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A COMPARISON OF MAZUR'S k AND AREA UNDER THE CURVE FOR DESCRIBING STEEP DISCOUNTERS

Jin H. Yoon^{1,2}, Richard De La Garza II¹, Thomas F. Newton¹, Robert Suchting², Matthew T. Weaver³, Gregory S. Brown^{1,4}, Yasmine Omar^{1,5}, and Ilana Haliwa²

¹Baylor College of Medicine, Menninger Department of Psychiatry and Behavioral Sciences, Houston, TX 77030

³Mercyhurst University, Department of Psychology, Erie, PA 16546

Abstract

Delay discounting describes how a reward loses value as a function of increasing delay to its receipt and has been reliably associated with a variety of vulnerable populations including those with substance use disorders (SUDs). Two commonly used models to assess delay discounting in the field of SUDs include log k derived from Mazur's hyperbolic equation and area under the curve (AUC). In the current study, we compared log k with AUC on delay discounting data obtained from non-treatment seeking, cocaine- and methamphetamine-dependent volunteers. We specifically chose this population in order to obtain a distribution of relatively steep discounters. The results show that the relationship between AUC and log k is better described by a quadratic rather than a linear function. In other words, changes in discounting, as measured by AUC and log k , are reflected differently across a range of obtained responses. Additionally, the distribution of AUC values was skewed, which appears to be more likely among populations exhibiting greater discounting. Finally, closer examination of indifference points revealed that AUC was almost perfectly predicted by the area from the two longest delays, with relatively less input from shorter delays. Given these results, researchers should exercise additional caution when deciding which method to assess discounting data and how final results are to be interpreted, particularly when dealing with relatively high rates of discounting. High rates of discounting are likely in populations with impulsive disorders such as those with SUDs.

Keywords

delay discounting; area under the curve; impulsivity; cocaine; methamphetamine

Delay discounting describes how a reward loses value as a function of increasing delay to its receipt (Bickel & Marsch, 2001). As such, delay discounting provides a useful framework for assessing when people may choose a smaller, more immediate reward (*e.g.*, consuming a drug) over a larger, delayed one (*e.g.*, improved health, financial stability, etc). This decision

Corresponding Author: Jin H. Yoon; jin.ho.yoon@uth.tmc.edu; Phone: 713-486-2796.

²Current Affiliation: University of Texas Health Science Center at Houston, Department of Psychiatry and Behavioral Sciences, Center for Neurobehavioral Research on Addictions, Houston, TX 77054.

⁴Current Affiliation: University of Texas at Austin, Austin, TX 78731.

⁵Current Affiliation: Rutgers the State University of New Jersey, Department of Psychology, Piscataway Township, NJ 08854

model is of interest as it appears to be relevant across a number of vulnerable populations. For example, greater discounting has been noted among those with substance use disorders (SUDs), pathological gambling, and obesity to name a few (Bickel, Jarmolowicz, Mueller, Koffarnus, & Gatchalian, 2012; Bickel & Marsch, 2001; Epstein, Salvy, Carr, Dearing, & Bickel, 2010; MacKillop et al., 2011; Perry & Carroll, 2008). The number of delay discounting papers has increased dramatically over the last decade. A simple search in PubMed using the keyword “delay discounting” resulted in 273 findings in the year 2015, compared to 25 in 2005.

Focusing on those with SUDs as an exemplar, greater delay discounting has been associated with drug use initiation, maintenance, and relapse. To summarize the primary findings: 1) greater DD (more impulsivity) has been reliably observed among those with SUDs compared to matched controls across a wide variety of drugs of abuse; and 2) studies suggests that discounting is predictive of initiation as well as relapse, albeit primarily in studies involving cigarette smoking (Bickel & Marsch, 2001; MacKillop et al., 2011; G. J. Madden & Bickel, 2010; Perry & Carroll, 2008; Reynolds, 2006).

In humans, discounting is typically assessed utilizing an adjusting-amount procedure in which participants are presented with choices between a smaller, more immediate reward and a larger reward following a fixed delay (G.J. Madden & Johnson, 2010). The distribution of delays is generally skewed, with shorter delays assessed more frequently relative to longer ones. This is done as relatively rapid changes in indifference points are observed at relatively shorter delay values, whereas changes become less pronounced out at relatively longer delay values, which is characteristic of hyperbolic functions.

