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Superiorization-based multi-energy CT image reconstruction

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Abstract

The recently-developed superiorization approach is efficient and robust for solving various constrained optimization problems. This methodology can be applied to multi-energy CT image reconstruction with the regularization in terms of the prior rank, intensity and sparsity model (PRISM). In this paper, we propose a superiorized version of the simultaneous algebraic reconstruction technique (SART) based on the PRISM model. Then, we compare the proposed superiorized algorithm with the Split-Bregman algorithm in numerical experiments. The results show that both the Superiorized-SART and the Split-Bregman algorithms generate good results with weak noise and reduced artefacts.

Keywords

x-ray spectral CT; image reconstruction; prior rank; intensity and sparsity model (PRISM); superiorization; Split-Bregman

1. Introduction

Dual- and multi-energy measurements in x-ray computed tomography (CT) have attracted great interest in recent years. Fast kVp switching (Langan *et al* 2010) and dual-source CT (Allmendinger 2014) equipped with conventional energy-integrating detectors are among the commercial dual-energy CT systems. Since Compton scattering and photoelectric are the two dominant effects in medical CT, dual-energy CT provides enough spectral information in most of current CT applications. However, the K-edge imaging for various contrast agents requires more than two energy spectra for material discrimination. While adding extra x-ray sources or kVp switching settings would definitely increase the system cost and mechanical complexity, the use of photon-counting detectors with energy-discriminating capability can solve this problem ideally.

Over the past years, photon-counting detectors are being rapidly developed for medical x-ray CT. Several prototypes of spectral CT/micro-CT systems with photon-counting detectors were reported for pre-clinical and clinical applications (Shikhaliev 2008, Walsh *et al* 2011, Wang *et al* 2011). Compared to traditional energy-integrating detectors, which record the x-ray transmission signal through biological tissues over the entire energy range (typically, 30–140 keV for medical CT), the photon-counting detectors use several energy thresholds and count each x-ray event into one of independent energy bins. This provides additional spectral information, enables K-edge CT (Feuerlein *et al* 2008, Pan *et al* 2012), and promises functional and molecular imaging (Fornaro *et al* 2011).

The multi-energy image reconstruction could be considered similar to the traditional CT image reconstruction. However, the data collected suffer from low signal-to-noise ratio (SNR) in each energy bin because of the radiation dose limitation, and the state-of-the-art photon-counting technology cannot work at a high counting rate required by clinical CT. Specifically, the detected x-ray flux is divided into multiple energy bins, which means even fewer photons and lower SNR in each bin relative to data collected using traditional current-integrating detectors (Heismann *et al* 2012). To overcome this problem, researchers proposed different regularizers for multi-energy/spectral CT image reconstruction. Among them, the compressed sensing (CS) based priors have been demonstrated to be effective. The key idea of the CS theory is to assume that most signals are sparse under an appropriate transformation. In CT image reconstruction, the total variation (TV) is one of the most widely used penalties for noise suppression (Tian *et al* 2011), sparser view (Sidky *et al* 2006, Jia *et al* 2010), and interior reconstruction (Yu and Wang 2009). For multi-energy image reconstruction, we proposed a prior rank, intensity and sparsity model (PRISM), which incorporates the low rank property among the multi-energy images in addition to other priors. Independently, other researchers utilized the low rank property in the gradient fields of multi-channel images since these images often share the same edge locations and common gradient directions (Lefkimiatis *et al* 2013, Holt 2014, Rigie and La Rivière 2015). Clearly, the multi-energy image reconstruction needs to handle high data noise and low SNR, and can benefit from utilizing sparse structures and data redundancy among different energy bins.

Adding regularizers to the data fidelity term forms a new objective function. The optimization of the objective function often falls into the general category of convex optimization when the regularizers are convex. Popular optimization algorithms include ADMM (Goldstein *et al* 2014, Sawatzky *et al* 2014), Split-Bregman (Goldstein and Osher 2009), the Chambolle–Pock primal-dual algorithm (Chambolle and Pock 2011), and others. All these algorithms have a good degree of similarity as discussed in Chambolle and Pock (2011), and have been demonstrated effective for regularized CT reconstruction.

