Published in final edited form as:

J Heat Transfer. 1979 August ; 101(3): 560–561. doi:10.1115/1.3451030.

Shielding of Heat Transfer from a Boundary

A. S. Popel¹

¹Departments of Chemical Engineering and Aerospace and Mechanical Engineering, University of Arizona, Tucson, AZ 85721, Mem. ASME.

Introduction

In some practical situations it is desirable to limit the heat transfer into a volume from the boundary surface, e.g., to maintain a prescribed temperature in the volume independent of variations of the surface temperature. One possible way to achieve this is to situate a control array of heat sources or sinks near the surface. In order to illustrate a practical application of the problem, consider a volume of material with constant thermoconductivity contained in a rectangular box with thermoisolating walls; the box is open from one side. Suppose that an experiment requires a controlled uniform temperature in the bulk of the volume, but the mass transfer with the ambient medium through the open surface must be retained. A possible experimental solution of this task would be situating an array of temperature-controlled strips below the surface which should not influence the mass transfer processes; at some depth below the strips the temperature distribution in the volume should be close to uniform.

In the present communication, the problem of heat transfer in the half-space containing a control one-dimensional periodic array of flat strips parallel to the surface is considered; and it is shown that an efficient shielding of heat transfer from the boundary can be achieved with this arrangement. A simple expression is derived for the temperature in the bulk of the volume in terms of the temperatures of the surface and the strips, and the geometrical parameters of the problem.

Analysis and Results

Consider the problem of steady-state heat transfer in the half-space bounded by a plane surface, and containing a periodic array of strips of infinite length; geometry of the problem is illustrated in Fig. 1(a) where the cross section perpendicular to the strips and to the bounding plane is shown. Let the temperature of the bounding surface be T_0 , and the temperature of the strips be T_1 where T_0 and T_1 are given constants. The problem is to determine the temperature distribution in the half-space. Since due to the symmetry of the problem the temperature distribution is two-dimensional and is spatially periodic, we can restrict ourselves to consideration of a semi-infinite strip $ABCDEF$ shown in Fig. 1(a) ($DE = a$, $DC = d$, $BC = b$). Introducing a coordinate system (x, y) with the origin located in the middle of DE , we can formulate the mathematical problem as follows:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \text{ for } -a/2 < x < a/2, y > 0 \quad (1)$$

$$T = T_0 \text{ at } y = 0, -a/2 \leq x \leq a/2 \quad (2)$$

$$T = T_1 \text{ at } x = a/2, b \leq y \leq d+b \quad (3)$$

$$\partial T / \partial x = 0 \text{ at } x = -a/2 \text{ and } x = a/2, 0 < y < d \text{ and } y > d+b \quad (4)$$

The problem may be solved by conformal mapping the semi-infinite strip $ABCDEF$ in the complex w -plane into a rectangle $B'C'D'E'$ in the w_2 -plane so that BC maps into $B'C'$ and DE into $D'E'$ (Fig. 1(b)). The transformation is given by the relationships [1]

$$w_1 = \frac{[\xi_1 + 1 - k(\xi_1 - 1)] \sin^2(\pi w/a) - 2\xi_1}{[\xi_1 - 1 - k(\xi_1 + 1)] \sin^2(\pi w/a) + 2\xi_1 k} \quad (5)$$

$$w_2 = \int_0^{w_1} (1 - t^2)^{-1/2} (1 - k^2 t^2)^{-1/2} dt \quad (6)$$

where $w = x + iy$, w_1 , and w_2 are complex variables, and

$$\xi_1 = \cosh^2 \frac{\pi d}{2a}, \xi_2 = \cosh^2 \frac{\pi(d+b)}{2a} \quad (7)$$

$$k = \frac{\xi_2(\xi_1 - 1)}{(\xi_2 - \xi_1) + \xi_1(\xi_2 - 1) + 2[\xi_1(\xi_2 - 1)(\xi_2 - \xi_1)]^{1/2}} \quad (8)$$