A common method for assessing delay discounting data is to fit the indifference points to a hyperbolic model (Mazur, 1987).

$$V = A / (1 + kD) \quad (1)$$

The equation describes how the value (V) of a reward of initial amount (A) decreases as a function of increasing delay (D) to receiving that reward. The free parameter k describes the rate of delay discounting with higher k values corresponding to greater discounting. While other models of discounting exist, Equation 1 is arguably the most commonly utilized model of discounting, particularly for research involving SUDs. For example, a meta-analysis of studies assessing DD among those with SUDs observed that 70% studies utilized Mazur’s hyperbolic equation (MacKillop et al., 2011). While the hyperbolic model continues to be widely used, some researchers have pointed to limitations of the model, including issues related to goodness of fit, and question whether it is the best method for presenting delay discounting data (Johnson & Bickel, 2008).

Area under the curve (AUC) is another commonly used method for assessing delay discounting data (Myerson, Green, & Warusawitharana, 2001). AUC is calculated by plotting indifference point values obtained at various delays, with value plotted on the y-axis and delay on the x-axis. Both axes are normalized such that potential values range from 0 to

1. The AUC for a section of the discounting curve is assessed by calculating the area of the trapezoid that consists of two temporally adjacent indifference points (V_1 , V_2) and their respective delays (D_1 , D_2).

$$AUC = (D_2 - D_1)((V_1 + V_2)/2) \quad (2)$$

Eq.(2) illustrates that by multiplying the base ($D_2 - D_1$) by the height ($(V_1 + V_2)/2$) of the trapezoid, one may obtain the AUC for that section of the discounting curve (Myerson et al., 2001). Implementing Eq.(2) for each subsequent section of the discounting curve and adding the area of the individual trapezoids together allows one to obtain the total AUC, which ranges from 0 to 1. AUC is theoretically neutral and AUC values are based directly on data points. Therefore, goodness of fit to an assumed mathematical model is not an issue as in the case of hyperbolic model. Another advantage of AUC cited in the original study by Myerson et al. is “the distribution of area measures, unlike distributions of estimates of the parameters, is not skewed. This means that one can use parametric statistics with area measures, whereas the parameter estimates require the use of nonparametric statistics.”

Both measures of delay discounting, k and AUC, continue to be widely used in the literature and have been demonstrated to differentiate between vulnerable populations vs. matched controls and be sensitive to a variety of experimental manipulations (Johnson & Bickel, 2008; MacKillop et al., 2011). While some studies even present both measures, to our knowledge, only one study has directly compared a range of k and AUC values obtained from the same individuals. In that study, Mitchell assessed delay discounting across several datasets (Mitchell, Wilson, & Karalunas, 2015). Of the three datasets, one included individuals with attention deficit hyperactivity disorder (ADHD), another included smokers and non-smokers (SMOKERS), and the third manipulated the magnitude of the delayed reward during the discounting task (AMOUNT). Parameters of the delay discounting assessment varied across and within the datasets. Overall, delays ranged from 0 to 365 days and the delayed monetary option ranged from \$10 to \$100. Both the range of delays and magnitude of monetary options have been used in other studies, but relatively larger delays and monetary values are also commonly used in the field. The study had a number of important findings. First, overall discounting was greater in the ADHD group, compared to the SMOKER group, followed by the AMOUNT group. Second, histograms of log-transformed k values normalized the distributions, which has been commonly reported in the literature. Interestingly, the distribution of AUC values was skewed for both the ADHD and AMOUNT groups on opposite sides of the distribution, although in the authors reported they were in the ranges appropriate for parametric statistics. This finding is of interest as one of the reported strengths of AUC is that it produces a normalized distribution (Myerson et al., 2001). Third, scatterplots of AUC vs logged k values revealed a monotonically decreasing relationship across all three datasets that became less pronounced at the ends of the distribution when discounting was very high or low. Specifically, the range of AUC values was decreased relative to the range of corresponding logged k values at the ends of the data continuum. This effect was most pronounced in the ADHD and AMOUNT groups, which also exhibited the lowest and highest distributions of discounting and also presented the