On the other hand, the recently-developed superiorization methodology provides a new framework for solving constrained optimization problems (Butnariu *et al* 2007, Davidi *et al* 2009, Censor *et al* 2010). This methodology tunes any iterative algorithm into its superiorized version that is not only compatible with the data fidelity term but also able to minimize the penalty term. Specifically, the basic algorithm that solves the data fidelity term remains the same but the intermediate solution is steered towards the direction to reduce the penalty term. Several studies were performed to superiorize the ML-EM algorithm for emission reconstruction (Garduno and Herman 2014, Luo and Zhou 2014). TV and the l_1 -norm of a wavelet transform were implemented as the regularizers. As far as CT image reconstruction is concerned, there are a number of efficient iterative algorithms, such as ART-like methods including ART (Gordon *et al* 1970) and SART (Andersen and Kak 1984), and statistical iterative methods (Bouman and Sauer 1996, Elbakri and Fessler 2002). One can refer to the review paper (Beister *et al* 2012) for more details of these algorithms. Superiorization of these well-developed algorithms would definitely make the evaluation of new regularizers easy and accelerate the transition from research to application.

In this work, we develop a Superiorized-SART method for multi-energy image reconstruction coupled with the PRISM prior. It should be noted that any iterative reconstruction algorithm may be superiorized as long as it is *strongly perturbation resilient* as proved in Censor *et al* (2014). We choose SART as the exemplary algorithm because of its cost-effectiveness and popularity. Also, PRISM is one of excellent options as a regularizer that can efficiently utilize the data sparsity and redundancy among multi-energy images.

The paper is organized as follows. In the next section, we will review the PRISM model. We will show that the SART algorithm can be superiorized, and how to superiorize SART with the PRISM prior. Also, we will briefly describe the Split-Bregman method for comparison, and explain our experimental design. In the third section, we will present the reconstruction results obtained using the two competing algorithms. In the fourth section, we will compare Superiorized-SART with SART and the Split-Bregman methods, and discuss image quality. Finally, we will conclude this paper.

2. Methods

2.1. Multi-energy imaging and PRISM model

As mentioned before, the photon-counting detectors put data in multiple energy bins across the entire x-ray spectrum. When every energy bin is small enough, the attenuation map of an object can be approximated as monochromic in each bin. Thus, in any energy bin the measurement can be modelled as a mono-energetic projection. In the k th energy bin, the measurement follows the Beer–Lambert Law:

$$I_k = I_{k,0} \exp(-Lx_k) \quad (1)$$

where I_k and $I_{k,0}$ are the measured and the incident photon intensities respectively; L stands for the row vector corresponding to the x-ray path defined by the imaging geometry, which is the same for all the energy bins; and x_k is a column vector representing the attenuation image in the k th energy bin. Taking the negative-log transform, we obtain the following matrix product:

$$G = [g_1, g_2, \dots, g_K] = A[x_1, x_2, \dots, x_K] = AX \quad (2)$$

where $g_k = -\ln \frac{I_k}{I_{k,0}}$ is a column vector representing a sinogram in the k th energy bin, A stands for the system matrix, and X is the spectral image vector with the components corresponding to the multiple energy bins respectively.

The original PRISM model assumes that the combined matrix of the multi-energy image vectors, $X = [x, x, \dots, x_K]$, is a superposition of a low rank matrix and a sparse matrix. The low rank component encourages correlative features, while the sparsity is associated with the compressibility of images. Several variants of the generic PRISM were proposed (Gao *et al*

2011). In this work, we only consider the rank and sparsity of the whole image matrix to have the same PRISM-oriented reconstruction problem as that in Chu *et al* (2013):

$$\text{minimize } \phi(X) = \|X\|_* + \lambda \sum_k TV(x_k), \text{ s.t. } AX = G \quad (3)$$

where $\phi(X)$ is the objective function, $\|\cdot\|_*$ denotes the nuclear norm, which is the summation of the singular values, $TV(\cdot)$ denotes the discrete total variation of an image with the following definition for an image denoted by $f(m, n)$,

$$TV(f) = \sum_m \sum_n \sqrt{(f_{m+1,n} - f_{m,n})^2 + (f_{m,n+1} - f_{m,n})^2}, \quad (4)$$

and the parameter λ determines the trade-off between the low rank and sparsity properties.