The latter parameter satisfies the condition $0 < k < 1$. The temperature distribution may be expressed in a simple form in terms of the variable w_2 :

$$T = \frac{T_1 - T_0}{2K(k)} \operatorname{Re} w_2 + \frac{T_1 + T_0}{2} \quad (9)$$

or in dimensionless form

$$t = \frac{T - T_0}{T_1 - T_0} = 1/2 \left(1 + \frac{\operatorname{Re} w_2}{K(k)} \right) \quad (10)$$

where $K(k)$ is the complete elliptic integral of the first kind [2], $K' = K(\sqrt{1 - k^2})$, and $\operatorname{Re} w_2$ designates the real part of the complex variable. Relationships (5–9) completely solve the problem.

Let us calculate the temperature T_∞ far away from the surfaces, i.e., at $y \rightarrow \infty$. It can be shown that the infinity in the w -plane corresponds to a finite point on the real axis in the w_1 -plane:

$$w_1^* = \frac{\xi_1 + 1 - k(\xi_1 - 1)}{\xi_1 - 1 - k(\xi_1 + 1)} \quad (11)$$

where $|w_1^*| > 1/k$. Using the solution (10) we can express the dimensionless temperature at the infinity in the form

$$t_\infty = \frac{T_\infty - T_0}{T_1 - T_0} = 1/2 \left[1 + (\operatorname{sgn} w_1^*) \frac{F(\arcsin k^{-1}|w_1^*|^{-1}, k)}{K(k)} \right] \quad (12)$$

where

$$F(\phi, k) = \int_0^\phi (1 - k^2 \sin^2 \theta)^{-1/2} d\theta \quad (13)$$

is the elliptic integral of the first kind. Relationships (12) imply $t_\infty \rightarrow 0$ as $t_\infty \rightarrow T_0$, $t_\infty \rightarrow 1$ as $T_\infty \rightarrow T_1$, and $\min(T_0, T_1) \leq T_\infty \leq \max(T_0, T_1)$. When $d/a \rightarrow 0$ the solution assumes a simpler form

$$t_\infty = 0.5 + \pi^{-1} \arcsin(1 - 2/\xi_2) \quad (14)$$

The quantity t_∞ given by (12) is plotted in Fig. 2 as a function of the parameters b/a and d/a . As d/a increases, the temperature T_∞ approaches the temperature of the strips T_1 even for small values of b/a .

These results make it possible to simply calculate one of the temperatures T_∞ , T_0 , T_1 if the other two are specified. To consider a numerical example, let $d/a = 0.5$, $b/a = 0.2$ then from Fig. 2 we find $t \approx 0.5$. Thus, for example, if the surface temperature is $T_0 = 0^\circ\text{C}$, and the temperature in the volume should be $T_\infty = 20^\circ\text{C}$, then the temperature of the strips has to be maintained at $T_1 = 40^\circ\text{C}$.

Acknowledgments

This work was supported in part by NIH Grant HL-17421.

Nomenclature

a, b, d	geometric parameters
F	elliptic integral of the first kind
k	elliptic modulus, equation (8)
K	complete elliptic integral of the first kind
T	temperature
T_0	temperature of the boundary
T_1	temperature of the strips
T_∞	temperature at the infinity
t	dimensionless temperature
t_∞	dimensionless temperature at the infinity
w, w_1, w_2	complex variables
x, y	rectangular coordinates
ξ_1, ξ_2	dimensionless parameters, equation (7)

References

1. Churchill, RV. Introduction to Complex Variables and Applications. McGraw-Hill; New York: 1948.
2. Abramowitz, M., Stegun, IA. Handbook of Mathematical Functions. Dover; New York: 1972. Ch. 17

(a) Geometry of the problem

(b) Rectangle in the complex w_2 -plane

Fig. 1.

Fig. 2.
Dimensionless temperature at the infinity as function of geometric parameters b/a and d/a