most skewed AUC histograms. Fourth, in the ADHD group, comparison between ADHD-diagnosed vs. undiagnosed groups revealed a significant difference in log k values, but not in AUC values. The authors concluded that the disparate results were due to the AUC indices being less sensitive to group differences presumably due to the presence of relatively more high discounting values. The authors concluded with a recommendation to use k values instead of or in conjunction with AUC values when the distribution has a high proportion of relatively high or low delay discounting values.

The shifting relationship between AUC and logged k values as discounting becomes greater (i.e., more impulsive) is noteworthy as populations of interest, such as those with SUDs, are likely to exhibit particularly high rates of discounting. Therefore, one goal of the current study was to build upon the findings by Mitchell et al by targeting a population likely to exhibit relatively high rates of discounting. Specifically, the current study focused on cocaine- and methamphetamine-dependent volunteers. Both groups have been observed to exhibit relatively high rates of discounting. For example, individuals with cocaine use disorder have been observed to have some of the highest rates of discounting in the literature to our knowledge (Coffey, Gudleski, Saladin, & Brady, 2003). Additionally, compared to those used in the Mitchell et al study, we assessed longer delays (up to 25 years) as well as larger monetary values (up to \$1,000), which are common values used in the literature. The second goal of the current study was to explore, in greater detail, possible reasons as to why these differences between AUC and log k values are observed. For example, we examined the potential influence of the skewed distribution of temporal delays (i.e., more frequent assessments at relatively shorter vs. longer delays), which is a common feature in delay discounting assessments.

METHODS

Participants

Participants (N=186) consisted of either cocaine- (N=115) or methamphetamine- (N=71) dependent individuals not seeking treatment for their substance use. Both groups are more likely to exhibit greater discounting compared to healthy controls (MacKillop et al., 2011; G. J. Madden & Bickel, 2010; Perry & Carroll, 2008; Reynolds, 2006). Participants ranged from 18 to 55 years old, reported regular use of their drug of choice (more than 3 times a week), reported no diagnosis (last two years) of psychiatric illness or medical problems, and were not currently taking any prescription medications. Note that the primary purpose for choosing this sample was to obtain a representative range of discounting values among individuals with SUDs, including relatively high delay discounting. Participants were enrolled in two separate studies assessing possible pharmacotherapies, modafinil and perindopril, for cocaine- and methamphetamine-dependence, respectively (Newton, De La Garza, & Grasing, 2010; Verrico et al., 2014). Delay discounting assessments were conducted prior to receiving any medication. All individuals provided informed consent and all procedures were approved by the Baylor College of Medicine and Michael E. DeBakey Veterans Affairs Medical Center Internal Review Boards.

Procedures

The delay discounting task was presented to participants via a laptop running the Windows® Operating System. Overall, the task took approximately 10 minutes to complete. Participants were instructed how to interact with the delay discounting program and informed that they would not receive any of the monetary amounts presented but to respond as if the choices were real. Details regarding the delay discounting task have been published elsewhere (García-Rodríguez, Secades-Villa, Weidberg, & Yoon, 2013). Overall, participants were presented with a choice between \$1,000 after a fixed delay versus various amounts of money available immediately using an adjusting-amounts procedure (Holt, Green, & Myerson, 2012). The delays values were 1 day, 1 week, 1 month, 6 months, 1 year, 5 years, and 25 years. Note that these monetary and delay values are representative of what is commonly assessed in the literature. The delays were presented in either an ascending or descending order for a given participant, and the order was counterbalanced across participants. The value of the immediate monetary option ranged from \$5 to \$995 in \$5 increments and was adjusted via a titrating procedure that honed in on the indifference point based on the participant's responses.