2.2. Superiorized SART with PRISM

In this subsection, we will superiorize the SART algorithm to solve equation (3). Suppose that the matrices in equation (3) have the following dimensions: $A \in R^{M \times N}$, $X \in R^{N \times K}$ and $G \in R^{M \times K}$. The SART algorithm solves the equation $AX = G$ by updating multi-energy images iteratively with

$$X_{j,:}^{(k+1)} = X_{j,:}^{(k)} + \frac{\omega}{\sum_{i=1}^M A_{ij}} \sum_{i=1}^M \frac{A_{ij}}{\sum_{j=1}^N A_{ij}} (G_{i,:} - [AX]_{i,:}) \quad (5)$$

for $k = 0, 1, \dots$, where ω is a relaxation parameter that adjusts the step-size. For simplicity, we also write the iterative procedure in a matrix form,

$$X^{(k+1)} = (I - \omega V^{-1} A^T W A) X^{(k)} + \omega V^{-1} A^T W G, \quad k=0, 1, \dots \quad (6)$$

where W is a diagonal matrix with diagonal elements $1/\sum_{j=1}^N A_{ij}$, and V is a diagonal matrix with diagonal elements $\sum_{i=1}^M A_{ij}$. To apply the superiorization method, we need to prove that the SART algorithm is strongly perturbation resilient.

Lemma 1—The SART iteration is non-expansive.

Proof—Based on the iteration procedure equation (6), we define the SART iteration operator $U: R^N \rightarrow R^N$,

$$U(X) = (I - \omega V^{-1} A^T W A) X - \omega V^{-1} A^T W G \quad (7)$$

From equation (7), we obtain

$$U(X) - U(Y) = (I - \omega V^{-1} A^T W A)(X - Y) \quad (8)$$

which can be rewritten as,

$$U(X) - U(Y) = V^{-\frac{1}{2}} \left(I - \omega V^{-\frac{1}{2}} A^T W A V^{-\frac{1}{2}} \right) V^{\frac{1}{2}} (X - Y) \quad (9)$$

From the definition of matrix norm, we have

$$\|U(X) - U(Y)\|_2 \leq \|V^{-\frac{1}{2}}\|_2 \left\| \left(I - \omega V^{-\frac{1}{2}} A^T W A V^{-\frac{1}{2}} \right) \right\|_2 \|V^{\frac{1}{2}}\|_2 \|X - Y\|_2 \quad (10)$$

Matrix V is diagonal with non-negative entries, thus $\|V^{-\frac{1}{2}}\|_2 \|V^{\frac{1}{2}}\|_2 = 1$. Matrix $V - A^T W A$ is positive semidefinite (Jiang and Wang 2003), so

$I - V^{-\frac{1}{2}} A^T W A V^{-\frac{1}{2}} = V^{-\frac{1}{2}} (V - A^T W A) V^{-\frac{1}{2}}$ is also positive semidefinite. Hence, the eigenvalues of $V^{-\frac{1}{2}} A^T W A V^{-\frac{1}{2}}$ is within $[0, 1]$. Then, we obtain

$$\left\| \left(I - \omega V^{-\frac{1}{2}} A^T W A V^{-\frac{1}{2}} \right) \right\|_2 < 1 \text{ for } 0 < \omega < 2 \quad (11)$$

Thus, the following inequality is true,

$$\|U(X) - U(Y)\|_2 \leq \|X - Y\|_2 \quad (12)$$

Equation (12) shows that the operator U is non-expansive. $\square$

Lemma 2—Define the proximity function as an indicator of how an X is compatible with the constraints,

$$\text{Prox}_C(X) := \|AX - G\|_2^2 \quad (13)$$

This proximity function is uniformly continuous.

Proof—From the triangle inequality, $\|\text{Prox}_C(X) - \text{Prox}_C(Y)\|_2 \leq \|A(X - Y)\|_2$. So, we have $\|\text{Prox}_C(X) - \text{Prox}_C(Y)\|_2 \leq \|A\|_2 \|X - Y\|_2$. Therefore, the proximity function is uniformly continuous. $\square$

Lemma 3—The SART algorithm is bounded convergent, which means that the image sequence generated by SART from X^0 converges to $Y(X^0)$ and that there exists a $\gamma \geq 0$ such that $\text{Prox}_C(Y(X^0)) - \gamma$ for every $X^0 \in \Omega$.

