mm. This phenomenon should be ascribed to the significantly increased stiffness coefficient in the equivalent mass-spring system, due to the increased volume of the body neck.

D. Effect of body head height on the band gap

Figures 6(a)–6(d) show the evolution of the band structure with the height of the body head ranging from $H = 2$ to 8 mm. We remark that the upper (lower) band-edge frequency decreases from 152 Hz (138 Hz) with $H = 2$ mm to 105 Hz (77 Hz) with $H = 8$ mm. In addition, the first band gap of the phononic crystal plate extends from $\delta f = 14$ Hz with $H = 2$ mm to $\delta f = 28$ Hz with $H = 8$ mm. Analogous to the diameter of the body head D , this phenomenon should also be ascribed to the increased effective mass due to the increased volume of the body head.

E. Effect of body neck height on the band gap

Figures 7(a)–7(d) show the evolution of the band structure with the height of the body neck ranging from $h = 2$ to 8 mm. Note that the upper (lower) band-edge frequency decreases from 153 Hz (130 Hz) with $h = 2$ mm to 81 Hz

(70 Hz) with $h = 8$ mm. In addition, the first band gap of the phononic crystal plate shrinks from $\delta f = 33$ Hz with $h = 2$ mm to $\delta f = 11$ Hz with $h = 8$ mm. On contrary to the height of the body head H , this phenomenon should also be ascribed to the decreased effective mass.

F. Energy transfer spectrum verification

In order to confirm the analysis of band structure, the transmissions through periodic multilayers with finite periods are calculated using the full-wave FEM simulations. The material parameters are chosen as follows: the density $\rho_1 = 7800 \text{ kg/m}^3$, the Young's modulus $E_1 = 2.1 \times 10^{11} \text{ N/m}^2$ and the Poisson's ratio $\gamma_1 = 0.29$ for steel; the density $\rho_2 = 1300 \text{ kg/m}^3$, the Young's modulus $E_2 = 117500 \text{ N/m}^2$ and the Poisson's ratio $\gamma_2 = 0.47$ for rubber. The computational domain is shown in Fig. 8(a), where the distance between the exciter (receiver) and the input side (output side) of the lattice is $l_0 = 40$ mm ($l_1 = 40$ mm). Without loss of generality, the phononic crystal lattice is composed of n -layer unit cells in the direction of wave propagation (x -direction), and is assumed to be periodic in the direction perpendicular to the wave propagation direction (y -direction). Thus, the external boundaries in x -direction are

FIG. 6. (Color online) (a) Variation of the band structures with the height of the body head $H =$ (a) 2 mm, (b) 4 mm, (c) 6 mm, and (d) 8 mm. The corresponding displacement fields of the eigenmodes are plotted in the insets of (a) to (d), respectively. (e) Complete phononic band gap extent as a function of H .

FIG. 7. (Color online) (a) Variation of the band structures with the height of the body neck $h =$ (a) 2 mm, (b) 4 mm, (c) 6 mm, and (d) 8 mm. The corresponding displacement fields of the eigenmodes are plotted in the insets of (a)–(d), respectively. (e) Complete phononic band gap extent as a function of h .

surrounded with perfect match layers (PML), which absorb elastic disturbances in the area and avoiding unnecessary reflection effects. The periodic boundary condition is applied at the boundaries in the y -direction, which simulates the infinite layers.¹³ In fact, the simulation model is analogous to the conventional experimental system of piezoelectricity-generated Lamb waves. A vertical harmonic force with the amplitude of 1 N is applied on the top surface of the plate. Elastic wave transmissions through the periodic plates are calculated, and normalized by that through homogeneous plates.

The normalized transmission spectra corresponding to Fig. 2(a) are plotted in Fig. 8(b) with $n = 3$ (dotted curve),

$n = 5$ (dashed curve), and $n = 10$ (solid curve). We note from Fig. 8(b) that the transmission gaps appear at 89–118 Hz for all the three case of periodic monolayer, which is in agreement with the complete phononic band gap in the band structure [as shown in Fig. 2(a)]. We also notice that the band gap become wider and deeper as n increases from 3 to 10. Only a few layers of unit cells in deep subwavelength scale can achieve significant vibration control performance, e.g., the transmission loss approaches -40 dB at $n = 5$ (the length of the sample is $5a = 5$ cm) while the longitudinal wavelength in steel is about 40 m at 90 Hz. From the applications point of view, the proposed phononic crystal plate structure provides significant advantages over conventional NVH technique, which achieve vibration control using a large area of pitch layer.

IV. EXPERIMENTAL VALIDATION

In order to further verify the low-frequency noise reduction performance of the phononic crystal structure, we carried out the low-frequency damping test in a vehicle at engine idling. The photograph of the test sample is shown in Figs. 9(c) and 9(e). The material and geometric parameters are consistent with the samples in Fig. 1. The metal parts of the phononic crystal structure are fabricated by the CNC (Computer numerical control) machinery, while the rubber parts are made using rubber mold with high precision to ensure the physical quality of the phononic crystal. After removing the plastic ceiling [as shown in Fig. 9(a)], an acceleration sensor (8688A50) [as shown in Figs. 9(b) and 9(d)] is placed in the middle of the roof and pasted on the roof plate for this experiment. In the case of engine idle and air conditioning, the low-frequency vibration spectrum of the steel roof without and with the phononic crystal are shown by the blue and red solid line in Fig. 10, respectively. When the measurement is completed, the vibration spectrum test containing the phononic crystals is carried out again [as shown in Fig. 9(e)]. A group of phononic crystal is composed of 20 units [as shown in Fig. 9(c)], a total of 16 groups [as shown in Fig. 9(d)]. Phononic crystals are arranged in the body of the steel roof [as shown in Fig. 9(c)], and the super glue is used connect the phononic crystal and steel roof. It is

FIG. 8. (Color online) (a) The plate geometry in the calculation of transmission power spectrum. Normalized transmission power spectra for (a) $n = 3$, (b) $n = 5$, (c) $n = 10$.

FIG. 9. (Color online) Photograph of (a) steel roof, (b) acceleration sensor, (c) phononic crystal, (d) experimental setup, and (e) experimental sample.

observed that low-frequency vibration are significantly attenuation in the frequency region from 90 to 117 Hz, confirming excellent attenuation performance (as shown in red box of Fig. 10). The experimental results clearly confirm the noise reduction effect in vehicles.

V. CONCLUSIONS

We have reported a phononic crystal plate that employs artificial local resonant units in deep-subwavelength scale to suppress the low-frequency vibration in the top roof of the vehicles, which remains great challenging in traditional NVH technique with conventional materials. The proposed structure can be made of rubber, steel and other common materials. We demonstrate through full-wave simulations that the band gap can be tuned from 67 to 220 Hz, which is difficult to achieve by using traditional damping materials such as pitch. The results verify that the band gap can be configured by using different materials or changing the local

geometric parameters. Finally, through the verification of the energy transfer spectra, the vibration and energy of the noise are significantly attenuated in the corresponding frequency regime of the band gap. The validity of the low frequency suppression of vibration by the proposed structure show promising potential applications in the design of NVH in vehicles.

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FIG. 10. (Color online) Measured transmission spectra of the vibration of car roof.

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