Data Analysis

Discounting data were first assessed for systematicity using the following two criteria: 1) indifference points were larger than the previous, shorter indifference point by 20%; and 2) the indifference point at the shortest delay was higher than the indifference point at the longest delay by 10% of the largest reward (e.g., 10% of \$1,000 = \$100)(Johnson & Bickel, 2008). Data were defined as being non-systematic if they failed to meet either of the criteria described above. A total of 12 subjects' data were found to be nonsystematic out of the initial 186. All subsequent data analyses are based on the data from the remaining 172 (92%) participants.

Obtained indifference points were fit to Equations 1 and 2 using the curve-fitting program GraphPad Prism 6 (GraphPad Software, San Diego, CA). Statistical analyses were conducted using either Sigmaplot 12 (Systat Software, Inc, San Jose, CA), R (www.r-project.org), or Mplus (Muthen & Muthen, Los Angeles, CA). Histograms of k , $\log k$, and AUC were generated using Sigmaplot's recommended settings. Statistical analyses were performed on log-transformed k values as the distribution of k values is skewed (Yoon & Higgins, 2008; Yoon et al., 2007). Results were considered significant at $p < .05$.

Generalized linear modeling was used to investigate the nature of the relationship between $\log k$ and AUC. A model comparison approach examined the impact of including iteratively higher-order polynomial term in the equation. In addition to inspection of polynomial term significance, model fit was compared using the Akaike information criteria (AIC). When comparing among competing models, the AIC provides a measure of relative model fit, whereby lower values indicate a better-fitting model (Burnham & Anderson, 2002; Burnham, Anderson, & Huyvaert, 2011). Log k was modeled as the predictor variable and AUC was modeled as the outcome variable in these equations. All models were fit in R using the `glm()` function and in Mplus with bootstrapping to countenance the skewed distribution of AUC.

RESULTS

Figure 1 shows the distribution of k , $\log k$, AUC, and $\log$ AUC values. As has been commonly observed in the literature, the distribution of k values is highly skewed and normalized following log-transformation. In contrast, the distribution of AUC values is also highly skewed, which is not commonly reported in the literature. Note that we increased the number of bins to 50 but the shape of the distribution remained the same (data not shown). The distribution of $\log$ AUC values is also presented. However, to our knowledge, this is not standard practice in the field as a cited benefit of AUC is that it has a normal distribution (Myerson et al., 2001).

Figure 2 shows a scatterplot of $\log k$ values and corresponding AUC values. The Spearman correlation between AUC and $\log k$ values was -0.79 , indicating a strong negative relationship. Overall, the distribution of points became less pronounced at the right side where discounting was relatively greater. In order to assess to correspondence between $\log k$ and AUC values, the distribution for $\log k$ and AUC values was divided in half and classified as being either on the “high” or “low” end of each distribution. For the majority of participants, $\log k$ and AUC values shared the same classification (i.e., both high or both low) as indicated by the filled symbols in Figure 2. However, when a mismatch in classification was observed, they were on the right side of figure, where discounting was highest as illustrated by the open symbols.

Generalized linear modeling was utilized to assess the linearity of the relationship between $\log k$ and AUC. Two models were fit: one including a linear effect of $\log k$ and other with both a linear and a quadratic (i.e., nonlinear) effect of $\log k$. In each model, the linear and nonlinear effects were statistically reliable ($p < 0.001$); however, the model including a quadratic effect of $\log k$ demonstrated superior fit (via lower AIC and RMSE values; Table 1). A scatterplot of $\log k$ vs. AUC values with fit lines for the linear (straight dashed line) and quadratic (solid curved line) models is provided in Figure 2.

Figure 3 (top) illustrates the relationship between AUC when using all indifference points (Total AUC) and AUC when using a portion of the indifference points (Partial AUC) for a given participant. As expected, r and r^2 values are relatively low when comparing Total AUC vs. the Partial AUC from 1 to 7 days. Values for r and r^2 increased as more and more indifference points are used to assess the Partial AUC. The Partial AUC from 1 day to 25 years is not included as the graph would be perfect line. However, the highest r and r^2 values were observed when comparing the Total AUC with the Partial AUC from 5 to 25 years, which represent the last two delays assessed. The same comparison was done for $\log k$ values (Figure 3, bottom). Similar to what was observed when comparing Total and Partial AUCs, r and r^2 increased as the number of indifference points included in the Partial $\log k$ increased. In contrast, the lowest r and r^2 values were observed when the Total $\log k$ was compared with the Partial $\log k$ from 5 to 25 years. Additionally, excluding the comparison using the 5 to 25 year indifference points, r and r^2 values were higher for each $\log k$ comparison relative to its AUC counterpart.