Proof—From iterative equation (6), we obtain

$$\|X^{(k+1)} - X^{(k)}\|_2 \leq \|(I - \omega V^{-1} A^T W A)\|_2 \|X^{(k+1)} - X^{(k)}\|_2 \quad (14)$$

From equation (11), there is a constant $0 < q < 1$, such that,

$$\|X^{(k+1)} - X^{(k)}\|_2 \leq q \|X^{(k+1)} - X^{(k)}\|_2 \quad (15)$$

Therefore, we have

$$\|X^{(k+1)} - X^{(k)}\|_2 \leq q^k \|X^{(1)} - X^{(0)}\|_2 \quad (16)$$

According to the Cauchy convergence criterion, the image sequence $\{X^{(k)}\}$ is convergent from an initial image. We denote its limit as $Y(X^{(0)})$. Furthermore, from triangle inequality, we have

$$\|AY(X^{(0)}) - G\|_2 \leq \|AX^{(k)} - G\|_2 + \|A\|_2 \|Y(X^{(0)}) - X^{(k)}\|_2 \quad (17)$$

From equation (6), we obtain,

$$\|AX^{(k)} - G\|_2 \leq \|I - \omega AV^{-1} A^T W\|_2 \|AX^{(k-1)} - G\|_2 \quad (18)$$

Matrix $W^{-1} - AV^{-1} A^T$ is positive semidefinite (Jiang and Wang 2003), therefore, the inequality hold $\|I - \omega AV^{-1} A^T W\|_2 < 1$, for $0 < \omega < 2$. Thus, we have

$$\|AX^{(k)} - G\|_2 < \|AX^{(k-1)} - G\|_2 < \dots < \|AX^{(0)} - G\|_2 \quad (19)$$

From equations (17) and (19), we have

$$\|AY(X^{(0)} - G)\|_2 < \|AX^{(0)} - G\|_2 + \|A\|_2 \|Y(X^{(0)} - X^{(k)})\|_2 \quad (20)$$

Due to the image sequence generated by $\{X^{(k)}\}$ converges to $Y(X^{(0)})$, there exists a constant $\gamma \geq 0$ such that $\text{Prox}_C(Y(X^{(0)})) = \gamma$. Thus, the SART algorithm is bounded convergent.

Based on the above three lemmas and theorem 1 in Herman *et al* (2012), we have the following corollary:

Corollary—The SART algorithm is strongly perturbation resilient (Definition 4.2 in Censor *et al* (2014)).

This corollary guarantees that the SART iteration can be superiorized, and the image sequence generated by the superiorized SART is convergent.

Now, let us superiorize the SART algorithm subject to the PRISM prior. We follow the same algorithm flowchart as in Censor *et al* (2014) and Garduno and Herman (2014). The superiorized SART algorithm is in table 1.

In the algorithm, we choose $\eta_l = 0.999^l$ and $N = 3$. A key step is to find a non-ascending matrix for the objective function $\phi(X)$ at $X^{k,n}$ so that $\phi(X^{(\text{iter})}) + \beta_{\text{iter}} \nu^{(\text{iter})} = \phi(\mu^{(\text{iter})})$. For our objective function, the total variation of a single image is defined in equation (4), which is not differentiable without regularization. To make it differentiable and avoid singularity, we added a small constant ε to equation (4)

$$TV(f) = \sum_m \sum_n \sqrt{(f_{m+1,n} - f_{m,n})^2 + (f_{m,n+1} - f_{m,n})^2 + \varepsilon^2} \quad (21)$$

and computing its gradient by

$$\begin{aligned} \frac{\partial TV(f)}{\partial f_{m,n}} = & \frac{2f_{m,n} - f_{m+1,n} - f_{m,n+1}}{\sqrt{(f_{m+1,n} - f_{m,n})^2 + (f_{m,n+1} - f_{m,n})^2 + \varepsilon^2}} \\ & + \frac{f_{m,n} - f_{m-1,n}}{\sqrt{(f_{m,n} - f_{m-1,n})^2 + (f_{m-1,n+1} - f_{m-1,n})^2 + \varepsilon^2}} \\ & + \frac{f_{m,n} - f_{m,n-1}}{\sqrt{(f_{m+1,n} - f_{m,n-1})^2 + (f_{m,n} - f_{m,n-1})^2 + \varepsilon^2}} \end{aligned} \quad (22)$$