DISCUSSION

One goal of the current study was to conduct a systematic replication of the study by Mitchell et al. (2015) using a population that exhibits greater discounting (i.e., individuals with either cocaine- or methamphetamine-dependence) as well as utilizing larger monetary rewards and longer delay values, which are commonly presented in the literature. Comparison of the distribution of AUC and log k values in Figure 2 of the current manuscript with those observed in the Mitchell et al. study shows that we did succeed in capturing relatively greater discounting. With this population of relatively steep discounters, we observed that the distribution of AUC values was highly skewed to the left, where discounting was relatively steeper (Figure 1). In the Mitchell et al. ADHD study, there was evidence of a similar skew, although less pronounced. Interestingly, in the AMOUNT study, the skew was in the opposite direction. Note that discounting was relatively shallow in this group. Taken together, these results suggest that the distribution of AUC values will become more and more skewed among populations with relatively steep, as illustrated in Figure 1 of the current study and in the ADHD study by Mitchell et al., or shallow discounting as in the AMOUNT study by Mitchell and colleagues. To our knowledge, no other studies have noted this relationship between relatively steep or shallow discounting and the distribution of AUC values. This finding has important implications as one supposed strength of AUC is that it has a normal distribution (Myerson et al., 2001). While having a skewed distribution does not eliminate the utility of using AUC to assess discounting in of itself, these results do highlight the importance of not assuming that normality of the data is a given. Indeed, our results suggest that a skewed AUC distribution is predicted with either relatively high or low discounting populations. Therefore, at the very least, we recommend assessing the distribution of AUC values in order to prevent the inappropriate application of certain statistical tests.

Related to the first goal, a second finding of the current study was that the distribution of log k and corresponding AUC values was better described by a quadratic vs. a linear function. These results are in line with those reported by Mitchell et al. (2015) while providing further statistical support. This finding appears to be the consequence of decreasing changes in AUC values compared to corresponding log k values with relatively higher rates of discounting (i.e., the right end of distribution in Fig. 2). This observation has potentially important implications as delay discounting tasks are typically administered to populations that exhibit high levels of discounting, including those with SUDs (Bickel & Marsch, 2001; MacKillop et al., 2011; G. J. Madden & Bickel, 2010; Perry & Carroll, 2008; Reynolds, 2006; Tziortzis, Mahoney, Kalechstein, Newton, & De La Garza, 2011). Therefore, if the population of interest exhibits a relatively high degree of discounting, the current results suggest that using AUC vs. log k may produce disparate results. Specifically, it would not be surprising to observe significant differences using k values, but not AUC values. Indeed, this exact result was observed in the ADHD group in the Mitchell et al. study, which had relatively high rates of discounting. While the current study does not provide evidence of further disparate results, we believe that future studies dealing with relatively high or low discounting populations should present results using both AUC and log k as measures of discounting. Additionally, it would be illuminating to review results from past studies dealing with

similar populations in which significant differences were not observed using AUC. Note that this does not necessarily imply that k is a more useful measure due to this apparent increased sensitivity. Such an argument would have to demonstrate that increased sensitivity is also associated with changes in other relevant measures such as drug use severity or latency to relapse.