The nuclear norm is non-differentiable, and we use one of its sub-gradients

$$U_r V_r^T \in \partial \|X\|_* \quad (23)$$

where U_r, V_r are the truncated first $r = \text{rank}(\mu)$ columns. Thus, we have

$$v = \lambda \sum_k \frac{\partial TV(X_k)}{\partial X_{m,n}^k} + U_r V_r^T \in \partial_X \phi(X) \quad (24)$$

Then, we choose $\nu / \|\nu\|_F$ as the non-ascending vector to minimize the objective function.

2.3. Split-Bregman method

For comparison, we briefly describe the Split-Bregman method. To do this, we make the replacements $D \leftarrow X$, $B_1 \leftarrow \nabla_m X$, and $B_2 \leftarrow \nabla_n X$, in which ∇_m and ∇_n denote the derivative operators along the row and column directions respectively. Then, the split formulation of equation (3) becomes

$$\min_{\mu, B_1, B_2, D} \lambda \| (B_1, B_2) \|_2 + \| D \|_* \frac{\mu}{2} \| AX - G \|_F^2 + \frac{\lambda_1}{2} \| D - X - U \|_F^2 + \frac{\lambda_2}{2} (\| B_1 - \nabla_m X - W_1 \|_F^2 + \| B_2 - \nabla_n X - W_2 \|_F^2) \quad (25)$$

where λ is the same as that in equation (3), μ is the trade-off between the data fidelity term and the PRISM prior, λ_1 and λ_2 are the Lagrange multipliers. This problem is solved alternatively using the algorithm in table 2. In this algorithm, Step 4 is solved using the conjugate-gradient (CG) method; Step 5 is solved using the singular value thresholding method; and Step 6 is updated using the generalized shrinkage method. One can refer to Goldstein and Osher (2009) for more details.

3. Numerical simulation

3.1. Experimental design

To evaluate the proposed algorithm, we performed numerical tests in a fan-beam geometry. We simulated a few-view projection with only 60 projection angles uniformly distributed over a range from 0° to 360° . The x-ray source was placed 45 cm away from the iso-center of the imaging plane, while the detector array was 45 cm away from the iso-center on the opposite side. The detector array consisted of 512 elements, which were equi-spatially distributed, covering 90 cm in total length.

We used a clinical dual-energy CT dataset to construct realistic numerical phantoms. The datasets were acquired on a GE Discovery CT750 scanner in Shanghai Ruijin Hospital, Shanghai, China, with an approval for retrospective research use. The water and bone material bases were extracted from the DECT datasets. Then, an additional Gadolinium base was added, whose K-edge is at 50.207 keV. Figure 1 shows the three material bases and a mono-energetic image at 70 keV. Poly-energetic projections were simulated via calculating mono-energetic images from the material bases and synthesizing single energy projections in various narrow energy bins. In our simulation, the detector element was operated in 8 energy bins with thresholds 40, 50, 60, 70, 80, 90, and 105 keV respectively, as shown in figure 2. The total number of photons from the x-ray source was 5×10^6 towards each detector cell. We ignored the detector imperfectness such as charge sharing, pile-up and dead-time, and added Poisson noise to the ideally measured numbers.

3.2. Representative results

The simulated few-view noisy datasets were reconstructed using the following four algorithms respectively: filtered back-projection (FBP), SART without PRISM, Split-Bregman-PRISM, and superiorized SART-PRISM. The FBP reconstruction was used as the initial guess for the iterative methods. To obtain the ground truth, we generated 1080 projections without noise and reconstructed the benchmark image using the FBP algorithm.

As far as the parameter selection is concerned for the iterative algorithms, we tested different sets of the parameters empirically and chose the best one. Table 3 shows the chosen parameters. The Split-Bregman has an inner loop for CG iteration, which was stopped when

the inner iteration converged or the maximum number of iterations was reached. Both the Superiorized-SART and Split-Bregman iterations were terminated when 2-norm errors between the truth and reconstruction results went below 1.5 (the truth of multi-energy images has a norm of 22.06), which was empirically chosen to indicate a good image quality in this pilot study. As a result, the Superiorized-SART ran 1665 iterations and the Split-Bregman ran 88 iterations. Since the superiorization method is based on the SART algorithm, we ran the SART algorithm for the same number of iterations.

Figure 3 shows a region-of-interest (ROI) of the images in the 1st, 4th, and 8th energy bins respectively. Also, we computed the objective function of each reconstructed multi-energy images and the time cost of each algorithm, as shown in table 4. Figure 4 plots the singular values of the reconstructed images on the logarithmic scale. Figure 5 plots the peak-to-noise ratio (PSNR) of reconstructed images at each energy bin.

4. Discussions

As shown in figure 3, the FBP reconstruction produced a pretty strong noise and streak artefacts in the images due to the limitations in the numbers of photons and projection angles. Yet, all the iterative algorithms demonstrated a good degree of noise suppression and artefacts reduction. Interestingly, we would like to compare the Superiorized-SART with its original version. In figure 3, it is seen that the images via superiorization preserved some streaks from the SART results. However, noise was effectively smoothened out by the TV penalty. This phenomenon was also reflected in the singular value plots. The SART results had almost the same main singular values as the ground truth, but it also contained substantial components caused by noise and artefacts. On contrary, the superiorization made significant improvements. This is predicable since the Superiorized-SART used the same equation as that for SART to update the image, yet it steered the image sequences to minimize the objective function. Regarding the computational cost, the superiorization method increased computational time compared to the original SART algorithm. One reason is that we were repeatedly computing the objective function at Step 12 in table 1, which requires SVD decomposition. One possible method to accelerate the Superiorized-SART is to implement an efficient SVD algorithm, which we would like to suggest the algorithm in Avron *et al* (2012). On the other hand, we could decrease the frequency for the objective function evaluation. Actually, in the simulation we found that the convergence of the iterative process was slow at the beginning but got improved steadily as it went on. Based on this observation, we suggest to increase the step size of $\mathbf{l}$ at the early stage so that the iterative process can be brought into the steady stage sooner and at a reduced computational cost.

In this study, we stopped the Superiorized-SART and Split-Bregman algorithms when they reached the similar reconstruction results so that we can perform a fair comparison of these two algorithms in terms of their computational costs and convergence speeds. Regarding the stopping criteria in real life, most iterative algorithms are terminated after a certain number of iterations or after running for long enough time, which are empirically determined. For the proposed Superiorized-SART algorithm, the stopping criteria can be similarly expressed.

For a given class of images, we could choose an upper bound of iteration number to indicate that the objective function has already reduced to a sufficiently low level.

Encouragingly, the Superiorized-SART algorithm provided a result visibly comparable to the Split-Bregman algorithm. Both the algorithms converged to rather small objective function values close to the ground truth. It should be noted that we only tested a few sets of parameters for the Split-Bregman algorithm. The reported results were the best among all our tests, but may not be the optima. Further evaluation and improvement should be desirable.

5. Conclusion

We have proposed a Superiorized-SART algorithm to optimize the PRISM-based objective function for multi-energy image reconstruction. The proposed algorithm provides a practical solution for CT image reconstruction. It modifies the image sequence generated by SART to fit the PRISM model gradually. We have proved that the SART algorithm is *strongly perturbation resilient* so that the superiorization method is justified in this context. The numerical simulation results have demonstrated satisfactory image quality improved with the superiorization methodology in comparison with the Split-Bregman algorithm. Favourably, the Superiorized-SART algorithm is much easier in implementation and parameter tuning. In brief, the Superiorized-SART algorithm works as an alternative to the original Split-Bregman method for PRISM-based multi-energy CT.

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Figure 1. Three material bases. (a) Water, (b) bone, (c) Gadolinium, and (d) the monoenergy image at 70 keV.

Figure 2. X-ray spectrum covering 8 energy bins from 30 to 120 keV. The thresholds of the energy bins are 40, 50, 60, 70, 80, 90, and 105 keV respectively.

Figure 3.