The second goal of the current study was to ascertain why AUC may be less sensitive at corresponding higher values of k . Suspecting the skewed distribution of delay values as a potential influence, we compared the relationship between Total AUC and a number of Partial AUCs using different indifference points. An analogous comparison was done using $\log k$. Overall, the results revealed that Partial AUC provided better r and r^2 values as more and more indifference points from 1 day to 5 years were used to assess Partial AUC. However, these were all inferior to the r and r^2 values when the Partial AUC just used indifference points from 5 to 25 years, the last two delays. Such results seem to be at odds with the primary assumptions of the discounting procedure. Specifically, the reason that the distribution of delays is typically skewed is that changes in the discounting curve are more likely to occur at shorter, rather than longer delays. However, in the case of AUC, what occurs at the shorter delays appears to be largely irrelevant in the final calculation. Indeed, the current results suggest that simply assessing indifference points at 5 and 25 year delays should provide a value nearly identically correlated to the total AUC. Note that we do not actually recommend only assessing later indifference points. In contrast to the above findings, comparisons between Partial $\log k$ and Total $\log k$ showed increasing r and r^2 values when the Partial $\log k$ incorporated more indifference points from 1 day to 25 years. Additionally, r and r^2 values for $\log k$ were superior to corresponding comparison for AUC, except when the Partial $\log k$ utilized indifference points from 5 to 25 years. Overall, these results suggest that $\log k$ is relatively more influenced by what is occurring at shorter delays relative to AUC.

At this point, we would like to reiterate that we are not stating that $\log k$ is a superior measure of discounting over AUC. However, we believe that the results of the current study point to limitations of AUC that may not be widely known by the scientific community. It is important to realize that AUC was developed to help address limitations noted by the community regarding Mazur's hyperbolic community such as poor fit to the data and skewed distribution of k values (Myerson et al., 2001). Reconciling the differential usage of AUC and Mazur's k is problematic. Each has arguments in favor of their use: AUC is considered a more impartial measure of discounting, as it is directly derived from the data, whereas k is superior for capturing the variance in discounting when the distribution of AUC values is skewed (as in the present sample of relatively steep discounters). To alleviate this issue, we suggest two potential alternatives for exploring discounting data: 1) fitting the indifference points to a reference probability distribution using generalized linear mixed modeling (GLMM)(Payne, 2013); and 2) using alternative AUC methods (Borges, Kuang, Milhorn, & Yi, 2016). Both examples are discussed below. Note that this is not exhaustive list of possible alternatives, but we believe that each method has certain strengths that suggest exploring them as methods for assessing discounting has merit.

In the present sample of relatively steep discounting cocaine and methamphetamine users, generalized linear mixed modeling was used to longitudinally fit the delay discounting indifference points using PROC GLIMMIX in SAS v. 9.3. Generalized linear mixed models are useful for fitting hierarchical data (each participant has seven data points) with a range of reference probability distributions (e.g., normal, lognormal, Poisson). The raw indifference points are continuous, range from 0 to 1, and positively skewed; as such, the beta distribution provided a logical choice for modeling the outcome. This distribution provided demonstrably better fit against competing alternative distributions (Table 2). Generalized linear mixed modeling is a powerful, more broadly-applicable alternative to fitting delay discounting data than the summary measures provided by Mazur's k or AUC.

Alternative approaches to calculating AUC was recently proposed by Borges et al., (2016). In the study, the authors emphasize the disparate influence of the portions of the AUC from the larger delays. They provide two methods to minimize this influence by transforming the delays into either a log scale (AUC_{log}) or ordinal scale (AUC_{ord}). Note these suggestions are congruent with our findings in Figure 1, in which a log transformation of the skewed AUC distribution results in a more normal distribution. As part of that study, the authors compared results from previously published studies from their group in which significant differences were found using k (Matusiewicz, Carter, Landes, & Yi, 2013; Yi, Carter, & Landes, 2012; Yi & Landes, 2012). For all three studies, both AUC_{log} and AUC_{ord} successfully replicated results in which significant differences were observed. Relevant to the current study, AUC was also assessed and failed to replicate the significant differences observed with k , AUC_{log} , and AUC_{ord} in two out of three studies.

A limitation of the current study is that we do not assess other measures of DD such as the hyperboloid model (Green & Myerson, 2004). However, Mazur's hyperbolic function and the AUC are arguably the two most commonly used measures of DD. Therefore, we believe the results of the current study are relevant across the broad discounting literature. For example, both the Mitchell et al., (2015) study and the Borges et al., (2016) study focused on comparisons of k and AUC, as well as variations on AUC in the Borges et al. study. Second, the current study utilized hypothetical rather than real monetary rewards. However, previous studies have demonstrated that participants respond similarly during DD tasks for both real and hypothetical monetary values (Johnson & Bickel, 2002; Lawyer, Schoepflin, Green, & Jenks, 2011). Third, the most important limitation of the current study is that we did not demonstrate differences in clinical outcomes. However, based on the findings of different outcomes presented in just the Mitchell et al., (2015) and Borges et al., (2016) studies, we believe that such outcomes are clearly not isolated. Also, the results of our analysis support that such outcomes are more likely in populations with greater discounting.