ROI comparison of the reconstructed images. The 1st, 2nd, and 3rd rows are the images in the 1st, 4th, and 8th energy bins, respectively. From left to right are the ground truth, FBP, SART, Superiorized-SART, and Split-Bregman results, respectively.

Figure 4. Singular values of the reconstructed images versus the truth on the logarithmic scale.

Figure 5.
PSNR of the reconstructed images at each energy bin.

Table 1

Superiorized Version of the SART Algorithm.

1.	Set $\text{iter} = 0$, $X^{\text{iter}} = X^0$, $l = -1$
2.	Repeat until a stopping criterion is satisfied
3.	Set $n = 0$
4.	Set $Y^{\text{iter},n} = X^{\text{iter}}$
5.	While $n < N$
6.	Set $v^{\text{itel},n}$ to be a non-ascending matrix for $\phi(X)$ at $Y^{\text{iter},n}$
7.	Set loop = true
8.	While loop
9.	Set $l = l + 1$
10.	Set $\beta_{\text{iter},n} = \eta_l$
11.	Set $Z = Y^{\text{iter},n} + \beta_{\text{iter},n} v^{\text{itel},n}$
12.	If $\phi(Z) < \phi(Y^{\text{iter},n})$
13.	Set $n = n + 1$
14.	Set $Y^{\text{iter},n} = Z$
15.	Set loop = false
16.	Set $X^{\text{iter}+1} = \text{SART}(X^{\text{iter},n})$
17.	Set $\text{iter} = \text{iter} + 1$

Table 2

Split-Bregman method with PRISM.

1.	Set $\text{iter} = 0, X^{\text{iter}} = D^{\text{iter}} = B_1^{\text{iter}} = B_2^{\text{iter}} = 0$
2.	Set $V^{\text{iter}} = W_1^{\text{iter}} - W_2^{\text{iter}} = 0$
3.	Repeat until a stopping criterion is satisfied
4.	Update $X^{\text{iter}+1} := \min_x \frac{\mu}{2} \ AX - G\ _F^2 + \frac{\lambda_1}{2} \ D^{\text{iter}} - X - U^{\text{iter}}\ _F^2 + \frac{\lambda_2}{2} (\ B_1^{\text{iter}} - \nabla_m X^{\text{iter}} - W_1^{\text{iter}}\ _F^2 + \ B_2^{\text{iter}} - \nabla_n X^{\text{iter}} - W_2^{\text{iter}}\ _F^2)$
5.	Update $D^{\text{iter}+1} := \min_D \ D\ _* + \frac{\lambda_1}{2} \ D - X^{\text{iter}+1} - U\ _F^2$
6.	Update $(B_1^{\text{iter}+1}, B_2^{\text{iter}+1}) := \min_{B_1, B_2} \lambda (\ B_1, B_2\ _2 + \frac{\lambda_2}{2} (\ B_1 - \nabla_m X^{\text{iter}+1} - W_1^{\text{iter}}\ _F^2 + \ B_2 - \nabla_n X^{\text{iter}+1} - W_2^{\text{iter}}\ _F^2))$
7.	Update $U^{\text{iter}+1} := U^{\text{iter}} + X^{\text{iter}+1} - D^{\text{iter}+1}$
8.	Update $W_1^{\text{iter}+1} := W_1^{\text{iter}} + \nabla_m X^{\text{iter}+1} - B_1^{\text{iter}+1}$
9.	Update $W_2^{\text{iter}+1} := W_2^{\text{iter}} + \nabla_n X^{\text{iter}+1} - B_2^{\text{iter}+1}$
10.	Set $\text{iter} = \text{iter} + 1$

Table 3

Parameters chosen for SART, superiorized-SART and Split-Bregman iterations.

SART		Superiorized-SART		Split-Bregman	
Iterations	1665	Iterations	1665	Iterations	88
ω	1	ω	1		
		λ	0.01	λ	0.01
				μ	5
				λ_1	0.02
				λ_2	0.02
				CG iterations	100

Table 4

Computational cost and objective function values.

	Truth	SART	Superiorized-SART	Split-Bregman
Time		292 s	2472 s	2022 s
$\phi(X)$	37.4961	46.1200	36.8321	38.0060