In summary, the current study extends on the findings of Mitchell et al., (2015) by examining a population with relatively greater discounting as well as assessing larger monetary and longer delay parameters. Additionally, the current study provides novel information by providing a closer examination as to how different sections of the discounting curve influence the final AUC and log k values. Overall, we believe the current study highlights a number of considerations for future delay discounting research. First, where applicable, studies should begin to report results from a number of delay discounting

outcomes so that the field may begin to assess which measure of discounting is the most useful for identifying replicable and reliable differences in discounting associated with pertinent measures (e.g., severity of drug use, relapse risk, etc.). Second, it would be helpful to re-examine previously published and non-published data sets to assess whether congruent outcomes are observed when using log k , AUC, or any other relevant methods for assessing discounting, particularly for relatively high discounting populations. Third, at the very least, the results of the current study as well those previously reported in the Mitchell et al., (2015) and Borges et al., (2016) studies all highlight that closer examination of the data is warranted, particularly when utilizing AUC with relatively steep discounters.

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Compliance with Ethical Standards

- None of the authors on the current manuscript have any conflict of interest.
- All procedures were in accordance to the ethical standards of the Baylor College of Medicine and Michael E. DeBakey Veterans Affairs Medical Center and approved by both institutions' respective Internal Review Boards.
- All individuals in the current study provided informed consent.
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Figure 1.
Distribution of k , $\log k$, AUC, and $\log AUC$ values.

Figure 2.

Scatterplot of individual $\log k$ values on the x-axis and corresponding AUC values on the y-axis. Closed and open symbols represent data points with matching vs. not matching classifications, respectively. The straight dashed line and curved solid line represent best fit curves from the linear and quadratic models, respectively.

Figure 3.

Scatterplot of AUC (top) and log k (bottom) values on the x-axis plotted against corresponding partial AUC and log k values calculated from using an increasing number of indifference points from 1 to 7 days (left column) to 1 day to 5 years (5th column). The far right column utilizes indifference points from 5 years to 25 years. The range for both the x- and y-axis for all log k figures is -6 to 1 . The range for the x-axis in all AUC figures is 0 to 1 . The range of the y-axis for AUC figures is from 0 to the maximum possible AUC value for the assessed indifference points. For example, the maximum possible area for 1 to 7 days would be 8.8×10^{-4} , whereas the maximum possible area for 5 to 25 years would be $.80$. Straight lines represent the best fits using linear regression.

Table 1

Summary of results for the linear ($AUC = \log k$) and quadratic ($AUC = \log k + \log k^2$) models using generalized linear modeling.

	Standard		95% Bootstrap				RMSE	AIC	R ²
	Estimate	Error	t value	p value	CI				
<u>Linear Model</u>									
Intercept	-0.1365	0.0171	-7.983	< 0.0001	-0.186	-0.085			
Log <i>k</i>	-0.1544	0.0085	-18.135	< 0.0001	-0.182	-0.121			
							0.1035	-286.22	0.659
<u>Quadratic Model</u>									
Intercept	0.0514	0.0168	3.053	0.0026	0.019	0.091			
Log <i>k</i>	0.0725	0.0161	4.492	< 0.0001	0.036	0.132			
Log <i>k</i> ²	0.0535	0.0036	14.985	< 0.0001	0.045	0.073			
							0.0678	-429.62	0.854

Table 2

Summary of results using Generalized Linear Mixed Modeling Distribution fits.

Distribution	-2 * Log Likelihood	AIC	BIC
Beta	-844.86	-836.86	-824.27
Gamma	-99.51	-91.51	-78.92
Normal	608.61	616.61	629.20